Chapter 2

Acute Angles and Right Triangles

Section 2.1 Trigonometric Functions of Acute Angles

For Exercises 1−6, refer to the Function Values of Special Angles chart on page 52 of the text.

1. C; $\sin 30° = \frac{1}{2}$

2. H; $\cos 45° = \frac{\sqrt{2}}{2}$

3. B; $\tan 45° = 1$

4. G; $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2$

5. E; $\csc 60° = \frac{1}{\sin 60°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$
$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

6. A; $\cot 30° = \frac{\cos 30°}{\sin 30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

7. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{21}{29}$
$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{20}{29}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{21}{20}$

8. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{45}{53}$
$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{28}{53}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{45}{28}$

9. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{n}{p}$
$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{m}{p}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{n}{m}$

10. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{k}{z}$
$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{y}{z}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{k}{y}$

11. $a = 5, b = 12$
$c^2 = a^2 + b^2 \Rightarrow c^2 = 5^2 + 12^2 \Rightarrow c^2 = 169 \Rightarrow$
$c = 13$
$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{12}{13}$
$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{5}{13}$
$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{12}{5}$
$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{5}{12}$
$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{13}{5}$
$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{13}{12}$

12. $a = 3, b = 4$
$c^2 = a^2 + b^2 \Rightarrow c^2 = 3^2 + 4^2 \Rightarrow c^2 = 25 \Rightarrow$
$c = 5$
$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{4}{5}$
$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{3}{5}$
$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{4}{3}$
$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{3}{4}$
$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{5}{3}$
$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{5}{4}$

13. $a = 6, c = 7$

$c^2 = a^2 + b^2 \Rightarrow 7^2 = 6^2 + b^2 \Rightarrow$
$49 = 36 + b^2 \Rightarrow 13 = b^2 \Rightarrow \sqrt{13} = b$

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\sqrt{13}}{7}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{6}{7}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{\sqrt{13}}{6}$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{6}{\sqrt{13}} = \frac{6}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$$
$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{7}{6}$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{7}{\sqrt{13}} = \frac{7}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{7\sqrt{13}}{13}$$

14. $b = 7, c = 12$

$c^2 = a^2 + b^2 \Rightarrow 12^2 = a^2 + 7^2 \Rightarrow$
$144 = a^2 + 49 \Rightarrow 95 = a^2 \Rightarrow \sqrt{95} = a$

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{7}{12}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{\sqrt{95}}{12}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{7}{\sqrt{95}} = \frac{7}{\sqrt{95}} \cdot \frac{\sqrt{95}}{\sqrt{95}} = \frac{7\sqrt{95}}{95}$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{\sqrt{95}}{7}$$
$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{12}{\sqrt{95}} = \frac{12}{\sqrt{95}} \cdot \frac{\sqrt{95}}{\sqrt{95}} = \frac{12\sqrt{95}}{95}$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{12}{7}$$

15. $a = 3, c = 10$

$c^2 = a^2 + b^2 \Rightarrow 10^2 = 3^2 + b^2 \Rightarrow$
$100 = 9 + b^2 \Rightarrow 91 = b^2 \Rightarrow \sqrt{91} = b$

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\sqrt{91}}{10}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{3}{10}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{\sqrt{91}}{3}$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{3}{\sqrt{91}} = \frac{3}{\sqrt{91}} \cdot \frac{\sqrt{91}}{\sqrt{91}} = \frac{3\sqrt{91}}{91}$$
$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{10}{3}$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{10}{\sqrt{91}} = \frac{10}{\sqrt{91}} \cdot \frac{\sqrt{91}}{\sqrt{91}} = \frac{10\sqrt{91}}{91}$$

16. $b = 8, c = 11$

$c^2 = a^2 + b^2 \Rightarrow 11^2 = a^2 + 8^2 \Rightarrow$
$121 = a^2 + 64 \Rightarrow 57 = a^2 \Rightarrow \sqrt{57} = a$

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{8}{11}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{\sqrt{57}}{11}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{8}{\sqrt{57}} = \frac{8}{\sqrt{57}} \cdot \frac{\sqrt{57}}{\sqrt{57}} = \frac{8\sqrt{57}}{57}$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{\sqrt{57}}{8}$$
$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{11}{\sqrt{57}} = \frac{11}{\sqrt{57}} \cdot \frac{\sqrt{57}}{\sqrt{57}} = \frac{11\sqrt{57}}{57}$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{11}{8}$$

17. $a = 1, c = 2$

$c^2 = a^2 + b^2 \Rightarrow 2^2 = 1^2 + b^2 \Rightarrow$
$4 = 1 + b^2 \Rightarrow 3 = b^2 \Rightarrow \sqrt{3} = b$

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\sqrt{3}}{2}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{1}{2}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{\sqrt{3}}{1} = \sqrt{3}$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

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$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{2}{1} = 2$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

18. $a = \sqrt{2},\ c = 2$

$$c^2 = a^2 + b^2 \Rightarrow 2^2 = \sqrt{2}^2 + b^2 \Rightarrow 4 = 2 + b^2 \Rightarrow 2 = b^2 \Rightarrow \sqrt{2} = b$$
$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\sqrt{2}}{2}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{\sqrt{2}}{2}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$
$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

19. $b = 2,\ c = 5$

$$c^2 = a^2 + b^2 \Rightarrow 5^2 = a^2 + 2^2 \Rightarrow 25 = a^2 + 4 \Rightarrow 21 = a^2 \Rightarrow \sqrt{21} = a$$
$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{2}{5}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{\sqrt{21}}{5}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{b}{a} = \frac{2}{\sqrt{21}} = \frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$
$$\cot B = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{a}{b} = \frac{\sqrt{21}}{2}$$
$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{a} = \frac{5}{\sqrt{21}} = \frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$
$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{b} = \frac{5}{2}$$

20. $\sin\theta = \cos(90° - \theta); \cos\theta = \sin(90° - \theta);$
$\tan\theta = \cot(90° - \theta); \cot\theta = \tan(90° - \theta);$
$\sec\theta = \csc(90° - \theta); \csc\theta = \sec(90° - \theta)$

21. $\cos 30° = \sin(90° - 30°) = \sin 60°$

22. $\sin 45° = \cos(90° - 45°) = \cos 45°$

23. $\csc 60° = \sec(90° - 60°) = \sec 30°$

24. $\cot 73° = \tan(90° - 73°) = \tan 17°$

25. $\sec 39° = \csc(90° - 39°) = \csc 51°$

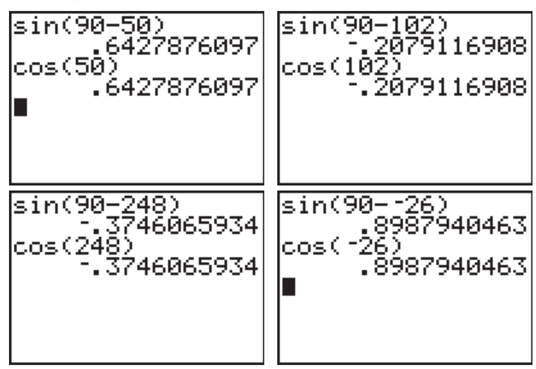
26. $\tan 25.4° = \cot(90° - 25.4°) = \cot 64.6°$

27. $\sin 38.7° = \cos(90° - 38.7°) = \cos 51.3°$

28. $\cos(\theta + 20°) = \sin[90° - (\theta + 20°)] = \sin(70° - \theta)$

29. $\sec(\theta + 15°) = \csc[90° - (\theta + 15°)] = \csc(75° - \theta)$

30. Using $\theta = 50°, 102°, 248°,$ and $-26°$, we see that $\sin(90° - \theta)$ and $\cos\theta$ yield the same values.



For exercises 31−40, if the functions in the equations are cofunctions, then the equations are true if the sum of the angles is 90º.

31.
$$\tan\alpha = \cot(\alpha + 10°)$$
$$\alpha + (\alpha + 10°) = 90°$$
$$2\alpha + 10° = 90°$$
$$2\alpha = 80° \Rightarrow \alpha = 40°$$

32.
$$\cos\theta = \sin(2\theta - 30°)$$
$$\theta + 2\theta - 30° = 90°$$
$$3\theta - 30° = 90°$$
$$3\theta = 120° \Rightarrow \theta = 40°$$

33. $\sin(2\theta+10°)=\cos(3\theta-20°)$
$(2\theta+10°)+(3\theta-20°)=90°$
$5\theta-10°=90°$
$5\theta=100°\Rightarrow\theta=20°$

34. $\sec(\beta+10°)=\csc(2\beta+20°)$
$(\beta+10°)+(2\beta+20°)=90°$
$3\beta+30°=90°$
$3\beta=60°\Rightarrow\beta=20°$

35. $\tan(3\beta+4°)=\cot(5\beta-10°)$
$(3\beta+4°)+(5\beta-10°)=90°$
$8\beta-6°=90°$
$8\beta=96°\Rightarrow\beta=12°$

36. $\cot(5\theta+2°)=\tan(2\theta+4°)$
$(5\theta+2°)+(2\theta+4°)=90°$
$7\theta+6°=90°$
$7\theta=84°\Rightarrow\theta=12°$

37. $\sin(\theta-20°)=\cos(2\theta+5°)$
$(\theta-20°)+(2\theta+5°)=90°$
$3\theta-15°=90°$
$3\theta=105°\Rightarrow\theta=35°$

38. $\cos(2\theta+50°)=\sin(2\theta-20°)$
$(2\theta+50°)+(2\theta-20°)=90°$
$4\theta+30°=90°$
$4\theta=60°\Rightarrow\theta=15°$

39. $\sec(3\beta+10°)=\csc(\beta+8°)$
$(3\beta+10°)+(\beta+8°)=90°$
$4\beta+18°=90°$
$4\beta=72°\Rightarrow\beta=18°$

40. $\csc(\beta+40°)=\sec(\beta-20°)$
$(\beta+40°)+(\beta-20°)=90°$
$2\beta+20°=90°$
$2\beta=70°\Rightarrow\beta=35°$

41. $\sin 50°>\sin 40°$
In the interval from 0º to 90º, as the angle increases, so does the sine of the angle, so sin 50º > sin 40º is true.

42. $\tan 28°\le\tan 40°$
In the interval from 0º to 90º, as the angle increases, the tangent of the angle increases, so tan 40º > tan 28º $\Rightarrow \tan 28°\le\tan 40°$ is true.

43. $\sin 46°<\cos 46°$
Using the cofunction identity, $\cos 46°=\sin(90°-46°)=\sin 44°$. In the interval from 0º to 90º, as the angle increases, so does the sine of the angle, so $\sin 46°<\sin 44°\Rightarrow\sin 46°<\cos 46°$ is false.

44. $\cos 28°<\sin 28°$
Using the cofunction identity, $\sin 28°=\cos(90°-28°)=\cos 62°$. In the interval from 0º to 90º, as the angle increases, the cosine of the angle decreases, so $\cos 28°<\cos 62°\Rightarrow\cos 28°<\sin 28°$ is false.

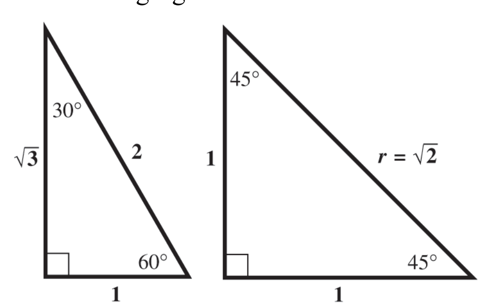
45. $\tan 41°<\cot 41°$
Using the cofunction identity, $\cot 41°=\tan(90°-41°)=\tan 49°$. In the interval from 0º to 90º, as the angle increases, the tangent of the angle increases, so $\tan 41°<\tan 49°\Rightarrow\tan 41°<\cot 41°$ is true.

46. $\cot 30°<\tan 40°$
Using the cofunction identity, $\cot 30°=\tan(90°-30°)=\tan 60°$. In the interval from 0º to 90º, as the angle increases, the tangent of the angle increases, so $\tan 60°<\tan 40°\Rightarrow\cot 30°<\cot 40°$ is false.

47. $\sec 60°>\sec 30°$
In the interval from 0º to 90º, as the angle increases, the cosine of the angle decreases, so the secant of the angle increases. Thus, $\sec 60°>\sec 30°$ is true.

48. $\csc 20°<\csc 30°$
In the interval from 0º to 90º, as the angle increases, sine of the angle increases, so cosecant of the angle decreases. Thus $\csc 20°<\csc 30°$ is false.

Use the following figures for exercises 49−64.



49. $\tan\ 30°=\dfrac{\text{side opposite}}{\text{side adjacent}}=\dfrac{1}{\sqrt{3}}$
$=\dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}$

50. $\cot 30° = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

51. $\sin 30° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{1}{2}$

52. $\cos 30° = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$

53. $\sec 30° = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{2}{\sqrt{3}}$
$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

54. $\csc 30° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{2}{1} = 2$

55. $\csc 45° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{\sqrt{2}}{1} = \sqrt{2}$

56. $\sec 45° = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{\sqrt{2}}{1} = \sqrt{2}$

57. $\cos 45° = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$
$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

58. $\cot 45° = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{1}{1} = 1$

59. $\tan 45° = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{1}{1} = 1$

60. $\sin 45° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$
$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

61. $\sin 60° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$

62. $\cos 60° = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{1}{2}$

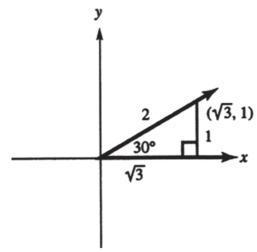
63. $\tan 60° = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

64. $\csc 60° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{2}{\sqrt{3}}$
$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

65. Because $\sin 60° = \frac{\sqrt{3}}{2}$ and 60° is between 0° and 90°, A = 60°.

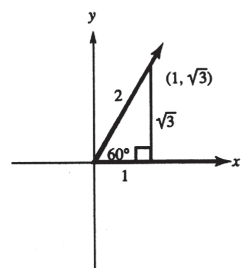
66. 0.7071067812 is a rational *approximation* for the exact value $\frac{\sqrt{2}}{2}$ (an irrational value).

67.



The line passes through (0, 0) and $(\sqrt{3}, 1)$. The slope is change in *y* over the change in *x*. Thus, $m = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ and the equation of the line is $y = \frac{\sqrt{3}}{3}x$.

68.

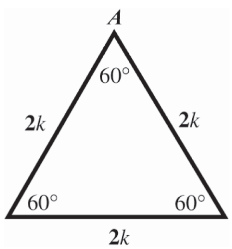


The line passes through $(0,0)$ and $(1,\sqrt{3})$. The slope is change in *y* over the change in *x*. Thus, $m = \frac{\sqrt{3}}{1} = \sqrt{3}$ and the equation of the line is $y = \sqrt{3}x$.

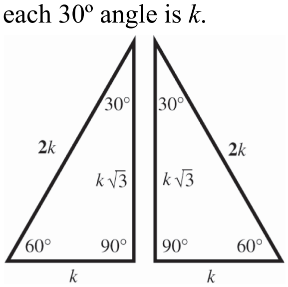
69. One point on the line $y = \sqrt{3}x$ is the origin $(0,0)$. Let (x, y) be any other point on this line. Then, by the definition of slope, $m = \frac{y-0}{x-0} = \frac{y}{x} = \sqrt{3}$, but also, by the definition of tangent, $\tan\theta = \sqrt{3}$. Because $\tan 60° = \sqrt{3}$, the line $y = \sqrt{3}x$ makes a 60° angle with the positive *x*-axis (See exercise 68).

70. One point on the line $y = \frac{\sqrt{3}}{3}x$, is the origin $(0,0)$. Let (x, y) be any other point on this line. Then, by the definition of slope, $m = \frac{y-0}{x-0} = \frac{y}{x} = \frac{\sqrt{3}}{3}$, but also, by the definition of tangent, $\tan\theta = \frac{\sqrt{3}}{3}$. Because $\tan 30° = \frac{\sqrt{3}}{3}$, the line $y = \frac{\sqrt{3}}{3}x$ makes a $30°$ angle with the positive *x*-axis. (See Exercise 67).

71. (a) Each of the angles of the equilateral triangle has measure $\frac{1}{3}(180°) = 60°$.



(b) The perpendicular bisects the opposite side so the length of each side opposite each 30º angle is *k*.



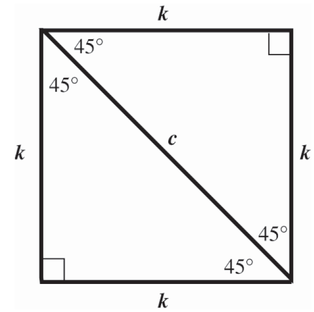
(c) Let *x* = the length of the perpendicular. Then apply the Pythagorean theorem.

$$x^2 + k^2 = (2k)^2 \Rightarrow x^2 + k^2 = 4k^2 \Rightarrow$$
$$x^2 = 3k^2 \Rightarrow x = \sqrt{3}k$$

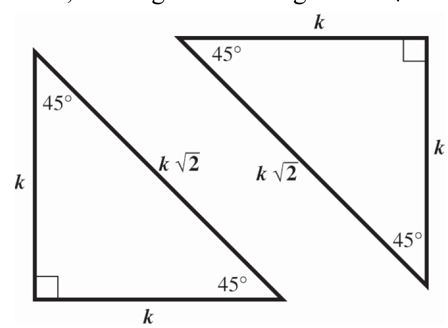
The length of the perpendicular is $\sqrt{3}k$.

(d) In a 30º-60º right triangle, the hypotenuse is always <u>2</u> times as long as the shorter leg, and the longer leg has a length that is <u>$\sqrt{3}$</u> times as long as that of the shorter leg. Also, the shorter leg is opposite the <u>30º</u> angle, and the longer leg is opposite the <u>60º</u> angle.

72. (a) The diagonal forms two isosceles right triangles. Each angle formed by a side of the square and the diagonal measures 45º.

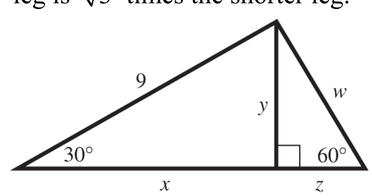


(b) By the Pythagorean theorem, $k^2 + k^2 = c^2 \Rightarrow 2k^2 = c^2 \Rightarrow c = \sqrt{2}k$. Thus, the length of the diagonal is $\sqrt{2}k$.



(c) In a 45º-45º right triangle, the hypotenuse has a length that is <u>$\sqrt{2}$</u> times as long as either leg.

73. Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle first to the triangle on the left to find the values of *y* and *x*, and then to the triangle on the right to find the values of *z* and *w*. In the $30° - 60°$ right triangle, the side opposite the $30°$ angle is $\frac{1}{2}$ the length of the hypotenuse. The longer leg is $\sqrt{3}$ times the shorter leg.



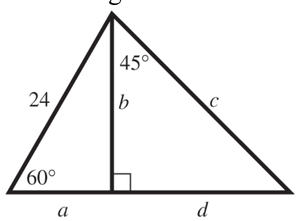
Thus, we have

$$y = \frac{1}{2}(9) = \frac{9}{2} \text{ and } x = y\sqrt{3} = \frac{9\sqrt{3}}{2}$$

$$y = z\sqrt{3}, \text{ so } z = \frac{y}{\sqrt{3}} = \frac{\frac{9}{2}}{\sqrt{3}} = \frac{9\sqrt{3}}{6} = \frac{3\sqrt{3}}{2},$$

$$\text{and } w = 2z, \text{ so } w = 2\left(\frac{3\sqrt{3}}{2}\right) = 3\sqrt{3}$$

74. Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle first to the triangle on the left to find the values of a and b. In the $30° - 60°$ right triangle, the side opposite the $30°$ angle is $\frac{1}{2}$ the length of the hypotenuse. The longer leg is $\sqrt{3}$ times the shorter leg.

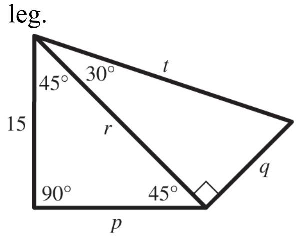


$a = \frac{1}{2}(24) = 12$ and $b = a\sqrt{3} = 12\sqrt{3}$

Apply the relationships between the lengths of the sides of a $45° - 45°$ right triangle next to the triangle on the right to find the values of d and c. In the $45° - 45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg. $d = b = 12\sqrt{3}$ and

$c = d\sqrt{2} = \left(12\sqrt{3}\right)\left(\sqrt{2}\right) = 12\sqrt{6}$

75. Apply the relationships between the lengths of the sides of a $45° - 45°$ right triangle to the triangle on the left to find the values of p and r. In the $45° - 45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg.



Thus, we have $p = 15$ and $r = p\sqrt{2} = 15\sqrt{2}$

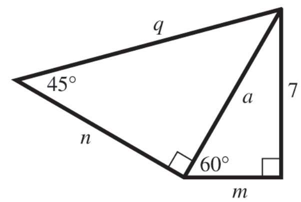
Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle next to the triangle on the right to find the values of q and t. In the $30° - 60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle).

Thus, we have $r = q\sqrt{3} \Rightarrow$

$$q = \frac{r}{\sqrt{3}} = \frac{15\sqrt{2}}{\sqrt{3}} = \frac{15\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{6} \text{ and}$$

$$t = 2q = 2\left(5\sqrt{6}\right) = 10\sqrt{6}$$

76. Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle first to the triangle on the right to find the values of m and a. In the $30° - 60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle).



Thus, we have

$$7 = m\sqrt{3} \Rightarrow m = \frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3} \text{ and}$$

$$a = 2m \Rightarrow a = 2\left(\frac{7\sqrt{3}}{3}\right) = \frac{14\sqrt{3}}{3}$$

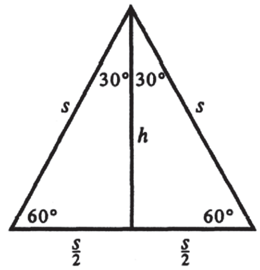
Apply the relationships between the lengths of the sides of a $45° - 45°$ right triangle next to the triangle on the left to find the values of n and q. In the $45° - 45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg. Thus, we have $n = a = \frac{14\sqrt{3}}{3}$ and

$$q = n\sqrt{2} = \left(\frac{14\sqrt{3}}{3}\right)\sqrt{2} = \frac{14\sqrt{6}}{3}.$$

77. Because $A = \frac{1}{2}bh$, we have

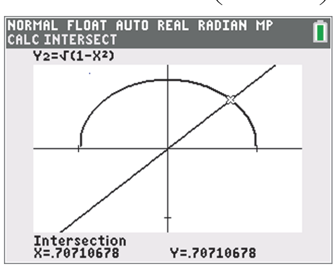
$$A = \frac{1}{2} \cdot s \cdot s = \frac{1}{2}s^2 \text{ or } A = \frac{s^2}{2}.$$

78. Let h be the height of the equilateral triangle. h bisects the base, s, and forms two 30°–60° right triangles.



The formula for the area of a triangle is $A = \frac{1}{2}bh.$ In this triangle, $b = s$. The height h of the triangle is the side opposite the 60° angle in either 30°–60° right triangle. The side opposite the 30° angle is $\frac{s}{2}$. The height is $\sqrt{3} \cdot \frac{s}{2}. = \frac{s\sqrt{3}}{2}$ So the area of the entire triangle is $A = \frac{1}{2}s\left(\frac{s\sqrt{3}}{2}\right) = \frac{s^2\sqrt{3}}{4}.$

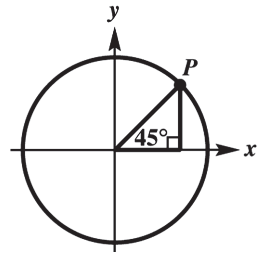
79. Graph the equations in the window [–1.5, 1.5] × [–1.2, 1.2]. The point of intersection is (0.70710678, 0.70710678). This corresponds to the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right).$



These coordinates are the sine and cosine of 45°.

80. Yes, the third angle can be found by subtracting the given acute angle from 90°, and the remaining two sides can be found using a trigonometric function involving the known angle and side.

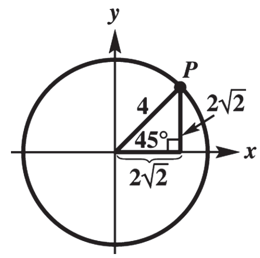
81.



82. $\sin 45° = \frac{y}{4} \Rightarrow y = 4\sin 45° = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}$

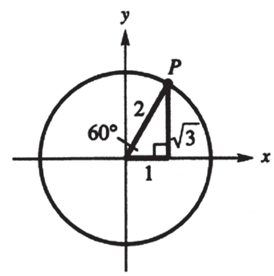
and

$\cos 45° = \frac{x}{4} \Rightarrow x = 4\cos 45° = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}$



83. The legs of the right triangle provide the coordinates of P, $\left(2\sqrt{2}, 2\sqrt{2}\right)$.

84.



$\sin 60° = \frac{y}{2} \Rightarrow y = 2\sin 60° = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

and $\cos 60° = \frac{x}{2} \Rightarrow x = 2\cos 60° = 2 \cdot \frac{1}{2} = 1$

The legs of the right triangle provide the coordinates of P. P is $\left(1, \sqrt{3}\right)$.

Section 2.2 Trigonometric Functions of Non-Acute Angles

1. The value of sin 240° is negative because 240° is in quadrant III. The reference angle is 60°, and the exact value of sin 240° is $-\frac{\sqrt{3}}{2}$.

2. The value of cos 390° is positive because 390° is in quadrant I. The reference angle is 30°, and the exact value of cos 390° is $\frac{\sqrt{3}}{2}$.

3. The value of tan (–150°) is positive because –150° is in quadrant III. The reference angle is 30° and the exact value of tan (–150°) is $\frac{\sqrt{3}}{3}$.

4. The value of sec 135° is negative because 135° is in quadrant II. The reference angle is 45°, and the exact value of sec 135° is $-\sqrt{2}$.

5. C; $180° - 98° = 82°$
(98° is in quadrant II)

6. F; $212° - 180° = 32°$
(212° is in quadrant III)

7. A; $-135° + 360° = 225°$ and $225° - 180° = 45°$
(225° is in quadrant III)

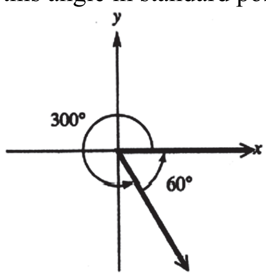
8. B; $-60° + 360° = 300°$ and $360° - 300° = 60°$
(300° is in quadrant IV)

9. D; $750° - 2 \cdot 360° = 30°$
(30° is in quadrant I)

10. B; $480° - 360° = 120°$ and $180° - 120° = 60°$
(120° is in quadrant II)

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
11.	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
12.	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
13.	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
14.	120°	$\frac{\sqrt{3}}{2}$	$\cos 120° = -\cos 60° = -\frac{1}{2}$	$-\sqrt{3}$	$\cot 120° = -\cot 60° = -\frac{\sqrt{3}}{3}$	$\sec 120° = -\sec 60° = -2$	$\frac{2\sqrt{3}}{3}$
15.	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\tan 135° = -\tan 45° = -1$	$\cot 135° = -\cot 45° = -1$	$-\sqrt{2}$	$\sqrt{2}$
16.	150°	$\sin 150° = \sin 30° = \frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$\cot 150° = -\cot 30° = -\sqrt{3}$	$\sec 150° = -\sec 30° = -\frac{2\sqrt{3}}{3}$	2
17.	210°	$-\frac{1}{2}$	$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\sec 210° = -\sec 30° = -\frac{2\sqrt{3}}{3}$	−2
18.	240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\tan 240° = \tan 60° = \sqrt{3}$	$\cot 240° = \cot 60° = \frac{\sqrt{3}}{3}$	−2	$-\frac{2\sqrt{3}}{3}$

19. To find the reference angle for 300°, sketch this angle in standard position.



The reference angle is $360° - 300° = 60°$. Because 300° lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \frac{1}{2}$

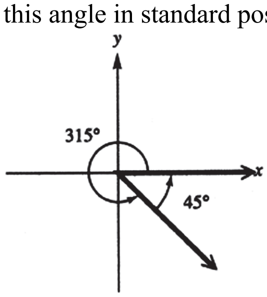
$\tan 300° = -\tan 60° = -\sqrt{3}$

$\cot 300° = -\cot 60° = -\frac{\sqrt{3}}{3}$

$\sec 300° = \sec 60° = 2$

$\csc 300° = -\csc 60° = -\frac{2\sqrt{3}}{3}$

20. To find the reference angle for 315°, sketch this angle in standard position.



The reference angle is $360° - 315° = 45°$. Because 315° lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin 315° = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 315° = \cos 45° = \frac{\sqrt{2}}{2}$

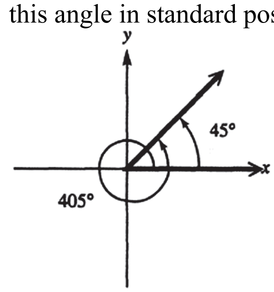
$\tan 315° = -\tan 45° = -1$

$\cot 315° = -\cot 45° = -1$

$\sec 315° = \sec 45° = \sqrt{2}$

$\csc 315° = -\csc 45° = -\sqrt{2}$

21. To find the reference angle for 405°, sketch this angle in standard position.



The reference angle for 405° is $405° - 360° = 45°$. Because 405° lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 45° See the Function Values of Special Angles table on page 52.)

$\sin 405° = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos 405° = \cos 45° = \frac{\sqrt{2}}{2}$

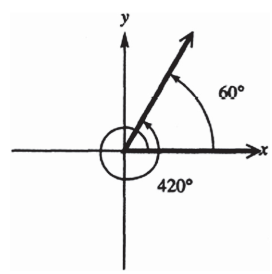
$\tan 405° = \tan 45° = 1$

$\cot 405° = \cot 45° = 1$

$\sec 405° = \sec 45° = \sqrt{2}$

$\csc 405° = \csc 45° = \sqrt{2}$

22. To find the reference angle for 420º, sketch this angle in standard position.



The reference angle for 420º is $420º - 360º = 60º$. Because 420º lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 60°. See the Function Values of Special Angles table on page 52.)

$\sin(420°) = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos(420°) = \cos 60° = \frac{1}{2}$

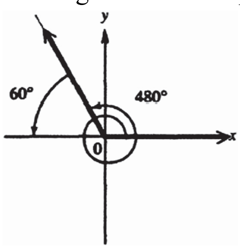
$\tan(420°) = \tan 60° = \sqrt{3}$

(*continued on next page*)

(*continued*)

$\cot(420°) = \cot 60° = \frac{\sqrt{3}}{3}$
$\sec(420°) = \sec 60° = 2$
$\csc(420°) = \csc 60° = \frac{2\sqrt{3}}{3}$

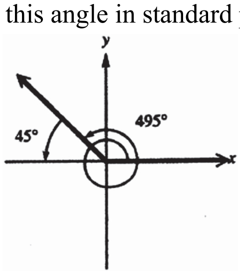
23. To find the reference angle for 480º, sketch this angle in standard position.



480º is coterminal with 480º − 360º = 120º. The reference angle is 180º − 120º = 60º. Because 480º lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$\sin(480°) = \sin 60° = \frac{\sqrt{3}}{2}$
$\cos(480°) = -\cos 60° = -\frac{1}{2}$
$\tan(480°) = -\tan 60° = -\sqrt{3}$
$\cot(480°) = -\cot 60° = -\frac{\sqrt{3}}{3}$
$\sec(80°) = -\sec 60° = -2$
$\csc(480°) = \csc 60° = \frac{2\sqrt{3}}{3}$

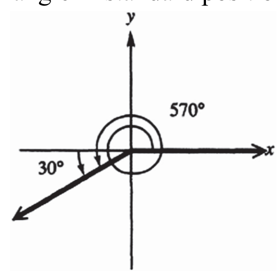
24. To find the reference angle for 495º, sketch this angle in standard position.



495º is coterminal with 495º − 360º = 135º. The reference angle is 180º − 135º = 45º. Because 495° lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$\sin 495° = \sin 45° = \frac{\sqrt{2}}{2}$
$\cos 495° = -\cos 45° = -\frac{\sqrt{2}}{2}$
$\tan 495° = -\tan 45° = -1$
$\cot 495° = -\cot 45° = -1$
$\sec 495° = -\sec 45° = -\sqrt{2}$
$\csc 495° = \csc 45° = \sqrt{2}$

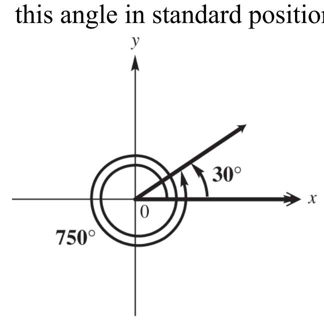
25. To find the reference angle for 570º sketch this angle in standard position.



570° is coterminal with 570º − 360º = 210º. The reference angle is 210º − 180º = 30º. Because 570º lies in quadrant III, the sine, cosine, secant, and cosecant are negative.

$\sin 570° = -\sin 30° = -\frac{1}{2}$
$\cos 570° = -\cos 30° = -\frac{\sqrt{3}}{2}$
$\tan 570° = \tan 30° = \frac{\sqrt{3}}{3}$
$\cot 570° = \cot 30° = \sqrt{3}$
$\sec 570° = -\sec 30° = -\frac{2\sqrt{3}}{3}$
$\csc 570° = -\csc 30° = -2$

26. To find the reference angle for 750°, sketch this angle in standard position.



750° is coterminal with 30° because $750° - 2 \cdot 360° = 750° - 720° = 30°$. Because 750° lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 30°.

$\sin 750° = \sin 30° = \frac{1}{2}$
$\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$
$\tan 750° = \tan 30° = \frac{\sqrt{3}}{3}$

(*continued on next page*)

(*continued*)

$\cot 750° = \cot 30° = \sqrt{3}$
$\sec 750° = \sec 30° = \frac{2\sqrt{3}}{3}$
$\csc 750° = \csc 30° = 2$

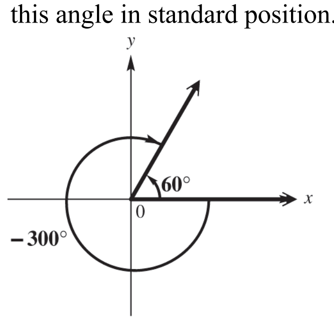
27. 1305º is coterminal with $1305° - 3 \cdot 360° = 1305° = 1080° = 225°$. The reference angle is 225º − 180º = 45º. Because 1305º lies in quadrant III, the sine, cosine, and secant and cosecant are negative.

$\sin 1305° = -\sin 45° = -\frac{\sqrt{2}}{2}$
$\cos 1305° = -\cos 45° = -\frac{\sqrt{2}}{2}$
$\tan 1305° = \tan 45° = 1$
$\cot 1305° = \cot 45° = 1$
$\sec 1305° = -\sec 45° = -\sqrt{2}$
$\csc 1305° = -\csc 45° = -\sqrt{2}$

28. 1500º is coterminal with $1500° - 4 \cdot 360° = 1500° - 1440° = 60°$. Because 1500º lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 60°.

$\sin(420°) = \sin 60° = \frac{\sqrt{3}}{2}$
$\cos(420°) = \cos 60° = \frac{1}{2}$
$\tan(420°) = \tan 60° = \sqrt{3}$
$\cot(420°) = \cot 60° = \frac{\sqrt{3}}{3}$
$\sec(420°) = \sec 60° = 2$
$\csc(420°) = \csc 60° = \frac{2\sqrt{3}}{3}$

29. To find the reference angle for $-300°$, sketch this angle in standard position.



The reference angle for $-300°$ is $-300° + 360° = 60°$. Because $-300°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for 60°. See the Function Values of Special Angles table on page 52.)

$\sin(-300°) = \sin 60° = \frac{\sqrt{3}}{2}$
$\cos(-300°) = \cos 60° = \frac{1}{2}$
$\tan(-300°) = \tan 60° = \sqrt{3}$
$\cot(-300°) = \cot 60° = \frac{\sqrt{3}}{3}$
$\sec(-300°) = \sec 60° = 2$
$\csc(-300°) = \csc 60° = \frac{2\sqrt{3}}{3}$

30. −390° is coterminal with $-390° + 2 \cdot 360° = -390° + 720° = 330°$. The reference angle is $360° - 330° = 30°$. Because −390° lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$\sin(-390°) = -\sin 30° = -\frac{1}{2}$
$\cos(-390°) = \cos 30° = \frac{\sqrt{3}}{2}$
$\tan(-390°) = -\tan 30° = -\frac{\sqrt{3}}{3}$
$\cot(-390°) = -\cot 30° = -\sqrt{3}$
$\sec(-390°) = \sec 30° = \frac{2\sqrt{3}}{3}$
$\csc(-390°) = -\csc 30° = -2$

31. –510° is coterminal with $-510° + 2 \cdot 360° = -510° + 720° = 210°$. The reference angle is $210° - 180° = 30°$. Because –510° lies in quadrant III, the sine, cosine, and secant and cosecant are negative.

$\sin(-510°) = -\sin 30° = -\frac{1}{2}$
$\cos(-510°) = -\cos 30° = -\frac{\sqrt{3}}{2}$
$\tan(-510°) = \tan 30° = \frac{\sqrt{3}}{3}$
$\cot(-510°) = \cot 30° = \sqrt{3}$
$\sec(-510°) = -\sec 30° = -\frac{2\sqrt{3}}{3}$
$\csc(-510°) = -\csc 30° = -2$

32. −1020º is coterminal with $-1020° + 3 \cdot 360° = -1020° + 1080° = 60°$. Because −1020º lies in quadrant I, the values of all of its trigonometric functions will be positive, so these values will be identical to the trigonometric function values for $60°$.

$$\sin(420°) = \sin 60° = \frac{\sqrt{3}}{2}$$
$$\cos(420°) = \cos 60° = \frac{1}{2}$$
$$\tan(420°) = \tan 60° = \sqrt{3}$$
$$\cot(420°) = \cot 60° = \frac{\sqrt{3}}{3}$$
$$\sec(420°) = \sec 60° = 2$$
$$\csc(420°) = \csc 60° = \frac{2\sqrt{3}}{3}$$

33. −1290° is coterminal with $-1290° + 4 \cdot 360° = -1290° + 1440° = 150°$. The reference angle is $180° - 150° = 30°$. Because −1290° lies in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$$\sin 2670° = \sin 30° = \frac{1}{2}$$
$$\cos 2670° = -\cos 30° = -\frac{\sqrt{3}}{2}$$
$$\tan 2670° = -\tan 30° = -\frac{\sqrt{3}}{3}$$
$$\cot 2670° = -\cot 30° = -\sqrt{3}$$
$$\sec 2670° = -\sec 30° = -\frac{2\sqrt{3}}{3}$$
$$\csc 2670° = \csc 30° = 2$$

34. −855º is coterminal with $-855° + 3 \cdot 360° = -855° + 1080° = 225°$. The reference angle is 225º − 180º = 45º. Because −855º lies in quadrant III, the sine, cosine, and secant and cosecant are negative.

$$\sin(-855°) = -\sin 45° = -\frac{\sqrt{2}}{2}$$
$$\cos(-855°) = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan(-855°) = \tan 45° = 1$$
$$\cot(-855°) = \cot 45° = 1$$
$$\sec(-855°) = -\sec 45° = -\sqrt{2}$$
$$\csc(-855°) = -\csc 45° = -\sqrt{2}$$

35. −1860º is coterminal with $-1860° + 6 \cdot 360° = -1860° + 2160° = 300°$. The reference angle is 360º − 300º = 60º. Because −1860º lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$$\sin(-1860°) = -\sin 60° = -\frac{\sqrt{3}}{2}$$
$$\cos(-1860°) = \cos 60° = \frac{1}{2}$$
$$\tan(-1860°) = -\tan 60° = -\sqrt{3}$$
$$\cot(-1860°) = -\cot 60° = -\frac{\sqrt{3}}{3}$$
$$\sec(-1860°) = \sec 60° = 2$$
$$\csc(-1860°) = -\csc 60° = -\frac{2\sqrt{3}}{3}$$

36. −2205º is coterminal with $-2205° + 7 \cdot 360° = -2205° + 2520° = 315°$. The reference angle is 360º − 315º = 45º. Because −2205º lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$$\sin(-2205°) = -\sin 45° = -\frac{\sqrt{2}}{2}$$
$$\cos(-2205°) = \cos 45° = \frac{\sqrt{2}}{2}$$
$$\tan(-2205°) = -\tan 45° = -1$$
$$\cot(-2205°) = -\cot 45° = -1$$
$$\sec(-2205°) = \sec 45° = \sqrt{2}$$
$$\csc(-2205°) = -\csc 45° = -\sqrt{2}$$

37. Because $1305°$ is coterminal with an angle of $1305° - 3 \cdot 360° = 1305° - 1080° = 225°$, it lies in quadrant III. Its reference angle is $225° - 180° = 45°$. Because the sine is negative in quadrant III, we have

$$\sin 1305° = -\sin 45° = -\frac{\sqrt{2}}{2}.$$

38. Because $1500°$ is coterminal with an angle of $1500° - 4 \cdot 360° = 1500° - 1440° = 60°$, it lies in quadrant I. Because $1500°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so

$$\sin 1500° = \sin 60° = \frac{\sqrt{3}}{2}.$$

39. Because $-510°$ is coterminal with an angle of $-510° + 2 \cdot 360° = -510° + 720° = 210°,$ it lies in quadrant III. Its reference angle is $210° - 180° = 30°.$ The cosine is negative in quadrant III, so
$\cos(-510°) = -\cos 30° = -\frac{\sqrt{3}}{2}.$

40. Because $-1020°$ is coterminal with an angle of $-1020° + 3 \cdot 360° = -1020° + 1080° = 60°,$ it lies in quadrant I. Because $-1020°$ lies in quadrant I, the values of all of its trigonometric functions will be positive, so $\tan(-1020°) = \tan 60° = \sqrt{3}.$

41. Because −855º is coterminal with $-855° + 3 \cdot 360° = -855° + 1080° = 225°$, it lies in quadrant III. Its reference angle is 225º − 180º = 45º. The cosecant is negative in quadrant III, so
$\csc(-855°) = -\csc 45° = -\sqrt{2}.$

42. Because −495º is coterminal with an angle of $-495° + 2 \cdot 360° = -495° + 720° = 225°$, it lies in quadrant III. Its reference angle is $225° - 180° = 45°.$ Tthe secant is negative in quadrant III, so $\sec(-495°) = -\sec 45° = -\sqrt{2}.$

43. Because 3015º is coterminal with $3015° - 8 \cdot 360° = 3015° - 2880° = 135°$, it lies in quadrant II. Its reference angle is 180º − 135º = 45º. The tangent is negative in quadrant II, so $\tan 3015° = -\tan 45° = -1.$

44. Because 2280º is coterminal with $2280° - 6 \cdot 360° = 2280° - 2160° = 120°$, it lies in quadrant II. Its reference angle is 180º − 120º = 60º. The cotangent is negative in quadrant II, so
$\cot 2280° = -\cot 60° = -\frac{\sqrt{3}}{3}.$

45. $\sin^2 120° + \cos^2 120° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$
$= \frac{3}{4} + \frac{1}{4} = 1$

46. $\sin^2 225° + \cos^2 225° = \left(-\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2$
$= \frac{2}{4} + \frac{2}{4} = 1$

47. $2\tan^2 120° + 3\sin^2 150° - \cos^2 180°$
$= 2\left(-\sqrt{3}\right)^2 + 3\left(\frac{1}{2}\right)^2 - (-1)^2$
$= 2(3) + 3\left(\frac{1}{4}\right) - 1 = \frac{23}{4}$

48. $\cot^2 135° - \sin 30° + 4\tan 45°$
$= (-1)^2 - \frac{1}{2} + 4(1) = 1 - \frac{1}{2} + 4 = \frac{9}{2}$

49. $\sin^2 225° - \cos^2 270° + \tan^2 60°$
$= \left(-\frac{\sqrt{2}}{2}\right)^2 + 0^2 + \left(\sqrt{3}\right)^2 = \frac{2}{4} + 3 = \frac{7}{2}$

50. $\cot^2 90° - \sec^2 180° + \csc^2 135°$
$= 0^2 - (-1)^2 + \left(\sqrt{2}\right)^2 = -1 + 2 = 1$

51. $\cos^2 60° + \sec^2 150° - \csc^2 210°$
$= \left(\frac{1}{2}\right)^2 + \left(-\frac{2\sqrt{3}}{3}\right)^2 - (-2)^2$
$= \frac{1}{4} + \frac{4}{3} - 4 = -\frac{29}{12}$

52. $\cot^2 135° + \tan^4 60° - \sin^4 180°$
$= (-1)^2 + \left(\sqrt{3}\right)^4 - 0^4 = 1 + 9 = 10$

53. $\cos(30° + 60°) \stackrel{?}{=} \cos 30° + \cos 60°$
Evaluate each side to determine whether this statement is true or false.
$\cos(30° + 60°) = \cos 90° = 0$ and
$\cos 30° + \cos 60° = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$
$0 \neq \frac{\sqrt{3}+1}{2},$ so the statement is false.

54. $\sin 30° + \sin 60° \stackrel{?}{=} \sin(30° + 60°)$
Evaluate each side to determine whether this statement is true or false.
$\sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$ and
$\sin(30° + 60°) = \sin 90° = 1$
Because $\frac{1+\sqrt{3}}{2} \neq 1,$ the given statement is false.

55. $\cos 60° \stackrel{?}{=} 2\cos 30°$

Evaluate each side to determine whether this statement is true or false.

$\cos 60° = \frac{1}{2}$ and $2\cos 30° = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

Because $\frac{1}{2} \neq \sqrt{3}$, the statement is false.

56. $\cos 60° \stackrel{?}{=} 2\cos^2 30° - 1$

Evaluate each side to determine whether this statement is true or false.

$\cos 60° = \frac{1}{2}$ and

$$2\cos^2 30° - 1 = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2\left(\frac{3}{4}\right) - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

Because $\frac{1}{2} = \frac{1}{2}$, the statement is true.

57. $\sin^2 45° + \cos^2 45° \stackrel{?}{=} 1$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$$

Because 1 = 1, the statement is true.

58. $\tan^2 60° + 1 = \sec^2 60°$

Evaluate each side to determine whether this statement is true or false.

$\tan^2 60° + 1 = \left(\sqrt{3}\right)^2 + 1 = 3 + 1 = 4$ and $\sec^2 60° = 2^2 = 4$. Because 4 = 4, the statement is true.

59. $\cos(2\cdot 45)° \stackrel{?}{=} 2\cos 45°$

Evaluate each side to determine whether this statement is true or false.

$\cos(2\cdot 45)° = \cos 90° = 0$ and

$$2\cos 45° = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

Because $0 \neq \sqrt{2}$, the statement is false.

60. $\sin(2\cdot 30°) \stackrel{?}{=} 2\sin 30°\cdot\cos 30°$

Evaluate each side to determine whether this statement is true or false.

$$\sin(2\cdot 30°) = \sin 60° = \frac{\sqrt{3}}{2}$$

$$2\sin 30°\cdot\cos 30° = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$

Because $\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$, the statement is true.

61. $\sin\theta = \frac{1}{2}$

Because $\sin\theta$ is positive, θ must lie in quadrants I or II. Because one angle, namely $30°$, lies in quadrant I, that angle is also the reference angle, θ'. The angle in quadrant II will be $180° - \theta' = 180° - 30° = 150°$.

62. $\cos\theta = \frac{\sqrt{3}}{2}$

Because $\cos\theta$ is positive, θ must lie in quadrants I or IV. One angle, namely $30°$, lies in quadrant I, so that angle is also the reference angle, θ'. The angle in quadrant IV will be $360° - \theta' = 360° - 30° = 330°$.

63. $\tan\theta = -\sqrt{3}$

Because $\tan\theta$ is negative, θ must lie in quadrants II or IV. The absolute value of $\tan\theta$ is $\sqrt{3}$, so the reference angle, θ' must be $60°$. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant IV angle θ equals $360° - \theta' = 360° - 60° = 300°$.

64. $\sec\theta = -\sqrt{2}$

Because $\sec\theta$ is negative, θ must lie in quadrants II or III. The absolute value of $\sec\theta$ is $\sqrt{2}$, so the reference angle, θ' must be $45°$. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 45° = 225°$.

65. $\cos\theta = \frac{\sqrt{2}}{2}$

Because $\cos\theta$ is positive, θ must lie in quadrants I or IV. One angle, namely $45°$, lies in quadrant I, so that angle is also the reference angle, θ'. The angle in quadrant IV will be $360° - \theta' = 360° - 45° = 315°$.

66. $\cot\theta = -\dfrac{\sqrt{3}}{3}$
Because $\cot\theta$ is negative, θ must lie in quadrants II or IV. The absolute value of $\cot\theta$ is $\dfrac{\sqrt{3}}{3}$, so the reference angle, θ' must be $60°$. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant IV angle θ equals $360° - \theta' = 360° - 60° = 300°$.

67. $\csc\theta = -2$
Because $\csc\theta$ is negative, θ must lie in quadrants III or IV. The absolute value of $\csc\theta$ is 2, so the reference angle, θ', is $30°$. The angle in quadrant III will be $180° + \theta' = 180° + 30° = 210°$, and the quadrant IV angle is $360° - \theta' = 360° - 30° = 330°$.

68. $\sin\theta = -\dfrac{\sqrt{3}}{2}$
Because $\sin\theta$ is negative, θ must lie in quadrants III or IV. The absolute value of $\sin\theta$ is $\dfrac{\sqrt{3}}{2}$, so the reference angle, θ', is 60°. The angle in quadrant III will be $180° + \theta' = 180° + 60° = 240°$, and the quadrant IV angle is $360° - \theta' = 360° - 60° = 300°$.

69. $\tan\theta = \dfrac{\sqrt{3}}{3}$
Because $\tan\theta$ is positive, θ must lie in quadrants I or III. One angle, namely $30°$, lies in quadrant I, so that angle is also the reference angle, θ'. The angle in quadrant III will be $180° + \theta' = 180° + 30° = 210°$.

70. $\cos\theta = -\dfrac{1}{2}$
Because $\cos\theta$ is negative, θ must lie in quadrants II or III. The absolute value of $\cos\theta$ is $\dfrac{1}{2}$, so the reference angle, θ' must be $60°$. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 60° = 240°$.

71. $\csc\theta = -\sqrt{2}$
Because $\csc\theta$ is negative, θ must lie in quadrants III or IV. The absolute value of $\csc\theta$ is $\sqrt{2}$, so the reference angle, θ' must be $45°$. The quadrant III angle θ equals $180° + \theta' = 180° + 45° = 225°$. and the quadrant IV angle θ equals $360° - \theta' = 360° - 45° = 315°$.

72. $\cot\theta = -1$
Because $\cot\theta$ is negative, θ must lie in quadrants II or IV. The absolute value of $\cot\theta$ is 1, so the reference angle, θ' must be $45°$. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$. and the quadrant IV angle θ equals $360° - \theta' = 360° - 45° = 315°$.

73. 150º is in quadrant II, so the reference angle is 180º − 150º = 30º.

$$\cos 30° = \frac{x}{r} \Rightarrow x = r\cos 30° = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

and $\sin 30° = \dfrac{y}{r} \Rightarrow y = r\sin 30° = 6 \cdot \dfrac{1}{2} = 3$

Because 150º is in quadrant II, the x- coordinate will be negative. The coordinates of P are $\left(-3\sqrt{3}, 3\right)$.

74. 225º is in quadrant III, so the reference angle is 225º − 180º = 45º.

$$\cos 45° = \frac{x}{r} \Rightarrow x = r\cos 45° = 10 \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2}$$

and

$$\sin 45° = \frac{y}{r} \Rightarrow y = r\sin 45° = 10 \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2}$$

Because 225º is in quadrant III, both the x- and y-coordinates will be negative. The coordinates of P are $\left(-5\sqrt{2}, -5\sqrt{2}\right)$.

75. For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.
Because $(-0.8)^2 + (0.6)^2 = 0.64 + 0.36 = 1$, there is an angle θ for which $\cos\theta = 0.6$ and $\sin\theta = -0.8$. Because $\cos\theta > 0$ and $\sin\theta < 0$, θ lies in quadrant IV.

76. For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.
Because $\left(\frac{3}{4}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{9}{16} + \frac{4}{9} = \frac{145}{144} \neq 1$, there is no angle θ for which $\cos\theta = \frac{2}{3}$ and $\sin\theta = \frac{3}{4}$.

77. If θ is in the interval $(90°, 180°)$, then $90° < \theta < 180° \Rightarrow 45° < \frac{\theta}{2} < 90°$. Thus $\frac{\theta}{2}$ lies in quadrant I, and $\cos\frac{\theta}{2}$ is positive.

78. If θ is in the interval $(90°, 180°)$, then $90° < \theta < 180° \Rightarrow 45° < \frac{\theta}{2} < 90°$. Thus $\frac{\theta}{2}$ lies in quadrant I, and $\sin\frac{\theta}{2}$ is positive.

79. If θ is in the interval $(90°, 180°)$, then $90° < \theta < 180° \Rightarrow 270° < \theta + 180° < 360°$. Thus $\theta + 180°$ lies in quadrant IV, and $\sec(\theta + 180°)$ is positive.

80. If θ is in the interval $(90°, 180°)$, then $90° < \theta < 180° \Rightarrow 270° < \theta + 180° < 360°$. Thus $\theta + 180°$ lies in quadrant IV, and $\cot(\theta + 180°)$ is negative.

81. If θ is in the interval $(90°, 180°)$, then
$90° < \theta < 180° \Rightarrow -90° > -\theta > -180° \Rightarrow$
$-180° < \theta < -90°$
Because 180º is coterminal with $-180º + 360º = 180º$ and $-90º$ is coterminal with $-90º + 360º = 270º$, $-\theta$ lies in quadrant III, and $\sin(-\theta)$ is negative.

82. If θ is in the interval $(90°, 180°)$, then
$90° < \theta < 180° \Rightarrow -90° > -\theta > -180° \Rightarrow$
$-180° < \theta < -90°$
Because 180º is coterminal with $-180º + 360º = 180º$ and $-90º$ is coterminal with $-90º + 360º = 270º$, $-\theta$ lies in quadrant III, and $\cos(-\theta)$ is negative.

83. When an integer multiple of 360° is added to θ, the resulting angle is coterminal with θ. The sine values of coterminal angles are equal.

84. When an integer multiple of 360° is added to θ, the resulting angle is coterminal with θ. The cosine values of coterminal angles are equal.

85. The reference angle for 115° is 180°−115° = 65°. Because 115° is in quadrant II the cosine is negative. Sin θ decreases on the interval (90°, 180°) from 1 to 0. Therefore, sin 115° is closest to 0.9.

86. The reference angle for 115° is 180°−115° = 65°. Because 115° is in quadrant II the cosine is negative. Cos θ decreases on the interval (90°, 180°) from 0 to –1. Therefore, cos 115° is closest to −0.4.

87. When $\theta = 45°$, $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$. Sine and cosine are both positive in quadrant I and both negative in quadrant III. Because $\theta + 180° = 45° + 180° = 225°$, 45º is the quadrant I angle, and 225º is the quadrant III angle.

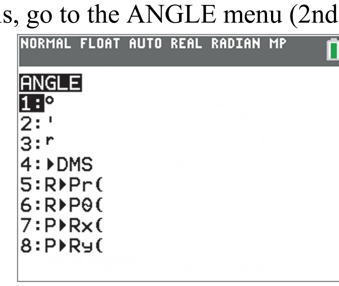
88. When $\theta = 45°$, $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$. Sine and cosine are opposites in quadrants II and IV. Thus, $180° - \theta = 180° - 45° = 135°$ in quadrant II, and $360° - \theta = 360° - 45° = 315°$ in quadrant IV.

Section 2.3 Approximations of Trigonometric Function Values

For Exercises 1−10, be sure your calculator is in degree mode.

1. J	**2.** B	**3.** E	**4.** F
5. D	**6.** I	**7.** H	**8.** A
9. G	**10.** C		

For Exercises 11−40, be sure your calculator is in degree mode. If your calculator accepts angles in degrees, minutes, and seconds, it is not necessary to change angles to decimal degrees. Keystroke sequences may vary on the type and/or model of calculator being used. Screens shown will be from a TI-84 Plus C calculator. To obtain the degree (°) and (′) symbols, go to the ANGLE menu (2nd APPS).

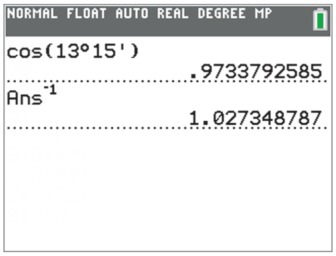


For Exercises 11−40, we include TI-84 screens only for those exercises involving cotangent, secant, and cosecant.

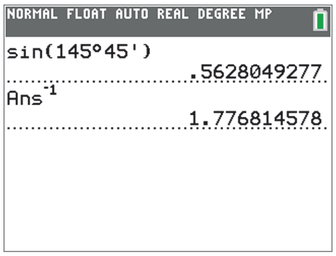
11. $\sin 38°42' \approx 0.625243$

12. $\cos 41°24' \approx 0.750111$

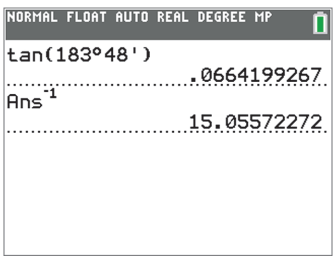
13. $\sec 13°15' \approx 1.027349$



14. $\csc 145°45' \approx 1.776815$



15. $\cot 183°48' \approx 15.055723$

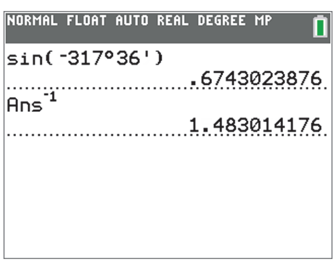


16. $\tan 421°30' \approx 1.841771$

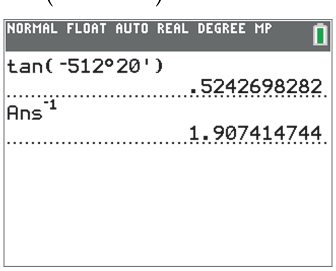
17. $\sin(-312°12') \approx 0.740805$

18. $\tan(-80°06') \approx -5.729742$

19. $\csc(-317°36') \approx 1.483014$



20. $\cot(-512°20') \approx 1.907415$



21. $\dfrac{1}{\cot 23.4°} = \tan 23.4° \approx 0.432739$

22. $\dfrac{1}{\sec 14.8°} = \cos 14.8° \approx 0.966823$

23. $\dfrac{\cos 77°}{\sin 77°} = \cot 77° \approx 0.230868$

24. $\dfrac{\sin 33°}{\cos 33°} = \tan 33° \approx 0.649408$

25. $\cot(90° - 4.72°) = \tan 4.72° \approx 0.082566$

26. $\cos(90° - 3.69°) = \sin 3.69° \approx 0.064358$

27. $\dfrac{1}{\csc(90° - 51°)} = \cos 51° \approx 0.629320$

28. $\dfrac{1}{\tan(90° - 22°)} = \dfrac{1}{\cot 22°}$
$= \tan 22° \approx 0.404026$

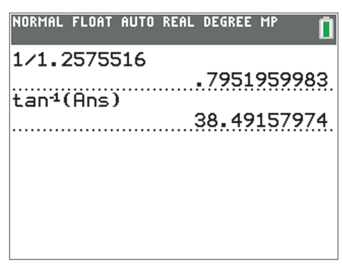
29. $\tan\theta = 1.4739716$
$\theta = \tan^{-1}(1.4739716) \approx 55.845496°$

30. $\tan\theta = 6.4358841$
$\theta = \tan^{-1}(6.4358841) \approx 81.168073°$

31. $\sin\theta = 0.27843196$
$\theta = \sin^{-1}(0.27843196) \approx 16.166641°$

32. $\sin\theta = 0.84802194 \Rightarrow$
$\theta = \sin^{-1}(0.84802194) \approx 57.997172°$

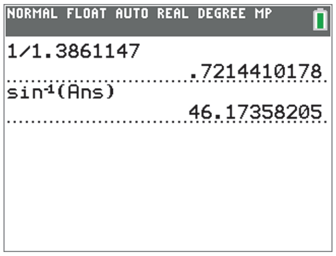
33. $\cot\theta = 1.2575516$
$\theta = \cot^{-1}(1.2575516) = \tan^{-1}\left(\dfrac{1}{1.2575516}\right)$
$\approx 38.491580°$



34. $\csc\theta = 1.3861147$

$\theta = \csc^{-1}(1.3861147) = \sin^{-1}\left(\frac{1}{1.3861147}\right)$

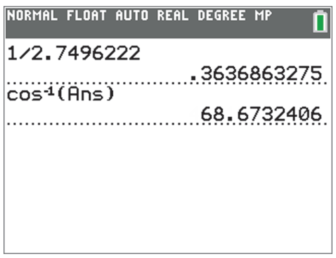
$\approx 46.173582°$



35. $\sec\theta = 2.7496222$

$\theta = \sec^{-1}(2.7496222) = \cos^{-1}\left(\frac{1}{2.7496222}\right)$

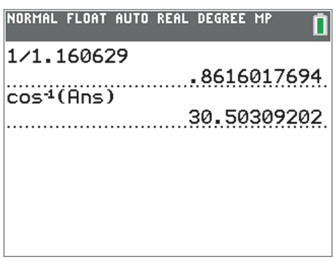
$\approx 68.673241°$



36. $\sec\theta = 1.1606249$

$\theta = \sec^{-1}(1.1606249) = \cos^{-1}\frac{1}{1.1606249}$

$\approx 30.502748°$



37. $\cos\theta = 0.70058013$

$\theta = \cos^{-1}(0.70058013) \approx 45.526434°$

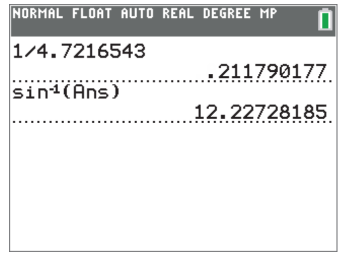
38. $\cos\theta = 0.85536428$

$\theta = \cos^{-1}(0.85536428) \approx 31.199998°$

39. $\csc\theta = 4.7216543 \Rightarrow$

$\theta = \csc^{-1}(4.7216543) = \sin^{-1}\left(\frac{1}{4.7216543}\right)$

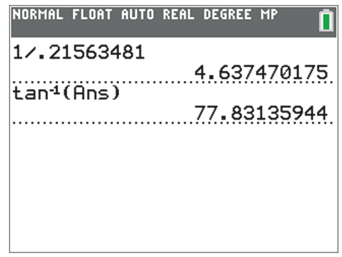
$\approx 12.227282°$



40. $\cot\theta = 0.21563481 \Rightarrow$

$\theta = \cot^{-1}(0.21563481) = \tan^{-1}\left(\frac{1}{0.21563481}\right)$

$\approx 77.831359°$



41. A common mistake is to have the calculator in radian mode, when it should be in degree mode (and vice verse).

42. If the calculator allows an angle θ where $0° \le \theta < 360°$, then we need to find an angle within this interval that is coterminal with 2000º by subtracting a multiple of 360º: $2000° - 5\cdot 360° = 2000° - 1800° = 200°$. Then find cos 200°. If the calculator is more restrictive on evaluating angles (such as $0° \le \theta < 90°$), then reference angles need to be used.

43. Find $\tan^{-1}(1.482560969)$.

$A = 56º$.

44. Find the sine of 22º.

$A = 0.3746065934º$

45. $\sin 35°\cos 55° + \cos 35°\sin 55° = 1$

46. $\cos 100°\cos 80° - \sin 100°\sin 80° = -1$

47. $\sin^2 36° + \cos^2 36° = 1$

48. $2\sin 25°13'\cos 25°13' - \sin 50°26' = 0$

49. $\cos 75°29'\cos 14°31' - \sin 75°29'\sin 14°31' = 0$

50. $\cos 28°14'\cos 61°46' - \sin 28°14'\sin 61°46' = 1$

51. $\sin 10° + \sin 10° \stackrel{?}{=} \sin 20°$

Using a calculator gives $\sin 10° + \sin 10° \approx 0.34729636$ and $\sin 20° \approx 0.34202014$. Thus, the statement is false.

52. $\cos 40° \stackrel{?}{=} 2\cos 20°$
Using a calculator gives
$\cos 40° \approx 0.76604444$ and
$2\cos 20° \approx 1.87938524.$ Thus, the statement is false.

53. $\sin 50° \stackrel{?}{=} 2\sin 25° \cos 25°$
Using a calculator gives $\sin 50° \approx 0.76604444$ and $2\sin 25° \cos 25° \approx 0.76604444.$ Thus, the statement is true.

54. $\cos 70° \stackrel{?}{=} 2\cos^2 35° - 1$
Using a calculator gives
$\cos 70° \approx 0.34202014$ and
$2\cos^2 35° - 1 \approx 0.34202014.$ Thus, the statement is true.

55. $\cos 40° \stackrel{?}{=} 1 - 2\sin^2 80°$
Using a calculator gives
$\cos 40° \approx 0.76604444$ and
$1 - 2\sin^2 80° \approx -0.93969262.$ Thus, the statement is false.

56. $2\cos 38°22' \stackrel{?}{=} \cos 76°44'$
Using a calculator gives
$2\cos 38°22' \approx 1.56810939$ and
$\cos 76°44' \approx 0.22948353.$ Thus, the statement is false.

57. $\sin 39°48' + \cos 39°48' \stackrel{?}{=} 1$
Using a calculator gives
$\sin 39°48' + \cos 39°48' \approx 1.40839322 \neq 1.$
Thus, the statement is false.

58. $\frac{1}{2}\sin 40° \stackrel{?}{=} \sin \frac{1}{2}(40°)$
Using a calculator gives
$\frac{1}{2}\sin 40° \approx 0.32139380$ and
$\sin \frac{1}{2}(40°) \approx 0.34202014.$
Thus, the statement is false.

59. $1 + \cot^2 42.5° \stackrel{?}{=} \csc^2 42.5°$
Using a calculator gives
$1 + \cot^2 42.5° \approx 2.1909542$ and
$\csc^2 42.5° \approx 2.1909542.$
Thus, the statement is true.

60. $\tan^2 72°25' + 1 \stackrel{?}{=} \sec^2 72°25'$
Using a calculator gives
$\tan^2 72°25' + 1 \approx 10.9577102$ and
$\sec^2 72°25' \approx 10.9577102.$
Thus, the statement is true.

61. $\cos(30° + 20°) \stackrel{?}{=} \cos 30° \cos 20° - \sin 30° \sin 20°$
Using a calculator gives
$\cos(30° + 20°) \approx 0.64278761$ and
$\cos 30° \cos 20° - \sin 30° \sin 20° \approx 0.64278761.$
Thus, the statement is true.

62. $\cos(30° + 20°) \stackrel{?}{=} \cos 30° + \cos 20°$
Using a calculator gives
$\cos(30° + 20°) \approx 0.64278761$ and
$\cos 30° + \cos 20° \approx 1.8057180.$ Thus, the statement is false.

63. $\sin\theta = 0.92718385$
$\sin\theta$ is positive in quadrants I and II.
$\sin^{-1}(0.92718385) = 68°$
The angle in quadrant II with the same sine is $180° - 68° = 112°.$

64. $\sin\theta = 0.52991926$
$\sin\theta$ is positive in quadrants I and II.
$\sin^{-1}(0.52991926) = 32°$
The angle in quadrant II with the same sine is $180° - 32° = 148°.$

65. $\cos\theta = 0.71933980$
$\cos\theta$ is positive in quadrants I and IV.
$\cos^{-1}(0.71933980) = 44°$
The angle in quadrant IV with the same cosine is $360° - 44° = 316°.$

66. $\cos\theta = 0.10452846$
$\cos\theta$ is positive in quadrants I and IV.
$\cos^{-1}(0.10452846) = 84°$
The angle in quadrant IV with the same cosine is $360° - 84° = 276°.$

67. $\tan\theta = 1.2348971$
$\tan\theta$ is positive in quadrants I and III.
$\tan^{-1}(1.2348971) = 51°$
The angle in quadrant III with the same tangent is $180° + 51° = 231°.$

68. $\tan\theta = 0.70020753$
$\tan\theta$ is positive in quadrants I and III.
$\tan^{-1}(0.70020753) = 35°$
The angle in quadrant III with the same tangent is $180° + 35° = 215°$.

69. $F = W\sin\theta$
$F = 2100\sin 1.8° \approx 65.96 \approx 70$ lb

70. $F = W\sin\theta$
$F = 2400\sin(-2.4°) \approx -100.5 \approx -100$ lb
F is negative because the car is traveling downhill.

71. $F = W\sin\theta$
$-130 = 2600\sin\theta \Rightarrow \dfrac{-130}{2600} = \sin\theta \Rightarrow$
$-0.05 = \sin\theta \Rightarrow \theta = \sin^{-1}(-0.05) \approx -2.9°$

72. $F = W\sin\theta$
$150 = 3000\sin\theta \Rightarrow \dfrac{150}{3000} = \sin\theta \Rightarrow$
$0.05 = \sin\theta \Rightarrow \theta = \sin^{-1}(0.05) \approx 2.9°$

73. $F = W\sin\theta$
$120 = W\sin(2.7°) \Rightarrow \dfrac{120}{\sin(2.7°)} = W \Rightarrow$
$W \approx 2547 \approx 2500$ lb

74. $F = W\sin\theta$
$-145 = W\sin(-3°) \Rightarrow \dfrac{-145}{\sin(-3°)} = W \Rightarrow$
$W \approx 2771 \approx 2800$ lb

75. $F = W\sin\theta$
$F = 2200\sin 2° \approx 76.77889275$ lb
$F = 2000\sin 2.2° \approx 76.77561818$ lb
The 2200-lb car on a 2° uphill grade has the greater grade resistance.

76.

θ	$\sin\theta$	$\tan\theta$	$\dfrac{\pi\theta}{180}$
0°	0.0000	0.0000	0.0000
0.5°	0.0087	0.0087	0.0087
1°	0.0175	0.0175	0.0175
1.5°	0.0262	0.0262	0.0262
2°	0.0349	0.0349	0.0349
2.5°	0.0436	0.0437	0.0436
3°	0.0523	0.0524	0.0524
3.5°	0.0610	0.0612	0.0611
4°	0.0698	0.0699	0.0698

(a) From the table, we see that if θ is small, $\sin\theta \approx \tan\theta \approx \dfrac{\pi\theta}{180}$.

(b) $F = W\sin\theta \approx W\tan\theta \approx \dfrac{W\pi\theta}{180}$

(c) $\tan\theta = \dfrac{4}{100} = 0.04$
$F \approx W\tan\theta = 2000(0.04) = 80$ lb

(d) Use $F \approx \dfrac{W\pi\theta}{180}$ from part (b). Let $\theta = 3.75$ and $W = 1800$.
$F \approx \dfrac{1800\pi(3.75)}{180} \approx 118$ lb

77. 45 mph = 66 ft/sec, $V = 66$, $\theta = 3°$, $g = 32.2$, $f = 0.14$

$$R = \frac{V^2}{g(f + \tan\theta)} = \frac{66^2}{32.2(0.14 + \tan 3°)} \approx 703 \text{ ft}$$

78. There are 5280 ft in one mile and 3600 sec in one min.

$$70 \text{ mph} = 70 \text{ mph} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 102\frac{2}{3} \text{ ft per sec} \approx 102.67 \text{ ft per sec}$$

$V = 102.67$, $\theta = 3°$, $g = 32.2$, $f = 0.14$

$$R = \frac{V^2}{g(f + \tan\theta)} \approx \frac{102.67^2}{32.2(0.14 + \tan 3°)} \approx 1701 \text{ ft}$$

79. Intuitively, increasing θ would make it easier to negotiate the curve at a higher speed much like is done at a race track. Mathematically, a larger value of θ (acute) will lead to a larger value for $\tan\theta$. If $\tan\theta$ increases, then the ratio determining R will *decrease*. Thus, the radius can be smaller and the curve sharper if θ is increased.

$$R = \frac{V^2}{g(f+\tan\theta)} = \frac{66^2}{32.2(0.14+\tan 4°)} \approx 644 \text{ ft}$$

$$R = \frac{V^2}{g(f+\tan\theta)} \approx \frac{102.67^2}{32.2(0.14+\tan 4°)} \approx 1559 \text{ ft}$$

As predicted, both values are less.

80. From Exercises 77 amd 78,

$R = \frac{V^2}{g(f+\tan\theta)}$. Solving for V we have

$$R = \frac{V^2}{g(f+\tan\theta)} \Rightarrow V^2 = Rg(f+\tan\theta) \Rightarrow$$
$$V = \sqrt{Rg(f+\tan\theta)}$$

$R = 1150$, $\theta = 2.1°$, $g = 32.2$, $f = 0.14$

$$V = \sqrt{Rg(f+\tan\theta)} = \sqrt{1150(32.2)(0.14+\tan 2.1°)} \approx 80.9 \text{ ft/sec}$$

80.9 ft/sec · 3600 sec/hr · 1 mi/5280 ft ≈ 55 mph,

The speed limit should be 55 mph.

81. **(a)** $\theta_1 = 46°$, $\theta_2 = 31°$, $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow c_2 = \frac{c_1\sin\theta_2}{\sin\theta_1} \Rightarrow$$
$$c_2 = \frac{(3\times10^8)(\sin 31°)}{\sin 46°} \approx 2\times10^8$$

Because c_1 is only given to one significant digit, c_2 can only be given to one significant digit. The speed of light in the second medium is about 2×10^8 m per sec.

(b) $\theta_1 = 39°$, $\theta_2 = 28°$, $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow c_2 = \frac{c_1\sin\theta_2}{\sin\theta_1} \Rightarrow$$
$$c_2 = \frac{(3\times10^8)(\sin 28°)}{\sin 39°} \approx 2\times10^8$$

Because c_1 is only given to one significant digit, c_2 can only be given to one significant digit. The speed of light in the second medium is about 2×10^8 m per sec.

82. **(a)** $\theta_1 = 40°, c_2 = 1.5\times10^8$ m per sec, and $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2\sin\theta_1}{c_1} \Rightarrow$$
$$\sin\theta_2 = \frac{(1.5\times10^8)(\sin 40°)}{3\times10^8} \Rightarrow$$
$$\theta_2 = \sin^{-1}\left[\frac{(1.5\times10^8)(\sin 40°)}{3\times10^8}\right] \approx 19°$$

(b) $\theta_1 = 62°$, $c_2 = 2.6\times10^8$ m per sec and $c_1 = 3\times10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2\sin\theta_1}{c_1} \Rightarrow$$
$$\sin\theta_2 = \frac{(2.6\times10^8)(\sin 62°)}{3\times10^8} \Rightarrow$$
$$\theta_2 = \sin^{-1}\left[\frac{(2.6\times10^8)(\sin 62°)}{3\times10^8}\right] \approx 50°$$

83. $\theta_1 = 90°$, $c_1 = 3\times10^8$ m per sec, and $c_2 = 2.254\times10^8$

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2\sin\theta_1}{c_1}$$
$$\sin\theta_2 = \frac{(2.254\times10^8)(\sin 90°)}{3\times10^8} = \frac{2.254\times10^8(1)}{3\times10^8} = \frac{2.254}{3} \Rightarrow$$
$$\theta_2 = \sin^{-1}\left(\frac{2.254}{3}\right) \approx 48.7°$$

84. $\theta_1 = 90° - 29.6° = 60.4°,\ c_1 = 3\times 10^8$ m per sec, and $c_2 = 2.254\times 10^8$ $c_1 = 3\times 10^8$ m per sec

$$\frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \sin\theta_2 = \frac{c_2\sin\theta_1}{c_1}$$

$$\sin\theta_2 = \frac{\left(2.254\times 10^8\right)\left(\sin 60.4°\right)}{3\times 10^8} = \frac{2.254}{3}\left(\sin 60.4°\right) \Rightarrow$$

$$\theta_2 = \sin^{-1}\left(\frac{2.254}{3}\left(\sin 60.4°\right)\right) \approx 40.8°$$

Light from the object is refracted at an angle of 40.8° from the vertical. Light from the horizon is refracted at an angle of 48.7° from the vertical. Therefore, the fish thinks the object lies at an angle of 48.7° – 40.8° = 7.9° above the horizon.

85. $V_1 = 55 \text{ mph} = 55 \text{ mph}\cdot\frac{1 \text{ hr}}{3600 \text{ sec}}\cdot\frac{5280 \text{ ft}}{1 \text{ mi}} = 80\frac{2}{3} \text{ ft per sec} \approx 80.67 \text{ ft per sec},$

$V_2 = 30 \text{ mph} = 30 \text{ mph}\cdot\frac{1 \text{ hr}}{3600 \text{ sec}}\cdot\frac{5280 \text{ ft}}{1 \text{ mi}} = 44 \text{ ft per sec}$

$\theta = 3.5°,\ K_1 = 0.4,\ K_2 = 0.02.$

$$D = \frac{1.05\left(V_1^2 - V_2^2\right)}{64.4\left(K_1 + K_2 + \sin\theta\right)} = \frac{1.05\left(80.67^2 - 44^2\right)}{64.4\left(0.4 + 0.02 + \sin 3.5°\right)} \approx 155 \text{ ft}$$

86. $V_1 \approx 80.67$ ft per sec, $V_2 = 44$ ft per sec, $\theta = -2°,\ K_1 = 0.4,$ and $K_2 = 0.02.$

$$D = \frac{1.05\left(V_1^2 - V_2^2\right)}{64.4\left(K_1 + K_2 + \sin\theta\right)} = \frac{1.05\left(80.67^2 - 44^2\right)}{64.4\left[0.4 + 0.02 + \sin(-2°)\right]} \approx 194 \text{ ft}$$

87. Negative values of θ require greater distances for slowing down than positive values.

88. Using the values for K_1 and K_2 from Exercise 73, determine V_2 when $D = 200,\ \theta = -3.5°,$

$V_1 = 90 \text{ mph} = 90 \text{ mph}\cdot\frac{1 \text{ hr}}{3600 \text{ sec}}\cdot\frac{5280 \text{ ft}}{1 \text{ mi}} = 132 \text{ ft per sec}$

$$D = \frac{1.05\left(V_1^2 - V_2^2\right)}{64.4\left(K_1 + K_2 + \sin\theta\right)}$$

$$200 = \frac{1.05\left(132^2 - V_2^2\right)}{64.4\left[0.4 + 0.02 + \sin(-3.5°)\right]}$$

$$200 = \frac{1.05\left(132^2\right) - 1.05V_2^2}{23.12}$$

$$200(23.12) = 18{,}295.2 - 1.05V_2^2$$

$$4624 = 18{,}295.2 - 1.05V_2^2$$

$$-13{,}671.2 = -1.05V_2^2$$

$$V_2^2 = \frac{-13{,}671.2}{-1.05}$$

$$V_2^2 = 13020.19048$$

$$V_2 \approx 114.106$$

$V_2 \approx 114$ ft/sec · 3600 sec/hr · 1 mi/5280 ft ≈ 78 mph

89. For Auto A, calculate $70\cdot\cos 10° \approx 68.94.$ Auto A's reading is approximately 69 mph. For Auto B, calculate $70\cdot\cos 20° \approx 65.78.$ Auto B's reading is approximately 66 mph.

90. The figure for this exercise indicates a right triangle. Because we are not considering the time involved in detecting the speed of the car, we will consider the speeds as sides of the right triangle. Given angle θ, $\cos\theta = \frac{r}{a}.$ Thus, the speed that the radar detects is $r = a\cos\theta.$ This confirms the "cosine effect" that reduces the radar reading.

91. $h = 1.9$ ft, $\alpha = 0.9°,\ \theta_1 = -3°,\ \theta_2 = 4°,$ $S = 336$ ft:

$$L = \frac{[4-(-3)]336^2}{200(1.9 + 336\tan .9°)} = 550 \text{ ft}$$

92. $h = 1.9$ ft, $\alpha = 1.5°,\ \theta_1 = -3°,\ \theta_2 = 4°,$ $S = 336$ ft:

$$L = \frac{[4-(-3)]336^2}{200(1.9 + 336\tan 1.5°)} = 369 \text{ ft}$$

Chapter 2 Quiz
(Sections 2.1−2.3)

1. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{24}{40} = \frac{3}{5}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{32}{40} = \frac{4}{5}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{24}{32} = \frac{3}{4}$

$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{32}{24} = \frac{4}{3}$

$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{40}{32} = \frac{5}{4}$

$\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{40}{24} = \frac{5}{3}$

2.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$

3. $\sin 30° = \frac{w}{36} \Rightarrow w = 36\sin 30° = 36\cdot\frac{1}{2} = 18$

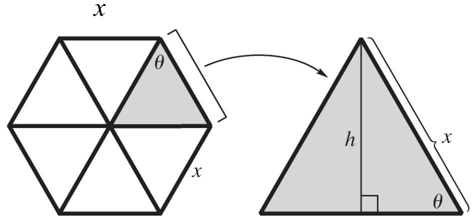
$\cos 30° = \frac{x}{36} \Rightarrow x = 36\cos 30° = 36\cdot\frac{\sqrt{3}}{2} = 18\sqrt{3}$

$\tan 45° = \frac{w}{y} \Rightarrow 1 = \frac{18}{y} \Rightarrow y = 18$

$\sin 45° = \frac{w}{z} \Rightarrow \frac{\sqrt{2}}{2} = \frac{18}{z} \Rightarrow z = \frac{36}{\sqrt{2}} = 18\sqrt{2}$

4. The height of one of the six equilateral triangles from the solar cell is

$\sin\theta = \frac{h}{x} \Rightarrow h = x\sin\theta$.



Thus, the area of each of the triangles is $\mathcal{A} = \frac{1}{2}bh = \frac{1}{2}x^2\sin\theta$. So, the area of the solar cell is $\mathcal{A} = 6\cdot\frac{1}{2}x^2\sin\theta = 3x^2\sin\theta$.

5. 180º − 135º = 45º, so the reference angle is 45º. The original angle (135º) lies in quadrant II, so the sine and cosecant are positive, while the remaining trigonometric functions are negative.

$\sin 135° = \frac{\sqrt{2}}{2}$; $\cos 135° = -\frac{\sqrt{2}}{2}$

$\tan 135° = -1$; $\cot 135° = -1$

$\sec 135° = -\sqrt{2}$; $\csc 135° = \sqrt{2}$

6. −150º is coterminal with 360º − 150º = 210º. This lies in quadrant III, so the reference angle is 210º − 180º = 30º. In quadrant III, the tangent and cotangent functions are positive, while the remaining trigonometric functions are negative.

$\sin(-150°) = -\sin 30° = -\frac{1}{2}$

$\cos(-150°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan(-150°) = \tan 30° = \frac{\sqrt{3}}{3}$

$\cot(-150°) = \cot 30° = \sqrt{3}$

$\sec(-150°) = -\sec 30° = -\frac{2\sqrt{3}}{3}$

$\csc(-150°) = -\csc 30° = -2$

7. 1020º is coterminal with 1020º − 720º = 300º. This lies in quadrant IV, so the reference angle is 360º − 300º = 60º. In quadrant IV, the cosine and secant are positive, while the remaining trigonometric functions are negative.

$\sin 1020° = -\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 1020° = \cos 60° = \frac{1}{2}$

$\tan 1020° = -\tan 60° = -\sqrt{3}$

$\cot 1020° = -\cot 60° = -\frac{\sqrt{3}}{3}$

$\sec 1020° = \sec 60° = 2$

$\csc 1020° = -\csc 60° = -\frac{2\sqrt{3}}{3}$

8. $\sin\theta = \frac{\sqrt{3}}{2}$

Because $\sin\theta$ is positive, θ must lie in quadrants I or II, and the reference angle, θ', is 60°. The angle in quadrant I is 60º, while the angle in quadrant II is $180° - \theta' = 180° - 60° = 120°$.

9. $\sec\theta = -\sqrt{2}$
Because $\sec\theta$ is negative, θ must lie in quadrants II or III. The absolute value of $\sec\theta$ is $\sqrt{2}$, so the reference angle, θ' must be $45°$. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 45° = 225°$.

10. $\sin 42°18' \approx 0.673013$

11. $\sec(-212°12') \approx -1.181763$

12. $\tan\theta = 2.6743210 \Rightarrow \theta \approx 69.497888°$

13. $\csc\theta = 2.3861147 \Rightarrow \theta \approx 24.777233°$

14. The statement is false.
$\sin(60° + 30°) = \sin 90° = 1$, while
$\sin 60° + \sin 30° = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$.

15. The statement is true. Using the cofunction identity, $\tan(90° - 35°) = \cot 35°$.

Section 2.4 Solutions and Applications of Right Triangles

1. B **2.** D **3.** A

4. F **5.** C **6.** E

7. 23.825 to 23.835

8. 28,999.5 to 29,000.5

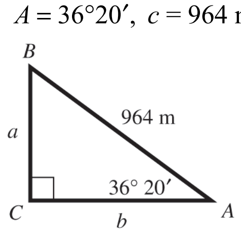
9. 8958.5 to 8959.5

10. No. Points scored in basketball are exact numbers and cannot take on values that are not whole numbers.

11. If h is the actual height of a building and the height is measure as 58.6 ft, then $|h - 58.6| \le \underline{0.05}$.

12. If w is the actual weight of a car and the weight is measure as 1542 lb, then $|w - 1542| \le \underline{0.5}$.

13. $A = 36°20'$, $c = 964$ m

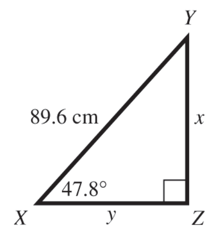


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$
$B = 90° - 36°20'$
$= 89°60' - 36°20' = 53°40'$

$\sin A = \frac{a}{c} \Rightarrow \sin 36°20' = \frac{a}{964} \Rightarrow$
$a = 964\sin 36°20' \approx 571$ m (rounded to three significant digits)

$\cos A = \frac{b}{c} \Rightarrow \cos 36°20' = \frac{b}{964} \Rightarrow$
$b = 964\cos 36°20' \approx 777$ m (rounded to three significant digits)

14. $X = 47.8°$, $z = 89.6$ cm

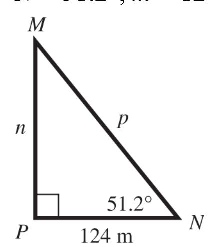


$Y + X = 90° \Rightarrow Y = 90° - X \Rightarrow$
$Y = 90° - 47.8° = 42.2°$

$\sin X = \frac{x}{z} \Rightarrow \sin 47.8° = \frac{x}{89.6} \Rightarrow$
$x = 89.6\sin 47.8° \approx 66.4$ cm (rounded to three significant digits)

$\cos X = \frac{y}{z} \Rightarrow \cos 47.8° = \frac{y}{89.6} \Rightarrow$
$y = 89.6\cos 47.8° \approx 60.2$ cm (rounded to three significant digits)

15. $N = 51.2°$, $m = 124$ m

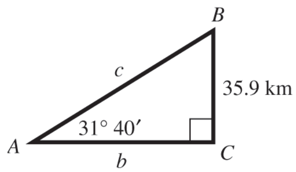


$M + N = 90° \Rightarrow M = 90° - N \Rightarrow$
$M = 90° - 51.2° = 38.8°$

$\tan N = \frac{n}{m} \Rightarrow \tan 51.2° = \frac{n}{124} \Rightarrow$
$n = 124\tan 51.2° \approx 154$ m (rounded to three significant digits)

$\cos N = \frac{m}{p} \Rightarrow \cos 51.2° = \frac{124}{p} \Rightarrow$
$p = \frac{124}{\cos 51.2°} \approx 198$ m (rounded to three significant digits)

16. $A = 31°40', \ a = 35.9$ km

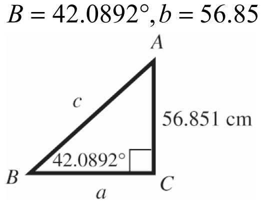


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$
$B = 90° - 31°40'$
$= 89°60' - 31°40' = 58°20'$

$\sin A = \dfrac{a}{c} \Rightarrow \sin 31°40' = \dfrac{35.9}{c} \Rightarrow$
$c = \dfrac{35.9}{\sin 31°40'} \approx 68.4$ km (rounded to three significant digits)

$\tan A = \dfrac{a}{b} \Rightarrow \tan 31°40' = \dfrac{35.9}{b} \Rightarrow$
$b = \dfrac{35.9}{\tan 31°40'} \approx 58.2$ km (rounded to three significant digits)

17. $B = 42.0892°, b = 56.851$

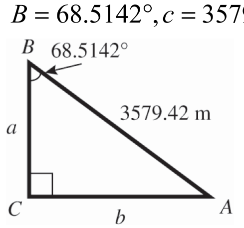


$A + B = 90° \Rightarrow A = 90° - B \Rightarrow$
$A = 90° - 42.0892° = 47.9108°$

$\sin B = \dfrac{b}{c} \Rightarrow \sin 42.0892° = \dfrac{56.851}{c} \Rightarrow$
$c = \dfrac{56.851}{\sin 42.0892°} \approx 84.816$ cm
(rounded to five significant digits)

$\tan B = \dfrac{b}{a} \Rightarrow \tan 42.0892° = \dfrac{56.851}{a} \Rightarrow$
$a = \dfrac{56.851}{\tan 42.0892°} \approx 62.942$ cm
(rounded to five significant digits)

18. $B = 68.5142°, c = 3579.42$

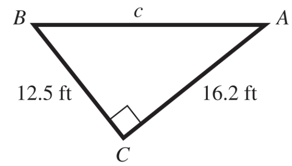


$A + B = 90° \Rightarrow A = 90° - B \Rightarrow$
$A = 90° - 68.5142° = 21.4858°$

$\sin B = \dfrac{b}{c} \Rightarrow \sin 68.5142° = \dfrac{b}{3579.42}$
$b = 3579.42 \sin 68.5142° \approx 3330.68$ m
(rounded to six significant digits)

$\cos B = \dfrac{a}{c} \Rightarrow \cos 68.5142° = \dfrac{a}{3579.42} \Rightarrow$
$a = 3579.42 \cos 68.5142° \approx 1311.04$ m
(rounded to six significant digits)

19. $a = 12.5, b = 16.2$

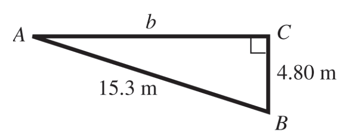


Using the Pythagorean theorem, we have
$a^2 + b^2 = c^2 \Rightarrow 12.5^2 + 16.2^2 = c^2 \Rightarrow$
$418.69 = c^2 \Rightarrow c \approx 20.5$ ft (rounded to three significant digits)

$\tan A = \dfrac{a}{b} \Rightarrow \tan A = \dfrac{12.5}{16.2} \Rightarrow$
$A = \tan^{-1} \dfrac{12.5}{16.2} \approx 37.6540°$
$\approx 37° + (0.6540 \cdot 60)' \approx 37°39' \approx 37°40'$
(rounded to three significant digits)

$\tan B = \dfrac{b}{a} \Rightarrow \tan B = \dfrac{16.2}{12.5} \Rightarrow$
$B = \tan^{-1} \dfrac{16.2}{12.5} \approx 52.3460°$
$\approx 52° + (0.3460 \cdot 60)' \approx 52°21'$
$\approx 52°20'$ (rounded to three significant digits)

20. $a = 4.80, c = 15.3$



Using the Pythagorean theorem, we have
$a^2 + b^2 = c^2 \Rightarrow 4.80^2 + b^2 = 15.3^2 \Rightarrow$
$4.80^2 + b^2 = 15.3^2$
$b^2 = 15.3^2 - 4.80^2 = 211.05$
$b \approx 14.5$ m (rounded to three significant digits)

$\sin A = \dfrac{a}{b} \Rightarrow \sin A = \dfrac{4.80}{15.3} \Rightarrow$
$A = \sin^{-1} \dfrac{4.80}{15.3} \approx 18.2839°$
$\approx 18° + (0.2839 \cdot 60)' \approx 18°17' \approx 18°20'$
(rounded to three significant digits)

(*continued on next page*)

(*continued*)

$$\cos B = \frac{a}{c} \Rightarrow \cos B = \frac{4.80}{15.3} \Rightarrow$$
$$B = \cos^{-1}\frac{4.80}{15.3} \approx 71.7161°$$
$$\approx 71° + (0.7161 \cdot 60)' \approx 71°43' \approx 71°40'$$

(rounded to three significant digits)

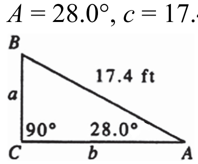
21. No. You need to have at least one side to solve the triangle. There are infinitely many similar right triangles satisfying the given conditions.

22. If we are given an acute angle and a side in a right triangle, the unknown part of the triangle requiring the least work to find is the other acute angle. It may be found by subtracting the given acute angle from 90°.

23. Answers will vary. If you know one acute angle, the other acute angle may be found by subtracting the given acute angle from 90°. If you know one of the sides, then choose two of the trigonometric ratios involving sine, cosine or tangent that involve the known side in order to find the two unknown sides.

24. If you know the lengths of two sides, the Pythagorean theorem can be used to find the length of the remaining side. Then an inverse trigonometric function can be used to find one of the acute angles. The other acute angle may be found by subtracting the calculated acute angle from 90°.

25. $A = 28.0°$, $c = 17.4$ ft

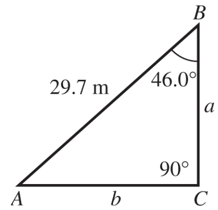


$$A + B = 90°$$
$$B = 90° - A$$
$$B = 90° - 28.0° = 62.0°$$

$$\sin A = \frac{a}{c} \Rightarrow \sin 28.0° = \frac{a}{17.4} \Rightarrow$$
$$a = 17.4 \sin 28.0° \approx 8.17 \text{ ft}$$ (rounded to three significant digits)

$$\cos A = \frac{b}{c} \Rightarrow \cos 28.00° = \frac{b}{17.4} \Rightarrow$$
$$b = 17.4 \cos 28.00° \approx 15.4 \text{ ft}$$ (rounded to three significant digits)

26. $B = 46.0°$, $c = 29.7$ m

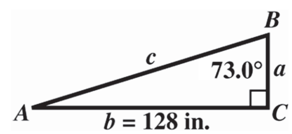


$$A + B = 90°$$
$$A = 90° - B$$
$$A = 90° - 46.0° = 44.0°$$

$$\cos B = \frac{a}{c} \Rightarrow \cos 46.0° = \frac{a}{29.7} \Rightarrow$$
$$a = 29.7 \cos 46.0° \approx 20.6 \text{ m}$$ (rounded to three significant digits)

$$\sin B = \frac{b}{c} \Rightarrow \sin 46.0° = \frac{b}{29.7} \Rightarrow$$
$$b = 29.7 \sin 46.0° \approx 21.4 \text{ m}$$ (rounded to three significant digits)

27. Solve the right triangle with $B = 73.0°$, $b = 128$ in. and $C = 90°$

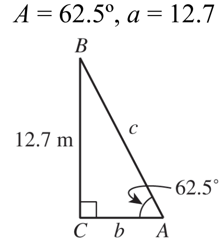


$$A + B = 90° \Rightarrow A = 90° - B \Rightarrow$$
$$A = 90° - 73.0° = 17.0°$$

$$\tan B° = \frac{b}{a} \Rightarrow \tan 73.0° = \frac{128}{a} \Rightarrow$$
$$a = \frac{128}{\tan 73.0°} \Rightarrow a = 39.1 \text{ in}$$ (rounded to three significant digits)

$$\sin B° = \frac{b}{c} \Rightarrow \sin 73.0° = \frac{128}{c} \Rightarrow$$
$$c = \frac{128}{\sin 73.0°} \Rightarrow c = 134 \text{ in}$$ (rounded to three significant digits)

28. $A = 62.5°$, $a = 12.7$ m



$$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$$
$$B = 90° - 62.5° = 27.5°$$

$$\tan A = \frac{a}{b} \Rightarrow \tan 62.5° = \frac{12.7}{b} \Rightarrow$$
$$b = \frac{12.7}{\tan 62.5°} \approx 6.61 \text{ m}$$ (rounded to three significant digits)

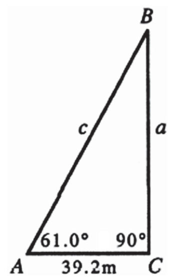
(*continued on next page*)

(*continued*)

$\sin A = \dfrac{a}{c} \Rightarrow \sin 62.5° = \dfrac{12.7}{c} \Rightarrow$

$c = \dfrac{12.7}{\sin 62.5°} \approx 14.3$ m (rounded to three significant digits)

29. $A = 61.0°$, $b = 39.2$ cm



$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$
$B = 90° - 61.0° = 29.0°$

$\tan A = \dfrac{a}{b} \Rightarrow \tan 61.0° = \dfrac{a}{39.2} \Rightarrow$

$a = 39.2 \tan 61.0 \approx 70.7$ cm

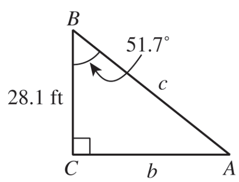
(rounded to three significant digits)

$\cos A = \dfrac{b}{c} \Rightarrow \cos 61.0° = \dfrac{39.2}{c} \Rightarrow$

$c = \dfrac{39.2}{\cos 61.0°} \approx 80.9$ cm

(rounded to three significant digits)

30. $B = 51.7°$, $a = 28.1$ ft



$A + B = 90° \Rightarrow B = 90° - B \Rightarrow$
$A = 90° - 51.7° = 38.3°$

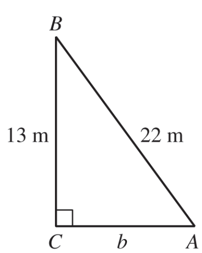
$\tan B = \dfrac{b}{a} \Rightarrow \tan 51.7° = \dfrac{b}{28.1} \Rightarrow$

$b = 28.1 \tan 51.7° \approx 35.6$ ft (rounded to three significant digits)

$\cos B = \dfrac{a}{c} \Rightarrow \cos 51.7° = \dfrac{28.1}{c} \Rightarrow$

$c = \dfrac{28.1}{\cos 51.7°} \approx 45.3$ ft (rounded to three significant digits)

31. $a = 13$ m, $c = 22$m



$c^2 = a^2 + b^2 \Rightarrow 22^2 = 13^2 + b^2 \Rightarrow$
$484 = 169 + b^2 \Rightarrow 315 = b^2 \Rightarrow b \approx 18$ m
(rounded to two significant digits)

We will determine the measurements of both A and B by using the sides of the right triangle. In practice, once you find one of the measurements, subtract it from $90°$ to find the other.

$\sin A = \dfrac{a}{c} \Rightarrow \sin A = \dfrac{13}{22} \Rightarrow$

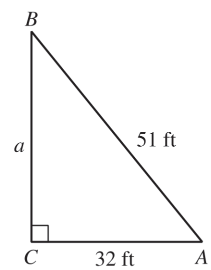
$A \approx \sin^{-1}\left(\dfrac{13}{22}\right) \approx 36.2215° \approx 36°$ (rounded to two significant digits)

$\cos B = \dfrac{b}{c} \Rightarrow \cos B = \dfrac{13}{22} \Rightarrow$

$B \approx \cos^{-1}\left(\dfrac{13}{22}\right) \approx 53.7784° \approx 54°$

(rounded to two significant digits)

32. $b = 32$ ft, $c = 51$ ft



$c^2 = a^2 + b^2 \Rightarrow 51^2 = a^2 + 32^2 \Rightarrow$
$2601 = a^2 + 1024 \Rightarrow 1577 = a^2 \Rightarrow a \approx 40$ ft
(rounded to two significant digits)

We will determine the measurements of both A and B by using the sides of the right triangle. In practice, once you find one of the measurements, subtract it from $90°$ to find the other.

$\cos A = \dfrac{b}{c} \Rightarrow \cos A = \dfrac{32}{51} \Rightarrow$

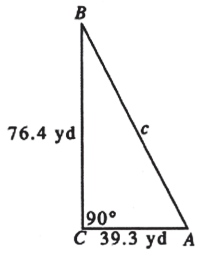
$A \approx \cos^{-1}\left(\dfrac{32}{51}\right) \approx 51.1377° \approx 51°$

(rounded to two significant digits)

$\sin B = \dfrac{b}{c} \Rightarrow \sin B = \dfrac{32}{51} \Rightarrow$

$B \approx \sin^{-1}\left(\dfrac{32}{51}\right) \approx 38.8623° \approx 39°$ (rounded to two significant digits)

33. $a = 76.4$ yd, $b = 39.3$ yd



$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2}$

$= \sqrt{(76.4)^2 + (39.3)^2} = \sqrt{5836.96 + 1544.49}$

$= \sqrt{7381.45} \approx 85.9$ yd (rounded to three significant digits)

We will determine the measurements of both A and B by using the sides of the right triangle. In practice, once you find one of the measurements, subtract it from $90°$ to find the other.

$\tan A = \dfrac{a}{b} \Rightarrow \tan A = \dfrac{76.4}{39.3} \Rightarrow$

$A \approx \tan^{-1}\left(\dfrac{76.4}{39.3}\right) \approx 62.7788°$

$\approx 62° + (0.7788 \cdot 60)' \approx 62°47' \approx 62°50'$

(rounded to three significant digits)

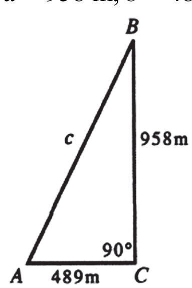
$\tan B = \dfrac{b}{a} \Rightarrow \tan B = \dfrac{39.3}{76.4} \Rightarrow$

$B \approx \tan^{-1}\left(\dfrac{39.3}{76.4}\right) \approx 27.2212°$

$\approx 27° + (0.2212 \cdot 60)' \approx 27°13' \approx 27°10'$

(rounded to three significant digits)

34. $a = 958$ m, $b = 489$ m



$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{958^2 + 489^2}$

$= \sqrt{917,764 + 239,121} = \sqrt{1,156,885}$

$\approx 1075.565887 \approx 1080$ m (rounded to three significant digits)

We will determine the measurements of both A and B by using the sides of the right triangle. In practice, once you find one of the measurements, subtract it from $90°$ to find the other.

$\tan A = \dfrac{a}{b} \Rightarrow \tan A = \dfrac{958}{489} \Rightarrow$

$A \approx \tan^{-1}\left(\dfrac{958}{489}\right) \approx 62.9585°$

$\approx 63° + (0.9585 \cdot 60)' \approx 62°58' \approx 63°00'$

(rounded to three significant digits)

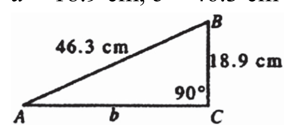
$\tan B = \dfrac{b}{a} \Rightarrow \tan B = \dfrac{489}{958} \Rightarrow$

$B \approx \tan^{-1}\left(\dfrac{489}{958}\right) \approx 27.0415°$

$\approx 27° + (0.0415 \cdot 60)' \approx 27°02' \approx 27°00'$

(rounded to three significant digits)

35. $a = 18.9$ cm, $c = 46.3$ cm



$c^2 = a^2 + b^2 \Rightarrow 46.3^2 = 18.9^2 + b^2 \Rightarrow$

$2143.69 = 357.21 + b^2 \Rightarrow 1786.48 = b^2 \Rightarrow$

$b \approx 42.3$ cm (rounded to three significant digits)

$\sin A = \dfrac{a}{c} \Rightarrow \sin A = \dfrac{18.9}{46.3} \Rightarrow$

$A \approx \sin^{-1}\left(\dfrac{18.9}{46.3}\right) \approx 24.09227°$

$\approx 24° + (0.09227 \cdot 60)' \approx 24°06' \approx 24°10'$

(rounded to three significant digits)

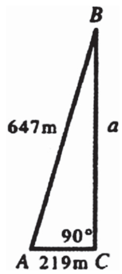
$\cos B = \dfrac{a}{c} \Rightarrow \cos B = \dfrac{18.9}{46.3} \Rightarrow$

$B \approx \cos^{-1}\left(\dfrac{18.9}{46.3}\right) \approx 65.9077°$

$\approx 65° + (0.9077 \cdot 60)' \approx 65°54' \approx 65°50'$

(rounded to three significant digits)

36. $b = 219$ cm, $c = 647$ m



$$\begin{aligned} c^2 &= a^2 + b^2 \\ 647^2 &= a^2 + 219^2 \\ 418{,}609 &= a^2 + 47{,}961 \\ 370{,}648 &= a^2 \\ a &\approx 609 \text{ m} \end{aligned}$$

(rounded to three significant digits)

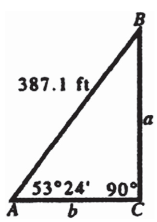
$$\cos A = \frac{b}{c} \Rightarrow \cos A = \frac{219}{647} \Rightarrow$$
$$A \approx \cos^{-1}\left(\frac{219}{647}\right) \approx 70.2154°$$
$$\approx 70° + (0.2154 \cdot 60)' \approx 70°13' \approx 70°10'$$

(rounded to three significant digits)

$$\sin B = \frac{b}{c} \Rightarrow \sin B = \frac{219}{647} \Rightarrow$$
$$B \approx \sin^{-1}\left(\frac{219}{647}\right) \approx 19.7846°$$
$$\approx 19° + (0.7846 \cdot 60)' \approx 19°47' \approx 19°50'$$

(rounded to three significant digits)

37. $A = 53°24'$, $c = 387.1$ ft

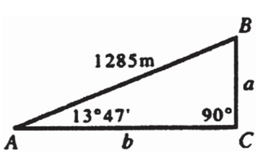


$$\begin{aligned} A + B &= 90° \\ B &= 90° - A \\ B &= 90° - 53°24' \\ &= 89°60' - 53°24' \\ &= 36°36' \end{aligned}$$

$$\sin A = \frac{a}{c} \Rightarrow \sin 53°24' = \frac{a}{387.1} \Rightarrow$$
$a = 387.1 \sin 53°24' \approx 310.8$ ft (rounded to four significant digits)

$$\cos A = \frac{b}{c} \Rightarrow \cos 53°24' = \frac{b}{387.1} \Rightarrow$$
$b = 387.1 \cos 53°24' \approx 230.8$ ft (rounded to four significant digits)

38. $A = 13°47'$, $c = 1285$ m

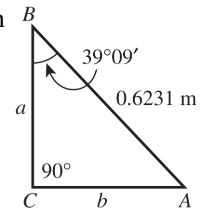


$$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$$
$$B = 90° - 13°47' = 89°60' - 13°47'$$
$$= 76°13'$$

$$\sin A = \frac{a}{c} \Rightarrow \sin 13°47' = \frac{a}{1285} \Rightarrow$$
$a = 1285 \sin 13°47' \approx 306.2$ m (rounded to four significant digits)

$$\cos A = \frac{b}{c} \Rightarrow \cos 13°47' = \frac{b}{1285} \Rightarrow$$
$b = 1285 \cos 13°47' \approx 1248$ m (rounded to four significant digits)

39. $B = 39°09'$, $c = 0.6231$ m

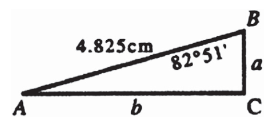


$$\begin{aligned} A + B &= 90° \\ B &= 90° - A \\ B &= 90° - 39°09' \\ &= 89°60' - 39°09' \\ &= 50°51' \end{aligned}$$

$$\sin B = \frac{b}{c} \Rightarrow \sin 39°09' = \frac{b}{0.6231} \Rightarrow$$
$b = 0.6231 \sin 39°09' \approx 0.3934$ m (rounded to four significant digits)

$$\cos B = \frac{a}{c} \Rightarrow \cos 39°09' = \frac{a}{0.6231} \Rightarrow$$
$a = 0.6231 \cos 39°09' \approx 0.4832$ m (rounded to four significant digits)

40. $B = 82°51'$, $c = 4.825$ cm

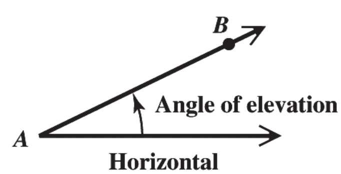


$$A + B = 90° \Rightarrow A = 90° - B \Rightarrow$$
$$A = 90° - 82°51' = 89°60' - 82°51'$$
$$= 7°09'$$

$$\sin B = \frac{b}{c} \Rightarrow \sin 82°51' = \frac{b}{4.825} \Rightarrow$$
$b = 4.825 \sin 82°51' \approx 4.787$ m (rounded to four significant digits)

$$\cos B = \frac{a}{c} \Rightarrow \cos 82°51' = \frac{a}{4.825} \Rightarrow$$
$a = 4.825 \cos 82°51' \approx 0.6006$ m (rounded to four significant digits)

41. If B is a point above point A as shown in the figure, the angle of elevation from A to B is the acute angle formed by the horizontal line through A and the line of sight from A to B.



42. No. An angle of elevation (and also an angle of depression) must be an acute angle.

43. Angles DAB and ABC are alternate interior angle formed by the transversal AB intersecting parallel lines AD and BC. Thus they have the same measure.

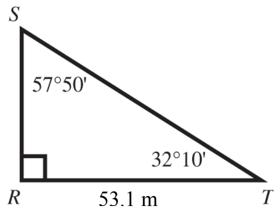
44. An angle of depression (and also an angle of elevation) is measured from a horizontal line to the line of sight and. Angle CAB is formed by the line of sight and vertical line AC.

45. $\sin 43°50' = \dfrac{d}{13.5}$

$d = 13.5 \sin 43°50' \approx 9.3496000$

The ladder goes up the wall 9.35 m. (rounded to three significant digits)

46. $T = 32°\ 10'$ and $S = 57°50'$



Because

$S + T = 32°\ 10' + 57°50' = 89°60' = 90°,$

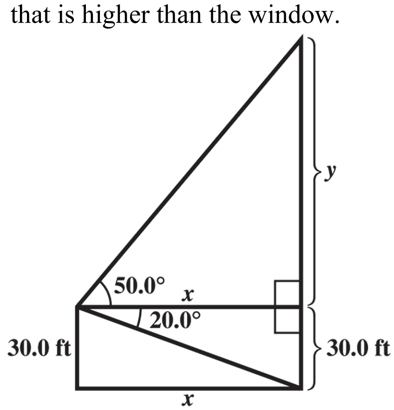
triangle RST is a right triangle. Thus, we have

$\tan 32°10' = \dfrac{RS}{53.1}$

$RS = 53.1 \tan 32°10' \approx 33.395727$

The distance across the lake is 33.4 m. (rounded to three significant digits)

47. Let x represent the horizontal distance between the two buildings and y represent the height of the portion of the building across the street that is higher than the window.



$\tan 20.0° = \dfrac{30.0}{x} \Rightarrow x = \dfrac{30.3}{\tan 20.0°} \approx 82.4$

$\tan 50.0° = \dfrac{y}{x} \Rightarrow$

$y = x \tan 50.0° = \left(\dfrac{30.0}{\tan 20.0°}\right) \tan 50.0°$

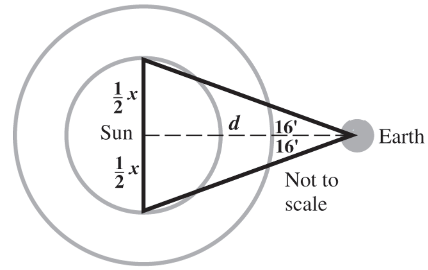
$\text{height} = y + 30.0$

$= \left(\dfrac{30.0}{\tan 20.0°}\right) \tan 50.0° + 30.0$

≈ 128.2295

The height of the building across the street is about 128 ft. (rounded to three significant digits)

48. Let x = the diameter of the sun.



The included angle is $32'$, so $\dfrac{1}{2}(32') = 16'$.

We will use this angle, d, and half of the diameter to set up the following equation.

$\dfrac{\frac{1}{2}x}{92{,}919{,}800} = \tan 16' \Rightarrow$

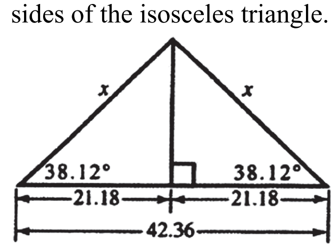
$x = 2(92{,}919{,}800)(\tan 16')$

$\approx 864{,}943.0189$

The diameter of the sun is about 864,900 mi. (rounded to four significant digits)

49. The altitude of an isosceles triangle bisects the base as well as the angle opposite the base. The two right triangles formed have interior angles which have the same measure. The lengths of the corresponding sides also have the same measure. The altitude bisects the base, so each leg (base) of the right triangles is $\frac{42.36}{2} = 21.18$ in.

Let x = the length of each of the two equal sides of the isosceles triangle.



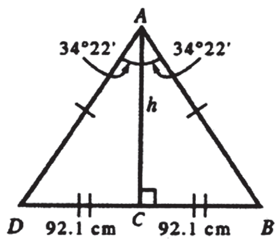
$\cos 38.12° = \dfrac{21.18}{x} \Rightarrow x \cos 38.12° = 21.18 \Rightarrow$

$x = \dfrac{21.18}{\cos 38.12°} \approx 26.921918$

The length of each of the two equal sides of the triangle is 26.92 in. (rounded to four significant digits)

50. The altitude of an isosceles triangle bisects the base as well as the angle opposite the base. The two right triangles formed have interior angles which have the same measure. The lengths of the corresponding sides also have the same measure. The altitude bisects the base, so each leg (base) of the right triangles are $\frac{184.2}{2} = 92.10$ cm. Each angle opposite to the base of the right triangles measures $\frac{1}{2}(68°44') = 34°22'$.

Let h = the altitude.



In triangle ABC,

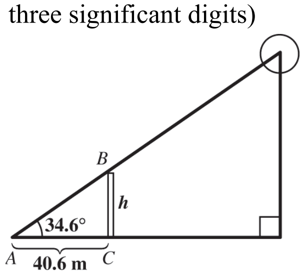
$$\tan 34°22' = \frac{92.10}{h} \Rightarrow h \tan 34°22' = 92.10 \Rightarrow$$
$$h = \frac{92.10}{\tan 34°22'} \approx 134.67667$$

The altitude of the triangle is 134.7 cm. (rounded to four significant digits)

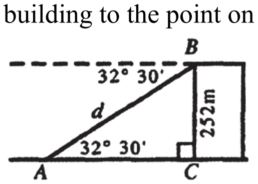
51. Let h represent the height of the tower. In triangle ABC we have

$$\tan 34.6° = \frac{h}{40.6}$$
$$h = 40.6 \tan 34.6° \approx 28.0081$$

The height of the tower is 28.0 m. (rounded to three significant digits)



52. Let d = the distance from the top B of the building to the point on the ground A.



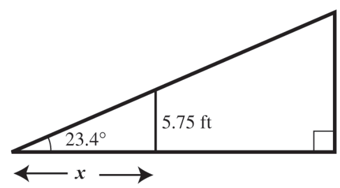
In triangle ABC,

$$\sin 32°30' = \frac{252}{d}$$
$$d = \frac{252}{\sin 32°30'} \approx 469.0121$$

The distance from the top of the building to the point on the ground is 469 m. (rounded to three significant digits)

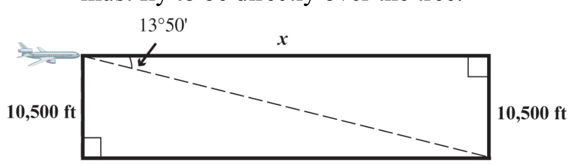
53. Let x = the length of the shadow.

$$\tan 23.4° = \frac{5.75}{x}$$
$$x \tan 23.4° = 5.75$$
$$x = \frac{5.75}{\tan 23.4°} \approx 13.2875$$



The length of the shadow is 13.3 ft. (rounded to three significant digits)

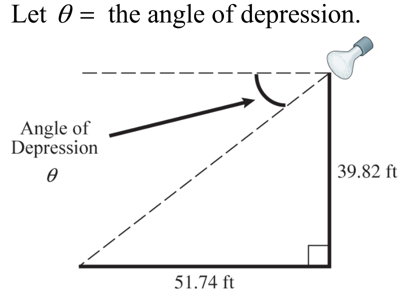
54. Let x = the horizontal distance that the plan must fly to be directly over the tree.



$$\tan 13°50' = \frac{10,500}{x}$$
$$x \tan 13°50' = 10,500$$
$$x = \frac{10,500}{\tan 13°50'} \approx 42,641.2351$$

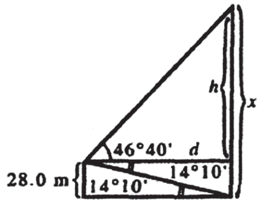
The horizontal distance that the plan must fly to be directly over the tree is 42,600 ft. (rounded to three significant digits)

55. Let $\theta =$ the angle of depression.



$$\tan \theta = \frac{39.82}{51.74} \Rightarrow \theta = \tan^{-1}\left(\frac{39.82}{51.74}\right)$$
$$\theta \approx 37.58° \approx 37°35'$$

56. Let x = the height of the taller building; h = the difference in height between the shorter and taller buildings; d = the distance between the buildings along the ground.



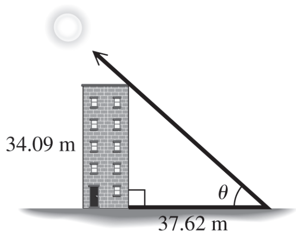
$$\frac{28.0}{d} = \tan 14°10' \Rightarrow 28.0 = d \tan 14°10' \Rightarrow$$
$$d = \frac{28.0}{\tan 14°10'} \approx 110.9262493 \text{ m}$$

(We hold on to these digits for the intermediate steps.) To find h, solve

$$\frac{h}{d} = \tan 46°40'$$
$$h = d \tan 46°40' \approx (110.9262493) \tan 46°40' \approx 117.5749$$

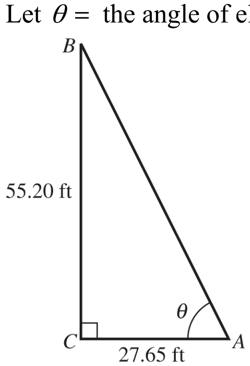
Thus, the value of h rounded to three significant digits is 118 m.
$x = h + 28.0 = 118 + 28.0 \approx 146\,\text{m}$, so the height of the taller building is 146 m.

57. Let θ = the angle of elevation of the sun.



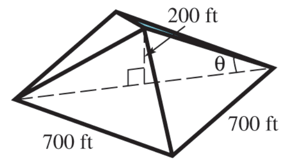
$$\tan\theta = \frac{34.09}{37.62} \Rightarrow \theta = \tan^{-1}\left(\frac{34.09}{37.62}\right)$$
$$\theta \approx 42.18°$$

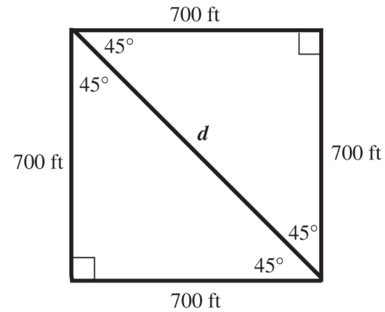
58. Let θ = the angle of elevation of the sun.



$$\tan\theta = \frac{55.20}{27.65} \Rightarrow \theta = \tan^{-1}\left(\frac{55.20}{27.65}\right)$$
$$\theta \approx 63.39°$$

59. In order to find the angle of elevation, θ, we need to first find the length of the diagonal of the square base. The diagonal forms two isosceles right triangles. Each angle formed by a side of the square and the diagonal measures 45˚.

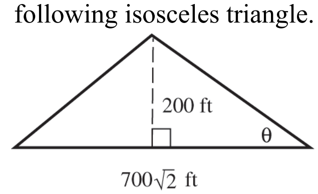


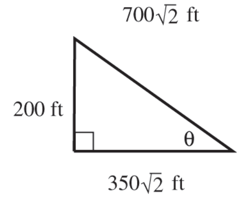


By the Pythagorean theorem,

$$700^2 + 700^2 = d^2 \Rightarrow 2 \cdot 700^2 = d^2 \Rightarrow$$
$$d = \sqrt{2 \cdot 700^2} \Rightarrow d = 700\sqrt{2}$$

Thus, length of the diagonal is $700\sqrt{2}$ ft. To to find the angle, θ, we consider the following isosceles triangle.





The height of the pyramid bisects the base of this triangle and forms two right triangles. We can use one of these triangles to find the angle of elevation, θ.

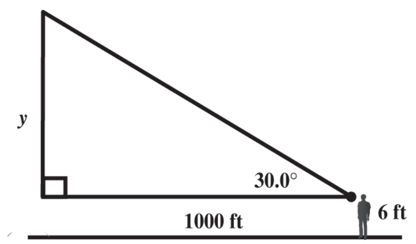
$$\tan\theta = \frac{200}{350\sqrt{2}} \Rightarrow$$
$$\theta \approx \tan^{-1}\left(\frac{200}{350\sqrt{2}}\right) \approx 22.0017$$

Rounding this figure to two significant digits, we have $\theta \approx 22°$.

60. Let y = the height of the spotlight (this measurement starts 6 feet above ground)

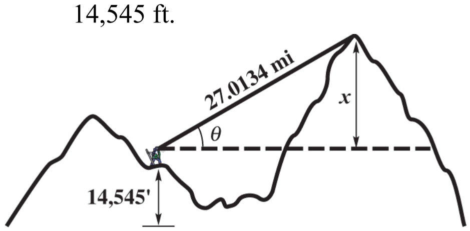
$$\tan 30.0° = \frac{y}{1000}$$
$$y = 1000 \cdot \tan 30.0° \approx 577.3502$$

Rounding this figure to three significant digits, we have $y \approx 577$.



However, the observer's eye-height is 6 feet from the ground, so the cloud ceiling is $577 + 6 = 583$ ft.

61. (a) Let x = the height of the peak above 14,545 ft.



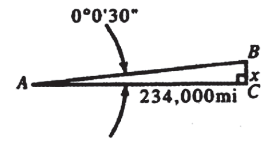
The diagonal of the right triangle formed is in miles, so we must first convert this measurement to feet. Because there are 5280 ft in one mile, we have the length of the diagonal is $27.0134(5280) = 142,630.752$. Find the value of x by solving $\sin 5.82° = \frac{x}{142,630.752}$.

$$x = 142,630.752 \sin 5.82° \approx 14,463.2674$$

Thus, the value of x rounded to five significant digits is 14,463 ft. Thus, the total height is about 14,545 + 14,463 = 29,008 ≈ 29,000 ft.

(b) The curvature of the earth would make the peak appear shorter than it actually is. Initially the surveyors did not think Mt. Everest was the tallest peak in the Himalayas. It did not look like the tallest peak because it was farther away than the other large peaks.

62. Let x = the distance from the assigned target.



In triangle ABC, we have

$$\tan 0°0'30'' = \frac{x}{234,000}$$
$$x = 234,000 \tan 0°0'30'' \approx 34.0339$$

The distance from the assigned target is 34.0 mi. (rounded to three significant digits)

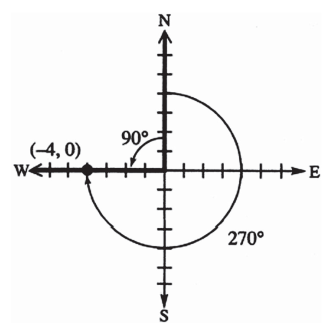
Section 2.5 Further Applications of Right Triangles

1. C **2.** D **3.** A **4.** G

5. B **6.** H **7.** F **8.** J

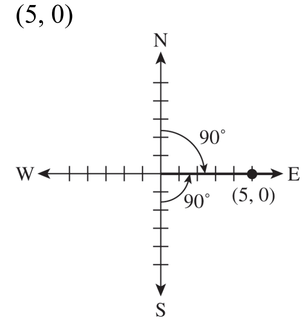
9. I **10.** E

11. (−4, 0)



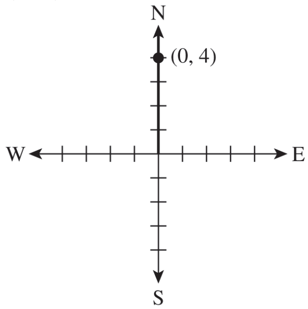
The bearing of the airplane measured in a clockwise direction from due north is 270°. The bearing can also be expressed as N 90° W, or S 90° W.

12. (5, 0)



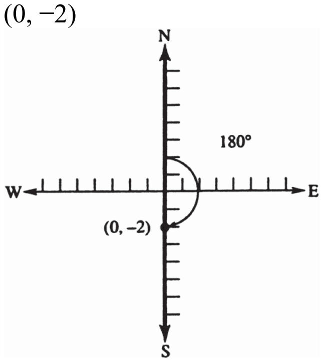
The bearing of the airplane measured in a clockwise direction from due north is 90°. The bearing can also be expressed as N 90° E, or S 90° E.

13. (0, 4)



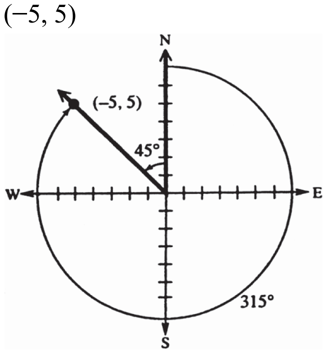
The bearing of the airplane measured in a clockwise direction from due north is 0°. The bearing can also be expressed as N 0° E or N 0º W.

14. (0, −2)



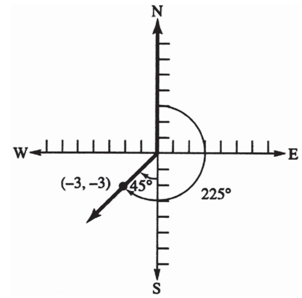
The bearing of the airplane measured in a clockwise direction from due north is 180°. The bearing can also be expressed as S 0° E or S 0º W.

15. (−5, 5)



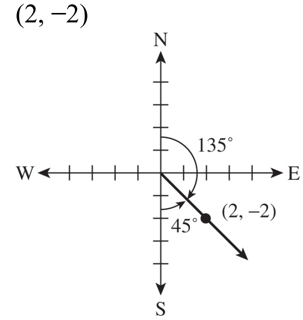
The bearing of the airplane measured in a clockwise direction from due north is 315°. The bearing can also be expressed as N 45° W.

16. (−3, −3)



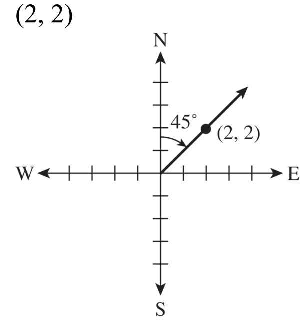
The bearing of the airplane measured in a clockwise direction from due north is 225°. The bearing can also be expressed as S 45° W.

17. (2, −2)



The bearing of the airplane measured in a clockwise direction from due north is 135°. The bearing can also be expressed as S 45° E.

18. (2, 2)

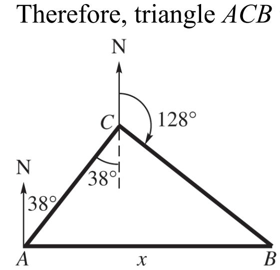


The bearing of the airplane measured in a clockwise direction from due north is 45°. The bearing can also be expressed as N 45° E.

19. Let x = the distance the plane is from its starting point. In the figure, the measure of angle ACB is

$$38° + (180° - 128°) = 38° + 52° = 90°.$$

Therefore, triangle ACB is a right triangle.



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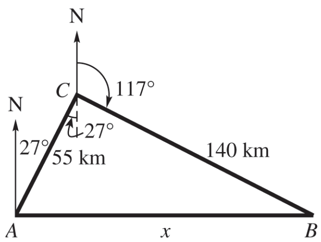
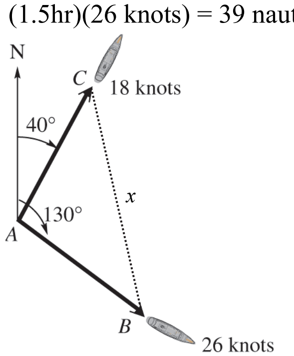
Because $d = rt$, the distance traveled in 1.5 hr is (1.5 hr)(110 mph) = 165 mi. The distance traveled in 1.3 hr is (1.3 hr)(110 mph) = 143 mi.
Using the Pythagorean theorem, we have
$x^2 = 165^2 + 143^2 \Rightarrow x^2 = 27,225 + 20,449 \Rightarrow$
$x^2 = 47,674 \Rightarrow x \approx 218.3438$
The plane is 220 mi from its starting point. (rounded to two significant digits)

20. Let x = the distance from the starting point.
In the figure, the measure of angle *ACB* is
$27° + (180° - 117°) = 27° + 63° = 90°.$
Therefore, triangle *ACB* is a right triangle.



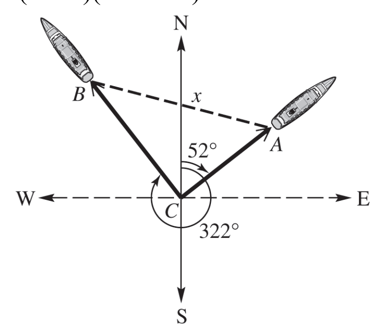
Applying the Pythagorean theorem, we have
$x^2 = 55^2 + 140^2 \Rightarrow x^2 = 3025 + 19,600 \Rightarrow$
$x^2 = 22,625 \Rightarrow x = \sqrt{22,625} \approx 150.4161$
The distance of the end of the trip from the starting point is about 150 km. (rounded to two significant digits)

21. Let x = distance the ships are apart.
In the figure, the measure of angle *CAB* is $130° - 40° = 90°$. Therefore, triangle *CAB* is a right triangle.
Because $d = rt$, the distance traveled by the first ship in 1.5 hr is (1.5 hr)(18 knots) = 27 nautical mi and the second ship is (1.5hr)(26 knots) = 39 nautical mi.



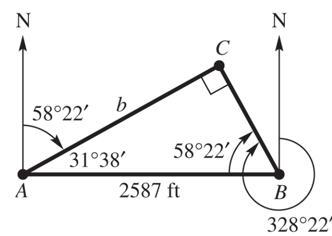
Applying the Pythagorean theorem, we have
$x^2 = 27^2 + 39^2 \Rightarrow x^2 = 729 + 1521 \Rightarrow$
$x^2 = 2250 \Rightarrow x = \sqrt{2250} \approx 47.4342$
The ships are 47 nautical mi apart (rounded to 2 significant digits).

22. Let x = distance the ships are apart.
In the figure, the measure of angle *BCA* is $360° - 322° + 52° = 90°$. Therefore, triangle *BCA* is a right triangle.
Because $d = rt$, the distance traveled by the first ship in 2.5 hr is (2.5 hr)(17 knots) = 42.5 nautical mi and the second ship is (2.5hr)(22 knots) = 55 nautical mi.



Applying the Pythagorean theorem, we have
$x^2 = 42.5^2 + 55^2 \Rightarrow x^2 = 4831.25 \Rightarrow$
$x = \sqrt{4831.25} \approx 69.5072$
The ships are 70 nautical mi apart (rounded to 2 significant digits).

23. Let b = the distance from dock *A* to the coral reef *C*.

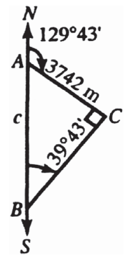


In the figure, the measure of angle *CAB* is $90° - 58°22' = 31°38'$, and the measure of angle *CBA* is $328°22' - 270° = 58°22'$.
Because $31°38' + 58°22' = 90°$, *ABC* is a right triangle.

$$\cos A = \frac{b}{2587}$$
$$\cos 31°38' = \frac{b}{2587} \Rightarrow b = 2587 \cos 31°38'$$
$$b \approx 2203 \text{ ft}$$

24. Let C = the location of the ship, and let c = the distance between the lighthouses.

$$m\angle BAC = 180° - 129°43' = 179°60' - 129°43' = 50°17'$$

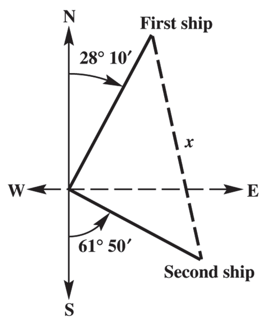


$50°17' + 39°43' = 90°,$ so we have a right triangle. Thus,

$$\sin 39°43' = \frac{3742}{c} \Rightarrow c \sin 39°43' = 3742 \Rightarrow$$
$$c = \frac{3742}{\sin 39°43'} \approx 5856.1020$$

The distance between the lighthouses is 5856 m (rounded to four significant digits).

25. Let x = distance between the two ships.



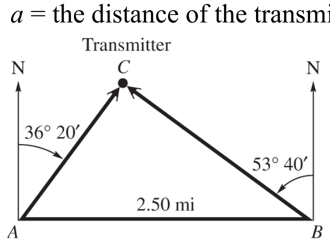
The angle between the bearings of the ships is $180° - (28°10' + 61°50') = 90°.$ The triangle formed is a right triangle. The distance traveled at 24.0 mph is (4 hr) (24.0 mph) = 96 mi. The distance traveled at 28.0 mph is (4 hr)(28.0 mph) = 112 mi.

Applying the Pythagorean theorem we have

$$x^2 = 96^2 + 112^2 \Rightarrow x^2 = 9216 + 12{,}544 \Rightarrow$$
$$x^2 = 21{,}760 \Rightarrow x = \sqrt{21{,}760} \approx 147.5127$$

The ships are 148 mi apart. (rounded to three significant digits)

26. Let C = the location of the transmitter; a = the distance of the transmitter from B.



The measure of angle CBA is $90° - 53°40' = 89°60' - 53°40' = 36°20'.$

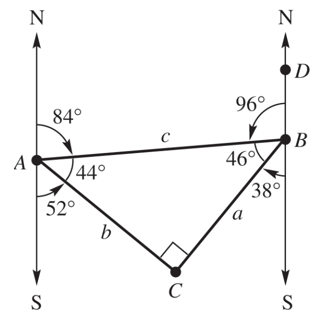
The measure of angle CAB is $90° - 36°20' = 89°60' - 36°20' = 53°40'.$

$A + B = 90°$, so $C = 90°$. Thus, we have

$$\sin A = \frac{a}{2.50} \Rightarrow \sin 53°40' = \frac{a}{2.50} \Rightarrow$$
$$a = 2.50 \sin 53°40' \approx 2.0140$$

The distance of the transmitter from B is 2.01 mi. (rounded to 3 significant digits)

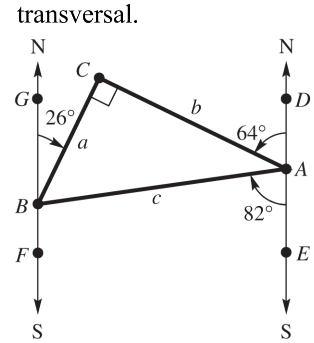
27. Let b = the distance from A to C and let c = the distance from A to B.



Because the bearing from A to B is N 84° E, the measure of angle ABD is 180° − 84° = 96°. The bearing from B to C is 38°, so the measure of angle ABC = 180° − (96° + 38°) = 46°. The bearing of A to C is 52°, so the measure of angle BAC is 180° − (52° + 84°) = 44°. The measure of angle C is 180° − (44° + 46°) = 90°, so triangle ABC is a right triangle. The distance from A to B, labeled c, is 2.4(250) = 600 miles.

$$\sin 46° = \frac{b}{c} = \frac{b}{600}$$
$$b = 600 \sin 46° \approx 430 \text{ mi}$$

28. The information in the example gives $m\angle DAC = 64°, m\angle EAB = 82°,$ and $m\angle GBC = 26°.$ The sum of the measures of angles EAB and FBA is 180º because they are interior angles on the same side of a transversal.



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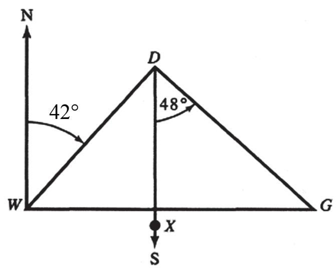
So
$m\angle FBA = 180° - m\angle EAB = 180° - 82° = 98°$.
$m\angle CAB = 180° - (64° + 82°) = 34°$ and
$m\angle ABC = 180° - (98° + 26°) = 56°$. Thus, angle C is a right angle.
It takes 1.8 hours at 350 mph to fly from A to B, so $AB = c = 1.8 \cdot 350 = 630$ mi. To find the distance from B to C, use $\cos B$.

$$\cos \angle ABC = \frac{a}{630} \Rightarrow a = 630 \cos 56° \approx 350 \text{ mi}$$

29. Draw triangle WDG with W representing Winston-Salem, D representing Danville, and G representing Goldsboro. Name any point X on the line due south from D.



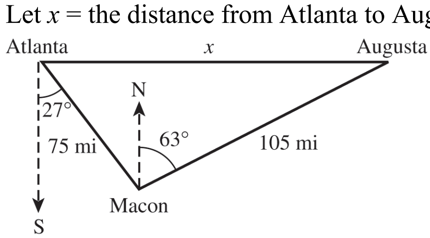
The bearing from W to D is 42° (equivalent to N 42° E), so angle WDX measures 42°. Because angle XDG measures 48°, the measure of angle D is 42° + 48° = 90°. Thus, triangle WDG is a right triangle.
Using $d = rt$ and the Pythagorean theorem, we have

$$WG = \sqrt{(WD)^2 + (DG)^2} = \sqrt{[65(1.1)]^2 + [65(1.8)]^2}$$

$$WG = \sqrt{71.5^2 + 117^2} = \sqrt{5112.25 + 13,689} = \sqrt{18,801.25} \approx 137$$

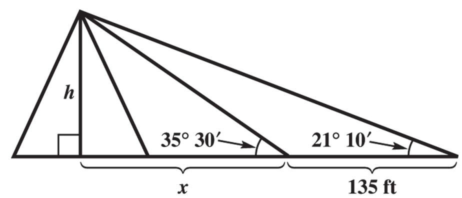
The distance from Winston-Salem to Goldsboro is approximately 140 mi. (rounded to two significant digits)

30. Let x = the distance from Atlanta to Augusta.



The line from Atlanta to Macon makes an angle of 27° + 63° = 90°, with the line from Macon to Augusta. Because $d = rt$, the distance from Atlanta to Macon is $60\left(1\frac{1}{4}\right) = 75$ mi. The distance from Macon to Augusta is $60\left(1\frac{3}{4}\right) = 105$ mi.
Use the Pythagorean theorem to find x:
$x^2 = 75^2 + 105^2 \Rightarrow x^2 = 5625 + 11,025 \Rightarrow$
$x^2 = 16,650 \approx 129.0349$
The distance from Atlanta to Augusta is 130 mi. (rounded to two significant digits)

31. Let x = the distance from the closer point on the ground to the base of height h of the pyramid.



In the larger right triangle, we have

$$\tan 21°10' = \frac{h}{135 + x} \Rightarrow h = (135 + x)\tan 21°10'$$

In the smaller right triangle, we have

$$\tan 35°30' = \frac{h}{x} \Rightarrow h = x \tan 35°30'.$$

Substitute for h in this equation, and solve for x to obtain the following.

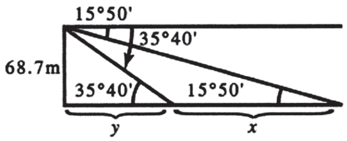
$$(135 + x)\tan 21°10' = x \tan 35°30'$$
$$135 \tan 21°10' + x \tan 21°10' = x \tan 35°30'$$
$$135 \tan 21°10' = x \tan 35°30' - x \tan 21°10'$$
$$135 \tan 21°10' = x(\tan 35°30' - \tan 21°10')$$
$$\frac{135 \tan 21°10'}{\tan 35°30' - \tan 21°10'} = x$$

Substitute for x in the equation for the smaller triangle.

$$h = \frac{135 \tan 21°10'}{\tan 35°30' - \tan 21°10'} \tan 35°30' \approx 114.3427$$

The height of the pyramid is 114 ft. (rounded to three significant digits)

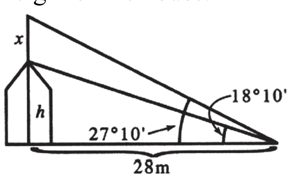
32. Let x = the distance traveled by the whale as it approaches the tower; y = the distance from the tower to the whale as it turns.



$$\frac{68.7}{y} = \tan 35°40' \Rightarrow 68.7 = y\tan 35°40' \Rightarrow$$
$$y = \frac{68.7}{\tan 35°40'} \text{ and } \frac{68.7}{x+y} = \tan 15°50'$$
$$68.7 = (x+y)\tan 15°50'$$
$$x+y = \frac{68.7}{\tan 15°50'} \Rightarrow x = \frac{68.7}{\tan 15°50'} - y$$
$$x = \frac{68.7}{\tan 15°50'} - \frac{68.7}{\tan 35°40'} \approx 146.5190$$

The whale traveled 147 m as it approached the lighthouse. (rounded to three significant digits)

33. Let x = the height of the antenna; h = the height of the house.



In the smaller right triangle, we have

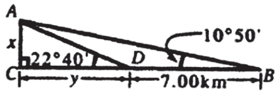
$$\tan 18°10' = \frac{h}{28} \Rightarrow h = 28\tan 18°10'.$$

In the larger right triangle, we have

$$\tan 27°10' = \frac{x+h}{28} \Rightarrow x + h = 28\tan 27°10' \Rightarrow$$
$$x = 28\tan 27°10' - h$$
$$x = 28\tan 27°10' - 28\tan 18°10'$$
$$\approx 5.1816$$

The height of the antenna is 5.18 m. (rounded to three significant digits)

34. Let x = the height of Mt. Whitney above the level of the road; y = the distance shown in the figure below.



In triangle ADC,

$$\tan 22°40' = \frac{x}{y} \Rightarrow y\tan 22°40' = x \Rightarrow$$
$$y = \frac{x}{\tan 22°40'}. \quad (1)$$

In triangle ABC

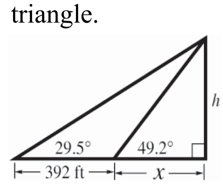
$$\tan 10°50' = \frac{x}{y+7.00}$$
$$(y+7.00)\tan 10°50' = x$$
$$y\tan 10°50' + 7.00\tan 10°50' = x$$
$$\frac{x - 7.00\tan 10°50'}{\tan 10°50'} = y \quad (2)$$

Setting equations 1 and 2 equal, we have

$$\frac{x}{\tan 22°40'} = \frac{x - 7.00\tan 10°50'}{\tan 10°50'}$$
$$x\tan 10°50' = x\tan 22°40' - 7.00(\tan 10°50')(\tan 22°40')$$
$$7.00(\tan 10°50')(\tan 22°40') = x\tan 22°40' - x\tan 10°50'$$
$$7.00(\tan 10°50')(\tan 22°40') = x(\tan 22°40' - \tan 10°50')$$
$$x = \frac{7.00(\tan 10°50')(\tan 22°40')}{\tan 22°40' - \tan 10°50'}$$
$$x \approx 2.4725$$

The height of the top of Mt. Whitney above road level is 2.47 km. (rounded to three significant digits)

35. Algebraic solution:
Let x = the side adjacent to 49.2º in the smaller triangle.



In the larger right triangle, we have

$$\tan 29.5° = \frac{h}{392+x} \Rightarrow h = (392+x)\tan 29.5°.$$

In the smaller right triangle, we have

$$\tan 49.2° = \frac{h}{x} \Rightarrow h = x\tan 49.2°.$$

Substituting, we have

$$x\tan 49.2° = (392+x)\tan 29.5°$$
$$x\tan 49.2° = 392\tan 29.5° + x\tan 29.5°$$
$$x\tan 49.2° - x\tan 29.5° = 392\tan 29.5°$$
$$x(\tan 49.2° - \tan 29.5°) = 392\tan 29.5°$$
$$x = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°}$$

Now substitute this expression for x in the equation for the smaller triangle to obtain

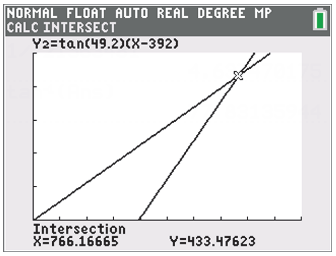
$$h = x\tan 49.2°$$
$$h = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°} \cdot \tan 49.2°$$

$\approx 433.4762 \approx 433$ ft (rounded to three significant digits.

(*continued on next page*)

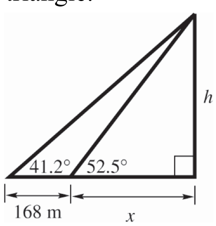
(*continued*)

Graphing calculator solution:
Graph $y_1 = (\tan 29.5°)x$ and
$y_2 = (\tan 29.5°)(x - 392)$ in the window
[0, 1000] × [0, 500]. Then find the intersection.



The height of the triangle is 433 ft (rounded to three significant digits.

36. Algebraic solution:
Let x = the side adjacent to 52.5º in the smaller triangle.



In the larger right triangle, we have

$$\tan 41.2° = \frac{h}{168 + x} \Rightarrow h = (168 + x)\tan 41.2°.$$

In the smaller right triangle, we have

$$\tan 52.5° = \frac{h}{x} \Rightarrow h = x \tan 52.5°.$$

Substituting, we have

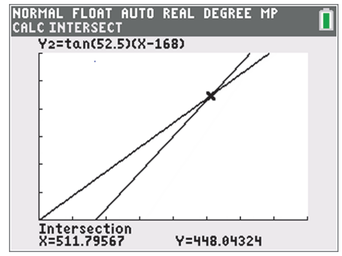
$$\begin{aligned} x \tan 52.5° &= (168 + x)\tan 41.2° \\ x \tan 52.5° &= 168 \tan 41.2° \\ &\quad + x \tan 41.2° \\ x \tan 52.5° - x \tan 41.2° &= 168 \tan 41.2° \\ x(\tan 52.5° - \tan 41.2°) &= 168 \tan 41.2° \\ x &= \frac{168 \tan 41.2°}{\tan 52.5° - \tan 41.2°} \end{aligned}$$

Now substitute this expression for x in the equation for the smaller triangle to obtain

$$\begin{aligned} h &= x \tan 52.5° \\ h &= \frac{168 \tan 41.2°}{\tan 52.5° - \tan 41.2°} \cdot \tan 52.5° \\ &\approx 448.0432 \approx 448 \text{ m (rounded to three} \end{aligned}$$

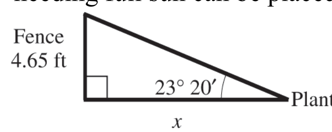
significant digits.

Graphing calculator solution:
Graph $y_1 = (\tan 41.2°)x$ and
$y_2 = (\tan 52.5°)(x - 168)$ in the window
[0, 800] × [0, 600]. Then find the intersection.



The height of the triangle is 448 m (rounded to three significant digits.

37. Let x = the minimum distance that a plant needing full sun can be placed from the fence.

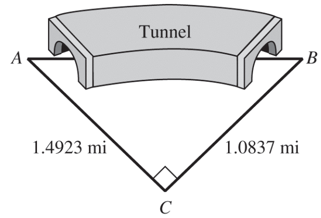


$$\tan 23°20' = \frac{4.65}{x} \Rightarrow x \tan 23°20' = 4.65 \Rightarrow$$

$$x = \frac{4.65}{\tan 23°20'} \approx 10.7799$$

The minimum distance is 10.8 ft. (rounded to three significant digits)

38.



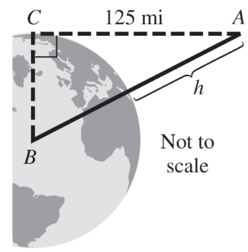
$$\tan A = \frac{1.0837}{1.4923} \approx 0.7261944649$$

$$\begin{aligned} A &\approx \tan^{-1}(0.7261944649) \approx 35.987° \\ &\approx 35°59.2' \approx 35°59'10'' \end{aligned}$$

$$\tan B = \frac{1.4923}{1.0837} \approx 1.377041617$$

$$\begin{aligned} B &\approx \tan^{-1}(1.377041617) \approx 54.013° \\ &\approx 54°00.8' \approx 54°00'50'' \end{aligned}$$

39. Let h = the minimum height above the surface of Earth so a pilot at A can see an object on the horizon at C.

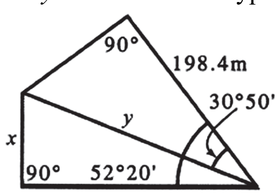


Using the Pythagorean theorem, we have

$$\left(4.00\times10^3+h\right)^2=\left(4.00\times10^3\right)^2+125^2$$
$$(4000+h)^2=4000^2+125^2$$
$$(4000+h)^2=16,000,000+15,625$$
$$(4000+h)^2=16,015,625$$
$$4000+h=\sqrt{16,015,625}\Rightarrow h=\sqrt{16,015,625}-4000\approx 4001.9526-4000=1.9526$$

The minimum height above the surface of Earth would be 1.95 mi. (rounded to 3 significant digits)

40. Let y = the common hypotenuse of the two right triangles.



$$\cos 30°50' = \frac{198.4}{y}\Rightarrow y=\frac{198.4}{\cos 30°50'}\approx 231.0571948$$

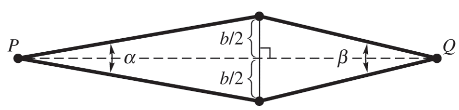
To find x, first find the angle opposite x in the right triangle:

$$52°20' - 30°50' = 51°80' - 30°50' = 21°30'$$

$$\sin 21°30' = \frac{x}{y}\Rightarrow \sin 21°30' \approx \frac{x}{231.0571948}\Rightarrow x\approx 231.0571948\ \sin 21°30' \approx 84.6827$$

The length x is approximate 84.7 m. (rounded)

41. (a)



$$PQ=d=\frac{b}{2}\cot\frac{\alpha}{2}+\frac{b}{2}\cot\frac{\beta}{2}=\frac{b}{2}\left(\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}\right)$$

(b) Using the result of part (a), let $\alpha = 37'48'', \beta = 42'03''$, and $b = 2.000$

$$d=\frac{b}{2}\left(\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}\right)\Rightarrow$$
$$d=\frac{2.000}{2}\left(\cot\frac{37'48''}{2}+\cot\frac{42'03''}{2}\right)=\cot 0.315°+\cot 0.3504166667°\approx 345.3951$$

The distance between the two points P and Q is about 345.4 cm.

42. $D = \dfrac{v^2 \sin\theta\cos\theta + v\cos\theta\sqrt{(v\sin\theta)^2 + 64h}}{32}$

All answers are rounded to four significant digits.

(a) $v = 44$ ft per sec and $h = 7$ ft, so $D = \dfrac{44^2 \sin\theta\cos\theta + 44\cos\theta\sqrt{(44\sin\theta)^2 + 64\cdot 7}}{32}$

If $\theta = 40°$, $D = \dfrac{1936\sin 40\cos 40 + 44\cos 40\sqrt{(44\sin 40)^2 + 448}}{32} \approx 67.00$ ft.

If $\theta = 42°$, $D = \dfrac{1936\sin 42\cos 42 + 44\cos 42\sqrt{(44\sin 42)^2 + 448}}{32} \approx 67.14$ ft

If $\theta = 45°$, $D = \dfrac{1936\sin 45\cos 45 + 44\cos 45\sqrt{(44\sin 45)^2 + 448}}{32} \approx 66.84$ ft

As θ increases, D increases and then decreases.

(b) $h = 7$ ft and $\theta = 42°$, so $D = \dfrac{v^2 \sin 42\cos 42 + v\cos 42\sqrt{(v\sin 42)^2 + 64\cdot 7}}{32}$

If $v = 43$, $D = \dfrac{43^2 \sin 42\cos 42 + 43\cos 42\sqrt{(43\sin 42)^2 + 448}}{32} \approx 64.40$ ft

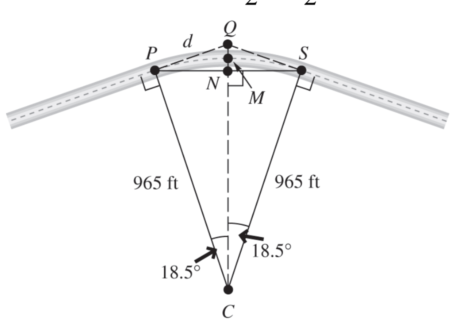
If $v = 44$, $D = \dfrac{44^2 \sin 42\cos 42 + 44\cos 42\sqrt{(44\sin 42)^2 + 448}}{32} \approx 67.14$ ft

If $v = 45$, $D = \dfrac{45^2 \sin 42\cos 42 + 45\cos 42\sqrt{(45\sin 42)^2 + 448}}{32} \approx 69.93$ ft

As v increases, D increases.

(c) The velocity affects the distance more. The shot-putter should concentrate on achieving as large a value of v as possible.

43. (a) If $\theta = 37°$, then $\dfrac{\theta}{2} = \dfrac{37°}{2} = 18.5°$.



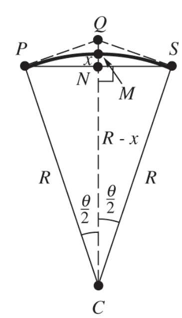
To find the distance between P and Q, d, we first note that angle QPC is a right angle. Hence, triangle QPC is a right triangle and we can solve

$$\tan 18.5° = \frac{d}{965}$$

$$d = 965\tan 18.5° \approx 322.8845$$

The distance between P and Q, is 320 ft. (rounded to two significant digits)

(b) We are dealing with a circle, so the distance between M and C is R. If we let x be the distance from N to M, then the distance from C to N will be $R - x$.



Triangle CNP is a right triangle, so we can set up the following equation.

$$\cos\frac{\theta}{2} = \frac{R-x}{R} \Rightarrow R\cos\frac{\theta}{2} = R - x \Rightarrow$$

$$x = R - R\cos\frac{\theta}{2} \Rightarrow x = R\left(1 - \cos\frac{\theta}{2}\right)$$

44. **(a)** $\theta \approx \frac{57.3S}{R} = \frac{57.3(336)}{600} = 32.088°$

$d = R\left(1 - \cos\frac{\theta}{2}\right)$

$= 600(1 - \cos 16.044°) \approx 23.3702$ ft

The distance is 23 ft. (rounded to two significant digits)

(b) $\theta \approx \frac{57.3S}{R} = \frac{57.3(485)}{600} = 46.3175°$

$d = R\left(1 - \cos\frac{\theta}{2}\right)$

$= 600(1 - \cos 23.15875°) \approx 48.3488$

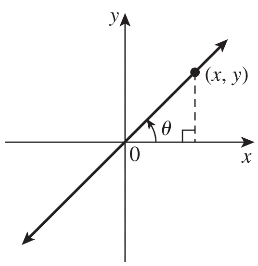
The distance is 48 ft. (rounded to two significant digits)

(c) The faster the speed, the more land needs to be cleared on the inside of the curve.

45. $y = \tan\theta(x - a) \Rightarrow y = \tan 35°(x - 25)$

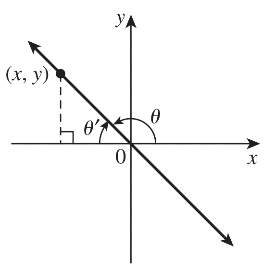
46. $y = \tan\theta(x - a) \Rightarrow y = \tan 15°(x - 5)$

47.



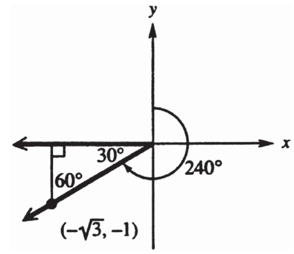
The line that bisects quadrants I and III passes through the origin, so $a = 0$. In addition, $\theta = 45°$ because the line bisects the angle formed by the axes. $\tan 45° = 1$, so an equation of the line is $y = \tan 45°(x)$ or $y = x$.

48.



The line that bisects quadrants II and IV passes through the origin, so $a = 0$. In addition, $\theta = 135°$ because the line bisects the angle formed by the axes and the reference angle θ' is 45°. $\tan 135° = -1$, so an equation of the line is $y = \tan 135°(x)$ or $y = -x$.

49. All points whose bearing from the origin is 240º lie in quadrant III.



The reference angle, θ', is 30º. For any point, (x, y) on the ray $\frac{x}{r} = -\cos\theta'$ and $\frac{y}{r} = -\sin\theta'$, where r is the distance from the point to the origin. Let $r = 2$, so

$\frac{x}{r} = -\cos\theta'$

$x = -r\cos\theta' = -2\cos 30° = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$

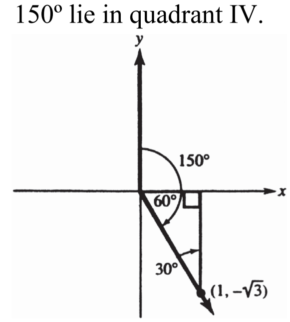
$\frac{y}{r} = -\sin\theta'$

$y = -r\sin\theta' = -2\sin 30° = -2 \cdot \frac{1}{2} = -1$

Thus, a point on the ray is $\left(-\sqrt{3}, -1\right)$. The ray contains the origin, the equation is of the form $y = mx$. Substituting the point $\left(-\sqrt{3}, -1\right)$, we have $-1 = m\left(-\sqrt{3}\right) \Rightarrow$

$m = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$. Thus, an equation of the ray is $y = \frac{\sqrt{3}}{3}x, x \le 0$ (because the ray lies in quadrant III).

50. All points whose bearing from the origin is 150º lie in quadrant IV.



The reference angle, θ', is 60º. For any point, (x, y) on the ray $\frac{x}{r} = \cos\theta'$ and $\frac{y}{r} = -\sin\theta'$, where r is the distance from the point to the origin. Let $r = 2$, so

$\frac{x}{r} = \cos\theta' \Rightarrow x = r\cos\theta' = 2\cos 60° = 2 \cdot \frac{1}{2} = 1$

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(*continued*)

$\frac{y}{r} = -\sin\theta'$

$y = -r\sin\theta' = -2\sin 60° = -2\cdot\frac{\sqrt{3}}{2} = -\sqrt{3}$

Thus, a point on the ray is $(1, -\sqrt{3})$.

The ray contains the origin, so the equation is of the form $y = mx$. Substituting the point $(1, -\sqrt{3})$, we have $-\sqrt{3} = m(-1) \Rightarrow m = -\sqrt{3}$.

Thus, an equation of the ray is $y = -\sqrt{3}x, x \geq 0$ (because the ray lies in quadrant IV).

Chapter 2 Review Exercises

1. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{60}{61}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{11}{61}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{60}{11}$

$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{11}{60}$

$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{61}{11}$

$\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{61}{60}$

2. $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{40}{58} = \frac{20}{29}$

$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{42}{58} = \frac{21}{29}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{40}{42} = \frac{20}{21}$

$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{42}{40} = \frac{21}{20}$

$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{58}{42} = \frac{29}{21}$

$\csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{58}{40} = \frac{29}{20}$

3. $\sin 4\beta = \cos 5\beta$

Sine and cosine are cofunctions, so the sum of the angles is 90º.

$4\beta + 5\beta = 90° \Rightarrow 9\beta = 90° \Rightarrow \beta = 10°$

4. $\sec(2\theta + 10°) = \csc(4\theta + 20°)$

Secant and cosecant are cofunctions, so the sum of the angles is 90º.

$(2\theta + 10°) + (4\theta + 20°) = 90°$

$6\theta + 30° = 90°$

$6\theta = 60° \Rightarrow \theta = 10°$

5. $\tan(5x + 11°) = \cot(6x + 2°)$

Tangent and cotangent are cofunctions, so the sum of the angles is 90º.

$(5x + 11°) + (6x + 2°) = 90°$

$11x + 13° = 90°$

$11x = 77° \Rightarrow x = 7°$

6. $\cos\left(\frac{3\theta}{5} + 11°\right) = \sin\left(\frac{7\theta}{10} + 40°\right)$

Sine and cosine are cofunctions, so the sum of the angles is 90º.

$\left(\frac{3\theta}{5} + 11°\right) + \left(\frac{7\theta}{10} + 40°\right) = 90°$

$\left(\frac{6\theta}{10} + 11°\right) + \left(\frac{7\theta}{10} + 40°\right) = 90°$

$\frac{13\theta}{10} + 51° = 90° \Rightarrow \frac{13\theta}{10} = 39°$

$\theta = 39°\cdot\frac{10}{13} = 30°$

7. $\sin 46° < \sin 58°$

In the interval from 0º to 90º, as the angle increases, so does the sine of the angle, so sin 46º < sin 58º is true.

8. $\cos 47° < \cos 58°$

In the interval from 0º to 90º, as the angle increases, the cosine of the angle decreases, so $\cos 47° < \cos 58°$ is false.

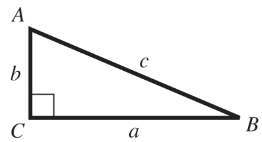
9. $\tan 60° \geq \cot 40°$

Using the cofunction identity, $\cot 40° = \tan(90° - 40°) = \tan 50°$. In quadrant I, the tangent function is increasing. Thus $\cot 40° = \tan 50° < \tan 60°$, and the statement is true.

10. $\csc 22° \leq \csc 68°$

In quadrant I, the cosecant function is decreasing. Thus $\csc 22° \geq \csc 68°$, and the statement is false.

11.



$\cos A = \frac{b}{c}$ and $\sin B = \frac{b}{c}$, so $\cos A = \sin B$.

This is an example of equality of cofunctions of complementary angles.

12. If $\theta = 135°$, $\theta' = 180° - 135° = 45°$.
If $\theta = -45°$, $\theta' = 45°$.
If $\theta = 300°$, $\theta' = 360° - 300° = 60°$.
If $\theta = 140°$, $\theta' = 180° - 140° = 40°$.

Of these reference angles, 40° is the only one which is not a special angle, so D, tan 140°, is the only one which cannot be determined exactly using the methods of this chapter.

13. 1020º is coterminal with $1020° - 2 \cdot 360° = 300°$. The reference angle is $360° - 300° = 60°$. Because 1020° lies in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$$\sin 1020° = -\sin 60° = -\frac{\sqrt{3}}{2}$$
$$\cos 1020° = \cos 60° = \frac{1}{2}$$
$$\tan 1020° = -\tan 60° = -\sqrt{3}$$
$$\cot 1020° = -\cot 60° = -\frac{\sqrt{3}}{3}$$
$$\sec 1020° = \sec 60° = 2$$
$$\csc 1020° = -\csc 60° = -\frac{2\sqrt{3}}{3}$$

14. A 120º angle lies in quadrant II, so the reference angle is $180° - 120° = 60°$. Because 120° is in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$$
$$\cos 120° = -\cos 60° = -\frac{1}{2}$$
$$\tan 120° = -\tan 60° = -\sqrt{3}$$
$$\cot 120° = -\cot 60° = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$
$$\sec 120° = -\sec 60° = -2$$
$$\csc 120° = \csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

15. –1470° is coterminal with $-1470° + 5 \cdot 360° = 330°$. This angle lies in quadrant IV. The reference angle is $360° - 330° = 30°$. Because –1470° is in quadrant IV, the sine, tangent, cotangent, and cosecant are negative.

$$\sin(-1470°) = -\sin 30° = -\frac{1}{2}$$
$$\cos(-1470°) = \cos 30° = \frac{\sqrt{3}}{2}$$
$$\tan(-1470°) = -\tan 30° = -\frac{\sqrt{3}}{3}$$
$$\cot(-1470°) = -\cot 30° = -\sqrt{3}$$
$$\sec(-1470°) = \sec 30° = \frac{2\sqrt{3}}{3}$$
$$\csc(-1470°) = -\csc 30° = -2$$

16. –225° is coterminal with –225° + 360° = 135°. This angle lies in quadrant II. The reference angle is $180° - 135° = 45°$. Because –225° is in quadrant II, the cosine, tangent, cotangent, and secant are negative.

$$\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$$
$$\cos(-225°) = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan(-225°) = -\tan 45° = -1$$
$$\cot(-225°) = -\cot 45° = -1$$
$$\sec(-225°) = -\sec 45° = -\sqrt{2}$$
$$\csc(-225°) = \csc 45° = \sqrt{2}$$

17. $\cos\theta = -\frac{1}{2}$

Because $\cos\theta$ is negative, θ must lie in quadrants II or III. The absolute value of $\cos\theta$ is $\frac{1}{2}$, so the reference angle, θ' must be $60°$. The quadrant II angle θ equals $180° - \theta' = 180° - 60° = 120°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 60° = 240°$.

18. $\sin\theta = -\frac{1}{2}$

Because $\sin\theta$ is negative, θ must lie in quadrants III or IV. The absolute value of $\sin\theta$ is $\frac{1}{2}$, so the reference angle, θ', is 30º. The angle in quadrant III will be $180° + \theta' = 180° + 30° = 210°$, and the quadrant IV angle is $360° - \theta' = 360° - 30° = 330°$.

19. $\sec\theta = -\frac{2\sqrt{3}}{3}$

Because $\sec\theta$ is negative, θ must lie in quadrants II or III. The absolute value of $\sec\theta$ is $\frac{2\sqrt{3}}{3}$, so the reference angle, θ' must be 30º. The quadrant II angle θ equals $180° - \theta' = 180° - 30° = 150°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 30° = 210°$.

20. $\cot\theta = -1$

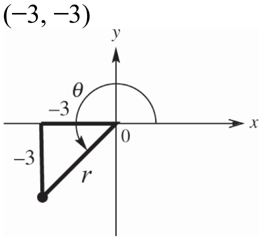
Because $\cot\theta$ is negative, θ must lie in quadrants II or IV. The absolute value of $\cot\theta$ is 1, so the reference angle, θ' must be 45°. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$. and the quadrant IV angle θ equals $360° - \theta' = 360° - 45° = 315°$.

21. $\tan^2 120° - 2\cot 240° = \left(-\sqrt{3}\right)^2 - 2\left(\frac{\sqrt{3}}{3}\right) = 3 - \frac{2\sqrt{3}}{3}$

22. $\cos 60° + 2\sin^2 30° = \frac{1}{2} + 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

23. $\sec^2 300° - 2\cos^2 150° = 2^2 - 2\left(-\frac{\sqrt{3}}{2}\right)^2 = 4 - \frac{3}{2} = \frac{5}{2}$

24. **(a)** $(-3, -3)$



The distance from the origin is r:

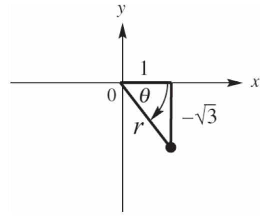
$r = \sqrt{x^2 + y^2} \Rightarrow r = \sqrt{(-3)^2 + (-3)^2} \Rightarrow$
$r = \sqrt{9+9} \Rightarrow r = \sqrt{18} \Rightarrow r = 3\sqrt{2}$

$\sin\theta = \frac{y}{r} = -\frac{3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = -\frac{3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{-3}{-3} = 1$

(b) $\left(1, -\sqrt{3}\right)$



The distance from the origin is r:

$r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + \left(-\sqrt{3}\right)^2}$
$= \sqrt{1+3} = \sqrt{4} = 2$

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{2}$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

For the exercises in this section, be sure your calculator is in degree mode.

25. $\sec 222°30' = \frac{1}{\cos 222°30'} \approx -1.356342$

26. $\sin 72°30' \approx 0.953717$

27. $\csc 78°21' = \frac{1}{\sin 78°21'} \approx 1.021034$

28. $\cot 305.6° = \frac{1}{\tan 305.6°} \approx -0.715930$

29. $\tan 11.7689° \approx 0.208344$

30. $\sec 58.9041° = \frac{1}{\cos 58.9041°} \approx 1.936213$

31. $\sin\theta = 0.8254121$
$\theta = \sin^{-1}(0.8254121) \approx 55.673870°$

32. $\cot\theta = 1.1249386$
$\theta = \cot^{-1}(1.1249386) = \tan^{-1}\left(\frac{1}{1.1249386}\right)$
$\approx 41.635092°$

33. $\cos\theta = 0.97540415$
$\theta = \cos^{-1}(0.97540415) \approx 12.733938°$

34. $\sec\theta = 1.2637891$
$\theta = \sec^{-1}(1.2637891) = \cos^{-1}\left(\frac{1}{1.2637891}\right)$
$\approx 37.695528°$

35. $\tan\theta = 1.9633124$
$\theta = \tan^{-1}(1.9633124) \approx 63.008286°$

36. $\csc\theta = 9.5670466$
$\theta = \csc^{-1}(9.5670466) = \sin^{-1}\left(\frac{1}{9.5670466}\right)$
$\approx 5.9998273°$

37. $\sin\theta = 0.73135370$
$\theta = \sin^{-1}(0.73135370) \approx 47°$
The value of $\sin\theta$ is positive in quadrants I and II, so the two angles in $[0°, 360°)$ are 47° and 180° – 47° = 133°.

38. $\tan\theta = 1.3763819$
$\theta = \tan^{-1}(1.3763819) \approx 54°$
The value of $\tan\theta$ is positive in quadrants I and III, so the two angles in $[0°, 360°)$ are 54° and 180° + 54° = 234°.

39. $\sin 50° + \sin 40° \stackrel{?}{=} \sin 90°$
Using a calculator gives $\sin 50° + \sin 40° = 1.408832053$ while $\sin 90° = 1$. Thus, the statement is false.

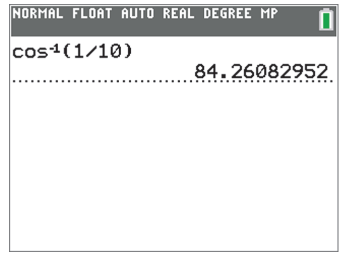
40. $1 + \tan^2 60° \stackrel{?}{=} \sec^2 60°$
$1 + \tan^2 60° = 1 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4$
$\sec^2 60° = 2^2 = 4$
Thus, the statement is true.

41. $\sin 240° \stackrel{?}{=} 2\sin 120° \cos 120°$
$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$ and
$2\sin 120° \cos 120° = 2\sin 60°(-\cos 60°)$
$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{3}}{2}$
Thus, the statement is true.

42. $\sin 42° + \sin 42° \stackrel{?}{=} \sin 84°$
Using a calculator gives $\sin 42° + \sin 42° = 1.338261213$ while $\sin 84° = 0.9945218954$. Thus, the statement is false.

43. No, this will result in an angle having tangent equal to 25. The function $\tan^{-1}$ is not the reciprocal of the tangent (the cotangent), but is the *inverse tangent* function. To find cot 25°, the student must find the reciprocal of tan 25°.
$\cot 25° = \frac{1}{\tan 25°} \neq \tan^{-1} 25°.$

44. We must find an angle having secant equal to 10, which means that its cosine is $\frac{1}{10}$.
To find $\sec^{-1} 10$, on a calculator, enter $\cos^{-1}\left(\frac{1}{10}\right)$.

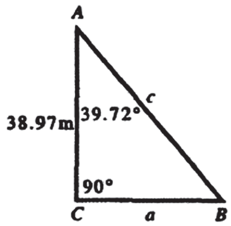


$\sec^{-1} 10 = \cos^{-1}\left(\frac{1}{10}\right) \approx 84.26082952°$

45. $A = 58°\ 30'$, $c = 748$
$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$
$B = 90° - 58°30' = 89°60' - 58°30'$
$= 31°30'$
$\sin A = \frac{a}{c} \Rightarrow \sin 58°30' = \frac{a}{748} \Rightarrow$
$a = 748 \sin 58°30' \approx 638$ (rounded to three significant digits)
$\cos A = \frac{b}{c} \Rightarrow \cos 58°30' = \frac{b}{748} \Rightarrow$
$b = 748 \cos 58°30' \approx 391$ (rounded to three significant digits)

46. $a = 129.7$, $b = 368.1$
$c = \sqrt{a^2 + b^2} \Rightarrow c = \sqrt{129.7^2 + 368.1^2}$
≈ 390.3 (rounded to four significant digits)
$\tan A = \frac{a}{b} = \frac{129.7}{368.1} \Rightarrow$
$A = \tan^{-1}\left(\frac{129.70}{368.10}\right) \approx 19.41°$
$\approx 19° + (0.41 \cdot 60)' \approx 19°25'$
$\tan B = \frac{b}{a} = \frac{368.1}{129.7} \Rightarrow$
$B = \tan^{-1}\left(\frac{368.1}{129.7}\right) \approx 70.59°$
$\approx 70° + (0.59 \cdot 60)' \approx 70°35'$

47. $A = 39.72°,\ b = 38.97$ m

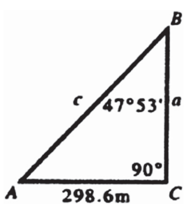


$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$
$B = 90° - 39.72° = 50.28°$

$\tan A = \dfrac{a}{b} \Rightarrow \tan 39.72° = \dfrac{a}{38.97} \Rightarrow$
$a = 38.97 \tan 39.72° \approx 32.38$ m (rounded to four significant digits)

$\cos A = \dfrac{b}{c} \Rightarrow \cos 39.72° = \dfrac{38.97}{c} \Rightarrow$
$c \cos 39.72° = 38.97 \Rightarrow$
$c = \dfrac{38.97}{\cos 39.72°} \approx 50.66$ m
(rounded to five significant digits)

48. $B = 47°53',\ b = 298.6$ m

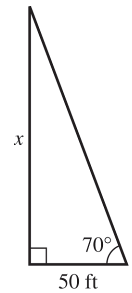


$A + B = 90° \Rightarrow A = 90° - B \Rightarrow$
$A = 90° - 47°53' = 89°60' - 47°53'$
$= 42°07'$

$\tan B = \dfrac{b}{a} \Rightarrow \tan 47°53' = \dfrac{298.6}{a} \Rightarrow$
$a \tan 47°53' = 298.6 \Rightarrow$
$a = \dfrac{298.6}{\tan 47°53'} \approx 270.0$ m
(rounded to four significant digits)

$\sin B = \dfrac{b}{c} \Rightarrow \sin 47°53' = \dfrac{298.6}{c} \Rightarrow$
$c \sin 47°53' = 298.6 \Rightarrow$
$c = \dfrac{298.6}{\sin 47°53'} \approx 402.5$ m
(rounded to four significant digits)

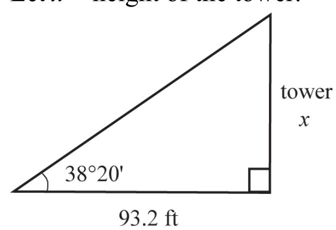
49. Let x = the height of the tree.



$\tan 70° = \dfrac{x}{50} \Rightarrow x = 50 \tan 70° \approx 137$ ft

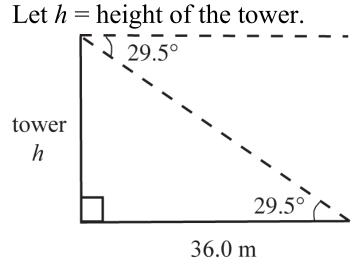
50. $r = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$
$\tan \theta = \dfrac{5}{12} \Rightarrow \theta = \tan^{-1}\left(\dfrac{5}{12}\right) \approx 23°$

51. Let x = height of the tower.



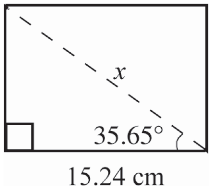
$\tan 38°20' = \dfrac{x}{93.2}$
$x = 93.2 \tan 38°20'$
$x \approx 73.6930$
The height of the tower is 73.7 ft. (rounded to three significant digits)

52. Let h = height of the tower.



$\tan 29.5° = \dfrac{h}{36.0}$
$h = 36.0 \tan 29.5° \approx 20.3678$
The height of the tower is 20.4 m. (rounded to three significant digits)

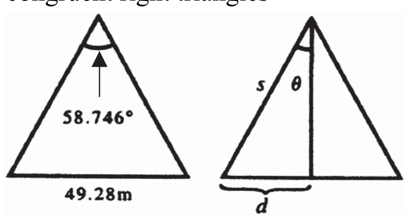
53. Let x = length of the diagonal



$$\cos 35.65° = \frac{15.24}{x}$$
$$x = \frac{15.24}{\cos 35.65°} \approx 18.7548$$

The length of the diagonal is 18.75 cm (rounded to three significant digits).

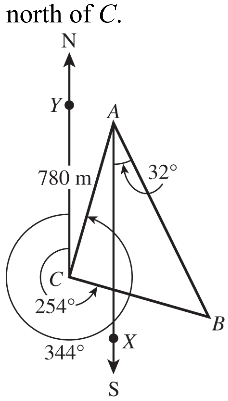
54. Let x = the length of the equal sides of an isosceles triangle.
Divide the isosceles triangle into two congruent right triangles



$$d = \frac{1}{2}(49.28) = 24.64 \text{ and}$$
$$\theta = \frac{1}{2}(58.746°) = 29.373°$$
$$\sin\theta = \frac{d}{s} \Rightarrow \sin 29.373° = \frac{24.64}{s} \Rightarrow$$
$$s = \frac{24.64}{\sin 29.373°} \approx 50.2352$$

Each side is 50.24 m long (rounded to 4 significant digits).

55. Draw triangle ABC and extend the north-south lines to a point X south of A and S to a point Y, north of C.

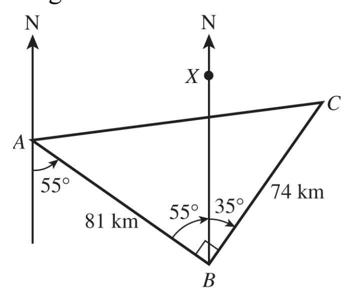


Angle $ACB = 344° - 254° = 90°$, so ABC is a right triangle.
Angle $BAX = 32°$ because it is an alternate interior angle to 32°.
Angle $YCA = 360° - 344° = 16°$
Angle $XAC = 16°$ because it is an alternate interior angle to angle YCA.
Angle $BAC = 32° + 16° = 48°$.
In triangle ABC,

$$\cos A = \frac{AC}{AB} \Rightarrow \cos 48° = \frac{780}{AB} \Rightarrow$$
$$AB\cos 48° = 780 \Rightarrow AB = \frac{780}{\cos 48°} \approx 1165.6917$$

The distance from A to B is 1200 m. (rounded to two significant digits)

56. Draw triangle ABC and extend north-south lines from points A and B. Angle ABX is 55° (alternate interior angles of parallel lines cut by a transversal have the same measure) so Angle ABC is 55° + 35° = 90°.

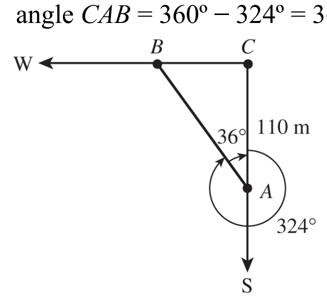


Angle ABC is a right angle, so use the Pythagorean theorem to find the distance from A to C.

$$(AC)^2 = 81^2 + 74^2 \Rightarrow (AC)^2 = 6561 + 5476 \Rightarrow$$
$$(AC)^2 = 12{,}037 \Rightarrow AC = \sqrt{12{,}037} \approx 109.7133$$

It is 110 km from A to C. (rounded to two significant digits)

57. Suppose A is the car heading south at 55 mph, B is the car heading west, and point C is the intersection from which they start. After two hours, using $d = rt$, $AC = 55(2) = 110$. Angle ACB is a right angle, so triangle ACB is a right triangle. The bearing of A from B is 324º, so angle $CAB = 360º - 324º = 36º$.



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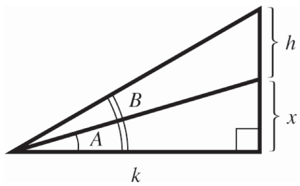
$$\cos\angle CAB = \frac{AC}{AB} \Rightarrow \cos 36° = \frac{110}{AB} \Rightarrow$$
$$AB = \frac{110}{\cos 36°} \approx 135.9675$$

The distance from A to B is about 140 mi (rounded to two significant digits).

58. Let x = the leg opposite angle A

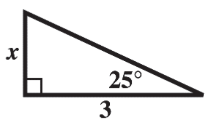


$\tan A = \dfrac{x}{k} \Rightarrow x = k\tan A$ and

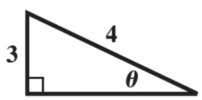
$\tan B = \dfrac{h+x}{k} \Rightarrow x = k\tan B - h$. So,

$$k\tan A = k\tan B - h$$
$$h = k\tan B - k\tan A = k(\tan B - \tan A)$$

59. Answers will vary. Sample answer: Find the value of x.



60. Answers will vary. Sample answer: Find the value of x.



61. $h = R\left(\dfrac{1}{\cos\left(\frac{180T}{P}\right)} - 1\right)$

(a) Let R = 3955 mi, T = 25 min, P = 140 min.

$$h = R\left(\frac{1}{\cos\left(\frac{180T}{P}\right)} - 1\right)$$
$$h = 3955\left(\frac{1}{\cos\left(\frac{180\cdot 25}{140}\right)} - 1\right) \approx 715.9424$$

The height of the satellite is approximately 716 mi.

(b) Let R = 3955 mi, T = 30 min, P = 140 min.

$$h = R\left(\frac{1}{\cos\left(\frac{180T}{P}\right)} - 1\right)$$
$$h = 3955\left(\frac{1}{\cos\left(\frac{180\cdot 30}{140}\right)} - 1\right) \approx 1103.6349$$

The height of the satellite is approximately 1104 mi.

62. **(a)** From the figure, we see that

$$\sin\theta = \frac{x_Q - x_P}{d} \Rightarrow x_Q = x_P + d\sin\theta.$$

Similarly, we have

$$\cos\theta = \frac{y_Q - y_P}{d} \Rightarrow y_Q = y_P + d\cos\theta.$$

(b) Let $(x_P, y_P) = (123.62, 337.95)$,

$\theta = 17°19'22''$, and d = 193.86 ft.

$$x_Q = x_P + d\sin\theta \Rightarrow$$
$$x_Q = 123.62 + 193.86\sin 17°19'22''$$
$$\approx 181.3427$$
$$y_Q = y_P + d\cos\theta \Rightarrow$$
$$y_Q = 337.95 + 193.86\cos 17°19'22''$$
$$\approx 523.0170$$

Thus, the coordinates of Q are (181.34, 523.02), rounded to five significant digits.

Chapter 2 Chapter Test

1. $\sin A = \dfrac{\text{side opposite}}{\text{hypotenuse}} = \dfrac{12}{13}$

$\cos A = \dfrac{\text{side adjacent}}{\text{hypotenuse}} = \dfrac{5}{13}$

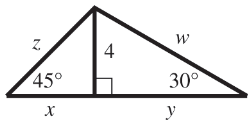
$\tan A = \dfrac{\text{side opposite}}{\text{side adjacent}} = \dfrac{12}{5}$

$\cot A = \dfrac{\text{side adjacent}}{\text{side opposite}} = \dfrac{5}{12}$

$\sec A = \dfrac{\text{hypotenuse}}{\text{side adjacent}} = \dfrac{13}{5}$

$\csc A = \dfrac{\text{hypotenuse}}{\text{side opposite}} = \dfrac{13}{12}$

2. Apply the relationships between the lengths of the sides of a $30°-60°$ right triangle first to the triangle on the right to find the values of *y* and *w*. In the $30°-60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle).



Thus, we have $y = 4\sqrt{3}$ and $w = 2(4) = 8$.

Apply the relationships between the lengths of the sides of a $45°-45°$ right triangle next to the triangle on the left to find the values of *x* and *z*. In the $45°-45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg. Thus, we have $x = 4$ and $z = 4\sqrt{2}$

3. $\sin(\theta + 15°) = \cos(2\theta + 30°)$

Sine and cosine are cofunctions, so the sum of the angles is 90º. So,

$$(\theta + 15°) + (2\theta + 30°) = 90°$$
$$3\theta + 45° = 90°$$
$$3\theta = 45° \Rightarrow \theta = 15°$$

4. (a) $\sin 24° < \sin 48°$

In the interval from 0º to 90º, as the angle increases, so does the sine of the angle, so sin 24º < sin 48º is true.

(b) $\cos 24° < \cos 48°$

In the interval from 0º to 90º, as the angle increases, so the cosine of the angle decreases, so cos 24º < cos 48º is false.

(c) $\cos(60° + 30°)$

$$= \cos 60° \cos 30° - \sin 60° \sin 30°$$

$$\cos(60° + 30°) = \cos 90° = 0$$

$$\cos 60° \cos 30° - \sin 60° \sin 30° = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{2}\right) = 0$$

Thus, the statement is true.

5. A 240º angle lies in quadrant III, so the reference angle is $240° - 180° = 60°$. Because 240° is in quadrant III, the sine, cosine, secant, and cosecant are negative.

$$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$$
$$\cos 240° = -\cos 60° = -\frac{1}{2}$$
$$\tan 240° = \tan 60° = \sqrt{3}$$
$$\cot 240° = \cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\sec 240° = -\sec 60° = -2$$
$$\csc 240° = -\csc 60° = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

6. –135° is coterminal with –135° + 360° = 225°. This angle lies in quadrant III. The reference angle is $225° - 180° = 45°$. Because –135° is in quadrant III, the sine, cosine, secant, and cosecant are negative.

$$\sin(-135°) = -\sin 45° = -\frac{\sqrt{2}}{2}$$
$$\cos(-35°) = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan(-135°) = \tan 45° = 1$$
$$\cot(-135°) = \cot 45° = 1$$
$$\sec(-135°) = -\sec 45° = -\sqrt{2}$$
$$\csc(-135°) = -\csc 45° = -\sqrt{2}$$

7. 990º is coterminal with $990° - 2 \cdot 360° = 270°$, which is the reference angle.

$$\sin 990° = \sin 270° = -1$$
$$\cos 990° = \cos 270° = 0$$
$$\tan 990° = \tan 270° \text{ undefined}$$
$$\cot 990° = \cot 270° = 0$$
$$\sec 990° = \sec 270° \text{ undefined}$$
$$\csc 990° = \csc 270° = -1$$

8. $\cos\theta = -\frac{\sqrt{2}}{2}$

Because $\cos\theta$ is negative, θ must lie in quadrant II or quadrant III. The absolute value of $\cos\theta$ is $\frac{\sqrt{2}}{2}$, so $\theta' = 45°$. The quadrant II angle θ equals $180° - \theta' = 180° - 45° = 135°$, and the quadrant III angle θ equals $180° + \theta' = 180° + 45° = 225°$.

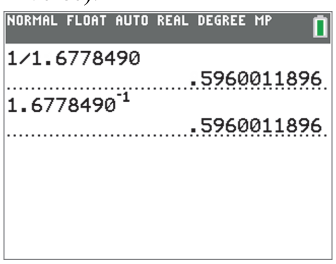
9. $\csc\theta = -\dfrac{2\sqrt{3}}{3}$

Because $\csc\theta$ is negative, θ must lie in quadrant III or quadrant IV. The absolute value of $\csc\theta$ is $\dfrac{2\sqrt{3}}{3}$, so $\theta' = 60°$. The quadrant III angle θ equals $180° + \theta' = 180° + 60° = 240°,$ and the quadrant IV angle θ equals $360° - \theta' = 360° - 60° = 300°.$

10. $\tan\theta = 1 \Rightarrow \theta = 45°$ or $\theta = 225°$

11. $\tan\theta = 1.6778490$

$\cot\theta = \dfrac{1}{\tan\theta} = (\tan\theta)^{-1}$, so we can use division or the inverse key (multiplicative inverse).



12. **(a)** $\sin 78°21' \approx 0.979399$

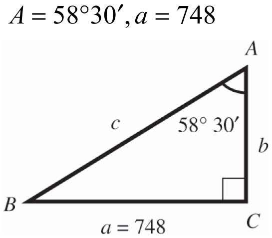
(b) $\tan 117.689° \approx -1.905608$

(c) $\sec 58.9041° = \dfrac{1}{\cos 58.9041°} \approx 1.936213$

13. $\sin\theta = 0.27843196$

$\theta = \sin^{-1}(0.27843196) \approx 16.166641°$

14. $A = 58°30', a = 748$



$A + B = 90° \Rightarrow B = 90° - A \Rightarrow$
$B = 90° - 58°30' = 31°30'$

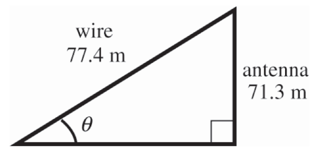
$\tan A = \dfrac{a}{b} \Rightarrow \tan 58°30' = \dfrac{748}{b} \Rightarrow$

$b = \dfrac{748}{\tan 58°30'} \approx 458$ (rounded to three significant digits

$\sin A = \dfrac{a}{c} \Rightarrow \sin 58°30' = \dfrac{748}{c} \Rightarrow$

$c = \dfrac{748}{\sin 58°30'} \approx 877$ (rounded to three significant digits

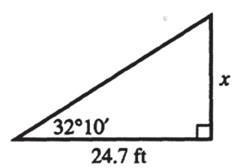
15. Let θ = the measure of the angle that the guy wire makes with the ground.



$\sin\theta = \dfrac{71.3}{77.4}$

$\theta = \sin^{-1}\left(\dfrac{71.3}{77.4}\right) \approx 67.1° \approx 67°10'$

16. Let x = the height of the flagpole.

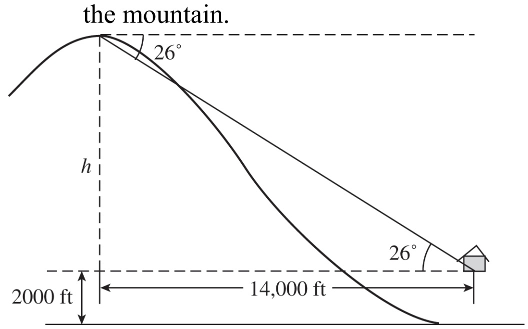


$\tan 32°10' = \dfrac{x}{24.7}$

$x = 24.7 \tan 32°10' \approx 15.5344$

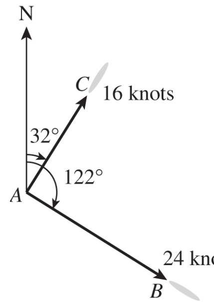
The flagpole is approximately 15.5 ft high. (rounded to three significant digits)

17. Let h = the height of the top of mountain above the cabin. Then 2000 + h = the height of the mountain.



$\tan 26° = \dfrac{h}{14,000} \Rightarrow h \approx 6800$ (rounded to two significant digits). Thus, the height of the mountain is about 6800 + 2000 = 8800 ft.

18. Let x = distance the ships are apart.
In the figure, the measure of angle CAB is $122° - 32° = 90°$. Therefore, triangle CAB is a right triangle.

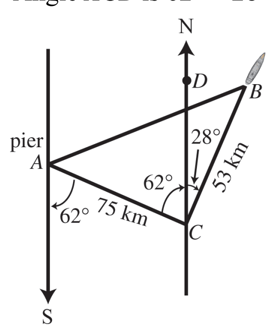


Because $d = rt$, the distance traveled by the first ship in 2.5 hr is
(2.5 hr)(16 knots) = 40 nautical mi and the second ship is
(2.5hr)(24 knots) = 60 nautical mi.
Applying the Pythagorean theorem, we have

$$x^2 = 40^2 + 60^2 \Rightarrow x^2 = 1600 + 3600 \Rightarrow$$
$$x^2 = 5200 \Rightarrow x = \sqrt{5200} \approx 72.111$$

The ships are 72 nautical mi apart (rounded to 2 significant digits).

19. Draw triangle ACB and extend north-south lines from points A and C. Angle ACD is 62° (alternate interior angles of parallel lines cut by a transversal have the same measure), so Angle ACB is 62° + 28° = 90°.

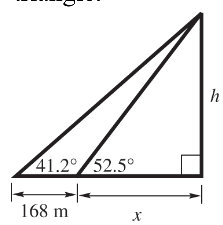


Angle ACB is a right angle, so use the Pythagorean theorem to find the distance from A to B.

$$(AB)^2 = 75^2 + 53^2 \Rightarrow (AB)^2 = 5625 + 2809 \Rightarrow$$
$$(AB)^2 = 8434 \Rightarrow AB = \sqrt{8434} \approx 91.8368$$

It is 92 km from the pier to the boat, rounded to two significant digits.

20. Let x = the side adjacent to 52.5° in the smaller triangle.



In the larger triangle, we have

$$\tan 41.2° = \frac{h}{168 + x} \Rightarrow h = (168 + x)\tan 41.2°.$$

In the smaller triangle, we have

$$\tan 52.5° = \frac{h}{x} \Rightarrow h = x \tan 52.5°.$$

Substitute for h in this equation to solve for x.

$$(168 + x)\tan 41.2° = x \tan 52.5°$$
$$168 \tan 41.2° + x \tan 41.2° = x \tan 52.5°$$
$$168 \tan 41.2° = x \tan 52.5° - x \tan 41.2°$$
$$168 \tan 41.2° = x(\tan 52.5° - \tan 41.2°)$$
$$\frac{168 \tan 41.2°}{\tan 52.5° - \tan 41.2°} = x$$

Substituting for x in the equation for the smaller triangle gives

$$h = x \tan 52.5°$$
$$h = \frac{168 \tan 41.2° \tan 52.5°}{\tan 52.5° - \tan 41.2°} \approx 448.0432$$

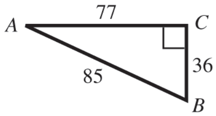
The height of the triangle is approximately 448 m (rounded to three significant digits).

Chapter 2

Acute Angles and Right Triangles

Section 2.1 Trigonometric Functions of Acute Angles

Classroom Example 1 (page 48)



$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{36}{85}$$
$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{77}{85}$$
$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{36}{77}$$
$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{77}{85}$$
$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{36}{85}$$
$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{77}{36}$$

Classroom Example 2 (page 49)

(a) $\sin 9° = \cos(90° - 9°) = \cos 81°$

(b) $\cot 76° = \tan(90° - 76°) = \tan 14°$

(c) $\csc 45° = \sec(90° - 45°) = \sec 45°$

Classroom Example 3 (page 50)

(a) $\cot(\theta - 8°) = \tan(4\theta + 13°)$
Cotangent and tangent are cofunctions, so the equation is true if the sum of the angles is 90º.
$\cot(\theta - 8°) = \tan(4\theta + 13°) \Rightarrow$
$(\theta - 8°) + (4\theta + 13°) = 90° \Rightarrow 5\theta + 5° = 90° \Rightarrow$
$5\theta = 85° \Rightarrow \theta = 17°$

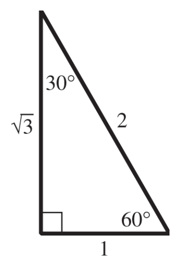
(b) $\sec(5\theta + 14°) = \csc(2\theta - 8°)$
Secant and cosecant are cofunctions, so the equation is true if the sum of the angles is 90º.
$\sec(5\theta + 14°) = \csc(2\theta - 8°) \Rightarrow$
$(5\theta + 14°) + (2\theta - 8°) = 90° \Rightarrow 7\theta + 6° = 90° \Rightarrow$
$7\theta = 84° \Rightarrow \theta = 12°$

Classroom Example 4 (page 51)

(a) $\tan 25° < \tan 23°$
In the interval from 0º to 90º, as the angle increases, the tangent of the angle increases. Thus $\tan 25° < \tan 23°$ is false.

(b) $\csc 44° < \csc 40°$
In the interval from 0º to 90º, as the angle increases, the sine of the angle increases, so the cosecant of the angle decreases. Thus, $\csc 44° < \csc 40°$ is true.

Classroom Example 5 (page 52)



$$\sin 30° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{1}{2}$$
$$\cos 30° = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$
$$\tan 30° = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\cot 30° = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$
$$\sec 30° = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$
$$\csc 30° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{2}{1} = 2$$

Section 2.2 Trigonometric Functions of Non-Acute Angles

Classroom Example 1 (page 57)

(a) 294º lies in quadrant IV. The reference angle is $360° - 294° = 66°$.

(b) $883° - 2 \cdot 360° = 163°$
The reference angle for 163º, and thus for 883º, is 180º − 163º = 17º.

Classroom Example 2 (page 58)

The reference angle for 225º is 225º − 180º = 45º. 225º lies in quadrant III, so the tangent and cotangent are positive while the sine, cosine, secant, and cosecant are negative.

$$\sin 225° = -\sin 45° = -\frac{\sqrt{2}}{2}$$
$$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan 225° = \tan 45° = 1$$
$$\cot 225° = \cot 45° = 1$$
$$\sec 225° = -\sec 45° = -\sqrt{2}$$
$$\csc 225° = -\csc 45° = -\sqrt{2}$$

Classroom Example 3 (page 59)

(a) Because −150º is coterminal with an angle of −150º + 360º = 210º, it lies in quadrant III, and its sine is negative. The reference angle is 210º − 180º = 30º. So $\sin(-150°) = -\sin 30° = -\frac{1}{2}$.

(b) Because 780º is coterminal with an angle of $780° - 2\cdot 360° = 60°$, it lies in quadrant I, and its cotangent is positive. The reference angle is 60º. So $\cot 780° = \cot 60° = \frac{\sqrt{3}}{3}$.

Classroom Example 4 (page 60)

$$\sin 45° = \frac{\sqrt{2}}{2}, \cos 135° = -\frac{\sqrt{2}}{2}, \tan 225° = 1$$
$$\sin^2 45° + 3\cos^2 135° - 2\tan 225°$$
$$= \left(\frac{\sqrt{2}}{2}\right)^2 + 3\left(-\frac{\sqrt{2}}{2}\right)^2 - 2(1)$$
$$= \frac{2}{4} + 3\left(\frac{2}{4}\right) - 2 = 0$$

Classroom Example 5 (page 60)

(a) $\sin 585° = \sin(585° - 360°) = \sin 225° = -\frac{\sqrt{2}}{2}$

(b) $\cot(-930°) = \cot(-930° + 3\cdot 360°) = \cot 150° = -\sqrt{3}$

Classroom Example 6 (page 61)

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$\sin\theta$ is negative, so θ must lie in quadrants III or IV. The absolute value of $\sin\theta$ is $\frac{\sqrt{3}}{2}$, so the reference angle θ' must be 60º. The angle in quadrant III is $60° + 180° = 240°$, and the angle in quadrant IV is $360° - 60° = 300°$.

Section 2.3 Finding Trigonometric Function Values Using a Calculator

Classroom Example 1 (page 64)

(a) $\tan 68°43' \approx 2.56707352$

(b) $\cos 193.622° \approx -0.97187064$

(c) $\csc 35.8471° \approx 1.70757967$

(d) $\sec(-287°) \approx 3.42030362$

Classroom Example 2 (page 65)

(a) $\cos\theta \approx 0.92118541 \Rightarrow \theta \approx 22.899999°$

(b) $\cot\theta \approx 1.4466474 \Rightarrow \theta \approx 34.654300°$

Classroom Example 3 (page 66)

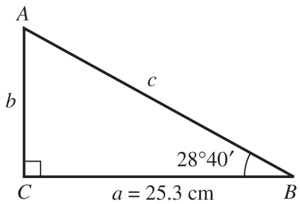
(a) $F = W\sin\theta = 5500\sin 3.9° \approx 370$ lb

(b) $F = W\sin\theta = 2800\sin(-4.8°) \approx -230$ lb
F is negative because the car is traveling downhill.

(c) $F = W\sin\theta \Rightarrow 288 = 2400\sin\theta \Rightarrow$
$\sin\theta = \frac{288}{2400} \Rightarrow \theta = \sin^{-1}\left(\frac{288}{2400}\right) \approx 6.89°$

Section 2.4 Solutions and Applications of Right Triangles

Classroom Example 1 (page 74)



$A + B = 90° \Rightarrow A + 28°40' = 90° \Rightarrow A = 61°20'$

$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{25.3}{AB}$$

$$\cos 28°40' = \frac{25.3}{AB}$$

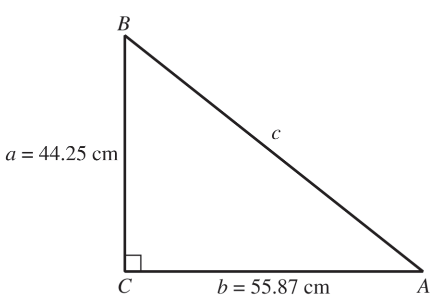
$$c = AB = \frac{25.3}{\cos 28°40'} \approx 28.8 \text{ cm}$$ (three significant digits)

$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{AC}{25.3}$$

$$\tan 28°40' = \frac{AC}{25.3}$$

$$b = AC = 25.3 \tan 28°40' \approx 13.8 \text{ cm}$$ (three significant digits)

Classroom Example 2 (page 75)



Find the length of the hypotenuse, AB, using the Pythagorean theorem:

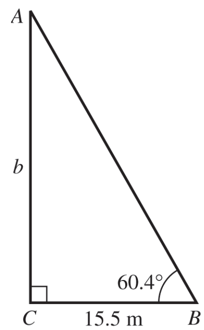
$$c = AB = \sqrt{AC^2 + BC^2} = \sqrt{55.87^2 + 44.25^2}$$
≈ 71.27 (four significant digits)

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{44.25}{55.87} \Rightarrow$$

$$A = \tan^{-1}\left(\frac{44.25}{55.87}\right) \approx 38.38° \text{ or } 38°23'$$

$$B = 90° - A \Rightarrow B = 90° - 38.38° = 51.62° \text{ or } 51°37'$$

Classroom Example 3 (page 76)



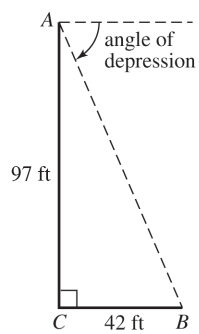
$m\angle B = 60.4°$

$$\tan B = \frac{b}{15.5} \Rightarrow \tan 60.4° = \frac{b}{15.5} \Rightarrow$$

$b = 15.5 \tan 60.4° \approx 27.3$ m

The tree is about 27.3 m tall.

Classroom Example 4 (page 76)

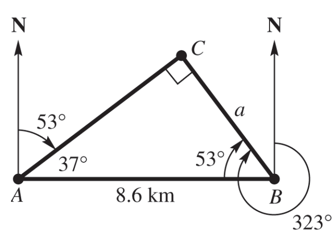


The angle of depression and angle B are alternate interior angles, so their measures are equal.

$$\tan B = \frac{97}{42} \Rightarrow B = \tan^{-1}\left(\frac{97}{42}\right) \Rightarrow \theta \approx 67°$$

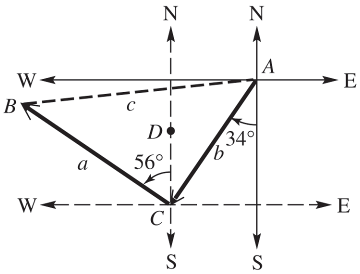
Section 2.5 Further Applications of Right Triangles

Classroom Example 1 (page 83)



$$\cos 53° = \frac{a}{8.6} \Rightarrow a = 8.6 \cos 53° \approx 5.2 \text{ km}$$

Classroom Example 2 (page 83)



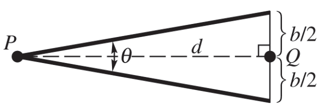
The port is at A. Because the NS lines are parallel, the measure of angle $ACD = 34°$. Therefore, the measure of angle $ACB = 34° + 56° = 90°$, making triangle ABC a right triangle. The ship traveled at 18 knots for 2.5 hours or $18(2.5) = 45$ nautical miles from A to C, labeled b in the figure. The ship traveled at 18 knots for 3.0 hours or $18(3.0) = 54$ nautical miles from C to B, labeled a in the figure. Use the Pythagorean theorem to find the distance from B to A, labeled c in the figure.

$a^2 + b^2 = c^2$

$54^2 + 45^2 = c^2 \Rightarrow c^2 = 4941 \Rightarrow c \approx 70$

The ship is about 70 nautical miles from port.

Classroom Example 3 (page 84)



(a) Find d when $\theta = 2°41'38''$ and $b = 3.5000$ cm

From the figure, we see that

$$\cot\frac{\theta}{2} = \frac{d}{\frac{b}{2}} \Rightarrow d = \frac{b}{2}\cot\frac{\theta}{2}.$$

Convert θ to decimal degrees:

$\theta = 2°41'38'' \approx 2.693888°$

Then, $d = \frac{b}{2}\cot\frac{\theta}{2} \Rightarrow$

$$d = \frac{3.5000}{2}\cot\frac{2.693888°}{2} \approx 74.42708031$$

≈ 74.427 cm

(b) If θ is $1''$ larger, then

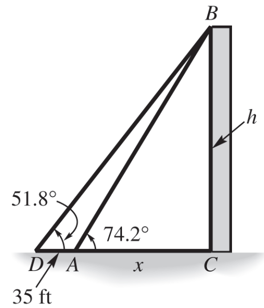
$\theta = 2°41'39'' \approx 2.694167°$

$$d = \frac{3.5000}{2}\cot\frac{2.694167°}{2} \approx 74.41940379$$

The difference is

$74.42708031 - 74.41940379 \approx 0.0076765$ cm.

Classroom Example 4 (page 84)



Algebraic solution:

There are two unknowns, the distance from the base of the building, x, and the height of the building, h.

In triangle ABC, $\tan 74.2° = \frac{h}{x} \Rightarrow h = x\tan 74.2°$.

In triangle BCD, $\tan 51.8° = \frac{h}{x+35} \Rightarrow$

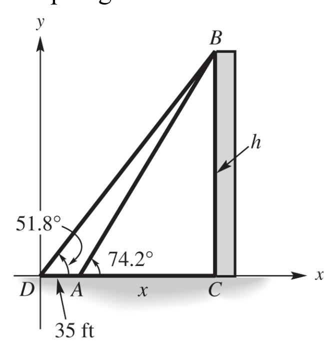
$h = (x+35)\tan 51.8°$.

Setting the two expressions equal, we have

$$\begin{aligned}
x\tan 74.2° &= (x+35)\tan 51.8° \\
x\tan 74.2° &= x\tan 51.8° + 35\tan 51.8° \\
x\tan 74.2° - x\tan 51.8° &= 35\tan 51.8° \\
x(\tan 74.2° - \tan 51.8°) &= 35\tan 51.8° \\
x &= \frac{35\tan 51.8°}{\tan 74.2° - \tan 51.8°}
\end{aligned}$$

$h = x\tan 74.2°$, so substitute the expression for x to find h: $h = \left(\frac{35\tan 51.8°}{\tan 74.2° - \tan 51.8°}\right)\tan 74.2° \approx 69$ ft

Graphing calculator solution:



Superimpose coordinate axes on the figure with D at the origin. Thus, the coordinates of A are (35, 0). The tangent of the angle between the x-axis and the graph of a line with equation $y = mx + b$ is the slope of the line. For line DB, $m = \tan 51.8°$. Because $b = 0$, the equation of line DB is $y_1 = x\tan 51.8°$. The equation of line AB is $y_2 = x\tan 74.2° + b$. Use the coordinates of A and the point-slope form to find the equation of AB.

(*continued on next page*)

(*continued*)

$$y_2 - y_1 = m\left(x_2 - x_1\right) \Rightarrow$$
$$y_2 - 0 = \tan 74.2°\left(x - 35\right) \Rightarrow$$
$$y_2 = \tan 74.2°\left(x - 35\right)$$

Graph y_1 and y_2 in the window [–10, 80] × [–10, 100], then find the point of intersection. The y-coordinate gives h.