2-1-1 [HD] Mass enters an open system with one inlet and one exit at a constant rate of 50 kg/min. At the exit, the mass flow rate is 60 kg/min. If the system initially contains 1000 kg of working fluid, determine (a) dm/dt treating the tank as a system and (b) the time when the system mass becomes 500 kg.

SOLUTION

(a) The mass balance equation will yield the rate of change in mass of this open system as it changes from state 1 (m = 1000 kg) to state 2 (m = 500 kg).

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = 50 - 60; \quad \Rightarrow \frac{dm}{dt} = -10 \ \frac{\text{kg}}{\text{min}}$$

(b) The change in mass is given by

$$\Delta m = m_2 - m_1; \quad \Rightarrow \Delta m = 500 - 1000; \quad \Rightarrow \Delta m = -500 \text{ kg};$$

Since $\frac{dm}{dt}$ is found to be constant, Δm can be rewritten as

$$\Delta m = \frac{dm}{dt}(t); \quad \Rightarrow t = \frac{\Delta m}{\left(\frac{dm}{dt}\right)}; \quad \Rightarrow t = \frac{-500}{-10}; \quad \Rightarrow t = 50 \ \text{min}$$

2-1-2 [HM] Steam enters an insulated tank through a valve. At a given instant, the mass of steam in the tank is found to be 10 kg, and the conditions at the inlet are measured as follows: A = 50 cm^2, V = 31 m/s, and ρ = 0.6454 kg/m^3. Determine (a) dm/dt treating the tank as a system. (b) Assuming the inlet conditions to remain unchanged, determine the mass of steam in the tank after 10 s.

SOLUTION

(a) $\frac{dm}{dt} = \dot{m}_1; \quad \Rightarrow \frac{dm}{dt} = \rho_1 A_1 V_1; \quad \Rightarrow \frac{dm}{dt} = (0.6454)(50\times10^{-4})(31);$

$\Rightarrow \frac{dm}{dt} = 0.1\ \frac{\text{kg}}{\text{s}}$

(b) $\Delta m = \frac{dm}{dt}(t); \quad \Rightarrow m_2 - m_1 = \frac{dm}{dt}(t); \quad \Rightarrow m_2 = (0.1)(10) + 10;$

$\Rightarrow m_2 = 1 + 10; \quad \Rightarrow m_2 = 11\ \text{kg}$

2-1-3 [HJ] Air is introduced into a piston-cylinder device through a 7 cm-diameter flexible duct. The velocity and specific volume at the inlet at a given instant are measured as 22 m/s and 0.1722 m^3/kg respectively. At the same time air jets out through a 1 mm-diameter leak with a density of 1.742 kg/m^3, the piston rises with a velocity of 10 cm/s, and the mass of the air in the device increases at a rate of 0.415 kg/s. Determine (a) the mass flow rate (*m*) of air at the inlet, and (b) the jet velocity. (c) If the jet pressure is 100 kPa, use the IG flow state daemon to determine the temperature of the jet at the exit.

SOLUTION

(a) $\dot{m}_1 = \rho_1 A_1 V_1 = \dfrac{A_1 V_1}{v_1} = \left(\dfrac{\pi D_1^2}{4}\right)\dfrac{V_1}{v_1} = 0.49167\ \dfrac{\text{kg}}{\text{s}}$

(b) $\dfrac{dm}{dt} = \dot{m}_1 - \dot{m}_2;$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 - \frac{dm}{dt}; \qquad \Rightarrow \dot{m}_2 = 0.0767\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_2 = \frac{A_2 V_2}{v_2};$$

$$\Rightarrow V_2 = \frac{\dot{m}_2 v_2}{A_2}; \qquad \Rightarrow V_2 = 56040\ \frac{\text{m}}{\text{s}}$$

(c) Launch the IG flow-state TESTcalc. Enter the known properties at the inlet and exit states (State-1 and State-2 respectively). T2 is calculated as part of State-2 as 200 K. The TEST-code can be found in the professional TEST site (www.thermofluids.net).

2-1-4 [HW] Air enters an open system with a velocity of 1 m/s and density of 1 kg/m^3 at 500 K through a pipe with a cross-sectional area of 10 cm^2 . The mass of air in the tank at a given instant is given by the expression $m = 5p/T$ where p is in kPa and T is in K. If the temperature in the tank remains constant at 500 K due to heat transfer, (a) determine the rate of increase of pressure in the tank. (b) What is the sign of heat transfer, positive (1) or negative (-1)?

SOLUTION

(a) $\frac{dm}{dt} = \dot{m}_1 - \cancel{\dot{m}_2}^{0}; \quad \Rightarrow \frac{dm}{dt} = \dot{m}_1;$

$$\Rightarrow \frac{d}{dt}\left(\frac{5p}{T}\right) = \rho_1 A_1 V_1;$$

$$\Rightarrow \frac{dp}{dt} = \left(\frac{T}{5}\right)\rho_1 A_1 V_1; \quad \Rightarrow \frac{dp}{dt} = \frac{500}{5}(1)(0.001)(1); \quad \Rightarrow \frac{dp}{dt} = 0.1\ \frac{\text{kPa}}{\text{s}}$$

(b) The flow work involved in pushing the mass into the system would transfer energy into the system in addition to the stored energy transported by the mass. To maintain a constant temperature (internal energy), heat must be transferred out of the system. Hence the sign of heat transfer must be negative by the WinHip sign convention.

2-1-5 [NR] A propane tank is being filled at a charging station (see figure in problem 2-1-2 [HM]). At a given instant the mass of the tank is found to increase at a rate of 0.4 kg/s. Propane from the supply line at state-1 has the following conditions: D = 5 cm, T = 25°C, and ρ = 490 kg/m^3. Determine the velocity of propane in the supply line.

SOLUTION

$$\frac{dm}{dt} = \dot{m}_1; \qquad \Rightarrow \dot{m}_1 = 0.4\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_1 = \rho_1 A_1 V_1;$$

$$\Rightarrow V_1 = \frac{\dot{m}_1}{\rho_1 A_1}; \qquad \Rightarrow V_1 = \frac{\dot{m}_1}{\rho_1}\left(\frac{4}{\pi D_1^2}\right); \qquad \Rightarrow V_1 = 0.416\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PC flow-state TESTcalc. Enter the known properties and press Calculate to obtain the velocity along with other properties of the flow state. TEST-code can be found in the TEST professional site at www.thermofluids.net.

2-1-6 [NO] Mass leaves an open system with a mass flow rate of $c*m$, where c is a constant and m is the system mass. If the mass of the system at t = 0 is m_0, derive an expression for the mass of the system at time t.

SOLUTION

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = 0 - cm; \quad \Rightarrow \frac{dm}{dt} = -cm;$$

$$\Rightarrow \int_{m_0}^{m} \frac{1}{m} dm = -c \int_{0}^{t} dt;$$

$$\Rightarrow \ln(m)\Big|_{m_0}^{m} = -ct\Big|_0^t; \quad \Rightarrow \ln(m) - \ln(m_0) = -ct; \quad \Rightarrow \ln\left(\frac{m}{m_0}\right) = -ct;$$

$$\Rightarrow \frac{m}{m_0} = e^{-ct}; \quad \Rightarrow m = m_0 e^{-ct}$$

$$\frac{m}{m_0} = e^{-ct}; \quad \Rightarrow \frac{m}{m_0} = 0.5;$$

$$\Rightarrow t = \frac{\ln(0.5)}{(-c)}; \quad \Rightarrow t = 69.3 \text{ s}$$

2-1-7 [NB] Water enters a vertical cylindrical tank of cross-sectional area 0.01 m^2 at a constant mass flow rate of 5 kg/s. It leaves the tank through an exit near the base with a mass flow rate given by the formula 0.2*h* kg/s, where *h* is the instantaneous height in m. If the tank is initially empty, (a) develop an expression for the liquid height *h* as a function of time *t*. (b) How long does it take for the water to reach a height of 10 m? Assume density of water to remain constant at 1000 kg/m^3.

SOLUTION

(a) $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \frac{dm}{dt} = 5 - \frac{h}{5};$

$$\frac{d\mathcal{V}}{dt} = \frac{\left(\frac{dm}{dt}\right)}{\rho_{water}}; \quad \Rightarrow \frac{d\mathcal{V}}{dt} = \frac{5}{1000} - \frac{h}{5000}; \quad \text{and} \quad \frac{dh}{dt} = \frac{\left(\frac{d\mathcal{V}}{dt}\right)}{A}; \quad \Rightarrow \frac{dh}{dt} = 100\frac{d\mathcal{V}}{dt};$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{2} - \frac{h}{50};$$

$$\Rightarrow \int_0^t dt = t = \int_0^h \frac{1}{\frac{1}{2} - \frac{h}{50}} dh;$$

Using the substitution method where

$$U = \frac{1}{2} - \frac{h}{50}; \quad \text{and} \quad dU = -\frac{1}{50} dh; \quad \Rightarrow dh = -50 dU;$$

$$\Rightarrow t = -50 \int_{\frac{1}{2}}^{\frac{1}{2}-\frac{h}{2}} \frac{1}{U} dU; \quad \Rightarrow t = -50\left[\ln\left(\frac{1}{2} - \frac{h}{50}\right) - \ln\left(\frac{1}{2}\right)\right];$$

$$\Rightarrow t = -50 \ln\left(\frac{\frac{1}{2} - \frac{h}{50}}{\frac{1}{2}}\right); \quad \Rightarrow t = -50 \ln\left(1 - \frac{h}{25}\right);$$

$$\Rightarrow h = 25\left(1 - e^{-0.02t}\right)$$

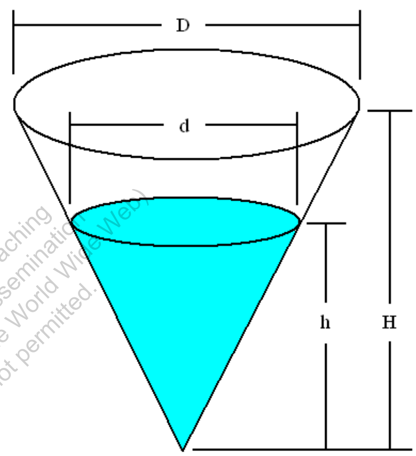
(b) $10 = 25\left(1 - e^{-0.02t}\right); \quad \Rightarrow e^{-0.02t} = 1 - \frac{10}{25};$

$$\Rightarrow -0.02t = \ln\left(\frac{15}{25}\right); \quad \Rightarrow t = 25.5 \text{ s}$$

2-1-8 [NS] A conical tank of base diameter *D* and height *H* is suspended in an inverted position to hold water. A leak at the apex of the cone causes water to leave with a mass flow rate of *c*sqrt(h)*, where *c* is a constant and *h* is the height of the water level from the leak at the bottom.(a) Determine the rate of change of height *h*. (b) Express *h* as a function of time *t* and other known constants, ρ (constant density of water), *D*, *H*, and *c* if the tank were completely full at *t* = 0. (c) If D= 1 m, H=1 m, $\rho = 1000$ kg/m3, and c= 1 kg/(s.m$^{1/2}$), how long does it take for the tank to empty?

SOLUTION

$$V = \frac{1}{12}\pi d^2 h;$$



Using similar triangles to express d in terms of D, H, and h

$$\frac{D}{H} = \frac{d}{h}; \quad \Rightarrow d = \frac{Dh}{H}; \quad \text{and} \quad V = \frac{\pi D^2 h^3}{12H^2};$$

$$m = V\rho; \quad \Rightarrow m = \frac{\pi D^2 h^3 \rho}{12H^2};$$

The derivative of the mass equation produces $\frac{dh}{dt}$

$$\frac{dm}{dt} = \left(\frac{\pi D^2 h^2 \rho}{4H^2}\right)\frac{dh}{dt};$$

From the mass balance equation

$$\dot{m}_e = -\frac{dm}{dt}; \quad \Rightarrow \dot{m}_e = -\left(\frac{\pi D^2 h^2 \rho}{4H^2}\right)\frac{dh}{dt};$$

(a) To solve for the rate of change in height, $\frac{dh}{dt}$, replace $\dot{m}_e$ with $c\sqrt{h}$

$$\frac{dh}{dt} = -\frac{4cH^2}{\pi D^2 h^{3/2}\rho}$$

(b) It is necessary to separate the variables in order to solve for the height h

$$\frac{dh}{dt} = -\frac{4cH^2}{\pi D^2 h^{3/2}\rho}; \quad \Rightarrow h^{3/2}dh = -\frac{4cH^2}{\pi D^2 \rho}dt;$$

$$\Rightarrow \int_H^h h^{3/2}dh = -\frac{4cH^2}{\pi D^2\rho}\int_0^t dt; \quad \Rightarrow \frac{2}{5}h^{5/2}\Big|_H^h = -\frac{4cH^2}{\pi D^2\rho}(t)\Big|_0^t;$$

$$\Rightarrow \frac{2}{5}h^{5/2} - \frac{2}{5}H^{5/2} = -\left(\frac{4cH^2}{\pi D^2 \rho}\right)t;$$

$$h = \left[H^{5/2} - \left(\frac{10cH^2}{\pi D^2 \rho}\right)t\right]^{2/5}$$

(c) $0 = h;$ $\quad \Rightarrow 0 = \left[H^{5/2} - \left(\frac{10cH^2}{\pi D^2 \rho}\right)t\right]^{2/5};$

$$\Rightarrow H^{5/2} = \left(\frac{10cH^2}{\pi D^2 \rho}\right)t;$$

$$\Rightarrow t = \frac{\pi D^2 \rho \sqrt{H}}{10c}; \qquad \Rightarrow t = \frac{\pi(1)(1000)\sqrt{1}}{10(1)}; \qquad \Rightarrow t = 314\text{ s}$$

2-1-9 [NA] Steam enters a mixing chamber at 100 kPa, 20 m/s and a specific volume of 0.4 m^3/kg. Liquid water at 100 kPa and 25^oC enters the chamber through a separate duct with a flow rate of 50 kg/s and a velocity of 5 m/s. If liquid water leaves the chamber at 100 kPa, 43^oC, 5.58 m/s and a volumetric flow rate of 3.357 m^3/min, determine the port areas at (a) the inlets and (b) the exit. Assume liquid water density to be 1000 kg/m^3 and steady state operation.

SOLUTION

From the mass balance equation,

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3;$$

$$\text{Where } \dot{m}_2 = 50\ \frac{\text{kg}}{\text{s}}; \quad \text{and} \quad \dot{m}_3 = \rho_3 \dot{V}_3; \quad \Rightarrow \dot{m}_3 = (1000)\left(\frac{3.357}{60}\right); \quad \Rightarrow \dot{m}_3 = 55.95\ \frac{\text{kg}}{\text{s}};$$

$$\text{(a.1)} \quad \dot{m}_1 = \dot{m}_3 - \dot{m}_2; \quad \Rightarrow \dot{m}_1 = 55.95 - 50; \quad \Rightarrow \dot{m}_1 = 5.95\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_1 = \rho_1 A_1 \mathrm{V}_1; \quad \Rightarrow \dot{m}_1 = \frac{A_1 \mathrm{V}_1}{v_1};$$

$$A_1 = \frac{\dot{m}_1 v_1}{\mathrm{V}_1}; \quad \Rightarrow A_1 = \frac{(5.95)(0.4)}{20}; \quad \Rightarrow A_1 = 0.119\ \text{m}^2; \quad \Rightarrow A_1 = 1{,}190\ \text{cm}^2$$

$$\text{(a.2)} \quad \dot{m}_2 = \rho_2 A_2 \mathrm{V}_2;$$

$$A_2 = \frac{\dot{m}_2}{\rho_2 \mathrm{V}_2}; \quad \Rightarrow A_2 = \frac{50}{(1000)(5)}; \quad \Rightarrow A_2 = 0.01\ \text{m}^2; \quad \Rightarrow A_2 = 100\ \text{cm}^2$$

$$\text{(b)} \quad A_3 = \frac{\dot{V}_3}{\mathrm{V}_3}; \quad \Rightarrow A_3 = \frac{\left(\frac{3.357}{60}\right)}{5.58}; \quad \Rightarrow A_3 = 0.01\ \text{m}^2; \quad \Rightarrow A_3 = 100\ \text{cm}^2$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the two inlet states, State-1 and State-2, and the exit state, State-3 from the properties supplied. The areas are calculated as part of the flow states. The TEST-code for this problem can be found in the professional site at www.thermofluids.net.

2-1-10 [NH] The diameter of the ports in the accompanying figures are 10 cm, 5 cm, and 1 cm at port 1, 2, and 3 respectively. At a given instant, water enters the tank at port 1 with a velocity of 2 m/s and leaves through port 2 with a velocity of 1 m/s. Assuming air to be insoluble in water and density of water and air to remain constant at 1000 kg/m^3 and 1 kg/m^3 respectively, determine:

(a) the mass flow rate (m) of water in kg/s at the inlet,

(b) the rate of change of mass of water $(dm/dt)_W$ in the tank

(c) the rate of change of mass of air $(dm/dt)_A$ in the tank,

(d) the velocity of air port 3.

SOLUTION

Since the water and air do not mix and their densities remain constant, we can find the rate of increase of volume of water in the tank which must be equal to the rate at which air is expelled.

(a) $\dot{m}_1 = \rho_w A_1 V_1; \quad \Rightarrow \dot{m}_1 = (1000)\left(\frac{\pi}{4}(0.1)^2\right)(2); \quad \Rightarrow \dot{m}_1 = 15.708 \ \frac{\text{kg}}{\text{s}}$

(b) $\dot{m}_2 = \rho_w A_2 V_2; \quad \Rightarrow \dot{m}_2 = (1000)\left(\frac{\pi}{4}(0.05)^2\right)(1); \quad \Rightarrow \dot{m}_2 = 1.96 \ \frac{\text{kg}}{\text{s}};$

$\text{Accumulation rate } (\dot{m}) = \dot{m}_1 - \dot{m}_2; \quad \Rightarrow \dot{m} = 15.708 - 1.96; \quad \Rightarrow \dot{m} = 13.74 \ \frac{\text{kg}}{\text{s}}$

(c) Volume of water accumulating is same as volume of air coming out of port 3.

$\dot{m} = \rho_w \dot{V\!\!\!/}; \quad \Rightarrow \dot{V\!\!\!/} = \frac{\dot{m}}{\rho_w}; \quad \Rightarrow \dot{V\!\!\!/} = \frac{13.74}{1000}; \quad \Rightarrow \dot{V\!\!\!/} = 0.01374 \ \frac{\text{m}^3}{\text{s}};$

$\dot{m}_3 = \rho_a \dot{V\!\!\!/}; \quad \Rightarrow \dot{m}_3 = (1)(0.01374); \quad \Rightarrow \dot{m}_3 = 0.01374 \ \frac{\text{kg}}{\text{s}}$

(d) $\dot{m}_3 = \rho_a A_3 V_3; \quad \Rightarrow V_3 = \frac{\dot{m}_3}{A_3 \rho_a}; \quad \Rightarrow V_3 = \frac{0.01374}{\left(\frac{\pi}{4}(0.01)^2\right)(1)};$

$\Rightarrow V_3 = 174.94 \ \frac{\text{m}}{\text{s}}$

2-1-11 [NN] Air is pumped into and withdrawn from a 10 m^3 rigid tank as shown in the accompanying figure. The inlet and exit conditions are as follow. Inlet: v_1 = 2 m^3/kg, V_1 = 10 m/s, A_1 = 0.01 m^2; Exit: v_2 = 5 m^3/kg, V_2 = 5 m/s, A_2 = 0.015 m^2. Assuming the tank to be uniform at all time with the specific volume and pressure related through $p*v$ = 9.0 (kPa-m^3), determine the rate of change of pressure in the tank.

SOLUTION

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e; \qquad \Rightarrow \frac{dm}{dt} = \frac{A_1V_1}{v_1} - \frac{A_2V_2}{v_2}; \qquad \Rightarrow \frac{dm}{dt} = \frac{(0.01)10}{2} - \frac{(0.015)5}{5};$$

$$\Rightarrow \frac{dm}{dt} = 0.035 \ \frac{\text{kg}}{\text{s}};$$

$$\frac{d\rho}{dt} = \frac{d}{dt}\left(\frac{m}{\forall}\right); \qquad \Rightarrow \frac{d\rho}{dt} = \frac{1}{\forall}\frac{dm}{dt}; \qquad \Rightarrow \frac{d\rho}{dt} = \frac{0.035}{10}; \qquad \Rightarrow \frac{d\rho}{dt} = 0.0035 \ \frac{\text{kg}}{\text{m}^3\text{s}};$$

$$pv = 9.0; \qquad \Rightarrow p = 9.0(\rho);$$

Taking the derivative of the pressure equation gives

$$\frac{dp}{dt} = 9.0\frac{d\rho}{dt}; \qquad \Rightarrow \frac{dp}{dt} = (9.0)(0.0035); \qquad \Rightarrow \frac{dp}{dt} = 0.0315 \ \frac{\text{kPa}}{\text{s}}$$

2-1-12 [NE] A gas flows steadily through a circular duct of varying cross-section area with a mass flow rate of 10 kg/s. The inlet and exit conditions are as follows. Inlet: V_1 = 400 m/s, A_1 = 179.36 cm^2; Exit: V_2 = 584 m/s, v_2 = 1.1827 m^3/kg. (a) Determine the exit area. (b) Do you find the increase in velocity of the gas accompanied by an increase in flow area counter-intuitive? Why?

SOLUTION

(a) $$\frac{dm}{dt}^{0} = \dot{m}_i - \dot{m}_e; \quad \Rightarrow \dot{m}_i = \dot{m}_e = \dot{m} = 10 \ \frac{\text{kg}}{\text{s}};$$

$$\dot{m} = \frac{A_2 V_2}{v_2}; \quad \Rightarrow \dot{m} = 10 \ \frac{\text{kg}}{\text{s}};$$

$$\Rightarrow A_2 = \frac{10(v_2)}{V_2}; \quad \Rightarrow A_2 = \frac{10(1.1827)}{584}; \quad \Rightarrow A_2 = 0.0202517 \text{ m}^2;$$

$$\Rightarrow A_2 = 202.52 \text{ cm}^2$$

(b) The increase in velocity and flow area may seem counter intuitive because we are used to flow of constant-density fluids such as water in which case an increase in area must accompany a decrease in velocity as required by the mass balance equation.

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the two states partially after selecting Custom gas. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-13 [NI] A pipe with a diameter of 10 cm carries nitrogen with a velocity of 10 m/s and specific volume 5 m^3/kg into a chamber. Surrounding the pipe, in an annulus of outer diameter 20 cm, is a flow of hydrogen entering the chamber at 20 m/s with a specific volume of 1 m^3/kg . The mixing chamber operates at steady state with a single exit of diameter 5 cm. If the velocity at the exit is 62 m/s, determine: (a) the mass flow rate in kg/s at the exit, (b) the specific volume of the mixture at the exit in m^3/kg, and (c) the apparent molar mass in kg/kmol of the mixture at the exit.

SOLUTION

$$\dot{m}_{N_2} = \frac{A_{N_2} V_{N_2}}{v_{N_2}}; \quad \Rightarrow \dot{m}_{N_2} = \frac{\left(\frac{\pi}{4}(0.1)^2\right)(10)}{5}; \quad \Rightarrow \dot{m}_{N_2} = 0.0157 \ \frac{\text{kg}}{\text{s}};$$

$$A_{H_2} = \frac{\pi}{4}\left[(0.2)^2 - (0.1)^2\right]; \quad \Rightarrow A_{H_2} = 0.0236 \text{ m}^2;$$

$$\dot{m}_{H_2} = \frac{A_{H_2} V_{H_2}}{v_{H_2}}; \quad \Rightarrow \dot{m}_{H_2} = \frac{(0.0236)(20)}{1}; \quad \Rightarrow \dot{m}_{H_2} = 0.4712 \ \frac{\text{kg}}{\text{s}};$$

(a) $$\dot{m}_e = \dot{m}_{H_2} + \dot{m}_{N_2}; \quad \Rightarrow \dot{m}_e = 0.4712 + 0.0157; \quad \Rightarrow \dot{m}_e = 0.4869 \ \frac{\text{kg}}{\text{s}}$$

(b) $$v_e = \frac{A_e V_e}{\dot{m}_e}; \quad \Rightarrow v_e = \frac{\left(\frac{\pi}{4}(0.05)^2\right)(62)}{0.4869}; \quad \Rightarrow v_e = 0.25 \ \frac{\text{m}^3}{\text{kg}}$$

(c) $$\bar{M} = \frac{\dot{m}}{\dot{n}}; \quad \Rightarrow \bar{M} = \frac{\dot{m}_{N_2} + \dot{m}_{H_2}}{\dot{n}_{N_2} + \dot{n}_{H_2}}; \quad \Rightarrow \bar{M} = \frac{\dot{m}_{N_2} + \dot{m}_{H_2}}{\left(\dot{m}_{N_2} / \bar{M}_{N_2}\right) + \left(\dot{m}_{H_2} / \bar{M}_{H_2}\right)};$$

$$\Rightarrow \bar{M} = \frac{0.4869}{(0.0157/28) + (0.4712/2)}; \quad \Rightarrow \bar{M} = 2.06 \ \frac{\text{kg}}{\text{kmol}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the three states partially after selecting Custom gas. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-14 [NL] A pipe with a diameter of 15 cm carries hot air with a velocity 200 m/s and temperature 1000 K into a chamber. Surrounding the pipe, in an annulus of outer diameter 20 cm, is a flow of cooler air entering the chamber at 10 m/s with a temperature of 300 K . The mixing chamber operates at steady state with a single exit of diameter 20 cm. If the air exits at 646 K and the specific volume of air is proportional to the temperature (in K), determine: (a) the exit velocity in m/s and (b) the d*m*/d*t* for the mixing chamber in kg/s.

SOLUTION

$v = cT$ (Given)

$$\dot{m}_h = \frac{A_h V_h}{v_h}; \quad \Rightarrow \dot{m}_h = \frac{A_h V_h}{cT_h}; \quad \Rightarrow \dot{m}_h = \frac{\left(\frac{\pi}{4}(0.15)^2\right)(200)}{c(1000)}; \quad \Rightarrow \dot{m}_h = \frac{0.00353}{c}\ \frac{\text{kg}}{\text{s}};$$

$$A_c = \frac{\pi}{4}\left[(0.2)^2 - (0.15)^2\right]; \quad \Rightarrow A_c = 0.0137\ \text{m}^2;$$

$$\dot{m}_c = \frac{A_h V_h}{v_c}; \quad \Rightarrow \dot{m}_c = \frac{A_c V_c}{cT_c}; \quad \Rightarrow \dot{m}_c = \frac{(0.0137)(10)}{c(300)}; \quad \Rightarrow \dot{m}_c = \frac{0.000458}{c}\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_e = \dot{m}_h + \dot{m}_c; \quad \Rightarrow \dot{m}_e = \frac{0.00353}{c} + \frac{0.000458}{c}; \quad \Rightarrow \dot{m}_e = \frac{0.00399}{\text{c}}\ \frac{\text{kg}}{\text{s}};$$

(a) $$\dot{m}_e = \frac{A_e V_e}{v_e}; \quad \Rightarrow V_e = \frac{\dot{m}_e v_e}{A_e}; \quad \Rightarrow V_e = \frac{\left(\frac{0.00399}{\not{c}}\right)(\not{c})(646)}{0.0314}; \quad \Rightarrow V_e = 82.09\ \frac{\text{m}}{\text{s}}$$

(b) At steady state, the global state of the mixing chamber does not change with time. Therefore, its mass remains constant and

$$\frac{dm}{dt} = 0\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the three states partially after selecting Custom gas. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-15 [NG] Steam enters a turbine through a duct of diameter 0.25 m at 10 MPa, 600°C and 100 m/s. It exits the turbine through a duct of 1 m diameter at 400 kPa and 200°C. For steady state operation, determine (a) the exit velocity and (b) the mass flow rate of steam through the turbine. Use the PC flow-state daemon to obtain the density of steam at the inlet and exit ports. (c) What-if Scenario: What would the exit velocity be if the exit area were equal to the inlet area?

SOLUTION

$$A_1 = \frac{\pi(0.25)^2}{4}; \quad \Rightarrow A_1 = 0.049 \text{ m}^3; \quad \text{and} \quad A_2 = \frac{\pi(1)^2}{4}; \quad \Rightarrow A_2 = 0.7854 \text{ m}^3;$$

Using the surface state PC model for steam

$$\rho_1 = 26.0624 \ \frac{\text{kg}}{\text{m}^3}; \qquad \rho_2 = 1.87193 \ \frac{\text{kg}}{\text{m}^3};$$

At steady state

$$\frac{dm}{dt} = 0 = m_1 - m_2; \quad \Rightarrow m_1 = m_2;$$

(a) $\rho_1 A_1 \text{v}_1 = \rho_2 A_2 \text{v}_2;$

$$\Rightarrow v_2 = \frac{\rho_1 A_1 \text{v}_1}{\rho_2 A_2}; \quad \Rightarrow v_2 = \frac{(26.0624)(0.049)(100)}{(1.87193)(0.7854)}; \quad \Rightarrow v_2 = 86.9 \ \frac{\text{m}}{\text{s}}$$

(b) $\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho_1 A_1 \text{v}_1;$

$$\dot{m} = (26.0624)(0.049)(100); \quad \Rightarrow \dot{m} = 127.7 \ \frac{\text{kg}}{\text{s}}$$

(c) To find v_2 when $A_2 = A_1$

$$\rho_1 A_1 \text{v}_1 = \rho_2 A_1 \text{v}_2; \quad \Rightarrow \text{v}_2 = \frac{\rho_1 \cancel{A_1} \text{v}_1}{\rho_2 \cancel{A_1}}; \quad \Rightarrow \text{v}_2 = \frac{\rho_1 \text{v}_1}{\rho_2};$$

$$\Rightarrow \text{v}_2 = \frac{(26.0624)(100)}{(1.87193)}; \quad \Rightarrow \text{v}_2 = 1392.27 \ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-16 [NZ] Steam enters a turbine with a mass flow rate of 10 kg/s at 10 MPa, 600°C and 30 m/s. It exits the turbine at 45 kPa, 30 m/s and a quality of 0.9. Assuming steady-state operation, determine (a) the inlet area and (b) the exit area. Use the PC flow-state daemon.

SOLUTION

$$\dot{m}_i = \frac{A_i \mathrm{V}_i}{v_i}; \qquad \Rightarrow A_i = \frac{\dot{m}_i v_i}{\mathrm{V}_i};$$

Using the superheated vapor table, $v_{i(10\text{ MPa, }600^\circ\text{C})} = 0.03837\ \frac{\text{m}^3}{\text{kg}}$;

(a) $A_i = \frac{10(0.03837)}{30}; \qquad \Rightarrow A_i = 0.01279\ \text{m}^2$

(b) Using the saturation pressure table for steam,

$$v_{f(45\text{ kPa})} = 0.00103\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{g(45\text{kPa})} = 3.5846\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{e(45\text{ kPa, }0.9)} = v_{f(45\text{ kPa})}(1-x) + v_{g(45\text{ kPa})}(x); \quad \Rightarrow v_{e(45\text{ kPa, }0.9)} = 0.00103(1-0.9) + 3.5846(0.9);$$

$$\Rightarrow v_{e(45\text{ kPa, }0.9)} = 3.226\ \frac{\text{m}^3}{\text{kg}};$$

At steady state

$$\dot{m}_i = \dot{m}_e = 10\ \frac{\text{kg}}{\text{s}};$$

$$\dot{m}_e = \frac{A_e \mathrm{V}_e}{v_e}; \qquad \Rightarrow A_e = \frac{\dot{m}_e v_e}{\mathrm{V}_e}; \qquad \Rightarrow A_e = \frac{10(3.226)}{30}; \qquad \Rightarrow A_e = 1.075\ \text{m}^2$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-17 [NK] Refrigerant R-134 enters a device as saturated liquid at 500 kPa with a velocity of 10 m/s and a mass flow rate of 2 kg/s. At the exit the pressure is 150 kPa and the quality is 0.2. If the exit velocity is 65 m/s, determine the (a) inlet and (b) exit areas. Use the PC flow-state daemon.

SOLUTION

(a) From the problem statement, $v_i = v_{f(500\text{ kPa})}$

Using the saturation pressure table for R-134,

$$v_{f(500\text{kPa})} = 0.000806\ \frac{\text{m}^3}{\text{kg}};$$

$$\dot{m}_i = \frac{A_i V_i}{v_i}; \quad \Rightarrow A_i = \frac{\dot{m}_i v_i}{V_i}; \quad \Rightarrow A_i = \frac{2(0.000806)}{10}; \quad \Rightarrow A_i = 0.000161\text{ m}^2;$$

$$\Rightarrow A_i = 1.61\text{ cm}^2$$

(b) Using the saturation pressure table for R134,

$$v_{f(150\text{ kPa})} = 7.4\times10^{-4}\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{g(150\text{ kPa})} = 0.13149\ \frac{\text{m}^3}{\text{kg}};$$

$$v_{e(150\text{ kPa, }0.2)} = v_{f(150\text{ kPa})}(1-x) + v_{g(150\text{ kPa})}(x);$$

$$\Rightarrow v_{e(150\text{ kPa, }0.2)} = (7.4\times10^{-4})(1-0.2) + 0.13149(0.2); \quad \Rightarrow v_{e(150\text{ kPa, }0.2)} = 0.02689\ \frac{\text{m}^3}{\text{kg}};$$

At steady state, $\dot{m}_i = \dot{m}_e = 2\ \frac{\text{kg}}{\text{s}};$

$$\dot{m}_e = \frac{A_e \text{V}_e}{v_e}; \quad \Rightarrow A_e = \frac{\dot{m}_e v_e}{\text{V}_e}; \quad \Rightarrow A_e = \frac{2(0.02689)}{65};$$

$$\Rightarrow A_e = 0.000827\text{ m}^2; \quad \Rightarrow A_e = 8.27\text{ cm}^2$$

TEST Solution:

Launch the PC flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-18 [NP] Air enters a 0.5m diameter fan at 25^{o}C, 100 kPa and is discharged at 28^{o}C, 105 kPa and a volume flow rate of 0.8 m^3/s. Determine for steady-state operation, (a) the mass flow rate of air in kg/min and (b) the inlet and (c) exit velocities. Use the PG flow-state daemon.

SOLUTION

From the ideal gas property table for air,

$$\rho_{i(100\text{ kPa, }25^o\text{C})} = 1.169\ \frac{\text{kg}}{\text{m}^3};$$

$$\rho_{e(105\text{ kPa, }28^o\text{C})} = 1.215\ \frac{\text{kg}}{\text{m}^3};$$

$$A_i = A_e;$$

$$A = \frac{\pi(d)^2}{4}; \quad \Rightarrow A = \frac{\pi(0.5)^2}{4}; \quad \Rightarrow A = 0.19635\text{ m}^3;$$

(a) At steady state,

$$\dot{m} = \dot{m}_i = \dot{m}_e = \dot{V}_e\rho_e;$$

$$\dot{m} = (0.8)(1.215); \quad \Rightarrow \dot{m} = 0.97193\ \frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m} = 58.3\ \frac{\text{kg}}{\text{min}}$$

(b) $\dot{m} = \rho_i A_i \text{v}_i;$

$$\text{v}_i = \frac{\dot{m}}{\rho_i A_i}; \quad \Rightarrow \text{v}_i = \frac{0.97193}{(1.1687)(0.19635)}; \quad \Rightarrow \text{v}_i = 4.23\ \frac{\text{m}}{\text{s}}$$

(c) $$\text{v}_e = \frac{\dot{V}_e}{A}; \quad \Rightarrow \text{v}_e = \frac{0.8}{0.19635}; \quad \Rightarrow \text{v}_e = 4.07\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-1-19 [NU] Air enters a nozzle, which has an inlet area of 0.1 m^2, at 200 kPa, 500^{o}C and 10 m/s. At the exit the conditions are 100 kPa and 443^{o}C. If the exit area is 35 cm^2, determine the steady-state exit velocity. Use the IG flow-state daemon.

SOLUTION

From the ideal gas property table for air,

$$\rho_{i(200\text{ kPa, }500^o\text{C})} = 0.9014\ \frac{\text{kg}}{\text{m}^3};$$

$$\rho_{e(100\text{ kPa, }443^o\text{C})} = 0.4866\ \frac{\text{kg}}{\text{m}^3};$$

At steady state,

$$\dot{m}_i = \dot{m}_e; \quad \Rightarrow \rho_i A_i \text{v}_i = \rho_e A_e \text{v}_e;$$

$$\text{v}_e = \frac{\rho_i A_i \text{v}_i}{\rho_e A_e}; \quad \Rightarrow \text{v}_e = \frac{(0.9014)(0.1)(10)}{(0.4866)(0.0035)}; \quad \Rightarrow \text{v}_e = 529.3\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the PG flow-state TESTcalc. Evaluate the inlet and exit states from the given conditions. The TEST-code for this solution can be found in the professional site at www.thermofluids.net.

2-2-1 [NX] A 20 kg block of solid cools down by transferring heat at a rate of 1 kW to the surroundings. Determine the rate of change of (a) stored energy and (b) internal energy of the block.

SOLUTION

(a) For this closed system:

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}}; \quad \Rightarrow \frac{dE}{dt} = (-1) - 0; \quad \Rightarrow \frac{dE}{dt} = -1 \text{ kW}$$

(b) $$\frac{dU}{dt} = \frac{d(E - \text{KE} - \text{PE})}{dt}; \quad \Rightarrow \frac{dU}{dt} = \frac{dE}{dt} - \cancelto{0}{\frac{d(\text{KE})}{dt}} - \cancelto{0}{\frac{d(\text{PE})}{dt}};$$

$$\Rightarrow \frac{dU}{dt} = -1 \text{ kW}$$

2-2-2 [KE] A rigid chamber contains 100 kg of water at 500 kPa, 100°C. A paddle wheel stirs the water at 1000 rpm with a torque of 100 N-m. while an internal electrical resistance heater heats the water, consuming 10 amps of current at 110 Volts. Because of thin insulation, the chamber loses heat to the surroundings at 27°C at a rate of 1.2 kW. Determine the rate at which the stored energy of the system changes.

SOLUTION

For this closed system:

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}}; \qquad \Rightarrow \frac{dE}{dt} = \left(-\dot{Q}_{\text{loss}}\right) - \left(-\dot{W}_{\text{in,sh}} - \dot{W}_{\text{in,el}}\right);$$

$$\Rightarrow \frac{dE}{dt} = \dot{W}_{\text{in,sh}} + \dot{W}_{\text{in,el}} - \dot{Q}_{\text{loss}};$$

$$\Rightarrow \frac{dE}{dt} = 2\pi NT + \frac{VI}{1000} - \dot{Q}_{\text{loss}}; \qquad \Rightarrow \frac{dE}{dt} = 2\pi\left(\frac{1000}{60}\right)\left(\frac{100}{1000}\right) + \frac{(110)(10)}{1000} - 1.2;$$

$$\Rightarrow \frac{dE}{dt} = 10.372 \text{ kW}$$

2-2-3 [NC] A closed system interacts with its surroundings and the following data are supplied: W_{sh} = -10 kW, W_{el} = 5 kW, Q = -5 kW. (a) If there are no other interactions, determine dE/dt. (b) Is this system necessarily steady (yes: 1; no: 0)?

SOLUTION

(a) $\frac{dE}{dt} = \dot{Q} - \dot{W}_{ext};$

$$\Rightarrow \frac{dE}{dt} = (-5) - (-10 + 5); \qquad \Rightarrow \frac{dE}{dt} = 0$$

(b) For a system, $\frac{dE}{dt} = 0$. The reverse, however, is not necessarily true. Therefore, the system is not necessarily steady (no: 0).

2-2-4 [QY] An electric bulb consumes 500 W of electricity. After it is turned on, the bulb becomes warmer and starts losing heat to the surroundings at a rate of 5*t* (*t* in seconds) watts until the heat loss equals the electric power input. (a) Plot the change in stored energy of the bulb with time. (b) How long does it take for the bulb to reach steady state?

SOLUTION

(a) $\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}}; \qquad \Rightarrow \frac{dE}{dt} = \left(-\frac{5t}{1000}\right) - \left(-\frac{500}{1000}\right); \quad [\text{kW}]$

$$\Rightarrow \frac{dE}{dt} = 0.5 - \frac{t}{200};$$

$$\Rightarrow dE = 0.5dt - \frac{t}{200}dt; \quad [\text{kW}]$$

Integrating,

$$\Rightarrow E = c + 0.5t - \frac{t^2}{400};$$

If $E = E(0)$ at $t = 0$, $c = E(0)$, then

$$E(t) = E(0) + 0.5t - \frac{t^2}{400};$$

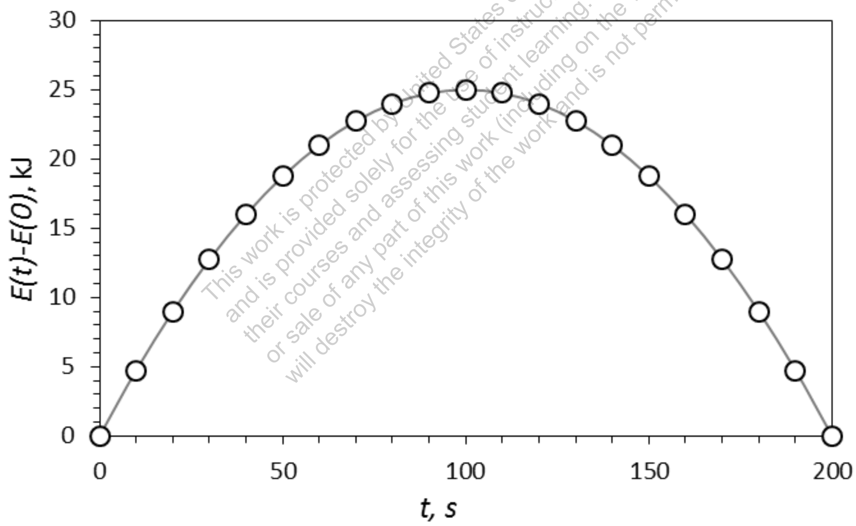


Fig. 1 Plot showing how the stored energy of the system changes with time, reaching steady state after 200 s.

(b) At $t = 200$ s, $\frac{dE}{dt} = 0$. The system (most likely) reaches its steady state at that time.

2-2-5 [NV] Suppose the specific internal energy in kJ/kg of the solid in problem 2-2-1 [NX] is related to its temperature through $u = 0.5T$, where T is the temperature of the solid in Kelvin, determine the rate of change of temperature of the solid. Assume the density of the solid to be 2700 kg/m^3.

SOLUTION

$$\frac{dU}{dt} = -1 \text{ kW};$$

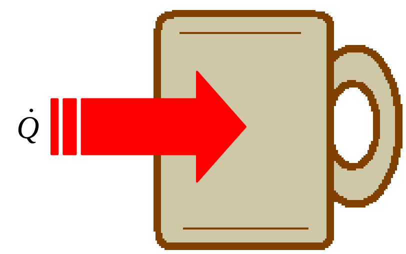
$$\frac{dU}{dt} = \frac{d(mu)}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{du}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{d(0.5T)}{dt}; \quad \Rightarrow \frac{dU}{dt} = (0.5)m\frac{dT}{dt};$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{(0.5)m}\frac{dU}{dt}; \quad \Rightarrow \frac{dT}{dt} = \frac{1}{(0.5)(20)}(-1); \quad \Rightarrow \frac{dT}{dt} = -0.1\ \frac{\text{K}}{\text{s}}$$

2-2-6 [NQ] A cup of coffee is heated in a microwave oven. If the mass of coffee (modeled as liquid water) is 0.2 kg and the rate of heat transfer is 0.1 kW, (a) determine the rate of change of internal energy (u). (b) Assuming the density of coffee to be 1000 kg/m^3 and the specific internal energy in kJ/kg to be related to temperature through $u = 4.2T$, where T is in Kelvin, determine how long it takes for the temperature of the coffee to increase by 20^{o}C.

SOLUTION

Since there is no movement of this system



(a) $$\frac{d\cancel{E}^{U}}{dt} = \dot{Q} - \cancel{\dot{W}_{ext}}^{0};$$

$$\Rightarrow \frac{dE}{dt} = \frac{dU}{dt} = 0.1 \text{ kW}$$

(b) $$\frac{d\cancel{E}^{U}}{dt} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \cancel{\dot{W}_{ext}}^{0}; \quad \Rightarrow \frac{d(mu)}{dt} = \dot{Q}; \quad \Rightarrow \frac{du}{dt} = \frac{\dot{Q}}{m};$$

$$\Rightarrow \dot{u} = \frac{\dot{Q}}{m}; \quad \Rightarrow 4.2\dot{T} = \frac{\dot{Q}}{m}; \quad \Rightarrow 4.2\frac{\Delta T}{\Delta t} = \frac{\dot{Q}}{m};$$

$$\frac{dU}{dt} = 0.1 \text{ kW};$$

$$\frac{dU}{dt} = \frac{d(mu)}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{du}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{d(4.2T)}{dt}; \quad \Rightarrow \frac{dU}{dt} = (4.2)m\frac{dT}{dt};$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{(4.2)m}\frac{dU}{dt}; \quad \Rightarrow \frac{dT}{dt} = \frac{1}{(4.2)(0.2)}(0.1); \quad \Rightarrow \frac{dT}{dt} = \frac{1}{8.4}; \quad \left[\frac{\text{K}}{\text{s}}\right]$$

Therefore,

$$\Delta T = \frac{\Delta t}{8.4}; \quad [\text{K}]$$

$$\Rightarrow \Delta t = (20)(8.4); \quad \Rightarrow \Delta t = 168 \text{ s}$$

2-2-7 [NT] At a given instant a closed system is loosing 0.1 kW of heat to the outside atmosphere. A battery inside the system keeps it warm by powering a 0.1 kW internal heating lamp. A shaft transfers 0.1 kW of work into the system at the same time. Determine:

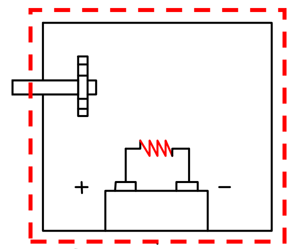
(a) the rate of external work transfer (include sign),

(b) the rate of heat transfer (include sign), and

(c) dE/dt of the system.

SOLUTION

Since the internal battery does not cross the boundary, shaft work and heat loss are the only energy transfers. Using appropriate signs (WinHip),



(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{sh}}; \quad \Rightarrow \dot{W}_{\text{ext}} = -0.1 \text{ kW}$

(b) $\dot{Q} = -0.1 \text{ kW}$

(c) $\dfrac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}}; \quad \Rightarrow \dfrac{dE}{dt} = (-0.1) - (-0.1); \quad \Rightarrow \dfrac{dE}{dt} = 0 \text{ kW}$

2-2-8 [TR] A semi-truck of mass 20,000 lb accelerates from 0 to 75 m/h (1 m/h = 0.447 m/s) in 10 seconds. (a) What is the change in kinetic energy of the truck in 10 seconds? (b) If PE and *U* of the truck can be assumed constant, what is the average value of *dE/dt* of the truck in kW during this period? (c) If 30% of the heat released from the combustion of diesel (heating value of diesel is 40 MJ/kg) is converted to kinetic energy, determine the average rate of fuel consumption in kg/s.

SOLUTION

(a) $V_f = (75)(0.447); \quad \Rightarrow V_f = 33.5\ \frac{\text{m}}{\text{s}};$

$$m = \frac{20{,}000}{2.2}; \quad \Rightarrow m = 9072\ \text{kg};$$

$$\Delta\text{KE} = \frac{m\left(V_f^{\,2} - V_b^{\,2}\right)}{2000}; \quad \Rightarrow \Delta\text{KE} = \frac{(9072)\left(33.5^2\right)}{2000}; \quad \Rightarrow \Delta\text{KE} = 5098\ \text{kJ}$$

(b) $\frac{dE}{dt} = \frac{d(U + \text{KE} + \text{PE})}{dt}; \quad \Rightarrow \frac{dE}{dt} = \cancelto{0}{\frac{dU}{dt}} + \frac{d(\text{KE})}{dt} + \cancelto{0}{\frac{d(\text{PE})}{dt}}; \quad \Rightarrow \frac{dE}{dt} = \frac{d(\text{KE})}{dt};$

$$\Rightarrow \left(\frac{dE}{dt}\right)_{\text{avg}} = \left(\frac{d(\text{KE})}{dt}\right)_{\text{avg}}; \quad \Rightarrow \left(\frac{dE}{dt}\right)_{\text{avg}} = \frac{\Delta\text{KE}}{\Delta t};$$

$$\Rightarrow \left(\frac{dE}{dt}\right)_{\text{avg}} = \frac{5098}{10}; \quad \Rightarrow \left(\frac{dE}{dt}\right)_{\text{avg}} = 509.8\ \text{kW}$$

(c) Assuming the net heat transfer due to fuel burning goes entirely into increasing the KE of the truck,

$$\left(\frac{dE}{dt}\right)_{\text{avg}} = \dot{Q} - \cancelto{0}{\dot{W}_{\text{ext}}}; \quad \Rightarrow \left(\frac{dE}{dt}\right)_{\text{avg}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}};$$

$$\dot{Q}_{\text{in}} = \frac{1}{0.3}\frac{d(\text{KE})}{dt}; \quad [\text{kW}]$$

$$\Rightarrow \dot{m}_F = \frac{\dot{Q}_{\text{in}}}{\text{Heating value}}; \quad \Rightarrow \dot{m}_F = \frac{1}{0.3}\frac{d(\text{KE})}{dt}\frac{1}{40000};$$

$$\Rightarrow \dot{m}_F = \frac{509.8}{(0.3)(40000)}; \quad \Rightarrow \dot{m}_F = 0.0425\ \frac{\text{kg}}{\text{s}}$$

TEST Solution:

Use the SL system state TESTcalc. Instructions and TEST-code can be found (linked under the problem statement) in the Problems module of the professional TEST site at www.thermofluids.net.

2-2-9 [NY] An insulated tank contains 50 kg of water, which is stirred by a paddle wheel at 300 rpm while transmitting a torque of 0.1 kN-m. At the same time, an electric resistance heater inside the tank operates at 110 V, drawing a current of 2 A. Determine the rate of heat transfer after the system achieves steady state.

SOLUTION

$$\dot{W}_{\text{sh,in}} = 2\pi\dot{n}T; \quad \Rightarrow \dot{W}_{\text{sh,in}} = 2\pi(300/60)0.1; \quad \Rightarrow \dot{W}_{\text{sh,in}} = 3.1416 \text{ kW};$$

$$\dot{W}_{\text{el,in}} = \frac{VI}{1000}; \quad \Rightarrow \dot{W}_{\text{el,in}} = -\frac{(110)(2)}{1000}; \quad \Rightarrow \dot{W}_{\text{el,in}} = 0.220 \text{ kW};$$

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = \left(-\dot{W}_{\text{sh,in}}\right) + \left(-\dot{W}_{\text{el,in}}\right);$$

$$\dot{Q} = -3.1416 - 0.22; \quad \Rightarrow \dot{Q} = -3.3616 \text{ kW}$$

2-2-10 [NF] A drill rotates at 4000 rpm while transmitting a torque of 0.012 kN-m. Determine the rate of change of stored energy of the block initially.

SOLUTION

$$\frac{dE}{dt} = \cancel{\dot{Q}}^{0} - \dot{W}_{\text{ext}}; \quad \Rightarrow \frac{dE}{dt} = -\dot{W}_{\text{ext}}; \quad \Rightarrow \frac{dE}{dt} = -\left(-\dot{W}_{\text{sh,in}}\right);$$

$$\Rightarrow \frac{dE}{dt} = 2\pi \frac{4000}{60}(0.012); \quad \Rightarrow \frac{dE}{dt} = 5 \text{ kW}$$

2-2-11 [ND] A 20 kg slab of aluminum is raised by a rope and pulley, arranged vertically, at a constant speed of 10 m/min. At the same time the block absorbs solar radiation at a rate of 0.2 kW. Determine the rate of change of (a) potential energy (PE), (b) internal energy (*U*), and (c) stored energy (*E*).

SOLUTION

(a) $$\frac{d}{dt}(\mathrm{PE}) = \frac{d}{dt}\left(\frac{mgz}{1000}\right); \quad \Rightarrow \frac{d}{dt}(\mathrm{PE}) = \frac{mg}{1000}\frac{dz}{dt}; \quad \Rightarrow \frac{d}{dt}(\mathrm{PE}) = \frac{(20)(9.81)}{1000}\left(\frac{10}{60}\right);$$

$$\Rightarrow \frac{d}{dt}(\mathrm{PE}) = 0.0327 \text{ kW}$$

(b) $$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dU}{dt} + \cancelto{0}{\frac{d}{dt}(\mathrm{KE})} + \frac{d}{dt}(\mathrm{PE}) = \dot{Q}_{\text{rad}} - \left(-\dot{W}_{\text{pull}}\right);$$

$$\Rightarrow \frac{dU}{dt} + \frac{d}{dt}(\mathrm{PE}) = \dot{Q}_{\text{rad}} - (-FV);$$

$$\Rightarrow \frac{dU}{dt} + \frac{d}{dt}(\mathrm{PE}) = \dot{Q}_{\text{rad}} + FV;$$

$$\Rightarrow \frac{dU}{dt} + \frac{d}{dt}(\mathrm{PE}) = \dot{Q}_{\text{rad}} + \frac{mg}{1000}V;$$

$$\Rightarrow \frac{dU}{dt} + 0.0327 = 0.2 + 0.0327;$$

$$\Rightarrow \frac{dU}{dt} = 0.2 \text{ kW}$$

(c) $$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}}; \quad \Rightarrow \frac{dE}{dt} = \dot{Q}_{\text{rad}} + FV; \quad \Rightarrow \frac{dE}{dt} = 0.2 + 0.0327; \quad \Rightarrow \frac{dE}{dt} = 0.2327 \text{ kW}$$

2-2-12 [NM] An external force *F* is applied to a rigid body of mass *m*. If its internal and potential energy remain unchanged, show that an energy balance on the body reproduces Newton's law of motion.

SOLUTION

We assume that there is no heat transfer.

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}}$$

$$\Rightarrow \cancel{\frac{dU}{dt}}^{0} + \frac{d}{dt}(\text{KE}) + \frac{d}{dt}(\text{PE}) = \cancel{\dot{Q}}^{0} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{d}{dt}(\text{KE}) + \cancel{\frac{d}{dt}(\text{PE})}^{0} = -\dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{d}{dt}(\text{KE}) = -(-FV);$$

$$\Rightarrow \frac{d}{dt}\left(\frac{mV^2}{2000}\right) = FV;$$

$$\Rightarrow \frac{m}{2000}\frac{dV^2}{dt} = FV;$$

$$\Rightarrow \frac{m}{2000}\frac{dV^2}{dV}\frac{dV}{dt} = FV;$$

$$\Rightarrow \frac{m}{2000}(2V)a = FV;$$

$$\Rightarrow \frac{ma}{1000} = F \qquad [\text{kN}]$$

$$\Rightarrow F = ma \qquad [\text{N}]$$

2-2-13 [NJ] An insulated block with a mass of 100 kg is acted upon by a horizontal force of 0.02 kN. Balanced by frictional forces, the body moves at a constant velocity of 2 m/s. Determine (a) the rate of change of stored energy in the system and (b) power transferred by the external force. (c) How do you account for the work performed by the external force?

SOLUTION

(a) $$\frac{dE}{dt} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \dot{Q} - \dot{W}_{\text{ext}};$$

For an insulated system with no mass transfer, the energy equation becomes

$$\frac{dE}{dt} = -\dot{W}_{\text{ext}}; \qquad \Rightarrow \frac{dE}{dt} = -\left(-\dot{W}_{\text{in}} + \dot{W}_{\text{out,friction}}\right); \qquad \Rightarrow \frac{dE}{dt} = \dot{W}_{\text{in}} - \dot{W}_{\text{out,friction}};$$

However, because the frictional force is equal to the applied force (to keep the body form accelerating) $\dot{W}_{\text{in}} = \dot{W}_{\text{out,friction}}$;

Therefore,

$$\frac{dE}{dt} = \dot{W}_{\text{in}} - \dot{W}_{\text{out,friction}}; \qquad \Rightarrow \frac{dE}{dt} = 0$$

(b) $$\dot{W}_{\text{in}} = FV; \qquad \Rightarrow \dot{W}_{\text{in}} = (0.02)(2); \qquad \Rightarrow \dot{W}_{\text{in}} = 40\ \text{W}$$

(c) The work done by the external force is transferred out of the system through frictional work. The frictional work is transferred to internal energy raising the temperature of the resisting material at the interface.

2-2-14 [NW] Do an energy analysis of a pendulum bob to show that the sum of its kinetic and potential energies remain constant. Assume internal energy to remain constant, neglect viscous friction and heat transfer. What-if Scenario: Discuss how the energy equation would be affected if viscous friction is not negligible.

SOLUTION

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \cancel{\frac{dU}{dt}}^{0} + \frac{d}{dt}(\text{KE}) + \frac{d}{dt}(\text{PE}) = \cancel{\dot{Q}}^{0} - \cancel{\dot{W}_{\text{ext}}}^{0};$$

$$\Rightarrow \frac{d}{dt}(\text{KE+PE}) = 0;$$

$$\Rightarrow \text{KE+PE} = \text{constant}$$

What-if Scenario:
Work done to overcome viscous friction would be a transfer of work from the pendulum to the surroundings, that is, $\dot{W}_{\text{ext}}$ must be negative.
Therefore, the sum of KE and PE would decrease over time

2-2-15 [ER] A rigid insulated tank contains 2 kg of a gas at 300 K and 100 kPa. A 1 kW internal heater is turned on. (a) Determine the rate of change of total stored energy (dE/dt). (b) If the internal energy of the gas is related to the temperature by $u = 1.1T$ (kJ/kg), where T is in Kelvin, determine the rate of temperature increase.

SOLUTION

(a) With no mass or heat transfer

$$\frac{dE}{dt} = -\dot{W}_{ext}; \quad \Rightarrow \frac{dE}{dt} = -\left(-\dot{W}_{\text{el,in}}\right); \quad \Rightarrow \frac{dE}{dt} = -\left(-1\ \text{kW}\right); \quad \Rightarrow \frac{dE}{dt} = 1\ \text{kW}$$

(b) With no change in KE or PE

$$\frac{dE}{dt} = \frac{dU}{dt} = 1\ \text{kW};$$

$$\Rightarrow \frac{dU}{dt} = \frac{d\left(mu\right)}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{du}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{d\left(1.1T\right)}{dt}; \quad \Rightarrow \frac{dU}{dt} = \left(1.1\right)m\frac{dT}{dt};$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{\left(1.1\right)m}\frac{dU}{dt}; \quad \Rightarrow \frac{dT}{dt} = \frac{1}{\left(1.1\right)\left(2\right)}\left(1\right); \quad \Rightarrow \frac{dT}{dt} = 0.455\ \frac{\text{K}}{\text{s}}$$

2-2-16 [EO] A 10 m^3 rigid tank contains air at 200 kPa and 150^{o}C. A 1 kW internal heater is turned on. Determine the rate of change of (a) stored energy (b) temperature and (c) pressure of air in the tank. Use the IG system state daemon. (Hint: Evaluate state-2 with stored energy incremented by the amount added in a small time interval, say, 0.1 s)

TEST Solution:

Let us evaluate two neighboring states, State-1 for the initial state and State-2 after a small interval, say 0.1 s.

Launch the IG system-state TESTcalc. Select air and evaluate State-1 from the given information. From State-2, m2=m1, Vol2=Vol1 (rigid tank). The energy equation for this closed system provides the missing property u2=u1+0.1/m1.

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dU}{dt} + \cancel{\frac{d}{dt}(\text{KE})}^{0} + \cancel{\frac{d}{dt}(\text{PE})}^{0} = \cancel{\dot{Q}}^{0} - \left(-\dot{W}_{\text{el,in}}\right); \ [\text{kW}]$$

$$\Rightarrow dU = \dot{W}_{\text{el,in}}dt; \ [\text{kJ}]$$

$$\Rightarrow mdu = \dot{W}_{\text{el,in}}dt;$$

$$\Rightarrow m\Delta u = \dot{W}_{\text{el,in}}\Delta t;$$

$$\Rightarrow u_2 - u_1 = \frac{\dot{W}_{\text{el,in}}\Delta t}{m}; \ \left[\frac{\text{kJ}}{\text{kg}}\right]$$

$$\Rightarrow u_2 = u_1 + \frac{\dot{W}_{\text{el,in}}\Delta t}{m};$$

$$\Rightarrow u_2 = u_1 + \frac{0.1}{m_1};$$

(a) In the I/O panel, evaluate =m1*(e2-e1)/0.1 = 1 kW

(b) In the I/O panel, evaluate =m1*(T2-T1)/0.1 = $0.0824 \ \frac{\text{K}}{\text{s}}$

(c) In the I/O panel, evaluate =m1*(p2-p1)/0.1 = $0.0389 \ \frac{\text{kPa}}{\text{s}}$

2-2-17 [EB] A 10 m^3 rigid tank contains steam with a quality of 0.5 at 200 kPa. A 1 MW internal heater is turned on. Determine the rate of change of (a) stored energy, (b) temperature and (c) pressure of steam in the tank. Use the PC system state daemon. (Hint: Evaluate state-2 with stored energy incremented by the amount added in a small time interval, say, 0.1 s)

TEST Solution:

Let us evaluate two neighboring states, State-1 for the initial state and State-2 after a small interval, say 0.1 s.

Launch the PC system-state TESTcalc. Select H2O and evaluate State-1 from the given information. From State-2, m2=m1, Vol2=Vol1 (rigid tank). The energy equation for this closed system provides the missing property u2=u1+0.1*1000/m1.

$$\frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dU}{dt} + \cancelto{0}{\frac{d}{dt}(\text{KE})} + \cancelto{0}{\frac{d}{dt}(\text{PE})} = \cancelto{0}{\dot{Q}} - \left(-\dot{W}_{\text{el,in}}\right); \; [\text{kW}]$$

$$\Rightarrow dU = \dot{W}_{\text{el,in}} dt; \; [\text{kJ}]$$

$$\Rightarrow m du = \dot{W}_{\text{el,in}} dt;$$

$$\Rightarrow m\Delta u = \dot{W}_{\text{el,in}} \Delta t;$$

$$\Rightarrow u_2 - u_1 = \frac{\dot{W}_{\text{el,in}} \Delta t}{m}; \; \left[\frac{\text{kJ}}{\text{kg}}\right]$$

$$\Rightarrow u_2 = u_1 + \frac{\dot{W}_{\text{el,in}} \Delta t}{m} = u_1 + \frac{0.1}{m_1};$$

(a) In the I/O panel, evaluate =m1*(e2-e1)/0.1 = 1000 kW

(b) In the I/O panel, evaluate =m1*(T2-T1)/0.1 = $1.1 \frac{\text{K}}{\text{s}}$

(c) In the I/O panel, evaluate =m1*(p2-p1)/0.1 = $5.16 \frac{\text{kPa}}{\text{s}}$

2-2-18 [ES] A piston-cylinder device containing air at 200 kPa loses heat at a rate of 0.5 kW to the surrounding atmosphere. At a given instant, the piston which has a cross-sectional area of 0.01 m^2 moves down with a velocity of 1 cm/s. Determine the rate of change of stored energy in the gas.

SOLUTION

$$\frac{dE}{dt} = \underbrace{\sum \dot{m}_i j_i}_{0} - \underbrace{\sum \dot{m}_e j_e}_{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - \left(-\dot{W}_{\text{B,in}}\right);$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} + p_i \frac{d\forall}{dt};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} + p_i A \frac{dx}{dt};$$

$$\Rightarrow \frac{dE}{dt} = -0.5 + (200)(0.01)(0.01);$$

$$\Rightarrow \frac{dE}{dt} = -0.48 \text{ kW}$$

2-2-19 [EA] A piston-cylinder device contains a gas, which is heated at a rate of 0.5 kW from an external source. At a given instant the piston, which has an area of 10 cm^2 moves up with a velocity of 1 cm/s. (a) Determine the rate of change of stored energy (dE/dt) in the gas. Assume atmospheric pressure to be 101 kPa and the piston to be weightless. Also, neglect friction. (b) What-if Scenario: How would the answer change if the piston were locked in its original position with a pin.

SOLUTION

(a) Since the piston is weightless, $p_i = p_0 = 101$ kPa

$$\frac{dE}{dt} = \underbrace{\sum \dot{m}_i j_i}_{0} - \underbrace{\sum \dot{m}_e j_e}_{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{B}};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - p_o \frac{d\forall}{dt};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - p_0 A \frac{dx}{dt};$$

$$\Rightarrow \frac{dE}{dt} = 0.5 - (101)(0.001)(0.01);$$

$$\Rightarrow \frac{dE}{dt} = 0.499 \text{ kW}$$

(b) If the piston were locked in its initial position, then there would be no boundary work done by the system.

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - \underbrace{\dot{W}_{\text{B}}}_{0};$$

$$\Rightarrow \frac{dE}{dt} = 0.5 - 0;$$

$$\Rightarrow \frac{dE}{dt} = 0.5 \text{ kW}$$

2-2-20 [EH] A gas trapped in a piston-cylinder device is heated (as shown in figure of problem 2-2-19 [EA]) from an initial temperature of 300 K. The initial load on the massless piston of area 0.2 m^2 is such that the initial pressure of the gas is 200 kPa. When the temperature of the gas reaches 600 K, the piston velocity is measured as 0.5 m/s and dE/dt is measured as 30 kW. At that instant, determine

(a) the rate of external work transfer,

(b)the rate of heat transfer, and

(c) the load (in kg) on the piston at that instant. Assume the ambient atmospheric pressure to be 100 kPa.

SOLUTION

(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{B}};\quad \Rightarrow \dot{W}_{\text{ext}} = pA\dfrac{dx}{dt};\quad \Rightarrow \dot{W}_{\text{ext}} = (200)(0.2)(0.5);\quad \Rightarrow \dot{W}_{\text{ext}} = 20\text{ kW}$

(b) $$\frac{dE}{dt} = \underbrace{\sum \dot{m}_i j_i}_{0} - \underbrace{\sum \dot{m}_e j_e}_{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dE}{dt} = \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = \frac{dE}{dt} + \dot{W}_{\text{ext}};\quad \Rightarrow \dot{Q} = 30 + 20;\quad \Rightarrow \dot{Q} = 50\text{ kW}$$

(c) Force balance

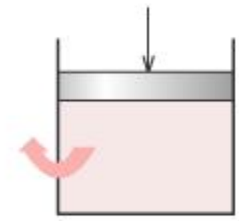
$$p_i A_p = p_o A_p + \frac{mg}{1000};\quad [\text{kPa}]$$

$$\Rightarrow m = \frac{(1000)A_p\left(p_i - p_o\right)}{g};\quad \Rightarrow m = \frac{0.2(200-100)(1000)}{9.81};\quad \Rightarrow m = 2038.74\text{ kg}$$

2-2-21 [EN] A piston-cylinder device is used to compress a gas by pushing the piston with an external force. During the compression process, heat is transferred out of the gas in such a manner that the stored energy in the gas remains unchanged. Also, the pressure is found to be inversely proportional to the volume of the gas. (a) Determine an expression for the heat transfer rate in terms of the instantaneous volume and the rate of change of pressure of the gas. (b) At a given instant, when the gas occupies a volume of 0.1 m3, the rate of change of pressure is found to be 1 kPa/s. Determine the rate of heat transfer.

SOLUTION

(a) $$\underset{0}{\frac{dE}{dt}} = \underbrace{\sum \dot{m}_i j_i}_{0} - \underbrace{\sum \dot{m}_e j_e}_{0} + \dot{Q} - \dot{W}_{\text{ext}};$$



$$\Rightarrow \dot{Q} = \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = \dot{W}_{\text{B}};$$

$$\Rightarrow \dot{Q} = p_i \frac{d\mathcal{V}}{dt};$$

Given $p_i \propto \frac{1}{\mathcal{V}}$; $\Rightarrow p_i \mathcal{V} = \text{c};$

$$\Rightarrow \dot{Q} = p_i \frac{d\mathcal{V}}{dt}; \qquad \Rightarrow \dot{Q} = p_i \frac{d\mathcal{V}}{dt} + \mathcal{V}\frac{dP_i}{dt} - \mathcal{V}\frac{dP_i}{dt};$$

$$\Rightarrow \dot{Q} = \underbrace{\frac{d}{dt}\left(p_i \mathcal{V}\right)}_{0 \text{ because } p_i\mathcal{V} \text{ is constant}} - \mathcal{V}\frac{dP_i}{dt};$$

$$\Rightarrow \dot{Q} = -\mathcal{V}\frac{dp_i}{dt}$$

(b) $$\dot{Q} = -\mathcal{V}\frac{dp_i}{dt}; \qquad \Rightarrow \dot{Q} = -(0.1)(1); \qquad \Rightarrow \dot{Q} = -0.1 \text{ kW}$$

2-2-22 [EE] A fluid flows steadily through a long insulated pipeline. Perform a mass and energy analysis to show that the flow energy *j* remains unchanged between the inlet and exit. What-if Scenario: How would this conclusion be modified if kinetic and potential energy changes were negligible?

SOLUTION

(a) At steady state, the mass balance equation is simplified as

$$\frac{dm}{dt}^{\cancel{}0,\text{steady state}} = \sum \dot{m}_i - \sum \dot{m}_e; \qquad \Rightarrow \dot{m}_i = \dot{m}_e;$$

With no external work or heat transfer,

$$\frac{dE}{dt}^{0,\text{steady state}} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \cancel{\dot{Q}}^0 - \cancel{\dot{W}_{\text{ext}}}^0;$$

$$\Rightarrow \cancel{\dot{m}_i} j_i = \cancel{\dot{m}_e} j_e;$$

$$\Rightarrow j_i = j_e$$

(b) With no negligible changes in kinetic or potential energy, the energy equation can be further simplified as:

$$j_i = j_e;$$

$$\Rightarrow h_i + \text{ke}_i + \text{pe}_i = h_e + \text{ke}_e + \text{pe}_e;$$

$$\Rightarrow h_i = h_e + \cancel{\Delta(\text{ke})}^0 + \cancel{\Delta(\text{pe})}^0;$$

$$\Rightarrow h_i = h_e$$

2-2-23 [EI] Water enters a constant-diameter, insulated, horizontal pipe at 500 kPa. Due to the presence of viscous friction the pressure drops to 400 kPa at the exit. At steady state, determine the changes in (a) specific kinetic energy (ke) and (b) specific internal energy (*u*) between the inlet and the exit. Assume water density to be 1000 kg/m^3. Use the SL flow state daemon to verify the answer.

SOLUTION

(a) At steady state

$$\dot{m}_i = \dot{m}_e = \dot{m};$$
$$\Rightarrow \rho A V_i = \rho A V_e;$$
$$\Rightarrow V_i = V_e;$$
$$\Delta \text{ke} = \text{ke}_e - \text{ke}_i; \qquad \Rightarrow \Delta \text{ke} = \frac{V_e^2}{2000} - \frac{V_i^2}{2000}; \qquad \Rightarrow \Delta \text{ke} = 0$$

(b) With no external work or heat transfer,

$$\underbrace{\frac{dE}{dt}}_{0} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e \underbrace{+\dot{Q} - \dot{W}_{\text{ext}}}_{0};$$
$$\Rightarrow \dot{m} j_i = \dot{m} j_e; \qquad \Rightarrow j_i = j_e;$$
$$\Rightarrow h_i + \text{ke}_i + \text{pe}_i = h_e + \text{ke}_e + \text{pe}_e;$$
$$\Rightarrow (u_i + p_i v) + ke_i + pe_i = (u_e + p_e v) + ke_e + pe_e;$$
$$\Rightarrow (u_i + p_i v) = (u_e + p_e v);$$
$$\Rightarrow u_e - u_i = p_i v - p_e v;$$
$$\Rightarrow \Delta u = p_i v - p_e v;$$
$$\Rightarrow \Delta u = (500)\left(\frac{1}{1000}\right) - (400)\left(\frac{1}{1000}\right); \qquad \Rightarrow \Delta u = 0.1 \ \frac{\text{kJ}}{\text{kg}}$$

TEST Solution:

Launch the SL flow state TESTCalc. Evaluate the inlet state, State-1, assigning an arbitrary temperature, area, and mass flow rate. For the exit, State-2, use mdot2=mdot1, A2=A1, j2=j1 and p2 = 400 kPa. The velocities in the two states must be set as unknown and calculated as part of the states. Verify the answer by evaluating the expression = u2-u1 in the I/O panel. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-24 [EL] Water flows steadily through a variable diameter insulated pipe. At the inlet, the velocity is 20 m/s and at the exit, the flow area is half of the inlet area. If the internal energy of water remains constant, determine the change in pressure between the inlet and exit. Assume water density to be 1000 kg/m^3.

SOLUTION

At steady state

$\dot{m}_i = \dot{m}_e = \dot{m};$

$$\Rightarrow \rho A V_i = \rho\left(\frac{1}{2}A\right)V_e; \qquad \Rightarrow V_i = \frac{1}{2}V_e; \qquad \Rightarrow V_e = 40\ \frac{\text{m}}{\text{s}};$$

At steady state, and with no external work or heat transfer,

$$\underbrace{\frac{dE}{dt}}_{0} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e \underbrace{+\dot{Q} - \dot{W}}_{0};$$

$$\Rightarrow \dot{m} j_i = \dot{m} j_e; \qquad \Rightarrow j_i = j_e;$$

$$\Rightarrow (u_i + p_i v) + ke_i + pe_i = (u_e + p_e v) + ke_e + pe_e;$$

With no change in potential energy,

$$\Rightarrow (u_i + p_i v) + ke_i = (u_e + p_e v) + ke_e;$$

With no change in internal energy,

$$\Rightarrow p_i v + \frac{V_i^2}{2000} = + p_e v + \frac{V_e^2}{2000}; \qquad \Rightarrow p_e - p_i = \frac{\left(\frac{V_i^2}{2000} - \frac{V_e^2}{2000}\right)}{v};$$

$$\Rightarrow \Delta p = \frac{\left(\frac{(20)^2}{2000} - \frac{(40)^2}{2000}\right)}{\left(\frac{1}{1000}\right)}; \Rightarrow \Delta p = -600\ \text{kPa}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state, State-1, from the given conditions and assigning an arbitrary temperature and area. For the exit, State-2, use mdot2=mdot1, A2=A1, j2=j1 and u2 = u1. The velocity at State-2 must be set as unknown to be calculated. Verify the answer by evaluating the expression = p2-p1 in the I/O panel. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-25 [EG] Oil enters a long insulated pipe at 200 kPa and 20 m/s. It exits at 175 kPa. Assuming steady flow, determine the changes in the following properties between the inlet and exit (a) *j*, (b) ke and (c) *h*. Assume oil density to be constant.

SOLUTION

(a) At steady state with no heat (insulated pipe) or external work transfer, the energy equation simplifies to:

$$\frac{dE}{dt} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \dot{Q} - \dot{W};$$

$$0 = \dot{m}_i j_i - \dot{m}_e j_e + 0 - 0;$$

$$\Rightarrow \dot{m} j_i = \dot{m} j_e;$$

$$j_i = j_e; \quad \Rightarrow \Delta j = 0$$

(b) $\dot{m}_i = \dot{m}_e;$

$$\Rightarrow \rho A V_i = \rho A V_e;$$

$$\Rightarrow V_i = V_e;$$

$$\Delta \text{ke} = \text{ke}_e - \text{ke}_i; \quad \Rightarrow \Delta \text{ke} = \frac{V_e^2}{2000} - \frac{V_i^2}{2000}; \quad \Rightarrow \Delta \text{ke} = 0$$

(c) $\Delta j = (\Delta h + \Delta ke + \Delta pe); \quad \Rightarrow 0 = (\Delta h + 0 + 0);$

$$\Delta h = 0$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state, State-1, from the given conditions and assigning an arbitrary temperature and area. For the exit, State-2, use mdot2=mdot1, A2=A1, j2=j1 and p2 = 175 kPa. Note that while enthalpy remains unchanged the temperature (and internal energy) increases slightly. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-26 [EZ] Nitrogen gas flows steadily through a pipe of diameter 10 cm. The inlet conditions are as follows: pressure 400 kPa, temperature 300 K and velocity 20 m/s. At the exit the pressure is 350 kPa (due to frictional losses). If the flow rate of mass and flow energy remain constant, determine (a) the exit temperature and (b) exit velocity. Use the IG (ideal gas) flow state daemon. (c) What-if Scenario: What would the exit temperature be if kinetic energy were neglected?

TEST Solution:

Launch the IG flow-state TESTCalc. Evaluate the inlet state, State-1, from the given conditions. For the exit, State-2, use mdot2=mdot1, A2=A1, j2=j1 and the given pressure. For State-3 (what-if scenario), use h3=h1 instead of j3=j1. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

(a) T2 = 299.9 K

(b) Vel2 = $22.85\ \frac{m}{s}$

(c) T3 = 300 K

2-2-27 [EK] A 5 cm diameter pipe discharges water into the open atmosphere at a rate of 20 kg/s at an elevation of 20 m. The temperature of water is 25°C and the atmospheric pressure is 100 kPa. Determine (a) $\dot{J}$, (b) $\dot{E}$, (c) $\dot{KE}$, (d) $\dot{H}$ and (e) $\dot{W}_F$. (f) How important is the flow work transfer compared to kinetic and potential energy carried by the mass? Use the SL flow state daemon.

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the exit state from the given conditions. Calculate the desired quantities in the I/O panel. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

(a) mdot1*j1 = -317319.5 kW

(b) mdot1*el = -317321.5 kW

(c) mdot1*(Vel1*Vel1)/2000 = 1.0437 kW

(d) mdot1*h1 = -317324.47 kW

(e) p1*A1*Vel1 = 2.0 kW

The negative values for the energy transport terms are due to the way zero energy is defined in TEST. Just like potential energy can be negative, energy can be negative in a relative scale.

2-2-28 [EP] Water enters a pipe at 90 kPa, 25°C and a velocity of 10 m/s. At the exit the pressure is 500 kPa and velocity is 12 m/s while the temperature remains unchanged. If the volume flow rate is 10 m^3/min both at the inlet and exit, determine the difference of flow rate of energy (J) between the exit and inlet. What-if Scenario: What would the answer if the exit velocity were 15 m/s instead?

TEST Solution:
Launch the SL flow-state TESTCalc. Evaluate the inlet and exit states from the given conditions. Calculate the desired quantities in the I/O panel. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

(a) mdot2*(j2-j1) = 72 kW

(b) mdot2*(j2-j1) = 78.7 kW

The negative values for the energy transport terms are due to the way zero energy is defined in TEST. Just like potential energy can be negative, energy can be negative in a relative scale.

2-2-29 [EU] Water at 1000 kPa, 25°C enters a 1-m-diameter horizontal pipe with a steady velocity of 10 m/s. At the exit the pressure drops to 950 kPa due to viscous resistance. Assuming steady-state flow, determine the rate of heat transfer ($\dot{Q}$) necessary to maintain a constant specific internal energy (u).

SOLUTION

At steady state, the mass balance equation is simplified as

$$\frac{dm}{dt} = \sum \dot{m}_i - \sum \dot{m}_e;$$

$$\Rightarrow \dot{m}_i = \dot{m}_e = \dot{m}; \qquad \Rightarrow \rho A V_i = \rho A V_e;$$

$$\Rightarrow V_i = V_e;$$

$$\Rightarrow \dot{m} = (1000)\left(\frac{\pi(1)^2}{4}\right)(10); \qquad \Rightarrow \dot{m} = 7854\ \frac{\text{kg}}{\text{s}};$$

$$\frac{dE}{dt} = \sum \dot{m} j_i - \sum \dot{m} j_e + \dot{Q} - \dot{W}_{\text{ext}};$$

With no external work, no change in ke or pe, at steady state,

$$\Rightarrow \dot{Q} = \dot{m}\left(j_e - j_i\right);$$

$$\Rightarrow \dot{Q} = \dot{m}\left[\left(u_e + p_e v + \text{ke}_e + \text{pe}_e\right) - \left(u_i + p_i v + \text{ke}_i + \text{pe}_i\right)\right];$$

$$\Rightarrow \dot{Q} = \dot{m}\left[\left(u_e - u_i\right) + p_e v - p_i v\right];$$

Since $u_e - u_i = \Delta u = 0$,

$$\Rightarrow \dot{Q} = \dot{m} v \left(p_e - p_i\right);$$

$$\Rightarrow \dot{Q} = (7854)\left(\frac{1}{1000}\right)(950 - 1000); \qquad \Rightarrow \dot{Q} = -392.7\ \text{kW}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet and exit states from the given conditions. Calculate =mdot1*(j2-j1) in the I/O panel to verify the manual result. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

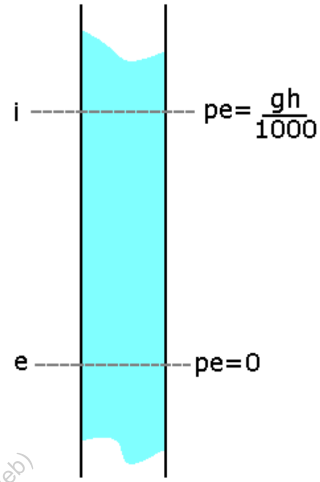
2-2-30 [EX] An incompressible fluid (constant density) flows steadily downward along a constant-diameter, insulated vertical pipe. Assuming internal energy remains constant, show that the pressure variation is hydrostatic.

SOLUTION

SOLUTION

$$\frac{dm}{dt} = \sum \dot{m}_i - \sum \dot{m}_e; \qquad \Rightarrow \dot{m}_i = \dot{m}_e = \dot{m};$$

$$\Rightarrow \rho A V_i = \rho A V_e; \qquad \Rightarrow V_i = V_e; \qquad \Rightarrow \Delta \mathrm{ke} = 0;$$



Steady state with no external work or heat transfer,

$$\underset{0}{\frac{dE}{dt}} = \sum \dot{m} j_i - \sum \dot{m} j_e \underbrace{+ \dot{Q} - \dot{W}}_{0}; \quad [\mathrm{kW}]$$

$$\Rightarrow \cancel{\dot{m}} j_i = \cancel{\dot{m}} j_e;$$

$$\Rightarrow j_i = j_e; \quad \left[\frac{\mathrm{kJ}}{\mathrm{kg}}\right]$$

$$\Rightarrow u_i + p_i v + \mathrm{pe}_i = u_e + p_e v + \mathrm{pe}_e;$$

$$\Rightarrow (p_e - p_i) v = \cancel{(u_i - u_e)}^{0} + (\mathrm{pe}_i - \mathrm{pe}_e);$$

$$\Rightarrow (p_e - p_i) v = \frac{g(z_i - z_e)}{1000};$$

$$\Rightarrow (p_e - p_i) = \frac{\rho g (z_i - z_e)}{1000}; \quad [\mathrm{kPa}]$$

The pressure variation is hydrostatic even in the presence of the flow. Note that the answer is independent of the direction of the flow.

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate an inlet state with arbitrary properties, say, 1000 kPa, 25 deg-C, 20 m/s, 50 m, and 0.1 m^2. At the exit, mdot1=mdot2, j2=j1, A2=A1. Let us assume z2=25 m. In the I/O panel, calculate the hydrostatic pressure difference 9.81*(z1-z2)*rho1/1000. It is exactly equal to p2-p1. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-31 [EC] An incompressible fluid (constant density) flows steadily through a converging nozzle. (a) Show that the specific flow energy remains constant if the nozzle is adiabatic. (b) Assuming internal energy remains constant and neglecting the inlet kinetic energy, obtain an expression for the exit velocity in terms of pressures at the inlet and exit and the fluid density. (c) For an inlet pressure of 300 kPa and exit pressure of 100 kPa, determine the exit velocity for a water nozzle.

SOLUTION

(a) $$\underbrace{\frac{dE}{dt}}_{0} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \underbrace{\dot{Q}}_{0,\,Adiabadic} - \underbrace{\dot{W}_{\text{ext}}}_{0};$$

$$\Rightarrow \dot{m}_i j_i = \dot{m}_e j_e;$$

At steady state, $\dot{m}_i = \dot{m}_e = \dot{m}$,

$$\Rightarrow \dot{m} j_i = \dot{m} j_e;$$

$$\Rightarrow j_i = j_e$$

(b) With no changes in pe, constant v, and by negligible ke_i

$$\Rightarrow j_i = j_e; \qquad \Rightarrow h_i + \text{ke}_i = h_e + \text{ke}_e;$$

$$\Rightarrow u_i + p_i v = u_e + p_e v + \text{ke}_e;$$

$$\Rightarrow p_i v = (u_e - u_i) + p_e v + \frac{(V_e)^2}{2000};$$

Since $u_e - u_i = \Delta u = 0$,

$$\Rightarrow V_e = \sqrt{(2000)v(p_i - p_e)}$$

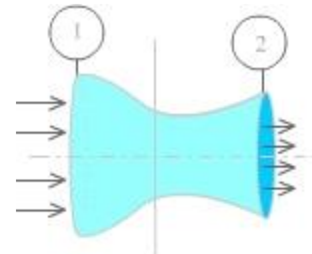
(c) $$V_e = \sqrt{(2000)v(p_i - p_e)}; \quad \Rightarrow V_e = \sqrt{(2000)\left(\frac{1}{1000}\right)(300-100)}; \qquad \Rightarrow V_e = 20\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state for water for the given pressure and arbitrarily assumed mdot1, T1, and a very small Vel1 (say 0.1 m/s). At the exit, set u2 = u1, mdot2=mdot1, j2=j1, A2=A1. To see the effect of inlet ke on exit velocity, change Vel1 and click the Super-Calculate button. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-32 [EV] Water flows steadily through an insulated nozzle. The following data is supplied: Inlet: p = 200 kPa, V = 10 m/s, z = 2 m; Exit: p = 100 kPa, z = 0. (a) Determine the exit velocity. Assume density of water to be 1000 kg/m^3. Also assume the internal energy remains constant. What-if Scenario: What would the exit velocity be if (b) change in potential energy or (c) inlet kinetic energy were neglected in the analysis?

SOLUTION



(a) $$\underset{0}{\frac{dE}{dt}} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \underset{0,\,Adiabadic}{\dot{Q}} - \underset{0}{\dot{W}_{\text{ext}}};$$

$$\Rightarrow \dot{m}_i j_i = \dot{m}_e j_e;$$

At steady state, $\dot{m}_i = \dot{m}_e = \dot{m}$,

$$\Rightarrow \not{\dot{m}} j_i = \not{\dot{m}} j_e; \qquad \Rightarrow j_i = j_e;$$

$$\Rightarrow h_i + \text{ke}_i + \text{pe}_i = h_e + \text{ke}_e + \text{pe}_e;$$

$$\Rightarrow (u_i + p_i v) + \text{ke}_i + \text{pe}_i = (u_e + p_i v) + \text{ke}_e + \text{pe}_e;$$

Since $u_e - u_i = \Delta u = 0$,

$$\Rightarrow \text{ke}_e = \frac{(V_e)^2}{2000} = p_i v + \text{ke}_i + \text{pe}_i - p_i v - \text{pe}_e;$$

$$\Rightarrow V_e = \sqrt{(2000)(p_i v + \text{ke}_i + \text{pe}_i - p_i v - \text{pe}_e)};$$

$$\Rightarrow V_e = \sqrt{(2000)\left(\frac{200}{1000} + \frac{(10)^2}{2000} + \frac{(9.81)(2)}{1000} - \frac{100}{1000} - \frac{(9.81)(0)}{1000}\right)}; \qquad \Rightarrow V_e = 18.42\ \frac{\text{m}}{\text{s}}$$

(b) $$V_e = \sqrt{(2000)\left(p_i v + \text{ke}_i - \underset{0}{\Delta\text{pe}} - p_i v\right)}; \qquad \Rightarrow v_e = \sqrt{(2000)(p_i v + \text{ke}_i - p_i v)};$$

$$\Rightarrow v_e = \sqrt{(2000)\left(\frac{200}{1000} + \frac{(10)^2}{2000} - \frac{100}{1000}\right)}; \qquad \Rightarrow v_e = 17.32\ \frac{\text{m}}{\text{s}}$$

(c) $$v_e = \sqrt{(2000)\left(p_i v + \underset{0}{\text{ke}_i} + pe_i - p_i v - \text{pe}_e\right)};$$

$$\Rightarrow v_e = \sqrt{(2000)\left(\frac{200}{1000} + \frac{(9.81)(2)}{1000} - \frac{100}{1000} - \frac{(9.81)(0)}{1000}\right)};$$

$$\Rightarrow v_e = 15.47\ \frac{\text{m}}{\text{s}}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state for water for the given pressure and arbitrarily assumed mdot1 and T1. At the exit, set u2 = u1, mdot2=mdot1, j2=j1, A2=A1. To see the effect of inlet ke on exit pe, change Vel1 or z1 to a small value and click the Super-Calculate button. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-33 [BNW] Water flowing steadily through a 2 cm diameter pipe at 30 m/s goes through an expansion joint to flow through a 4 cm diameter pipe. Assuming the internal energy remains constant, determine (a) the change in pressure as the water goes through the transition. (b) Also determine the displacement of a mercury column in mm for this change in pressure. Assume water density to be 997 kg/m^3.

SOLUTION

(a) $A_i = \pi \frac{(0.02)^2}{4}; \quad \Rightarrow A_i = 0.000314 \text{ m}^2;$

$A_e = \pi \frac{(0.02)^2}{4}; \quad \Rightarrow A_e = 0.001257 \text{ m}^2;$

At steady state $\dot{m}_i = \dot{m}_e;$

$$\Rightarrow \rho A_i V_i = \rho A_e V_e;$$

$$\Rightarrow V_e = \frac{\rho A_i}{\rho A_e} V_i; \quad \Rightarrow V_e = \frac{A_i}{A_e} V_i; \quad \Rightarrow V_e = \frac{0.000314}{0.001257}(30); \quad \Rightarrow V_e = 7.5 \frac{\text{m}}{\text{s}};$$

At steady state, and with no external work or heat transfer,

$$\underbrace{\frac{dE}{dt}}_{0} = \sum \dot{m} j_i - \sum \dot{m} j_e \underbrace{+\dot{Q} - \dot{W}}_{0};$$

$$\Rightarrow \dot{m} j_i = \dot{m} j_e; \quad \Rightarrow j_i = j_e; \quad \Rightarrow h_i + \text{ke}_i + \text{pe}_i = h_e + \text{ke}_e + \text{pe}_e;$$

With $\Delta u = 0$ and $\Delta \text{pe} = 0$, the specific flow energy balance equation becomes

$$\Rightarrow p_i v + \text{ke}_i = p_e v + \text{ke}_e;$$

$$\Rightarrow p_e - p_i = \frac{(\text{ke}_i - \text{ke}_e)}{v};$$

$$\Rightarrow \Delta p = \rho \left(\frac{(V_i)^2}{2000} - \frac{(V_e)^2}{2000} \right);$$

$$\Rightarrow \Delta p = (997) \left(\frac{(30)^2}{2000} - \frac{(7.5)^2}{2000} \right); \quad \Rightarrow \Delta p = 420.61 \text{ kPa}$$

(b) Using $1 \text{ kPa} \approx 7.5 \text{ mmHg}$

$$\Delta p_{\text{mmHg}} = (420.61 \text{ kPa})\left(7.5 \frac{\text{mmHg}}{\text{kPa}}\right); \quad \Rightarrow \Delta p_{\text{mmHg}} = 3154.58 \text{ mmHg}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state for water for the given velocity and flow area and arbitrarily assumed p1 and T1 (100 kPa and 25 °C). At the exit, set u2 = u1, mdot2=mdot1, j2=j1, and known area. The increase in pressure can be calculated in the I/O panel from =p2-p1. Verify that the answer is independent of the inlet pressure or temperature. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-34 [ET] An adiabatic work producing device works at steady state with the working fluid entering through a single inlet and leaving through a single exit. Derive an expression for the work output in terms of the flow properties at the inlet and exit. What-if Scenario: How would the expression for work simplify if changes in ke and pe were neglected?

SOLUTION

(a) The energy balance equation is given as

$$\underbrace{\frac{dE}{dt}}_{0,\text{Steady State}} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \underbrace{\dot{Q}}_{0,\text{Adiabadic}} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}_i j_i - \dot{m}_e j_e;$$

At steady state, $\dot{m}_i = \dot{m}_e = \dot{m},$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(j_i - j_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(u_i + p_i v + pe_i + ke_i\right) - \left(u_e + p_e v + pe_e + ke_e\right)\right]$$

(b) What-if Scenario: With no changes in *ke* or *pe*,

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \text{ke}_i + \text{pe}_i\right) - \left(h_e + \text{ke}_e + \text{pe}_e\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i - h_e\right) - \Delta\text{ke} - \Delta\text{pe}\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(h_i - h_e\right)$$

2-2-35 [EY] A pump is a device that raises the pressure of a liquid at the expense of external work. (a) Determine the pumping power necessary to raise the pressure of liquid water from 10 kPa to 2000 kPa at a flow rate of 1000 L/min. Assume density of water to be 1000 kg/m3 and neglect changes in specific internal, kinetic and potential energies. (b) What-if Scenario: What would the pumping power be if pe were not negligible and z_1 = -10, z_2 = 0?

SOLUTION

(a) At steady state, $\dot{m}_i = \dot{m}_e = \dot{m}$,

$$\dot{m} = \left(1000\ \frac{\text{L}}{\text{min}}\right)\left(0.001\ \frac{\text{m}^3}{\text{L}}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1}{60}\ \frac{\text{min}}{\text{s}}\right); \qquad \Rightarrow \dot{m} = 16.\bar{6}\ \frac{\text{kg}}{\text{s}};$$

$$\underset{\text{0,Steady State}}{\frac{dE}{dt}} = \sum \dot{m}j_i - \sum \dot{m}j_e + \underset{0}{\dot{Q}} - \dot{W}_{\text{ext}};$$

The energy equation becomes

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(j_i - j_e\right);$$

By neglecting changes in ke and pe,

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(h_i - h_e\right); \qquad \Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(u_i + p_i v\right) - \left(u_e + p_e v\right)\right];$$

By neglecting changes in specific internal energy

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}v\left(p_i - p_e\right); \qquad \Rightarrow \dot{W}_{\text{ext}} = \frac{\dot{m}}{\rho}\left(p_i - p_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = \frac{16.\bar{6}}{1000}\left(10 - 2000\right); \qquad \Rightarrow \dot{W}_{\text{ext}} = -33.17\ \text{kW};$$

The negative sign implies that the work is transferred into the system. The pumping power is the magnitude of that work transfer: $\dot{W}_{\text{P}} = -\dot{W}_{\text{ext}} = 33.17\ \text{kW}$.

(b) By including the changes in pe, the expression for work becomes

$$\dot{W}_{\text{ext}} = \dot{m}\left[\left(p_i v + \frac{gz_i}{1000}\right) - \left(p_e v + \frac{gz_e}{1000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = \left(16.\bar{6}\right)\left[\left(\frac{10}{1000} + \frac{(9.81)(-10)}{1000}\right) - \left(\frac{2000}{1000} + \frac{(9.81)(0)}{1000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = -34.8\ \text{kW};$$

$$\Rightarrow \dot{W}_{\text{pump}} = 34.8\ \text{kW}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state for water for the given volume flow rate, pressure and an arbitrarily assumed T1 (25 deg-C). At the exit, set p2, u2 = u1, mdot2=mdot1, and let Vel2 and z2 be at their default zero values. Calculate the pumping power in the I/O panel from = mdot1*(j2-j1). For the second part, change z2, calculate State-2 again, and evaluate the pumping power in the I/O panel. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-36 [EF] An adiabatic pump working at steady state raises the pressure of water from 100 kPa to 1 MPa, while the specific internal energy (u) remains constant. If the exit is 10 m above the inlet and the flow rate of water is 100 kg/s, (a) determine the pumping power. Neglect any change in kinetic energy (ke). Assume density of water to be 997 kg/m^3. (b) What-if Scenario: What would the pumping power be if any change in potential energy (pe) were also neglected?

SOLUTION

(a) At steady state, $\dot{m}_i = \dot{m}_e = \dot{m} = 100 \ \frac{kg}{s}$;

$$\underbrace{\frac{dE}{dt}}_{0,\text{Steady State}} = \sum \dot{m} j_i - \sum \dot{m} j_e + \underbrace{\dot{Q}}_{0} - \dot{W}_{\text{ext}};$$

The energy equation becomes

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}(j_i - j_e); \qquad \Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[(u_i + p_i v + ke_i + pe_i) - (u_e + p_e v + ke_e + pe_e)\right];$$

By neglecting changes in ke,

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[(u_i + p_i v + \text{pe}_i) - (u_e + p_e v + \text{pe}_e)\right]; \qquad \Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[(p_i v + \text{pe}_i) - (p_e v + \text{pe}_e)\right];$$

By choosing to reference pe from state e,

$$\Rightarrow \dot{W}_{\text{ext}} = (100)\left[\left(\frac{100}{997} + \frac{(9.81)(0)}{1000}\right) - \left(\frac{1000}{997} + \frac{(9.81)(10)}{1000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = -100.1 \text{ kW}; \text{ (WinHip: negative means work is going in)}$$

$$\Rightarrow \dot{W}_{\text{P}} = -\dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{P}} = 100.1 \text{ kW} \text{ (Pumping power is positive with an obvious direction of work transfer)}$$

(b) By neglecting changes in pe, the expression for $\dot{W}_{ext}$ becomes

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m} v (p_i - p_e); \qquad \Rightarrow \dot{W}_{\text{ext}} = \frac{\dot{m}}{\rho}(p_i - p_e);$$

$$\Rightarrow \dot{W}_{\text{ext}} = \frac{100}{997}(100 - 1000); \qquad \Rightarrow \dot{W}_{\text{P}} = -\dot{W}_{\text{ext}}; \qquad \Rightarrow \dot{W}_{\text{P}} = 90.3 \text{ kW}$$

TEST Solution:

Launch the SL flow-state TESTCalc. Evaluate the inlet state for water for the given volume flow rate, pressure and an arbitrarily assumed T1 (25 deg-C). At the exit, set p2, z2, u2 = u1, mdot2=mdot1, and let Vel2 be at its default zero value. Calculate the pumping power in the I/O panel from = mdot1*(j2-j1). For the second part, change z2, calculate State-2 again, and evaluate the pumping power in the I/O panel. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-37 [ED] Steam flows steadily through a single-flow device with a flow rate of 10 kg/s. It enters with an enthalpy of 3698 kJ/kg and a velocity of 30 m/s. At the exit, the corresponding values are 3368 kJ/kg and 20 m/s respectively. If the rate of heat loss from the device is measured as 100 kW, (a) determine the rate of work transfer. Neglect any change in potential energy. (b) What-if Scenario: What would the rate of work transfer be if the change in kinetic energy were also neglected?

SOLUTION

(a) At steady state, $\dot{m}_i = \dot{m}_e = \dot{m} = 10 \ \frac{\text{kg}}{\text{s}}$;

$$\underbrace{\frac{dE}{dt}}_{0,\text{Steady State}} = \sum \dot{m} j_i - \sum \dot{m} j_e + \dot{Q} - \dot{W}_{\text{ext}};$$

The energy equation becomes

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(j_i - j_e\right) + \dot{Q};$$

With no change in potential energy

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \text{ke}_i\right) - \left(h_e + \text{ke}_e\right)\right] + \dot{Q};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \frac{(V_i)^2}{2000}\right) - \left(h_e + \frac{(V_e)^2}{2000}\right)\right] + \dot{Q};$$

$$\Rightarrow \dot{W}_{\text{ext}} = (10)\left[\left(3698 + \frac{(30)^2}{2000}\right) - \left(3368 + \frac{(20)^2}{2000}\right)\right] - 100;$$

$$\Rightarrow \dot{W}_{\text{ext}} = 3202.5 \text{ kW}$$

(b) With no change in kinetic energy, the expression for $\dot{W}_{\text{ext}}$ becomes

$$\dot{W}_{\text{ext}} = \dot{m}\left(h_i - h_e\right) + \dot{Q};$$

$$\Rightarrow \dot{W}_{\text{ext}} = (10)\left(3698 - 3368\right) + 100; \qquad \Rightarrow \dot{W}_{\text{ext}} = 3200 \text{ kW}$$

TEST Solution:

Launch the PC flow-state TESTCalc. Evaluate the inlet and exit states, State-1 and State-2, from the given conditions. In the I/O panel, calculate the external work transfer as =mdot1*(j2-j1) - 100. The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-38 [EM] Steam enters an adiabatic turbine with a mass flow rate of 5 kg/s at 3 MPa, 600°C and 80 m/s. It exits the turbine at 40°C, 30 m/s and a quality of 0.9. Assuming steady-state operation, determine the shaft power produced by the turbine. Use the PC flow- state daemon to evaluate enthalpies at the inlet and exit.

SOLUTION

(a) At steady state, $\dot{m}_i = \dot{m}_e = \dot{m} = 5 \ \frac{\text{kg}}{\text{s}}$;

$$\underbrace{\frac{dE}{dt}}_{0,\text{Steady State}} = \sum \dot{m}j_i - \sum \dot{m}j_e + \cancel{\dot{Q}}^{0} - \dot{W}_{\text{ext}};$$

The energy equation becomes

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(j_i - j_e\right);$$

Neglecting any change in potential energy

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \text{ke}_i\right) - \left(h_e + \text{ke}_e\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \frac{(V_i)^2}{2000}\right) - \left(h_e + \frac{(V_e)^2}{2000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = (5)\left[\left(3682 + \frac{(80)^2}{2000}\right) - \left(2334 + \frac{(30)^2}{2000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = 6757 \text{ kW}$$

(b) With no change in kinetic energy, the expression for $\dot{W}_{\text{ext}}$ becomes

$$\dot{W}_{\text{ext}} = \dot{m}\left(h_i - h_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = (10)(3682 - 2334); \qquad \Rightarrow \dot{W}_{\text{ext}} = 6743.5 \text{ kW}$$

TEST Solution:

Launch the PC flow-state TESTCalc. Evaluate the inlet and exit states, State-1 and State-2, from the given conditions. In the I/O panel, calculate the external work transfer as =mdot1*(j2-j1). When ke is negligible, the external work is =mdot1*(h2-h1). The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

2-2-39 [EJ] A gas enters an adiabatic work consuming device at 300 K, 20 m/s, and leaves at 500 K, 40 m/s. (a) If the mass flow rate is 5 kg/s, determine the rate of work transfer. Neglect change in potential energy and assume the specific enthalpy of the gas to be related to its temperature in K through h = 1.005T. (b) What-if Scenario: By what percent would the answer change if the change in kinetic energy were also neglected?

SOLUTION

(a) At steady state, $\dot{m}_i = \dot{m}_e = \dot{m} = 5 \ \frac{\text{kg}}{\text{s}}$;

$$\underbrace{\frac{dE}{dt}}_{0,\text{Steady State}} = \sum \dot{m} j_i - \sum \dot{m} j_e + \underbrace{\dot{Q}}_{0,\text{ Adiabatic}} - \dot{W}_{\text{ext}};$$

The energy equation becomes

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(j_i - j_e\right);$$

With no change in potential energy

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \text{ke}_i\right) - \left(h_e + \text{ke}_e\right)\right];$$

Given: $h = 1.005T$;

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(1.005T_i + \frac{(V_i)^2}{2000}\right) - \left(1.005T_e + \frac{(V_e)^2}{2000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = (5)\left[\left(1.005(300) + \frac{(20)^2}{2000}\right) - \left(1.005(500) + \frac{(40)^2}{2000}\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = -1008 \text{ kW}$$

(b) With change in kinetic energy neglected,

$$\dot{W}_{\text{ext}} = \dot{m}\left(h_i - h_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(1.005T_i - 1.005T_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = (5)\left(1.005(300) - 1.005(500)\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = -1005 \text{ kW};$$

The percent change is given as:

$$\Rightarrow \frac{(-1008) - (-1005)}{-1008}(100) = 0.3\ \%$$

TEST Solution:

In the PG model (Chapter 3) enthalpy change is proportional to temperature change and the constant of proportionality is called c_p. Launch the PG flow-state TESTCalc and select 'Custom'. Evaluate the inlet and exit states, State-1 and State-2, from the given conditions and using c_p = 1.005 for each state. In the I/O panel, calculate the external work transfer as =mdot1*(j2-j1). When ke is negligible, the external work is

=mdot1*(h2-h1). The TEST-code for this problem can be found in the TEST-pro website at www.thermofliuds.net.

xxx2-2-40 [EW] A refrigerant is compressed by an adiabatic compressor operating at steady state to raise the pressure from 200 kPa to 750 kPa. The following data are supplied for the inlet and exit ports. Inlet: $v = 0.0835$ m^3/kg, $h = 182.1$ kJ/kg, $V = 30$ m/s; Exit: $v = 0.0244$ m^3/kg, $h = 205.4$ kJ/kg, $V = 40$ m/s. If the volume flow rate at the inlet is 3000 L/min, determine (a) the mass flow rate, (b) the volume flow rate at the exit and (c) the compressor power. (d) What-if Scenario: What would the power consumption be if the change in kinetic energy were neglected?

SOLUTION

(a) $$\dot{V} = \left(3000\ \frac{\text{L}}{\text{min}}\right)\left(\frac{1}{1000}\ \frac{\text{m}^3}{\text{L}}\right)\left(\frac{1}{60}\ \frac{\text{min}}{\text{s}}\right); \quad \Rightarrow \dot{V} = 0.05\ \frac{\text{m}^3}{\text{s}};$$

$$\dot{m}_i = \frac{\dot{V}_i}{v_i}; \quad \Rightarrow \dot{m}_i = \frac{0.05}{0.0835}; \quad \Rightarrow \dot{m}_i = 0.598\ \frac{\text{kg}}{\text{s}}$$

(b) At steady state

$$\dot{m}_i = \dot{m}_e = \dot{m};$$

$$\dot{V}_e = \dot{m}v_e; \quad \Rightarrow \dot{V}_e = (0.598)(0.0244); \quad \Rightarrow \dot{V}_e = 0.0146\ \frac{\text{m}^3}{\text{s}};$$

$$\Rightarrow \dot{V}_e = 876\ \frac{\text{L}}{\text{min}}$$

(c) $$\underbrace{\frac{dE}{dt}}_{\text{0,Steady State}} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \underbrace{\dot{Q}}_{\text{0, Adiabatic}} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(j_i - j_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left[\left(h_i + \text{ke}_i\right) - \left(h_e + \text{ke}_e\right)\right];$$

$$\Rightarrow \dot{W}_{\text{ext}} = (0.598)\left[\left(182.1 + \frac{(30)^2}{2000}\right) - \left(205.4 + \frac{(40)^2}{2000}\right)\right]; \quad \Rightarrow \dot{W}_{\text{ext}} = -14.2\ \text{kW};$$

While the external work transfer is negative, indicating work going in (WinHip), the compressor power automatically implies energy consumption and a sign is not necessary.

$$\Rightarrow \dot{W}_{\text{C}} = -\dot{W}_{\text{ext}} = 14.2\ \text{kW}$$

(d) With no change in ke, the external work can be calculated as

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{m}\left(h_i - h_e\right);$$

$$\Rightarrow \dot{W}_{\text{ext}} = (0.598)(182.1 - 205.4); \quad \Rightarrow \dot{W}_{\text{ext}} = -14\ \text{kW}$$

$$\Rightarrow \dot{W}_{\text{C}} = -\dot{W}_{\text{ext}} = 14\ \text{kW}$$

TEST Solution:

Because the identity of the fluid is not known, a TEST solution is not possible.

2-2-41 [IR] Two flows of equal mass flow rate, one at state-1 and another at state-2 enter an adiabatic mixing chamber and leave through a single port at state-3. Obtain an expression for the velocity and specific enthalpy at the exit. Assume negligible changes in ke and pe.

SOLUTION

At steady state

$\dot{m}_i = \dot{m}_e = \dot{m};$

(a) Where $\dot{m}_1 + \dot{m}_2 = \dot{m}_i$ and $\dot{m}_3 = \dot{m}_e$;

$$\Rightarrow \dot{m}_3 = \rho A_3 V_3 = 2\dot{m};$$

$$\Rightarrow V_3 = \frac{2\dot{m}}{\rho A_3}$$

(b) $$\underbrace{\frac{dE}{dt}}_{0,\text{Steady State}} = \sum \dot{m} j_i - \sum \dot{m} j_e + \underbrace{\dot{Q}}_{0,\text{ Adiabatic}} - \underbrace{\dot{W}_{\text{ext}}}_{0};$$

The energy equation becomes

$$\Rightarrow \dot{m}_3 j_3 = \dot{m}_1 j_1 + \dot{m}_2 j_2;$$

Since $\dot{m}_1 = \dot{m}_2$

$$\Rightarrow \dot{m} j_3 = \frac{\dot{m}}{2} j_1 + \frac{\dot{m}}{2} j_2;$$

$$\Rightarrow j_3 = (0.5)(j_1 + j_2);$$

With no changes in *ke* or *pe*

$$\Rightarrow h_3 = (0.5)(h_1 + h_2)$$

2-2-42 [IO] Air at 500 kPa, 30°C from a supply line is used to fill an adiabatic tank. At a particular moment during the filling process, the tank contains 0.2 kg of air at 200 kPa and 50°C. If the mass flow rate is 0.1 kg/s, and the specific enthalpy of air at the inlet is 297.2 kJ/kg, determine (a) the rate of flow energy into the tank, (b) the rate of increase of internal energy in the tank and (c) the rate of increase of specific internal energy if the specific internal energy in the tank is 224.5 kJ/kg at that instant. Assume the tank to be uniform at all times and neglect kinetic and potential energies.

SOLUTION

(a) Neglecting the ke and pe:

$$\dot{J}_i = \dot{m}_i j_i; \quad \Rightarrow \dot{J}_i = \dot{m}_i h_i; \quad \Rightarrow \dot{J}_i = (0.1)(297.2); \quad \Rightarrow \dot{J}_i = 29.72 \text{ kW}$$

(b) The energy equation is given as:

$$\frac{dE}{dt} = \sum \dot{m}_i j_i - \underbrace{\sum \dot{m}_e j_e}_{0} + \underbrace{\dot{Q}}_{0,\text{ Adiabatic}} - \underbrace{\dot{W}_{\text{ext}}}_{0};$$

$$\Rightarrow \frac{dE}{dt} = \dot{m}_i j_i; \quad \Rightarrow \frac{dE}{dt} = \dot{m}_i h_i; \quad \Rightarrow \frac{dE}{dt} = (0.1)(297.2); \quad \Rightarrow \frac{dE}{dt} = 29.72 \text{ kW};$$

$$\Rightarrow \frac{d(U + \text{KE} + \text{PE})}{dt} = 29.72 \text{ kW};$$

$$\Rightarrow \frac{dU}{dt} = 29.72 \text{ kW}$$

(c) $\frac{dU}{dt} = 29.72 \text{ kW};$

$$\Rightarrow \frac{d(mu)}{dt} = 29.72 \text{ kW};$$

$$\Rightarrow m\frac{du}{dt} + u\frac{dm}{dt} = 29.72 \text{ kW};$$

$$\Rightarrow m\frac{du}{dt} = 29.72 - (224.5)(0.1); \quad \Rightarrow m\frac{du}{dt} = 7.27 \text{ kW};$$

$$\Rightarrow \frac{du}{dt} = \frac{7.27}{0.2}; \quad \Rightarrow \frac{du}{dt} = 36.35 \frac{\text{kW}}{\text{kg}}$$

2-2-43 [IB] An insulated tank is being filled with a gas through a single inlet. At a given instant, the mass flow rate is measured as 0.5 kg/s and the enthalpy h as 400 kJ/kg. Negleting ke and pe, determine the rate of increase of stored energy in the system at that instant.

SOLUTION

The energy equation is given as:

$$\frac{dE}{dt} = \sum \dot{m}_i j_i - \underbrace{\sum \dot{m}_e j_e}_{0} + \underbrace{\dot{Q}}_{0,\ \text{Adiabatic}} - \underbrace{\dot{W}_{\text{ext}}}_{0};$$

$$\Rightarrow \frac{dE}{dt} = \dot{m}_i j_i; \quad \Rightarrow \frac{dE}{dt} = \dot{m}_i h_i; \quad \Rightarrow \frac{dE}{dt} = (0.5)(400); \quad \Rightarrow \frac{dE}{dt} = 200 \text{ kW}$$

2-2-44 [IS] Saturated steam at 200 kPa, which has a specific enthalpy (h) of 2707 kJ/kg is expelled from a pressure cooker at a rate of 0.1 kg/s. Determine the rate of heat transfer necessary to maintain a constant stored energy E in the cooker. Assume that there is sufficient liquid water in the cooker at all time to generate the saturated steam. Neglect kinetic and potential energy of the steam.

SOLUTION

The energy equation is given as:

$$\underset{0}{\frac{dE}{dt}} = \underbrace{\sum \dot{m}_i j_i}_{0} - \sum \dot{m}_e j_e + \dot{Q} - \underset{0}{\dot{W}_{\text{ext}}};$$

$$\Rightarrow \dot{Q} = \dot{m}_e j_e;$$

With no changes in ke or pe:

$$\Rightarrow \dot{Q} = \dot{m}_e h_e; \qquad \Rightarrow \dot{Q} = (0.1)(2707);$$

$$\Rightarrow \dot{Q} = 270.7 \text{ kW}$$

The positive sign indicates that heat must be added.

2-3-1 [IA] Heat is transferred from a TER at 1500 K to a TER at 300 K at a rate of 10 kW. Determine the rates at which entropy (a) leaves the TER at higher temperature and (b) enters the TER at lower temperature. (c) How do you explain the discontinuity in the result?

SOLUTION

The entropy balance equation is expressed as:

$$\underbrace{\frac{dS}{dt}}_{\substack{\text{Rate of increase}\\\text{of entropy for}\\\text{an open system.}}} = \underbrace{\sum_i \dot{m}_i s_i}_{\substack{\text{Entropy}\\\text{transported by}\\\text{mass flow in.}}} - \underbrace{\sum_e \dot{m}_e s_e}_{\substack{\text{Entropy}\\\text{transported by}\\\text{mass flow out.}}} + \underbrace{\frac{\dot{Q}}{T_B}}_{\substack{\text{Entropy}\\\text{transferred}\\\text{by heat.}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of}\\\text{generation of}\\\text{entropy inside}\\\text{the system}\\\text{boundary.}}}$$

For this analysis, we are only interested in the term that provides the rate of entropy transferred by heat.

(a) With a boundary chosen just inside of the 1500 K TER,

$$\frac{\dot{Q}_{\text{out}}}{T_B} = \frac{10}{1500}; \qquad \Rightarrow \frac{\dot{Q}_{\text{out}}}{T_B} = 0.00\overline{6}\ \frac{\text{kW}}{\text{K}}$$

(b) With a boundary chosen just inside of the 300 K TER,

$$\frac{\dot{Q}_{\text{in}}}{T_B} = \frac{10}{300}; \qquad \Rightarrow \frac{\dot{Q}_{\text{in}}}{T_B} = 0.0\overline{3}\ \frac{\text{kW}}{\text{K}}$$

(c) Entropy enters the 300 K TER at a higher rate than it leaves the 1500 K TER. This provides evidence that entropy generation is a result of temperature difference.

2-3-2 [IH] A wall separates a hot reservoir at 1000 K from a cold reservoir at 300 K. The temperature difference between the two reservoirs drive a heat transfer at the rate of 500 kW. If the wall maintains steady state, determine (a) $\dot{Q}$ (in kW), (b) $\dot{W}_{ext}$ (in kW), (c) d*S*/d*t* (in kW/K), and (d) $\dot{S}_{gen,univ.}$ (kW/K).

SOLUTION

(a) Treating the wall as the system, through which 500 kW of heat is transferred,

$$\dot{Q} = \dot{Q}_{in} - \dot{Q}_{out}; \quad \Rightarrow \dot{Q} = 500 - 500; \quad \Rightarrow \dot{Q} = 0 \text{ kW}$$

(b) There is not external (shaft, electrical, or boundary) work transfer.

$$\dot{W}_{ext} = 0 \text{ kW}$$

(c) Because the wall is at steady state

$$\frac{dS}{dt} = 0 \ \frac{\text{kW}}{\text{K}}$$

(d) An entropy balance on the system's universe produces:

$$\frac{dS}{dt} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}_{in}}{T_{hot}} - \frac{\dot{Q}_{out}}{T_{cold}} + \dot{S}_{gen,univ};$$

$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_{out}}{T_{cold}} - \frac{\dot{Q}_{in}}{T_{hot}}; \quad \Rightarrow \dot{S}_{gen,univ} = 500\left(\frac{1}{300} - \frac{1}{1000}\right); \quad \Rightarrow \dot{S}_{gen,univ} = 1.167 \ \frac{\text{kW}}{\text{K}}$$

2-3-3 [IN] A resistance heater operates inside a tank consuming 0.5 kW of electricity. Due to heat transfer to the ambient atmosphere at 300 K, the tank is maintained at a steady state. The surface temperature of the tank remains constant at 400 K. Determine the rate at which entropy (a) leaves the tank and (b) enters the tank's universe. (c) How does the system maintain steady state with regard to entropy?

SOLUTION

(a) The energy equation is given as

$$\frac{d\cancel{E}^{\,0,\text{ steady state}}}{dt} = \cancel{\dot{J}_{\text{net}}}^{\,0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow 0 = \left(-\dot{Q}_{\text{out}}\right) - \left(-\dot{W}_{\text{in}}\right);$$

$$\Rightarrow \dot{Q}_{\text{out}} = \dot{W}_{\text{in}} = 0.5 \text{ kW};$$

The entropy balance equation is expressed as

$$\underbrace{\frac{dS}{dt}}_{\substack{\text{Rate of increase}\\ \text{of entropy for}\\ \text{an open system.}}} = \underbrace{\sum_i \dot{m}_i s_i}_{\substack{\text{Entropy}\\ \text{transported by}\\ \text{mass flow in.}}} - \underbrace{\sum_e \dot{m}_e s_e}_{\substack{\text{Entropy}\\ \text{transported by}\\ \text{mass flow out.}}} + \underbrace{\frac{\dot{Q}}{T_B}}_{\substack{\text{Entropy}\\ \text{transferred}\\ \text{by heat.}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of}\\ \text{generation of}\\ \text{entropy inside}\\ \text{the system}\\ \text{boundary.}}}$$

For this analysis, we are only interested in the term that gives the rate of entropy transferred by heat. By creating a boundary just inside of the surface of the tank, the entropy leaving the tank is given by:

$$\frac{\dot{Q}}{T_B} = \frac{\dot{Q}_{\text{out}}}{T_B}; \quad \Rightarrow \frac{\dot{Q}}{T_B} = \frac{\dot{Q}_{\text{out}}}{T_{\text{tank}}}; \quad \Rightarrow \frac{\dot{Q}}{T_B} = \frac{0.5}{400}; \quad \Rightarrow \frac{\dot{Q}}{T_B} = 0.00125 \ \frac{\text{kW}}{\text{K}}$$

(b) By creating a boundary just outside of the surface of the tank

$$\frac{\dot{Q}}{T_B} = \frac{\dot{Q}_{\text{out}}}{T_{\text{surroundings}}}; \quad \Rightarrow \frac{\dot{Q}}{T_B} = \frac{0.5}{300}; \quad \Rightarrow \frac{\dot{Q}}{T_B} = 0.0016 \ \frac{\text{kW}}{\text{K}}$$

(c) System:

$$\frac{d\cancel{S}^{\,0,\text{ steady state}}}{dt} = \sum_i \cancel{\dot{m}_i}^{\,0} s_i - \sum_e \cancel{\dot{m}_e}^{\,0} s_e + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\left(-\dot{Q}_{\text{out}}\right)}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{0.5}{300}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.0016 \ \frac{\text{kW}}{\text{K}}$$

The system maintains steady state by generating entropy.

2-3-4 [IE] Heat is conducted through a slab of thickness 2 cm. The temperature varies linearly from 500 K on the left face to 300 K on the right face. If the rate of heat transfer is 2 kW, determine the rate of entropy transfer dS/dt (magnitude only) at the (a) left and (b) right faces. (c) Plot how the rate of entropy transfer varies from the left to the right face.

SOLUTION

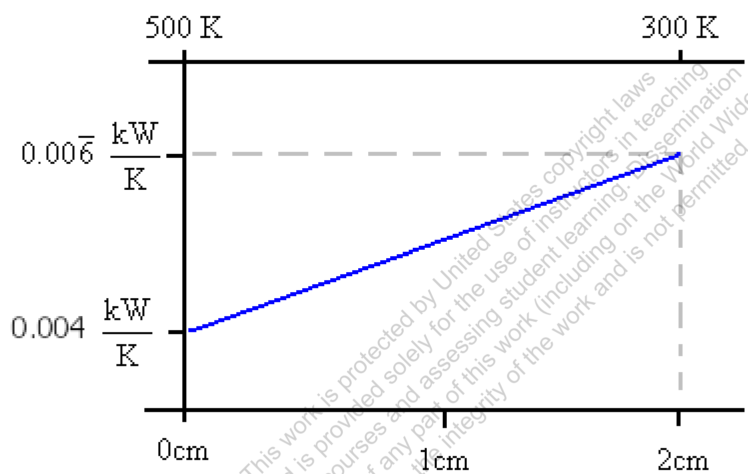
(a) On the left face,

$$\frac{\dot{Q}_{\text{left}}}{T_B} = \frac{2}{500}; \qquad \Rightarrow \frac{\dot{Q}_{\text{left}}}{T_B} = 0.004 \ \frac{\text{kW}}{\text{K}}$$

(b) On the right face,

$$\frac{\dot{Q}_{\text{right}}}{T_B} = \frac{2}{300}; \qquad \Rightarrow \frac{\dot{Q}_{\text{left}}}{T_B} = 0.00667 \ \frac{\text{kW}}{\text{K}}$$

(c)



2-3-5 [II] A 30 kg aluminum block cools down from its initial temperature of 500 K to the atmospheric temperature of 300 K. Determine the total amount of entropy transfer from the system's universe. Assume the specific internal energy of aluminum (in kJ/kg) is related to its absolute temperature (K) through *u* = *0.9T*.

SOLUTION

With no mass flow involved in this system:

$$\frac{dE}{dt} = \cancelto{0}{\dot{J}_{\text{net}}} + \dot{Q} - \cancelto{0}{\dot{W}_{\text{ext}}}; \quad \Rightarrow \frac{dU}{dt} = \left(-\dot{Q}_{\text{out}}\right);$$

$$\Rightarrow \frac{d\left(mu\right)}{dt} = -\dot{Q}_{\text{out}}; \quad \Rightarrow \frac{d\left(mu\right)}{dt} = -\dot{Q}_{\text{out}};$$

$$\Rightarrow m\frac{du}{dt} = -\dot{Q}_{\text{out}}; \quad \Rightarrow \left(0.9\right)m\frac{dT}{dt} = -\dot{Q}_{\text{out}}; \quad [\text{kW}]$$

$$\Rightarrow \left(0.9\right)mdT = -\dot{Q}_{\text{out}}dt; \quad [\text{kJ}]$$

$$\Rightarrow Q_{\text{out}} = -\left(0.9\right)m\Delta T; \quad \Rightarrow Q_{\text{out}} = -\left(0.9\right)\left(30\right)\left(300-500\right); \quad \Rightarrow Q_{\text{out}} = 5400 \text{ kJ};$$

The boundary for the system's universe passes through the surroundings. Therefore, the entropy transferred into the surroundings is:

$$\frac{Q_{\text{out}}}{T_B} = \frac{Q_{\text{out}}}{T_0}; \quad \Rightarrow \frac{Q_{\text{out}}}{T_B} = \frac{5400}{300}; \quad \Rightarrow \frac{Q_{\text{out}}}{T_B} = 18 \; \frac{\text{kJ}}{\text{K}}$$

2-3-6 [IL] Water is heated in a boiler from a source at 1800 K. If the heat transfer rate is 20 kW, (a) determine the rate of entropy transfer into the boiler's universe. (b) Discuss the consequences of reducing the source temperature with respect to the boiler size and entropy transfer.

SOLUTION

(a) By choosing a boundary just outside of the boiler's surface, the rate of entropy transfer from the reservoir is given as:

$$\frac{\dot{Q}_{\text{in}}}{T_B} = \frac{20}{1800}; \qquad \Rightarrow \frac{\dot{Q}_{\text{in}}}{T_B} = 0.011 \ \frac{\text{kW}}{\text{K}}$$

(b) As the reservoir temperature is decreased, the entropy transfer decreases. However, to maintain the same amount of heat transfer, the area of contact between the system and the reservoir must be increased resulting in the increase of system size.

2-3-7 [IG] An insulated tank contains 50 kg of water at 30°C, which is stirred by a paddle wheel at 300 rpm while transmitting a torque of 0.2 kNm. Determine (a) the rate of change of temperature (dT/dt) (b) the rate of change of total entropy (dS/dt) and (c) the rate of generation of entropy (S_{gen}) within the tank. Assume $s = 4.2 \ln T$ and $u = 4.2T$, where T is in Kelvin.

SOLUTION

(a) The energy equation is given as:

$$\frac{dE^{U}}{dt} = \sum \dot{m}_i j_i^{\,0} - \sum \dot{m}_e j_e^{\,0} + \dot{Q}^{0} - \dot{W}_{ext};$$

$$\Rightarrow \frac{dU}{dt} = -\dot{W}_{\text{ext}}; \quad \Rightarrow \frac{dU}{dt} = -\left(-\dot{W}_{\text{sh,in}}\right);$$

$$\Rightarrow \frac{dU}{dt} = \dot{W}_{\text{sh,in}}; \quad \Rightarrow \frac{dU}{dt} = 2\pi \frac{300}{60} 0.2; \quad \Rightarrow \frac{dU}{dt} = 6.28 \text{ kW};$$

$$\Rightarrow \frac{dU}{dt} = \frac{d(mu)}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{du}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{d(4.2T)}{dt};$$

$$\Rightarrow \frac{dU}{dt} = 4.2m\frac{dT}{dt}; \quad \Rightarrow \frac{dU}{dt} = 6.28 \text{ kW};$$

$$\Rightarrow \frac{dT}{dt} = \frac{6.28}{4.2(50)}; \quad \Rightarrow \frac{dT}{dt} = 0.03 \ \frac{\text{K}}{\text{s}}$$

(b) The entropy equation is given as:

$$\frac{dS}{dt} = \sum \dot{m}_i s_i^{\,0} - \sum \dot{m}_e s_e^{\,0} + \frac{\dot{Q}^{0}}{T_B} + \dot{S}_{gen};$$

$$\Rightarrow \frac{dS}{dt} = \dot{S}_{gen}; \quad \Rightarrow \text{Answers b and c will be the same}$$

$$\frac{dS}{dt} = \frac{d(ms)}{dt}; \quad \Rightarrow \frac{dS}{dt} = m\frac{ds}{dt}; \quad \Rightarrow \frac{dS}{dt} = 4.2m\frac{d(\ln T)}{dt}; \quad \Rightarrow \frac{dS}{dt} = \frac{4.2m}{T}\frac{dT}{dt};$$

$$\Rightarrow \dot{S}_{gen} = \frac{dS}{dt}; \quad \Rightarrow \dot{S}_{gen} = \frac{4.2m}{T}\frac{dT}{dt}; \quad \Rightarrow \dot{S}_{gen} = \frac{4.2(50)}{(273+30)}\frac{dT}{dt}; \quad \Rightarrow \dot{S}_{gen} = \frac{4.2(50)}{303}(0.03);$$

$$\Rightarrow \dot{S}_{gen} = 0.0208 \ \frac{\text{kW}}{\text{K}}$$

(c) $\dot{S}_{gen} = 0.0208 \ \frac{\text{kW}}{\text{K}}$

2-3-8 [IZ] A rigid insulated tank contains 1 kg of air at 300 K and 100 kPa. A 1 kW internal heater is turned on. Determine the rate of (a) entropy transfer into the tank (b) the rate of change of total entropy of the system and (c) the rate of generation of entropy within the tank. Assume $s = \ln T$ and $u = T$, where T is in Kelvin.

SOLUTION

(a) Entropy transfer into the system is given as:

$$\frac{\cancel{\dot{Q}}^0}{T_B} = 0 \ \frac{\text{kW}}{\text{K}}$$

(b) The energy equation produces:

$$\frac{d\cancel{E}^U}{dt} = \sum \cancel{\dot{m}_i j_i}^0 - \sum \cancel{\dot{m}_e j_e}^0 + \cancel{\dot{Q}}^0 - \dot{W}_{ext};$$

$$\Rightarrow \frac{dU}{dt} = -\dot{W}_{\text{ext}}; \quad \Rightarrow \frac{dU}{dt} = -\left(-\dot{W}_{\text{el,in}}\right); \quad \Rightarrow \frac{dU}{dt} = 1 \text{ kW};$$

$$\Rightarrow \frac{dU}{dt} = \frac{d(mu)}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{du}{dt}; \quad \Rightarrow \frac{dU}{dt} = m\frac{dT}{dt} = 1 \text{ kW};$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{m}; \quad \Rightarrow \frac{dT}{dt} = \frac{1}{1}; \quad \Rightarrow \frac{dT}{dt} = 1 \ \frac{\text{K}}{\text{s}};$$

Therefore,

$$\frac{dS}{dt} = \frac{d(ms)}{dt}; \quad \Rightarrow \frac{dS}{dt} = m\frac{ds}{dt}; \quad \Rightarrow \frac{dS}{dt} = m\frac{d(\ln T)}{dt}; \quad \Rightarrow \frac{dS}{dt} = \frac{m}{T}\frac{dT}{dt}; \quad \Rightarrow \frac{dS}{dt} = \frac{1}{300}(1);$$

$$\Rightarrow \frac{dS}{dt} = 0.0033 \ \frac{\text{kW}}{\text{K}}$$

(c) The entropy equation can be simplified as:

$$\frac{dS}{dt} = \sum \cancel{\dot{m}_i s_i}^0 - \sum \cancel{\dot{m}_e s_e}^0 + \frac{\cancel{\dot{Q}}^0}{T_B} + \dot{S}_{gen,univ};$$

$$\Rightarrow \dot{S}_{gen,univ} = \frac{dS}{dt}; \quad \Rightarrow \dot{S}_{gen,univ} = 0.0033 \ \frac{\text{kW}}{\text{K}}$$

2-3-9 [IK] A rigid tank contains 10 kg of air at 500 K and 100 kPa while the surroundings is at 300 K. A 2 kW internal heater keeps the gas hot by compensating the heat losses. At steady state, determine the rate of (a) heat transfer, (b) the rate of entropy generation (S_{gen}) inside the tank, and (c) the rate of entropy generation ($S_{gen,univ}$) in the system's universe.

SOLUTION

(a) $$\frac{dE}{dt}^{\,0,\text{ steady state}} = \dot{J}_{net}^{\,0} + \dot{Q} - \dot{W}_{ext} = \left(-\dot{Q}_{loss}\right) - \left(-\dot{W}_{el,in}\right);$$

$$\Rightarrow \dot{Q}_{loss} = \dot{W}_{el,in} = 2 \text{ kW};$$

$$\Rightarrow \dot{Q} = -2 \text{ kW}$$ (Negative sign indicates heat is being lost)

(b) Drawing the system boundary just inside the tank

$$\frac{dS}{dt}^{\,0,\text{ steady state}} = \sum_i \dot{m}_i^{\,0} s_i - \sum_e \dot{m}_e^{\,0} s_e + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

$$\Rightarrow \dot{S}_{gen} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{gen} = -\frac{\left(-\dot{Q}_{loss}\right)}{T_B}; \quad \Rightarrow \dot{S}_{gen} = \frac{\dot{Q}_{loss}}{T_B}; \quad \Rightarrow \dot{S}_{gen} = \frac{2}{500};$$

$$\Rightarrow \dot{S}_{gen} = 0.004 \ \frac{\text{kW}}{\text{K}}$$

(c) Drawing the boundary just outside the tank

$$\dot{S}_{gen,univ} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{gen,univ} = -\frac{\left(-\dot{Q}_{loss}\right)}{T_B}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_{loss}}{T_B}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{2}{300};$$

$$\Rightarrow \dot{S}_{gen,univ} = 0.0067 \ \frac{\text{kW}}{\text{K}}$$

2-3-10 [IP] A 10 m^3 insulated rigid tank contains 30 kg of wet steam with a quality of 0.9. An internal electric heater is turned on, which consumes electric power at a rate of 10 kW. After the heater is on for one minute, determine (a) the change in temperature, (b) the change in pressure and (c) the change in entropy of steam.

TEST Solution:

The energy equation produces:

$$\frac{d\not{E}^{U}}{dt} = \sum \not{\dot{m}_i j_i}^{0} - \sum \not{\dot{m}_e j_e}^{0} + \not{\dot{Q}}^{0} - \dot{W}_{ext};$$

$$\Rightarrow \frac{dU}{dt} = -\dot{W}_{\text{ext}}; \qquad \Rightarrow \frac{dU}{dt} = -\left(-\dot{W}_{\text{el,in}}\right); \qquad \Rightarrow \frac{dU}{dt} = 10 \text{ kW};$$

$$\Rightarrow \frac{dU}{dt} = \frac{d(mu)}{dt}; \qquad \Rightarrow \frac{dU}{dt} = m\frac{du}{dt}; \qquad \Rightarrow \frac{dU}{dt} = 1 \text{ kW};$$

$$\Rightarrow du = \frac{dt}{m}; \left[\frac{\text{kJ}}{\text{kg}}\right]$$

$$\Rightarrow u_2 - u_1 = \frac{60}{m}; \left[\frac{\text{kJ}}{\text{kg}}\right]$$

Launch the PC system-state TESTcalc. Evaluate the initial state, State-1, from the given properties. For the final state, State-2, use m2=m1, Vol2=Vol1, and u2=u1+10*60/m1. Obtain the answers in the I/O panel by evaluating expressions:

(a) =T2-T1 = 0.451°C

(b) =p2-p1 = 6.11 kPa

(c) =m1*(s2-s1) = 1.02 $\frac{\text{kJ}}{\text{K}}$

2-3-11 [BEC] One kg of air is trapped in a rigid chamber of volume 0.2 m^3 at 300 K. Because of electric work transfer, the temperature of air increases at a rate of 1^{o}C/s. Using the *IG system state daemon*, calculate: (a) the rate of change of stored energy, dE/dt, (b) the rate of external work transfer (include sign), (c) the rate of change of total entropy of the system, dS/dt, (d) the rate of entropy generation inside the system. (Hint: Evaluate two states separated by 1 s).

TEST Solution:

The energy equation produces:

$$\frac{d\cancel{E}^{U}}{dt} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \cancel{\dot{Q}}^{0} - \dot{W}_{ext};$$

$$\Rightarrow \frac{dE}{dt} = \frac{dU}{dt}; \qquad \Rightarrow \frac{dE}{dt} = -\dot{W}_{\text{ext}};$$

$$\Rightarrow \frac{dE}{dt} = -\left(-\dot{W}_{\text{el,in}}\right); \qquad \Rightarrow \frac{dE}{dt} = \dot{W}_{\text{el,in}}; \quad [\text{kW}]$$

Launch the IG system-state TESTcalc. Evaluate the initial state, State-1, from the given properties. Let State-2 represent the state after 1 s. The new state now has T2 = T1+1. Also, for the closed system m2=m1 and Vol2=Vol1. In the I/O panel evaluate part a and part c:

(a) $\frac{\text{dE}}{\text{dt}}$ = m1*(e2-e1)/1 = 0.717 kW

(b) From the energy equation:

$$\dot{W}_{\text{ext}} = -\frac{dE}{dt}; \qquad \Rightarrow \dot{W}_{\text{ext}} = -0.717 \text{ kW}$$

(c) $\frac{\text{dS}}{\text{dt}}$ = m1*(s2-s1)/1 = 0.00239 $\frac{\text{kW}}{\text{K}}$

(d) The entropy equation produces:

$$\frac{dS}{dt} = \sum_i \cancel{\dot{m}_i}^{0} s_i - \sum_e \cancel{\dot{m}_e}^{0} s_e + \frac{\cancel{\dot{Q}}^{0}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow \dot{S}_{\text{gen}} = \frac{dS}{dt}; \qquad \Rightarrow \dot{S}_{\text{gen}} = 0.00239 \ \frac{\text{kW}}{\text{K}}$$

2-3-12 [IU] A rigid cylindrical tank stores 100 kg of a substance at 500 kPa and 500 K while the outside temperature is 300 K. A paddle wheel stirs the system transferring shaft work at a rate of 0.5 kW. At the same time an internal electrical resistance heater transfers electricity at the rate of 1 kW. (a) Do an energy analysis to determine the rate of heat transfer in kW for the tank. (b) Determine the absolute value of the rate at which entropy leaves the internal system (at a uniform temperature of 500 K) in kW/K. (c) Determine the rate of entropy generation for the system's universe.

SOLUTION

(a) $$\frac{d\cancel{E}^{0}}{\cancel{dt}} = \cancel{\dot{J}_{\text{net}}}^{0} + \dot{Q} - \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -(1+0.5);$$

$$\Rightarrow \dot{Q} = -1.5\text{kW}$$

(b) Taking the system just inside the tank

$$\frac{\dot{Q}_{\text{loss}}}{T_{\text{B}}} = \frac{1.5}{500}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_{\text{B}}} = 0.003\ \frac{\text{kW}}{\text{K}}$$

(c) Application of the entropy equation over the system's universe produces:

$$\frac{d\cancel{S}^{0,\text{ steady state}}}{\cancel{dt}} = \sum_i \cancel{\dot{m}_i}^{0} s_i - \sum_e \cancel{\dot{m}_e}^{0} s_e + \frac{\dot{Q}}{T_{\text{B}}} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_{\text{B}}}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\left(-\dot{Q}_{\text{loss}}\right)}{T_{\text{B}}}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{\text{loss}}}{T_{\text{B}}}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{1.5}{300};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = 0.005\ \frac{\text{kW}}{\text{K}}$$

2-3-13 [IX] A 1 cm diameter insulated pipe carries a steady flow of water at a velocity of 30 m/s. The temperature increases from 300 K at the inlet to 301 K at the exit due to friction. If the specific entropy of water is related to its absolute temperature through $s = 4.2 \ln T$, determine the rate of generation of entropy within the pipe. Assume water density to be 1000 kg/m^3.

SOLUTION

At steady state $\dot{m}_i = \dot{m}_e = \dot{m}$;

$$\dot{m} = \rho A V; \qquad \Rightarrow \dot{m} = 1000\left[\pi \frac{(0.01)^2}{4}\right](30); \qquad \Rightarrow \dot{m} = 2.36\ \frac{\text{kg}}{\text{s}};$$

The entropy equation is given as

$$\frac{dS}{dt}^{\,0} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\dot{Q}^{\,0}}{T_B} + \dot{S}_{gen};$$

$$\Rightarrow \dot{S}_{gen} = \dot{m}(s_e - s_i);$$

$$\Rightarrow \dot{S}_{gen} = \dot{m}(4.2)(\ln T_e - \ln T_i);$$

$$\Rightarrow \dot{S}_{gen} = \dot{m}(4.2)\left(\ln \frac{T_e}{T_i}\right);$$

$$\Rightarrow \dot{S}_{gen} = (2.36)(4.2)\left(\ln \frac{301}{300}\right);$$

$$\Rightarrow \dot{S}_{gen} = 0.033\ \frac{\text{kW}}{\text{K}}$$

2-3-14 [IC] Liquid water (density 997 kg/m^3) flows steadily through a pipe with a volume flow rate of 30,000 L/min. Due to viscous friction, the pressure drops from 500 kPa at the inlet to 150 kPa at the exit. If the specific internal energy and specific entropy remain constant along the flow, determine (a) the rate of heat transfer and (b) the rate of entropy generation in and around the pipe. Assume the temperature of the surroundings to be 300 K.

SOLUTION

(a) At steady state $\dot{m}_i = \dot{m}_e = \dot{m}$;

$$\dot{m} = \rho A V; \quad \Rightarrow \dot{m} = \rho \dot{V}; \quad \Rightarrow \dot{m} = \left(997\frac{\text{kg}}{\text{m}^3}\right)\left(30{,}000\frac{\text{L}}{\text{min}}\right)\left(\frac{1}{1000}\frac{\text{m}^3}{\text{L}}\right)\left(\frac{1}{60}\frac{\text{min}}{\text{s}}\right);$$

$$\Rightarrow \dot{m} = 448.5\ \frac{\text{kg}}{\text{s}};$$

The energy balance equation is given as:

$$\frac{dE}{dt}^{\,0} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \dot{Q} - \dot{W}_{ext}^{\,0};$$

$$\Rightarrow \dot{Q} = \dot{m}\left(j_e - j_i\right); \quad \Rightarrow \dot{Q} = \dot{m}\left(\Delta j\right);$$

$$\Rightarrow \dot{Q} = \dot{m}\left(\Delta h + \Delta ke^{0} + \Delta pe^{0}\right); \quad \Rightarrow \dot{Q} = \dot{m}\left(+\Delta u^{0} + \Delta pv\right);$$

$$\Rightarrow \dot{Q} = \dot{m}\left(p_e v - p_i v\right);$$

$$\Rightarrow \dot{Q} = \frac{\dot{m}}{\rho}\left(p_e - p_i\right); \quad \Rightarrow \dot{Q} = \frac{498.5}{997}\left(150-500\right); \quad \Rightarrow \dot{Q} = -175\ \text{kW}$$

(b) The entropy equation is given as:

$$\frac{dS}{dt}^{\,0} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}\left(s_e - s_i\right) - \frac{\dot{Q}}{T_B};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}\Delta s^{0} - \frac{\dot{Q}}{T_B};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{-175}{300}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.583\ \frac{\text{kW}}{\text{K}}$$

2-3-15 [IV] An electric water heater works by passing electricity through an electrical resistance placed inside the flow of liquid water as shown in the accompanying animation. The specific internal energy and entropy of water are correlated to its absolute temperature through $u = 4.2T$ and $s = 4.2\ln T$ (T in K) respectively. Water enters the heater at 300 K with a flow rate of 5 kg/s. At the exit the temperature is 370 K. Assuming steady state operation, negligible changes in ke and pe, and negligible heat transfer, determine (a) electrical power consumption rate and (b) the rate of entropy generation within the heater. What-if Scenario: What would the answers be if the exit temperature were reduced by 20°C?

SOLUTION

(a) The energy balance equation is given as:

$$\frac{dE^{\,0}}{dt} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \dot{Q}^{\,0} - \dot{W}_{ext};$$

$$\Rightarrow \dot{W}_{ext} = -\dot{m}(j_i - j_e); \quad \Rightarrow \dot{W}_{ext} = -\dot{m}\Delta j; \quad \Rightarrow \dot{W}_{ext} = -\dot{m}\left(\Delta h + \Delta ke^{\,0} + \Delta pe^{\,0}\right);$$

$$\Rightarrow \dot{W}_{ext} = -\dot{m}\left(\Delta u + \Delta pv^{\,0}\right); \quad \Rightarrow \dot{W}_{ext} = -\dot{m}(u_e - u_i);$$

$$\Rightarrow \dot{W}_{ext} = -\dot{m}(u_e - u_i); \quad \Rightarrow \dot{W}_{ext} = -\dot{m}(4.2)(T_e - T_i); \quad \Rightarrow \dot{W}_{ext} = -(5)(4.2)(370 - 300);$$

$$\Rightarrow \dot{W}_{ext} = -1470 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\frac{dS^{\,0}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\dot{Q}^{\,0}}{T_B} + \dot{S}_{gen};$$

$$\Rightarrow \dot{S}_{gen} = \dot{m}(s_e - s_i); \quad \Rightarrow \dot{S}_{gen} = \dot{m}(4.2)(\ln T_e - \ln T_i);$$

$$\Rightarrow \dot{S}_{gen} = \dot{m}(4.2)\left(\ln \frac{T_e}{T_i}\right); \quad \Rightarrow \dot{S}_{gen} = (5)(4.2)\ln\left(\frac{370}{300}\right); \quad \Rightarrow \dot{S}_{gen} = 4.04 \ \frac{\text{kW}}{\text{K}}$$

(c.1) $\dot{W}_{ext} = -\dot{m}(4.2)(T_e - T_i); \quad \Rightarrow \dot{W}_{ext} = -(5)(4.2)((370-20)-300);$

$$\Rightarrow \dot{W}_{ext} = -1050 \text{ kW}$$

(c.2) $\dot{S}_{gen} = \dot{m}(4.2)\left(\ln \frac{T_e}{T_i}\right); \quad \Rightarrow \dot{S}_{gen} = (5)(4.2)\ln\left(\frac{(370-20)}{300}\right);$

$$\Rightarrow \dot{S}_{gen} = 3.237 \ \frac{\text{kW}}{\text{K}}$$

2-3-16 [IQ] An open system with only one inlet and one exit operates at steady state. Mass enters the system at a flow rate of 5 kg/s with the following properties: h = 3484 kJ/kg, s = 8.0871 kJ/kg-K and V = 20 m/s. At the exit the properties are as follows: h = 2611 kJ/kg, s = 8.146 kJ/kg-K and V = 25 m/s. The device produces 4313 kW of shaft work while rejecting some heat to the atmosphere at 25°C. (a) Do a mass analysis to determine the mass flow rate at the exit. (b) Do an energy analysis to determine the rate of heat transfer (include sign). (c) Do an entropy analysis to evaluate the rate of entropy generation in the system's universe.

SOLUTION

(a) Mass analysis:

$$\frac{\cancel{dm}^0}{\cancel{dt}} = \sum \dot{m}_i - \sum \dot{m}_e;$$

$$\Rightarrow \dot{m}_e = \dot{m}_i = \dot{m} = 5\ \frac{\text{kg}}{\text{s}}$$

(b) Energy analysis:

$$\frac{\cancel{dE}^0}{\cancel{dt}} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{m}\left(j_i - j_e\right) + \dot{W}_{\text{ext}}; \qquad \Rightarrow \dot{Q} = \dot{m}\Delta j + \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = \dot{m}\left(\Delta h + \Delta \text{ke} + \cancel{\Delta \text{pe}}^0\right) + \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = \dot{m}\left[\left(h_e + \frac{(V_e)^2}{2000}\right) - \left(h_i + \frac{(V_i)^2}{2000}\right)\right] + \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = (5)\left[\left(2611 + \frac{(25)^2}{2000}\right) - \left(3484 + \frac{(20)^2}{2000}\right)\right] + 4313;$$

$$\Rightarrow \dot{Q} = -51.4\ \text{kW}$$

(c) Entropy analysis:

$$\frac{\cancel{dS}^0}{\cancel{dt}} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}\left(s_e - s_i\right) + \frac{\dot{Q}}{T_B};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \dot{m}(8.146 - 8.0871) - \frac{51.4}{(273+25)}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.122\ \frac{\text{kW}}{\text{K}}$$

2-3-17 [IT] Steam flows steadily through a work-producing, adiabatic, single-flow device with a flow rate of 7 kg/s. At the inlet h = 3589 kJ/kg, s = 7.945 kJ/kg-K, and at the exit h = 2610 kJ/kg, s = 8.042 kJ/kg-K. If changes in ke and pe are negligible, determine (a) the work produced by the device, and (b) the rate of entropy generation within the device. What-if Scenario: What would the answer in part (b) be if the device lost 5 kW of heat from its surface at 200°C?

SOLUTION

(a) At steady state:

$$\dot{m}_i = \dot{m}_e = \dot{m};$$

The energy balance equation is given as:

$$\frac{d\cancel{E}^{0}}{\cancel{d}t} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \cancel{\dot{Q}}^{0} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = -\dot{m}\Delta j; \qquad \Rightarrow \dot{W}_{\text{ext}} = -\dot{m}\left(\Delta h + \cancel{\Delta ke}^{0} + \cancel{\Delta pe}^{0}\right); \qquad \Rightarrow \dot{W}_{\text{ext}} = -\dot{m}\Delta h;$$

$$\Rightarrow \dot{W}_{ext} = -\dot{m}\left(h_e - h_i\right); \qquad \Rightarrow \dot{W}_{\text{ext}} = -7\left(2610 - 3589\right); \qquad \Rightarrow \dot{W}_{\text{ext}} = 6853 \text{ kW}$$

(b) The entropy balance equation is given as

$$\frac{d\cancel{S}^{0}}{\cancel{d}t} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\cancel{\dot{Q}}^{0}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow \dot{S}_{\text{gen}} = \dot{m}\left(s_e - s_i\right); \qquad \Rightarrow \dot{S}_{\text{gen}} = (7)(8.042 - 7.945); \qquad \Rightarrow \dot{S}_{\text{gen}} = 0.679 \ \frac{\text{kW}}{\text{K}}$$

(c) In the presence of heat transfer, the entropy balance equation would become:

$$\Rightarrow \dot{S}_{\text{gen}} = \dot{m}\left(s_e - s_i\right) - \frac{\dot{Q}}{T_B}; \qquad \Rightarrow \Rightarrow \dot{S}_{\text{gen}} = (7)(8.042 - 7.945) - \frac{(-5)}{(273 + 200)};$$

$$\Rightarrow \dot{S}_{gen} = 0.69 \ \frac{\text{kW}}{\text{K}}$$

2-3-18 [IY] The following information are supplied at the inlet and exit of an adiabatic nozzle operating at steady state: Inlet: $V = 30$ m/s, $h = 976.2$ kJ/kg, $s = 6.149$ kJ/kg-K; Exit: $h = 825.5$ kJ/kg. Determine (a) the exit velocity and (b) the minimum specific entropy (s) possible at the exit.

SOLUTION

(a) At steady state:

$$\dot{m}_i = \dot{m}_e = \dot{m};$$

The energy balance equation is given as:

$$\frac{dE}{dt}^{0} = \sum \dot{m}_i j_i - \sum \dot{m}_e j_e + \dot{Q}^{0} - \dot{W}_{ext}^{0};$$

$$\Rightarrow \dot{m} j_i = \dot{m} j_e; \qquad \Rightarrow j_i = j_e;$$

Neglecting any change in potential energy:

$$\Rightarrow h_i + ke_i = h_e + ke_e; \qquad \Rightarrow h_i + \frac{(V_i)^2}{2000} = h_e + \frac{(V_e)^2}{2000};$$

$$\Rightarrow V_e = \sqrt{\left(h_i + \frac{(V_i)^2}{2000} - h_e\right)(2000)}; \qquad \Rightarrow V = \sqrt{\left(976.2 + \frac{(30)^2}{2000} - 825.5\right)(2000)};$$

$$\Rightarrow V = 550 \ \frac{\text{m}}{\text{s}}$$

(b) The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\dot{Q}^{0}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow \dot{S}_{\text{gen}} = \dot{m}(s_i - s_e);$$

$$\Rightarrow s_e = \frac{\dot{S}_{\text{gen}}}{\dot{m}} + s_i;$$

Since $\dot{S}_{\text{gen}} \geq 0$ (thermodynamic friction cannot be negative), the minimum value of the exit entropy is given as.

$$\Rightarrow s_{e,\min} = \frac{0}{\dot{m}} + 6.149; \qquad \Rightarrow s_{e,\min} = 6.149 \ \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

2-3-19 [IF] A refrigerant flows steadily through an insulated tube, its entropy increases from 0.2718 kJ/kg-K at the inlet to 0.3075 kJ/kg-K at the exit. If the mass flow rate of the refrigerant is 0.2 kg/s and the pipe is insulated, determine (a) the rate of entropy generation within the pipe. (b) What-if Scenario: What would the rate of entropy generation be if the mass flow rate doubled?

SOLUTION

(a) At steady state:

$$\dot{m}_i = \dot{m}_e = \dot{m};$$

The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \frac{\dot{Q}^{0}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow \dot{S}_{\text{gen}} = \dot{m}(s_e - s_i); \quad \Rightarrow \dot{S}_{\text{gen}} = (0.2)(0.3075 - 0.2718); \quad \Rightarrow \dot{S}_{\text{gen}} = 0.00714 \ \frac{\text{kW}}{\text{K}}$$

(b) By doubling the magnitude of the mass flow rate, the inlet and exit conditions do not change.

$$\Rightarrow \dot{S}_{\text{gen}} = (2\dot{m})(s_e - s_i); \quad \Rightarrow \dot{S}_{\text{gen}} = (2)(0.2)(0.3075 - 0.2718); \quad \Rightarrow \dot{S}_{\text{gen}} = 0.01428 \ \frac{\text{kW}}{\text{K}}$$

2-4-1 [ID] A tank contains 50 kg of water, which is stirred by a paddle wheel at 300 rpm while transmitting a torque of 0.2 kNm. After the tank achieves steady state, determine (a) the rate of heat transfer, (b) the rate of entropy transfer into the atmosphere and (c) the rate of entropy generation in the tank's universe. Assume the atmospheric temperature to be 25°C.

SOLUTION

(a) $\dot{W}_{ext} = -\dot{W}_{sh,in} = -2\pi NT; \quad \Rightarrow \dot{W}_{ext} = -2\pi \frac{300}{60} 0.2; \quad \Rightarrow \dot{W}_{ext} = -6.28 \text{ kW};$

The energy balance equation simplifies as:

$$\frac{dE^{\,0}}{dt} = \sum \dot{m}_i j_i^{\,0} - \sum \dot{m}_e j_e^{\,0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{loss}; \quad \Rightarrow \dot{Q} = \dot{W}_{ext}; \quad \Rightarrow \dot{Q} = -6.28 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\frac{dS^{\,0}}{dt} = \sum \dot{m}_i s_i^{\,0} - \sum \dot{m}_e s_e^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{loss}}{T_B} + \dot{S}_{gen}; \quad \Rightarrow \dot{S}_{gen} = \frac{\dot{Q}_{loss}}{T_B};$$

Since entropy is transferred by heat transfer to atmosphere:

$$\frac{\dot{Q}_{loss}}{T_B} = \frac{6.28}{(273+25)}; \quad \Rightarrow \frac{\dot{Q}_{loss}}{T_B} = 0.021 \frac{\text{kW}}{\text{K}}$$

(c) From the entropy balance equation:

$$\dot{S}_{gen} = \frac{\dot{Q}_{loss}}{T_B}; \quad \Rightarrow \dot{S}_{gen} = 0.021 \frac{\text{kW}}{\text{K}}$$

2-4-2 [IM] A tank contains 1 kg of air at 500 K and 500 kPa. A 1 kW internal heater operates inside the tank at steady state to make up for the heat lost to the atmosphere which is at 300 K. Determine (a) the rate of entropy transfer into the atmosphere, (b) the rate of entropy generation in the system's universe, and (c) the internal rate of entropy generation.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = -1\ \text{kW};$

The energy balance equation simplifies as:

$$\frac{dE}{dt}^{0} = \sum \dot{m}_i j_i^{\,0} - \sum \dot{m}_e j_e^{\,0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \qquad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \qquad \Rightarrow \dot{Q} = -1\ \text{kW};$$

The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i^{\,0} - \sum \dot{m}_e s_e^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_B} + \dot{S}_{\text{gen}}; \qquad \Rightarrow \dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{loss}}}{T_B};$$

Since entropy is transferred into the atmosphere by heat:

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{\dot{Q}_{\text{loss}}}{T_0}; \qquad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{1}{300}; \qquad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.00333\ \frac{\text{kW}}{\text{K}}$$

(b) From the entropy balance equation:

$$\dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{1}{300}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.00333\ \frac{\text{kW}}{\text{K}}$$

(c) By choosing a boundary just inside of the system's surface:

$$\dot{S}_{\text{gen, int}} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \qquad \Rightarrow \dot{S}_{\text{gen, int}} = \frac{1}{500}; \qquad \Rightarrow \dot{S}_{\text{gen, int}} = 0.002\ \frac{\text{kW}}{\text{K}}$$

2-4-3 [IJ] A rigid tank contains 1 kg of air initially at 300 K and 100 kPa. A 1 kW internal heater is turned on. After the tank achieves steady state, determine (a) the rate of heat transfer, (b) the rate of entropy transfer into the atmoshere and (c) the rate of entropy generation in the tank's universe. Assume the atmospheric temperature to be 0°C.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = -1 \text{ kW};$

The energy balance equation simplifies as:

$$\frac{dE}{dt}^{0} = \sum \dot{m}_i j_i^{0} - \sum \dot{m}_e j_e^{0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -1 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i^{0} - \sum \dot{m}_e s_e^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_B} + \dot{S}_{\text{gen,univ}}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_{\text{loss}}}{T_0};$$

Since entropy is only transferred by heat into the atmosphere,

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{\dot{Q}_{\text{loss}}}{T_0}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{1}{273}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.003663 \ \frac{\text{kW}}{\text{K}}$$

(c) From the entropy balance equation:

$$\dot{S}_{\text{gen, univ}} = \frac{\dot{Q}_{\text{loss}}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = \frac{\dot{Q}_{\text{loss}}}{T_0}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = \frac{1}{273}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.003663 \ \frac{\text{kW}}{\text{K}}$$

2-4-4 [IW] A closed chamber containing a gas is at steady state. The shaft transfers power at a rate of 2 kW to the paddle wheel and the electric lamp consumes electricity at a rate of 500 W. Using an energy balance determine (a) the rate of heat transfer. (b) If the surface temperature of the chamber is 400 K, determine the entropy generated within the chamber. (c) What-if Scenario: What would the entropy generation within the chamber be if the surface temperature increased to 500 K?

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{sh,in}} - \dot{W}_{\text{el,in}};\quad \Rightarrow \dot{W}_{\text{ext}} = (-2) + (-0.5);\quad \Rightarrow \dot{W}_{\text{ext}} = -2.5\text{ kW};$

The energy balance equation simplifies as:

$$\frac{dE}{dt}^{0} = \sum \dot{m}_i j_i^{0} - \sum \dot{m}_e j_e^{0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}};\quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}};\quad \Rightarrow \dot{Q} = -2.5\text{ kW}$$

(b) The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i^{0} - \sum \dot{m}_e s_e^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow 0 = \frac{-\dot{Q}_{\text{loss}}}{T_s} + \dot{S}_{\text{gen,int}};\quad \Rightarrow \dot{S}_{\text{gen,int}} = \frac{\dot{Q}_{\text{loss}}}{T_s};\quad \Rightarrow \dot{S}_{\text{gen,int}} = \frac{2.5}{400};$$

$$\Rightarrow \dot{S}_{\text{gen,int}} = 0.00625\ \frac{\text{kW}}{\text{K}}$$

(c) $\dot{S}_{\text{gen, int}} = \frac{\dot{Q}_{\text{loss}}}{T_s};\quad \Rightarrow \dot{S}_{\text{gen,int}} = \frac{2.5}{500};\quad \Rightarrow \dot{S}_{\text{gen,int}} = 0.005\ \frac{\text{kW}}{\text{K}}$

2-4-5 [LR] A copper block receives heat from two different sources: 5 kW from a source at 1500 K and 3 kW from a source at 1000 K. It loses heat to atmosphere at 300 K. Assuming the block to be at steady state, determine (a) the net rate of heat transfer in kW; (b) the rate of entropy generation in the system's universe. What-if Scenario: What would the entropy generation if the second source were also at 1500 K?

SOLUTION

(a) $$\frac{dE}{dt}^{\!\!\!\not\, 0} = \dot{J}_{\text{net}}^{\not\, 0} + \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{W}_{\text{ext}}^{\not\, 0};$$

$$\Rightarrow \dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_3 = (5+3); \quad \Rightarrow \dot{Q}_3 = 8 \text{ kW};$$

$$\Rightarrow \dot{Q}_{\text{net}} = \dot{Q}; \quad \Rightarrow \dot{Q}_{\text{net}} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3; \quad \Rightarrow \dot{Q}_{\text{net}} = 0 \text{ kW}$$

(b) $$\frac{dS}{dt}^{\!\!\!\not\, 0} = \dot{S}_{\text{net}}^{\not\, 0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{S}_{\text{gen,univ}};$$

$$\dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{8}{300} - \frac{5}{1500} - \frac{3}{1000};$$

$$\Rightarrow \dot{S}_{\text{gen}} = 0.0203 \ \frac{\text{kW}}{\text{K}}$$

2-4-6 [LO] An electric bulb consumes 500 W of electricity at steady state. The outer surface of the bulb is warmer than the surrounding atmosphere by 75°C. If the atmospheric temperature is 300 K, determine (a) the rate of heat transfer between the bulb (the system) and the atmosphere. Also determine the entropy generation rate (b) within the bulb, (c) in the system's universe and (d) in the immediate surroundings outside the bulb.

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = -0.5 \text{ kW};$

The energy balance equation simplifies as:

$$\cancel{\frac{dE}{dt}}^{0} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -0.5 \text{ kW}$$

(b) The entropy balance equation is given as:

$$\cancel{\frac{dS}{dt}}^{0} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\Rightarrow \dot{S}_{\text{gen,int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = -\frac{-0.5}{375}; \quad \Rightarrow \dot{S}_{\text{gen,int}} = 0.00133 \frac{\text{kW}}{\text{K}}$$

(c) $\dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_0}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{-0.5}{300}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.001667 \frac{\text{kW}}{\text{K}}$

(d) $\dot{S}_{\text{gen, surr}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen, surr}} = 0.001667 - 0.00133; \quad \Rightarrow \dot{S}_{\text{gen, surr}} = 0.00033 \frac{\text{kW}}{\text{K}}$

2-4-7 [LB] An electric heater consumes 2 kW of electricity at steady state to keep a house at 27^{o}C. The outside temperature is -10^{o}C. Taking the heater inside the house as the system, determine (a) the maximum possible energetic efficiency of the heater, (b) the rate of entropy generation in the heater's universe in W/K. (c) If the surface of the heater is at 150^{o}C, how much entropy is generated in the immediate surroundings of the the heater?

SOLUTION

(a) $\dot{W}_{\text{ext}} = -\dot{W}_{\text{el,in}} = 2 \text{ kW};$

$$\frac{dE}{dt}^{0} = \dot{J}_{\text{net}}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{out}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -2 \text{ kW};$$

$$\eta_{\text{I-max}} \equiv \frac{\text{Desired energy output}}{\text{Required energy input}}; \quad \Rightarrow \eta_{\text{I-max}} = \frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{el,in}}}; \quad \Rightarrow \eta_{\text{I-max}} = \frac{2}{2};$$

$$\Rightarrow \eta_{\text{I-max}} = 100\%$$

(b) $$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i^{0} - \sum \dot{m}_e s_e^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}};$$

$$\dot{S}_{\text{gen,univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = -\frac{-2}{273+27}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.00667 \frac{\text{kW}}{\text{K}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = 6.67 \frac{\text{W}}{\text{K}}$$

(c) $$\dot{S}_{\text{gen, univ}} = 6.67 \frac{\text{kW}}{\text{K}};$$

$$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-2000}{273+150}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = \frac{2000}{423}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 4.728 \frac{\text{W}}{\text{K}};$$

$$\dot{S}_{\text{gen,ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 6.67 - 4.728; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 1.94 \frac{\text{W}}{\text{K}}$$

2-4-8 [LS] An electric adaptor for a notebook computer (converting 110 volts to 19 volts) operates 10°C warmer than the surroundings, which is at 20°C. If the output current is measured at 3 amps and heat is lost from the adapter at a rate of 10 W, determine (a) the energetic efficiency of the device, (b) the rate of internal entropy generation and (c) the rate of external entropy generation.

SOLUTION

(a) $\dot{W}_{ext} = \dot{W}_{el,out} - \dot{W}_{el,in}; \quad \Rightarrow \dot{W}_{ext} = \frac{(19)(3)}{1000} - \dot{W}_{el,in}; \quad [\text{kW}]$

$\dot{Q} = -\dot{Q}_{loss}; \quad \Rightarrow \dot{Q} = -\frac{10}{1000}; \quad [\text{kW}]$

The energy balance equation is given as:

$$\frac{dE}{dt}^{0} = \sum \dot{m}_i j_i^{0} - \sum \dot{m}_e j_e^{0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \dot{W}_{ext} = \dot{Q}; \quad \Rightarrow \dot{W}_{el,out} - \dot{W}_{el,in} = -\dot{Q}_{loss};$$

$$\Rightarrow \dot{W}_{el,in} = \dot{W}_{el,out} + \dot{Q}_{loss}; \quad \Rightarrow \dot{W}_{el,in} = \frac{(3)(19)}{1000} + 0.01; \quad \Rightarrow \dot{W}_{el,in} = 0.067 \text{ kW};$$

$$\eta_I \equiv \frac{\text{Desired energy output}}{\text{Required energy input}}; \quad \Rightarrow \eta_I = \frac{\dot{W}_{el,out}}{\dot{W}_{el,in}}; \quad \Rightarrow \eta_I = \frac{\left[\frac{(3)(19)}{1000}\right]}{0.067}; \quad \Rightarrow \eta_I = 0.85;$$

$$\Rightarrow \eta_I = 85\%$$

(b) The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i^{0} - \sum \dot{m}_e s_e^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

$$\dot{S}_{gen,\,int} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{gen,\,int} = -\frac{-0.01}{(273+20+10)}; \quad \Rightarrow \dot{S}_{gen,\,int} = 0.000033 \ \frac{\text{kW}}{\text{K}};$$

$$\Rightarrow \dot{S}_{gen,\,int} = 0.033 \ \frac{\text{W}}{\text{K}}$$

(c) $\dot{S}_{gen,\,univ} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{gen,\,univ} = -\frac{-0.01}{(273+20)}; \quad \Rightarrow \dot{S}_{gen,\,univ} = 0.0000341 \ \frac{\text{kW}}{\text{K}};$

$$\Rightarrow \dot{S}_{gen,\,univ} = 0.034 \ \frac{\text{W}}{\text{K}};$$

$$\dot{S}_{gen,\,ext} = \dot{S}_{gen,\,univ} - \dot{S}_{gen,\,int}; \quad \Rightarrow \dot{S}_{gen,\,ext} = 0.034 - 0.033; \quad \Rightarrow \dot{S}_{gen,\,ext} = 0.001 \ \frac{\text{W}}{\text{K}}$$

2-4-9 [LA] At steady state, the input shaft of a gearbox rotates at 2000 rpm while transmitting a torque of 0.2 kN-m. Due to friction, 1 kW of power is dissipated into heat and the rest is delivered to the output shaft. If the atmospheric temperature is 300 K and the surface of the gearbox maintains a constant temperature of 350 K, determine (a) the rate of entropy transfer into the atmosphere, (b) the rate of entropy generation in the system's universe, (c) the rate of entropy generation within the gearbox and (d) the rate of entropy generation in the immediate surroundings.

SOLUTION

(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{sh,out}} - \dot{W}_{\text{sh,in}}; \quad [\text{kW}]$

$\dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = -1; \quad [\text{kW}]$

The entropy balance equation is given as:

$$\frac{dS}{dt}^{0} = \sum \dot{m}_i s_i^{\,0} - \sum \dot{m}_e s_e^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

The rate of entropy transfer into air is give as:

$$\frac{\dot{Q}_{\text{loss}}}{T_B} = \frac{1}{300}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 0.00333\ \frac{\text{kW}}{\text{K}}; \quad \Rightarrow \frac{\dot{Q}_{\text{loss}}}{T_B} = 3.33\ \frac{\text{W}}{\text{K}}$$

(b) From the entropy balance equation:

$$\dot{S}_{\text{gen, univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = -\frac{-1}{300}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.00333\ \frac{\text{kW}}{\text{K}}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 3.33\ \frac{\text{W}}{\text{K}}$$

(c) $$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-1}{350}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 0.002857\ \frac{\text{kW}}{\text{K}}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 2.857\ \frac{\text{W}}{\text{K}}$$

(d) $$\dot{S}_{\text{gen,ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 3.33 - 2.857; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 0.476\ \frac{\text{W}}{\text{K}}$$

2-4-10 [LH] A gearbox (a closed steady system that converts low-torque shaft power to high-torque shaft power) consumes 100 kW of shaft work Due to lack of proper lubrication, the frictional losses amounts to 5 kW, resulting in an output power of 95 kW. The surface of the grearbox is measured to be 350 K while the surrounding temperature is 300 K. Determine (a) the first law efficiency (η) of the gearbox, (b) the heat transfer rate (c) the rate of entropy generation in kW/K in the system's universe, and (d) the external rate of entropy generation (entropy that is generated in the immediate surroundings of the gear box).

SOLUTION

(a) $\dot{W}_{\text{ext}} = \dot{W}_{\text{sh,out}} - \dot{W}_{\text{sh,in}}; \quad \Rightarrow \dot{W}_{\text{ext}} = 95 - 100; \quad \Rightarrow \dot{W}_{\text{ext}} = -5 \text{ kW};$

$$\eta_{\text{I}} \equiv \frac{\text{Desired energy output}}{\text{Required energy input}}; \quad \Rightarrow \eta_{\text{I}} = \frac{\dot{W}_{\text{el,out}}}{\dot{W}_{\text{el,in}}}; \quad \Rightarrow \eta_{\text{I}} = \frac{95}{100}; \quad \Rightarrow \eta_{\text{I}} = 0.95;$$

$$\Rightarrow \eta_{\text{I}} = 95\%$$

(b) The energy balance equation produces:

$$\frac{d\cancel{E}^{0}}{\cancel{d}t} = \sum \cancel{\dot{m}_i j_i}^{0} - \sum \cancel{\dot{m}_e j_e}^{0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{Q} = -\dot{Q}_{\text{loss}}; \quad \Rightarrow \dot{Q} = \dot{W}_{\text{ext}}; \quad \Rightarrow \dot{Q} = -5 \text{ kW}$$

(c) $$\frac{d\cancel{S}^{0}}{\cancel{d}t} = \sum \cancel{\dot{m}_i s_i}^{0} - \sum \cancel{\dot{m}_e s_e}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen};$$

$$\dot{S}_{\text{gen, univ}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = -\frac{-5}{300}; \quad \Rightarrow \dot{S}_{\text{gen, univ}} = 0.01667 \ \frac{\text{kW}}{\text{K}}$$

(d) $$\dot{S}_{\text{gen, int}} = -\frac{\dot{Q}}{T_B}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = -\frac{-5}{350}; \quad \Rightarrow \dot{S}_{\text{gen, int}} = 0.01428 \ \frac{\text{kW}}{\text{K}};$$

$$\dot{S}_{\text{gen,ext}} = \dot{S}_{\text{gen, univ}} - \dot{S}_{\text{gen, int}}; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 0.01667 - 0.01428; \quad \Rightarrow \dot{S}_{\text{gen,ext}} = 0.00238 \ \frac{\text{kW}}{\text{K}}$$

2-4-11 [LN] A closed steady system receives 1000 kW of heat from a reservoir at 1000 K and 2000 kW of heat from a reservoir at 2000 K. Heat is rejected to the two reservoirs at 300 K and 3000 K, respectively. (a) Determine the maximum amount of heat that can be transferred to the reservoir at 3000 K. (b) The device clearly transfers heat to a high temperature TER without directly consuming external work. Is this a violation of the Clausius statement of the second law of thermodynamics?(1:Yes; 2:No)

SOLUTION

(a) Designating the reservoirs with the suffixes 1, 2, 3, and 4 respectively, the energy balance equation for the closed-steady system can be written as:

$$\frac{dE}{dt}^{0} = \dot{J}_{\text{net}}^{0} + \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{Q}_4 - \dot{W}_{\text{ext}}^{0};$$

$$\Rightarrow \dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_4;$$

$$\Rightarrow \dot{Q}_3 = 3000 - \dot{Q}_4;$$

The entropy balance equation simplifies as:

$$\frac{dS}{dt}^{0} = \dot{S}_{\text{net}}^{0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_4}{T_4} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \frac{\dot{Q}_3}{T_3} + \frac{\dot{Q}_4}{T_4} = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \dot{S}_{\text{gen,univ}};$$

Substituting $\dot{Q}_3$ form the energy equation,

$$\Rightarrow \frac{3000 - \dot{Q}_4}{300} + \frac{\dot{Q}_4}{3000} = \frac{1000}{1000} + \frac{2000}{2000} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow 10 - \frac{\dot{Q}_4}{300} + \frac{\dot{Q}_4}{3000} = 2 + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{Q}_4 = \frac{-8 + \dot{S}_{\text{gen,univ}}}{\left(\frac{1}{3000} - \frac{1}{300}\right)} = 2666.67 - 333.33\dot{S}_{\text{gen,univ}};$$

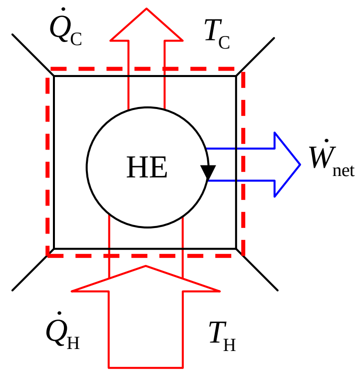
$$\Rightarrow \dot{Q}_{4,\max} = 2666.67 \text{ kW}$$

(b) 2: No. As long as entropy is not destroyed in the system's universe, the second law is not violated.

2-5-1 [LE] A steam power plant produces 500 MW of electricity with an overall thermal efficiency of 35%. Determine (a) the rate at which heat is supplied to the boiler, and (b) the waste heat that is rejected by the plant. (c) If the heating value of coal (heat that is realeased when 1 kg of coal is burned) is 30 MJ/kg, determine the rate of consumption of coal in tons(US)/day. Assume that 100% of heat released goes to the cycle. (d) ***What-if Scenario*** How would the fuel consumption rate change if the thermal efficiency were to increase to 40%?

SOLUTION

(a) The efficiency of a heat engine is defined as:



$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H};$$

$$\Rightarrow \dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \qquad \Rightarrow \dot{Q}_H = \frac{500}{0.35};$$

$$\Rightarrow \dot{Q}_H = 1429 \text{ MW}$$

(b) The energy balance equation is given as:

$$\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{J}_{net}}^{0} + \dot{Q} - \dot{W}_{ext} = \cancel{\dot{J}_{net}}^{0} + \left(\dot{Q}_H - \dot{Q}_C\right) - \left(\dot{W}_{net}\right);$$

$$\Rightarrow \dot{Q}_C = \dot{Q}_H - \dot{W}_{net}; \qquad \Rightarrow \dot{Q}_C = 1429 - 500; \qquad \Rightarrow \dot{Q}_C = 929 \text{ MW}$$

(c) The amount of coal (U.S. tons per day) is calculated as:

$$\dot{m}_F = \left(1429\frac{\text{MJ}}{\text{s}}\right)\left(\frac{1}{30}\frac{\text{kg}}{\text{MJ}}\right)\left(\frac{1}{907.2}\frac{\text{ton}}{\text{kg}}\right)\left(86400\frac{\text{s}}{\text{day}}\right); \qquad \Rightarrow \dot{m}_F = 4536.5\ \frac{\text{tons}}{\text{day}}$$

(d) With an increase in efficiency, a reduction in heat input is expected. T_H

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \qquad \Rightarrow \dot{Q}_H = \frac{500}{0.4}; \qquad \Rightarrow \dot{Q}_H = 1250 \text{ MW};$$

Since the fuel consumption is directly proportional to the heat input, the new rate of fuel consumption can be expressed as:

$$\dot{m}_F = (4536.5)\frac{0.35}{0.4}; \qquad \Rightarrow \dot{m}_F = 3969.4\ \frac{\text{tons}}{\text{day}}$$

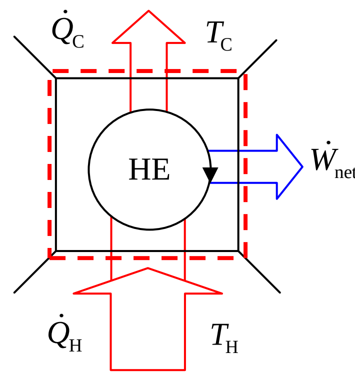
TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-2 [LI] A utility company charges its residential customers 12 cents/kW.h for electricity and $1.20 per Therm for natural gas. Fed up with the high cost of electricity, a customer decides to generate his own electricity by using a natural gas fired engine that has a thermal efficiency of 35%. Determine the fuel cost per kWh of electricity produced by the customer. Do you think electricity and natural gas are fairly priced by your utility company?

SOLUTION

The heat required to generate 1 kWh = 3.6 MJ of electricity can be calculated from the thermal efficiency.



$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \quad [\text{kW}]$$

$$\Rightarrow Q_H = \frac{W_{net}}{\eta_{th}}; \qquad \Rightarrow Q_H = \frac{3.6}{0.35};$$

$$\Rightarrow Q_H = 10.28 \text{ MJ};$$

Since 1 Therm = 105.5 MJ, the cost of heat to generate 1 kWh of electricity is:

$$\Rightarrow \frac{10.28}{105.5}(120) = 11.7 \text{ cents}$$

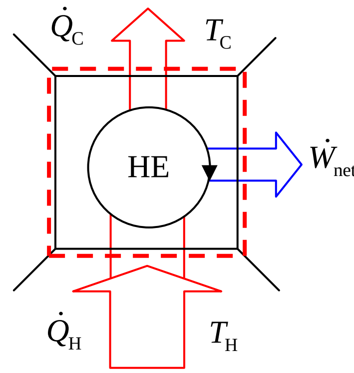
With only a 2.5% difference between the price of electricity and the cost to generate it, the electricity price seems to be fair.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-3 [LL] A sport utility vehicle with a thermal efficiency (η_{th}) of 20% produces 250 hp of engine output while traveling at a velocity of 80 mph. (a) Determine the rate of fuel consumption in kg/s if the heating value of the fuel is 43 MJ/kg. (b) If the density of the fuel is 800 kg/m^3, determine the fuel mileage of the vehicle in the unit of miles/gallon.

SOLUTION



Using the Engineering Converter TESTcalc we obtain:
250 hp = 186.4 kW;
80 mph = 35.76 m/s;
1 mile = 1609.3 m;
1 gallon = 0.003785 m^3;

(a) $\eta_{th} = \dfrac{\dot{W}_{net}}{\dot{Q}_H};$

$$\Rightarrow \dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}}; \quad \Rightarrow \dot{Q}_H = \frac{186.4}{0.2};$$

$$\Rightarrow \dot{Q}_H = 932 \text{ kW};$$

The fuel consumption rate is calculated as:

$$\left(932\frac{\text{kJ}}{\text{s}}\right)\left(\frac{1}{43000}\frac{\text{kg}}{\text{kJ}}\right) = 0.021677 \ \frac{\text{kg}}{\text{s}}$$

(b) 1 gallon of fuel is consumed in:

$$t = \frac{m_F}{\dot{m}_F}; \quad \Rightarrow t = \frac{\rho_F \mathcal{V}_F}{\dot{m}_F}; \quad \Rightarrow t = \frac{(800)(0.003785)}{0.021677}; \quad \Rightarrow t = 139.7 \text{ s};$$

The distance traveled by the vehicle in that time is:

$$x = Vt; \quad \Rightarrow x = (35.76)(139.7); \quad \Rightarrow x = 4995.2 \text{ m}; \quad \Rightarrow x = 3.1 \text{ mile};$$

Therefore, the fuel mileage $= 3.1 \ \dfrac{\text{miles}}{\text{gallon}}$

2-5-4 [LG] A truck engine consumes diesel at a rate of 30 L/h and delivers 65 kW of power to the wheels. If the fuel has a heating value of 43.5 MJ/kg and a density of 800 kg/m^3, determine (a) the thermal efficiency of the engine and (b) the waste heat rejected by the engine. (c) How does the engine discard the waste heat?

SOLUTION

(a) The heat input can be calculated as:

$$\dot{Q}_{\mathrm{H}} = \dot{m}_F; \quad (\text{Heating Value})$$

$$\Rightarrow \dot{Q}_{\mathrm{H}} = \rho_F \dot{V}_F; \quad \Rightarrow \dot{Q}_{\mathrm{H}} = (800)\left(\frac{30}{(1000)(3600)}\right)(43500); \quad \Rightarrow \dot{Q}_{\mathrm{H}} = 290 \text{ kW};$$

$$\eta_{th} = \frac{\dot{W}_{\mathrm{net}}}{\dot{Q}_{\mathrm{H}}}; \quad \Rightarrow \eta_{th} = \frac{65}{290}; \quad \Rightarrow \eta_{th} = 22.4\%$$

(b) $\dot{Q}_{\mathrm{C}} = \dot{Q}_{\mathrm{H}} - \dot{W}_{\mathrm{net}}; \quad \Rightarrow \dot{Q}_{\mathrm{C}} = 290 - 65; \quad \Rightarrow \dot{Q}_{\mathrm{C}} = 225 \text{ kW}$

(c) The waste heat is picked up by circulating water around the engine. The warm water is cooled in the radiator where the heat is ultimately rejected into the atmosphere. In cold climates, some of this waste heat can be recycled by reusing it to regulate the cab temperature.

2-5-5 [LZ] Determine the rate of coal consumption by a thermal power plant with a power output of 350 MW in tons/hr. The thermal efficiency (η_{th}) of the plant is 35% and the heating value of the coal is 30 MJ/kg.

SOLUTION

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}; \qquad \Rightarrow \dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}};$$

$$\Rightarrow \dot{Q}_H = \frac{350}{0.35}; \qquad \Rightarrow \dot{Q}_H = 1000 \text{ MW};$$

The rate of coal consumption can now be calculated as:

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating Value})}; \qquad \Rightarrow \dot{m}_F = \frac{1000}{30}\,\frac{\text{kg}}{\text{s}}; \qquad \Rightarrow \dot{m}_F = \frac{(1000)(3600)}{(30)(907.2)}\,\frac{\text{tons}}{\text{hr}};$$

$$\Rightarrow \dot{m}_F = 132.275\,\frac{\text{tons}}{\text{hr}}$$

2-5-6 [LK] In 2003 the United States generated 3.88 trillion kWh of electricity, 51% of which came from coal-fired power plants. (a) Assuming an average thermal efficiency of 34% and the heating value of coal as 30 MJ/kg, determine the coal consumption in 2003 in tons. (b) What-if Scenario: What would the coal consumption be if the average thermal efficiency were 35% instead?

SOLUTION

(a) Electricity generated from coal:

$$W_{net} = (0.51)(3.88\times10^{12}); \quad \Rightarrow W_{net} = 1.98\times10^{12} \text{ kWh}; \quad \Rightarrow W_{net} = (3.6)(1.98\times10^{12}) \text{ MJ};$$

$$\Rightarrow W_{net} = 7.128\times10^{12} \text{ MJ};$$

The required heat input:

$$Q_H = \frac{W_{net}}{\eta_{th}}; \quad \Rightarrow Q_H = \frac{7.128\times10^{12}}{0.34}; \quad \Rightarrow Q_H = 20.96\times10^{12} \text{ MJ};$$

The required coal consumed in 2003:

$$m_F = \frac{Q_H}{(\text{Heating Value})}; \quad \Rightarrow m_F = \frac{20.96\times10^{12}}{30} \text{ kg}; \quad \Rightarrow m_F = \frac{20.96\times10^{12}}{(30)(907.2)} \text{ tons};$$

$$\Rightarrow m_F = 7.7\times10^{8} \text{ tons}$$

(b) If the thermal efficiency was 35% instead:

$$\Rightarrow Q_H = \frac{W_{net}}{\eta_{th}}; \quad \Rightarrow Q_H = \frac{7.128\times10^{12}}{0.35}; \quad \Rightarrow Q_H = 20.365\times10^{12} \text{ MJ};$$

$$m_F = \frac{Q_H}{(\text{Heating Value})}; \quad \Rightarrow m_F = \frac{20.365\times10^{12}}{30} \text{ kg}; \quad \Rightarrow m_F = \frac{20.365\times10^{12}}{(30)(907.2)} \text{ tons};$$

$$\Rightarrow m_F = 7.48\times10^{8} \text{ tons}$$

2-5-7 [LP] Determine the fuel cost per kWh of electricity produced by a heat engine with a thermal efficiency of 40% if it uses diesel as the source of heat. The following data is supplied for diesel: price = \$2.00 per gallon; heating value = 42.8 MJ/kg; density = 850 kg/m^3.

SOLUTION

$$W_{\text{net}} = 1\text{ kWh}; \quad \Rightarrow W_{\text{net}} = 3.6\text{ MJ};$$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{th} = \frac{W_{\text{net}}}{Q_{\text{H}}};$$

$$\Rightarrow Q_{\text{H}} = \frac{W_{\text{net}}}{\eta_{th}}; \quad \Rightarrow Q_{\text{H}} = \frac{3.6}{0.4}; \quad \Rightarrow Q_{\text{H}} = 9\text{ MJ};$$

$$m_{\text{F}} = \frac{Q_H}{(\text{Heating value})}; \quad \Rightarrow m_{\text{F}} = \frac{9}{42.8}; \quad \Rightarrow m_{\text{F}} = 0.21\text{ kg};$$

$$V_{\text{F}} = \frac{m_F}{\rho_F}; \quad \Rightarrow V_{\text{F}} = \frac{0.21}{850}\text{ m}^3; \quad \Rightarrow V = \frac{0.21}{850}(264.2)\text{ gallon}; \quad \Rightarrow V = 0.0653\text{ gallon};$$

The fuel cost:

$$\Rightarrow (0.0653)(200) = 13.1\ \frac{\text{cents}}{\text{kWh}}$$

2-5-8 [LU] A gas turbine with a thermal efficiency (η_{th}) of 21% develops a power output of 8 MW. Determine (a) the fuel consumption rate in kg/min if the heating value of the fuel is 50 MJ/kg. (b) If the maximum temperature achieved during the combustion of diesel is 1700 K, determine the maximum thermal efficiency possible. Assume the atmospheric temperature to be 300 K.

SOLUTION

(a) $\dot{W}_{\text{net}} = 8 \text{ MW};$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{th}};$$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{8}{0.21}; \qquad \Rightarrow \dot{Q}_{\text{H}} = 38.09 \text{ MW};$$

$$\dot{m}_{\text{F}} = \frac{Q_H}{(\text{Heating value})}; \qquad \Rightarrow \dot{m}_{\text{F}} = \frac{38.09}{50}\ \frac{\text{kg}}{\text{s}}; \qquad \Rightarrow \dot{m}_{\text{F}} = \frac{38.09}{50}(60)\ \frac{\text{kg}}{\text{min}};$$

$$\Rightarrow \dot{m}_{\text{F}} = 45.72\ \frac{\text{kg}}{\text{min}}$$

(b) The maximum efficiency can be found using the Carnot efficiency model:

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H}; \qquad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{1700}; \qquad \Rightarrow \eta_{\text{th, Carnot}} = 0.8235;$$

$$\Rightarrow \eta_{\text{th, Carnot}} = 82.35\%$$

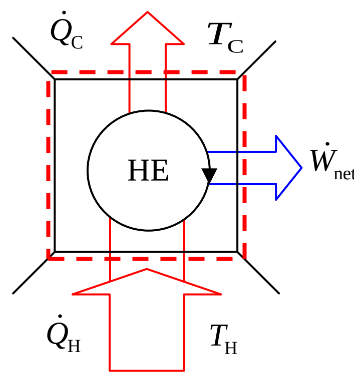
2-5-9 [LX] Two different fuels are being considered for a 1 MW (net output) heat engine which can operate between the highest temperature produced during the burning of the fuel and the atmospheric temperature of 300 K. Fuel A burns at 2500 K, delivering 50 MJ/kg (heating value) and costs $2 per kilogram. Fuel B burns at 1500 K, delivering 40 MJ/kg and costs $1.50 per kilogram. Determine the minimum fuel cost per hour for (a) fuel A and (b) fuel B.

SOLUTION

(a) For Fuel A: The minimum cost per hour will be achieved when the heat engine is at a maximum or Carnot efficiency.

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_{C,A}}{T_{H,A}}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{2500};$$

$$\Rightarrow \eta_{\text{th, Carnot}} = 0.88; \quad \Rightarrow \eta_{\text{th, Carnot}} = 88\%;$$



$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = \eta_{\text{th, Carnot}} = 0.88;$$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th, Carnot}}}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{1}{0.88}; \quad \Rightarrow \dot{Q}_{\text{H}} = 1.136 \text{ MW};$$

The cost per hour of fuel A can now be calculated as:

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{1.136}{50}\frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = \frac{(1.136)(3600)}{50}\frac{\text{kg}}{\text{h}};$$

$$\frac{\text{cost}}{\text{hour}} = \frac{(1.136)(3600)}{50}(2); \quad \Rightarrow \frac{\text{cost}}{\text{hour}} = \frac{\$163.58}{\text{h}}$$

(b) For Fuel B: The same process can be used to determine the cost per hour of fuel B.

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_{C,B}}{T_{H,B}}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{1500}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.8; \quad \Rightarrow \eta_{\text{th, Carnot}} = 80\%;$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = \eta_{\text{th, Carnot}} = 0.8;$$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{1}{0.8}; \quad \Rightarrow \dot{Q}_{\text{H}} = 1.25 \text{ MW};$$

The cost per hour of fuel B can now be calculated as:

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{1.25}{40}\frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = \frac{(1.25)(3600)}{40}\frac{\text{kg}}{\text{h}};$$

$$\frac{\text{cost}}{\text{hour}} = \frac{(1.25)(3600)}{40}(1.50); \quad \Rightarrow \frac{\text{cost}}{\text{hour}} = \frac{\$168.75}{\text{h}}$$

2-5-10 [LC] A heat engine receives heat from a source at 2000 K at a rate of 500 kW, and rejects the waste heat to a medium at 300 K. The net output from the engine is 300 kW. (a) Determine the maximum power that could be generated by the engine for the same heat input. (b) Determine the thermal efficiency of the engine.

SOLUTION

(a) The maximum efficiency is the Carnot efficiency given as:

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{300}{200}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.85; \quad \Rightarrow \eta_{\text{th, Carnot}} = 85\%;$$

$$\eta_{\text{th, Carnot}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H}} \eta_{\text{th, Carnot}};$$

$$\Rightarrow \dot{W}_{\text{net}} = (500)(0.85); \quad \Rightarrow \dot{W}_{\text{net}} = 425 \text{ kW}$$

(b) The actual thermal efficiency of this device can be calculated from the given data as:

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = \frac{300}{500}; \quad \Rightarrow \eta_{\text{th}} = 0.6; \quad \Rightarrow \eta_{\text{th}} = 60\%$$

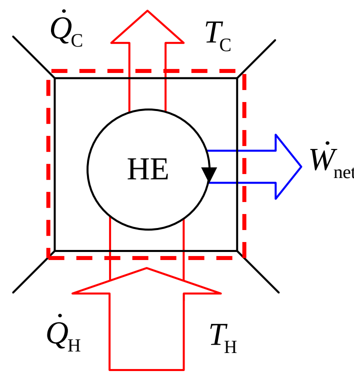
2-5-11 [LV] A Carnot heat engine with a thermal efficiency of 60% receives heat from a source at a rate of 3000 kJ/min, and rejects the waste heat to a medium at 300 K. Determine (a) the power that is generated by the engine and (b) the source temperature.

SOLUTION

(a) Knowing the efficiency and heat input of this Carnot heat engine, the power generated by this engine can be easily calculated.

$$\eta_{\text{th, Carnot}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \qquad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H}} \eta_{\text{th, Carnot}};$$

$$\Rightarrow \dot{W}_{\text{net}} = \left(\frac{3000}{60}\right)(0.6); \qquad \Rightarrow \dot{W}_{\text{net}} = 30 \text{ kW}$$



(b) Using the Carnot formula:

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H};$$

$$\Rightarrow T_H = \frac{T_C}{1-\eta_{\text{th, Carnot}}}; \qquad \Rightarrow T_H = \frac{300}{1-0.6}; \qquad \Rightarrow T_H = 750 \text{ K}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-12 [LQ] A heat engine operates between a reservoir at 2000 K and an ambient temperature of 300 K. It produces 10 MW of shaft power. If it has a thermal efficiency (η_{th}) of 40%, (a) determine the rate of fuel consumption in kg/h if the heating value of the fuel is 40 MJ/kg. (b) If the heat engine is replaced by the most efficient engine possible, what is the minimum possible fuel consumption rate for the same power output?

SOLUTION

(a) $\eta_{\text{th}} = \dfrac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = 0.4;$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{10}{0.4}; \quad \Rightarrow \dot{Q}_{\text{H}} = 25 \text{ MW};$$

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{25}{40}\frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = \frac{(25)(3600)}{40}\frac{\text{kg}}{\text{h}};$$

$$\Rightarrow \dot{m}_F = 2250 \ \frac{\text{kg}}{\text{h}}$$

(b) $\eta_{\max} = \eta_{\text{th,Carnot}}; \quad \Rightarrow \eta_{\max} = 1 - \dfrac{T_C}{T_H}; \quad \Rightarrow \eta_{\max} = 1 - \dfrac{300}{2000}; \quad \Rightarrow \eta_{\max} = 0.85;$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{10}{0.85}; \quad \Rightarrow \dot{Q}_{\text{H}} = 11.76 \text{ MW};$$

$$\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{11.76}{40}\frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = \frac{(11.76)(3600)}{40}\frac{\text{kg}}{\text{h}};$$

$$\Rightarrow \dot{m}_F = 1058.8 \ \frac{\text{kg}}{\text{h}}$$

TEST Solution:
Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-13 [LT] A heat engine, operating between two reservoirs at 1500 K and 300 K, produces an output of 100 MW. If the thermal efficiency of the engine is measured at 50%, determine

(a) The Carnot efficiency of the engine (in percent),

(b) the rate of heat transfer into the engine from the hot source in MW,

(c) the rate of fuel consumption (in kg/s) if the heating value of fuel is 40 MJ/kg.

SOLUTION

(a) $\eta_{\text{th,Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th,Carnot}} = 1 - \frac{300}{1500}; \quad \Rightarrow \eta_{\text{th,Carnot}} = 80\%$

(b) $\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{\text{th}} = 0.5;$

$$\Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{100}{0.5}; \quad \Rightarrow \dot{Q} = 200 \text{ MW}$$

(c) $\dot{m}_F = \frac{\dot{Q}_H}{(\text{Heating value})}; \quad \Rightarrow \dot{m}_F = \frac{200}{40}; \quad \Rightarrow \dot{m}_F = 5 \frac{\text{kg}}{\text{s}}$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-14 [LY] A heat engine receives heat from two reservoirs: 50 MW from a reservoir at 500 K and 100 MW from a reservoir at 1000 K. If it rejects 90 MW to atmosphere at 300 K, (a) determine the thermal efficiency of the engine. (b) Calculate the entropy generated in kW/K in the engine's universe.

SOLUTION

(a) $\dot{Q}_{\text{in}} = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_{\text{in}} = 50 + 100; \quad \Rightarrow \dot{Q}_{\text{in}} = 150 \text{ MW};$

$\dot{W}_{\text{net}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}; \quad \Rightarrow \dot{W}_{\text{net}} = 150 - 90; \quad \Rightarrow \dot{W}_{\text{net}} = 60 \text{ MW};$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}; \quad \Rightarrow \eta_{th} = \frac{60}{150}; \quad \Rightarrow \eta_{th} = 40\%$$

(b) $$\frac{dS}{dt}^{0} = \dot{S}_{\text{net}}^{0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_1}{T_1}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{90}{300} - \frac{100}{1000} - \frac{50}{500}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = 0.1 \frac{\text{MW}}{\text{K}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = 100 \frac{\text{kW}}{\text{K}}$$

2-5-15 [LF] A heat engine produces 1000 kW of power while receiving heat from two reservoirs: 1000 kW from a 1000 K source and 2000 kW from a 2000 K source. Heat is rejected to the atmosphere at 300 K. Determine (a) the waste heat (heat rejected) in kW. (b)the entropy generation rate in kW/K in the system's universe.

SOLUTION

(a) $\dot{Q}_{\text{in}} = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_{\text{in}} = 1000 + 2000; \quad \Rightarrow \dot{Q}_{\text{in}} = 3000 \text{ kW};$

$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{out}} = 3000 - 1000; \quad \Rightarrow \dot{Q}_{\text{out}} = 2000 \text{ kW}$

(b) $\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{S}_{\text{gen,univ}};$

$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_1}{T_1}; \quad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{2000}{300} - \frac{2000}{2000} - \frac{1000}{1000};$

$\Rightarrow \dot{S}_{\text{gen,univ}} = 4.67 \frac{\text{kW}}{\text{K}}$

2-5-16 [LD] A heat engine, operating between two reservoirs at 1500 K and 300 K, produces 150 kW of net power. If the rate of heat transfer from the hot reservoir to the engine is measured at 350 kW, determine (a) the thermal efficiency of the engine, (b) the rate of fuel consumption in kg/h to maintain the hot reservoir at steady state (assume the heating value of the fuel to be 45 MJ/kg), (c) the minimum possible fuel consumption rate (in kg/h) for the same output.

SOLUTION

(a) $\eta_{th} = \frac{W_{\text{net}}}{\dot{Q}_{\text{in}}}; \quad \Rightarrow \eta_{th} = \frac{150}{350}; \quad \Rightarrow \eta_{th} = 42.85\%$

(b) $\dot{m}_F = \frac{\dot{Q}_{\text{in}}}{\text{Heating value}}; \quad \Rightarrow \dot{m}_F = \frac{350}{45000}; \quad \Rightarrow \dot{m}_F = 0.00778 \frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_F = 28 \frac{\text{kg}}{\text{h}}$

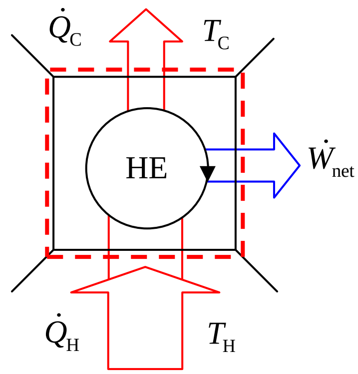
(c) $\eta_{\max} = \eta_{th,\text{Carnot}}; \quad \Rightarrow \eta_{\max} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\max} = 1 - \frac{300}{1500}; \quad \Rightarrow \eta_{\max} = 80\%;$

$Q_{\text{in,min}} = \frac{W_{\text{net}}}{\eta_{\max}}; \quad \Rightarrow Q_{\text{in,min}} = \frac{150}{0.8}; \quad \Rightarrow Q_{\text{in,min}} = 187.5 \text{ kW};$

$\dot{m}_{F,\min} = \frac{\dot{Q}_{in,\min}}{\text{Heating Value}}; \quad \Rightarrow \dot{m}_{F,\min} = \frac{187.5}{45000}; \quad \Rightarrow \dot{m}_{F,\min} = 0.00417 \frac{\text{kg}}{\text{s}}; \quad \Rightarrow \dot{m}_{F,\min} = 15 \frac{\text{kg}}{\text{h}}$

2-5-17 [LM] A solar-energy collector produces a maximum temperature of 100°C. The collected energy is used in a cyclic heat engine that operates in a 5°C environment. (a) What is the maximum thermal efficiency? (b) What-if Scenario: What would the maximum efficiency be if the collector were redesigned to focus the incoming light to enhance the maximum temperature to 400°C?

SOLUTION



(a) $\eta_{max} = \eta_{th,\ Carnot};\quad \Rightarrow \eta_{max} = 1 - \frac{T_C}{T_H};$

$$\Rightarrow \eta_{max} = 1 - \frac{5+273}{100+273};\quad \Rightarrow \eta_{max} = 0.255;$$

$$\Rightarrow \eta_{max} = 25.5\%$$

(b) With the maximum temperature increased to 400°C:

$$\eta_{th,\ Carnot} = 1 - \frac{T_C}{T_H};\quad \Rightarrow \eta_{th,\ Carnot} = 1 - \frac{5+273}{400+273};$$

$$\Rightarrow \eta_{th,\ Carnot} = 0.587;\quad \Rightarrow \eta_{th,\ Carnot} = 58.7\%$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-18 [LJ] The Ocean Thermal Energy Conversion (OTEC) system in Hawaii utilizes the surface water and deep water as thermal energy reservoirs. Assume the ocean temperature at the surface to be 20°C and at some depth to be 5°C; determine (a) the maximum possible thermal efficiency achievable by a heat engine. (b) What-if Scenario: What would the maximum efficiency be if the surface water temperature increased to 25°C?

SOLUTION

(a) $\eta_{\max} = \eta_{\text{th, Carnot}}; \quad \Rightarrow \eta_{\max} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\max} = 1 - \frac{5+273}{20+273}; \quad \Rightarrow \eta_{\max} = 0.0512;$

$\Rightarrow \eta_{\max} = 5.12\%$

(b) With the surface water temperature increased to 25°C:

$\eta_{\text{th, Carnot}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 1 - \frac{5+273}{25+273}; \quad \Rightarrow \eta_{\text{th, Carnot}} = 0.0671;$

$\Rightarrow \eta_{\text{th, Carnot}} = 6.71\%$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-19 [LW] You have been hired by a venture capitalist to evaluate a concept engine proposed by an inventor, who claims that the engine consumes 100 MW at a temperature of 500 K, rejects 40 MW at a temperature of 300 K, and delivers 50 MW of mechanical work. Does this claim violates the first law of thermodynamics (answer 1), second law (answer 2), both (answer 3), or none (answer 4)?

SOLUTION

A heat engine, by definition, must operate at steady state. The energy balance equation for this engine reduces to:

$$\frac{dE}{dt}^{0} = \sum \dot{m}_i j_i^{\,0} - \sum \dot{m}_e j_e^{\,0} + \dot{Q} - \dot{W}_{\text{ext}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \dot{Q};$$

$$\Rightarrow \dot{W}_{\text{net}} = \left(\dot{Q}_{\text{H}} - \dot{Q}_{\text{C}}\right); \qquad \Rightarrow \dot{W}_{\text{net}} = 100 - 40; \qquad \Rightarrow \dot{W}_{\text{net}} = 60 \text{ MW};$$

The engine, by producing only 50 MW of power violates the energy balance, that is, the first law of thermodynamics.

The entropy balance equation can be simplified as:

$$\frac{dS}{dt}^{0} = \dot{S}_{\text{net}}^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}; \qquad \Rightarrow \frac{dS}{dt}^{0} = \dot{S}_{\text{net}}^{\,0} + \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{\text{gen,univ}};$$

$$\Rightarrow \dot{S}_{\text{gen,univ}} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = \frac{40}{300} - \frac{100}{500}; \qquad \Rightarrow \dot{S}_{\text{gen,univ}} = -0.0667 \ \frac{\text{kW}}{\text{K}};$$

A negative entropy generation means that the engine violates the second law of thermodynamics.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat engine radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-20 [GR] A heat engine produces 40 kW of power while consuming 40 kW of heat from a source at 1200 K, 50 kW of heat from a source at 1500 K, and rejecting the waste heat to atmosphere at 300 K. Determine (a) the thermal efficiency of the engine. (b) What-if Scenario: What would the thermal efficiency be if all the irreversibility could be magically eliminated? Assume no change in heat input from the two sources.

SOLUTION

(a) $\dot{Q}_{\text{in}} = \dot{Q}_1 + \dot{Q}_2; \quad \Rightarrow \dot{Q}_{\text{in}} = 40 + 50; \quad \Rightarrow \dot{Q}_{\text{in}} = 90 \text{ kW};$

$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{out}} = 90 - 40; \quad \Rightarrow \dot{Q}_{\text{out}} = 50 \text{ kW};$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{th} = \frac{40}{90}; \quad \Rightarrow \eta_{th} = 0.444; \quad \Rightarrow \eta_{th} = 44.4\%$$

(b) The entropy balance equation for the reversible engine produces the heat rejected to atmosphere:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_{\text{out}}}{T_0} + \cancel{\dot{S}_{\text{gen}}}^{0,\ \text{reversible}};$$

$$\Rightarrow \dot{Q}_{\text{out}} = T_0\left(\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2}\right); \quad \Rightarrow \dot{Q}_{\text{out}} = 300\left(\frac{40}{1200} + \frac{50}{1500}\right); \quad \Rightarrow \dot{Q}_{\text{out}} = 20 \text{ kW};$$

With the heat input unchanged, the net work for this engine can be obtained from the energy balance equation:

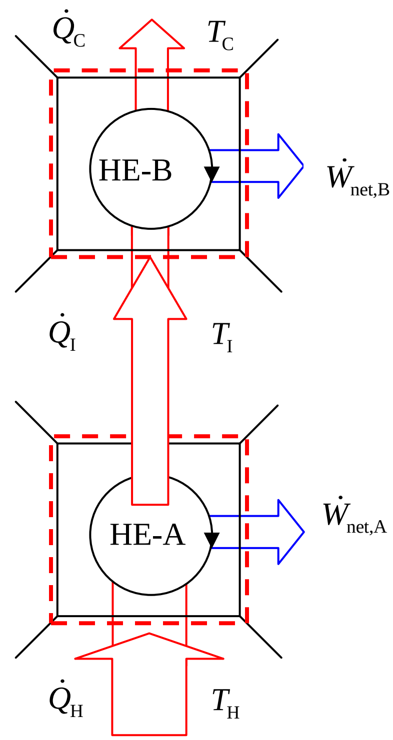
$\dot{W}_{\text{net}} = 90 - \dot{Q}_{\text{out}}; \quad \Rightarrow \dot{W}_{\text{net}} = 90 - 20; \quad \Rightarrow \dot{W}_{\text{net}} = 70 \text{ kW};$

The efficiency of this reversible engine:

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H}}}; \quad \Rightarrow \eta_{th} = \frac{70}{(40+50)}; \quad \Rightarrow \eta_{th} = 77.7\%$$

2-5-21 [GO] Two reversible engines A and B are arranged in series with the waste heat of engine A used to drive engine B. Engine A receives 200 MJ from a hot source at a temperature of 420°C. Engine B is in communication with a heat sink at a temperature of 4.4°C. If the work output of A is twice that of B, determine (a) the intermediate temperature between A and B, and (b) the thermal efficiency of each engine.

SOLUTION



(a) In terms of the unknown intermediate reservoir temperature T_I:

$$\dot{W}_{net,A} = \eta_{th,A}\dot{Q}_H; \quad \Rightarrow \dot{W}_{net,A} = \left(1-\frac{T_I}{T_H}\right)\dot{Q}_H;$$

$$\dot{W}_{net,B} = \eta_{th,B}\dot{Q}_I; \quad \Rightarrow \dot{W}_{net,B} = \left(1-\frac{T_C}{T_I}\right)\dot{Q}_I;$$

$$\Rightarrow \dot{W}_{net,B} = \left(1-\frac{T_C}{T_I}\right)\left(\dot{Q}_H - \dot{W}_{net,A}\right);$$

$$\dot{W}_{net,A} = 2\dot{W}_{net,B};$$

$$\Rightarrow \eta_{th,A}\dot{Q}_H = 2\eta_{th,B}\left(\dot{Q}_H - \dot{W}_{net,A}\right);$$

$$\Rightarrow \dot{Q}_H\left(2\eta_{th,B} - \eta_{th,A}\right) = 2\eta_{th,B}\dot{W}_{net,A};$$

$$\Rightarrow \dot{Q}_H\left(2\eta_{th,B} - \eta_{th,A}\right) = 2\eta_{th,B}\eta_{th,A}\dot{Q}_H;$$

$$\Rightarrow 2\left(1-\frac{T_C}{T_I}\right) - \left(1-\frac{T_I}{T_H}\right) = 2\left(1-\frac{T_C}{T_I}\right)\left(1-\frac{T_I}{T_H}\right);$$

$$\Rightarrow 1 - 2\frac{T_C}{T_I} + \frac{T_I}{T_H} = 2 - 2\frac{T_C}{T_I} - 2\frac{T_I}{T_H} + 2\frac{T_C}{T_H};$$

$$\Rightarrow 3\frac{T_I}{T_H} = 1 + 2\frac{T_C}{T_H}; \quad \Rightarrow T_I = \frac{T_H}{3} + \frac{2T_C}{3};$$

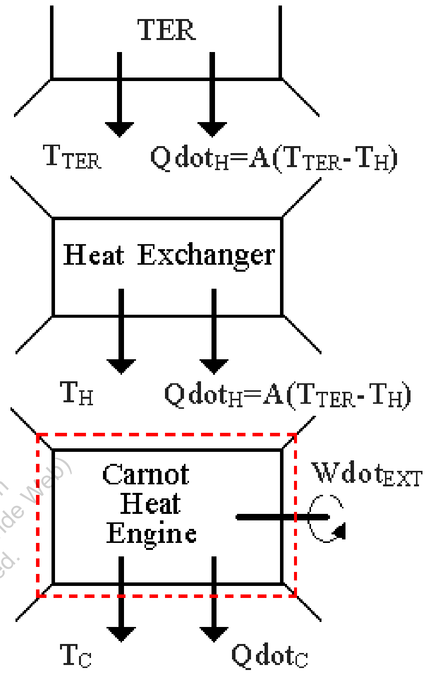
$$\Rightarrow T_I = \frac{(420+273)+2(4.4+273)}{3}; \quad \Rightarrow T_I = 415.9 \text{ K}$$

(b.1) $$\eta_{th,A} = \frac{\dot{W}_{net,A}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th,A} = 1-\frac{T_I}{T_H}; \quad \Rightarrow \eta_{th,A} = 1-\frac{415.9}{(420+273)}; \quad \Rightarrow \eta_{th,A} = 40\%$$

(b.2) $$\eta_{th,B} = \frac{\dot{W}_{net,B}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th,B} = 1-\frac{T_C}{T_I}; \quad \Rightarrow \eta_{th,B} = 1-\frac{(4.4+273)}{415.9}; \quad \Rightarrow \eta_{th,B} = 33.3\%$$

2-5-22 [GB] A Carnot heat engine receives heat from a TER at T_{TER} through a heat exchanger where the heat transfer rate is proportional to the temperature difference as $Q_H = A(T_{TER}-T_H)$. It rejects heat to a cold reservoir at T_C. If the heat engine is to maximize the work output, show that the high temperature in the cycle should be selected as $T_H = sqrt(T_{TER}T_C)$.

SOLUTION



$$\dot{W}_{net} = \dot{Q}_H \eta_{th,Carnot}; \qquad \Rightarrow \dot{W}_{net} = \dot{Q}_H\left(1-\frac{T_C}{T_H}\right);$$

$$\Rightarrow \dot{W}_{net} = \dot{Q}_H - \dot{Q}_H\frac{T_C}{T_H};$$

To increase $\dot{Q}_H = A(T_{TER} - T_H)$, T_H must be reduced, which reduces the thermal efficiency. To maximize $\dot{W}_{net}$, we express it as a function of T_H.

$$\dot{W}_{net} = \dot{Q}_H\left(1-\frac{T_C}{T_H}\right); \qquad \Rightarrow \dot{W}_{net} = A(T_{TER}-T_H)\left(1-\frac{T_C}{T_H}\right);$$

$$\Rightarrow \dot{W}_{net} = AT_{TER} - AT_H - AT_{TER}\frac{T_C}{T_H} + AT_C;$$

To optimize the work output, the derivative expression is set equal to zero.

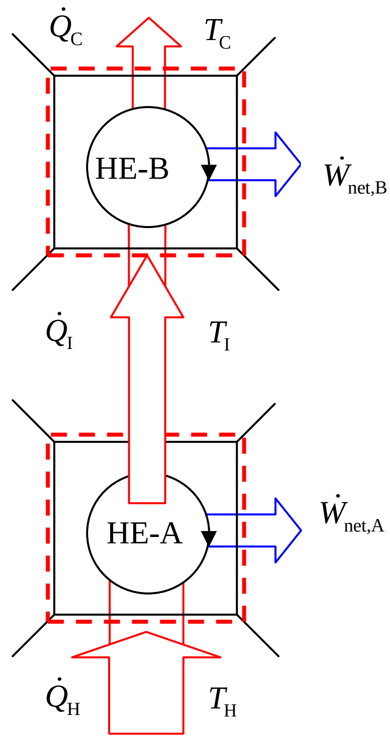
$$\frac{d\dot{W}_{net}}{dT_H} = 0 - A + \frac{AT_{TER}T_C}{(T_H)^2};$$

$$\Rightarrow 0 = -A + \frac{AT_{TER}T_C}{(T_H)^2};$$

$$\Rightarrow T_H = \sqrt{T_{TER}T_C}$$

2-5-23 [GS] Two Carnot engines operate in series. The first one receives heat from a TER at 2500 K and rejects the waste heat to another TER at a temperature *T*. The second engine receives this energy rejected by the first one, converts some of it to work, and rejects the rest to a TER at 300 K. If the thermal efficiency of both the engines are the same, (a) determine the temperature (*T*). What-if Scenario: (b) What would the temperature be if the two engines produced the same output instead?

SOLUTION



(a) Equating the Carnot efficiencies of these two heat engines yields the intermediate temperature *T*.

$$\eta_{\text{th, Carnot, A}} = \eta_{\text{th, Carnot, B}};$$

$$\Rightarrow 1-\frac{T_I}{T_H} = 1-\frac{T_B}{T_I};$$

$$\Rightarrow T_I = \sqrt{T_C T_H}; \qquad \Rightarrow T_I = \sqrt{(300)(2500)};$$

$$\Rightarrow T_I = 866 \text{ K}$$

(b) If the work output are the same, we obtain:

$$\dot{W}_{\text{net,A}} = \eta_{\text{th,A}}\dot{Q}_H; \qquad \Rightarrow \dot{W}_{\text{net,A}} = \left(1-\frac{T_I}{T_H}\right)\dot{Q}_H;$$

$$\dot{W}_{\text{net,B}} = \eta_{\text{th,B}}\dot{Q}_I; \qquad \Rightarrow \dot{W}_{\text{net,B}} = \left(1-\frac{T_C}{T_I}\right)\dot{Q}_I;$$

$$\Rightarrow \dot{W}_{\text{net,B}} = \left(1-\frac{T_C}{T_I}\right)\left(\dot{Q}_H - \dot{W}_{\text{net,A}}\right);$$

$$\dot{W}_{\text{net,A}} = \dot{W}_{\text{net,B}};$$

$$\Rightarrow \eta_{\text{th,A}}\dot{Q}_H = \eta_{\text{th,B}}\left(\dot{Q}_H - \dot{W}_{\text{net,A}}\right);$$

$$\Rightarrow \dot{Q}_H\left(\eta_{\text{th,B}} - \eta_{\text{th,A}}\right) = \eta_{\text{th,B}}\dot{W}_{\text{net,A}}; \qquad \Rightarrow \dot{Q}_H\left(\eta_{\text{th,B}} - \eta_{\text{th,A}}\right) = \eta_{\text{th,B}}\eta_{\text{th,A}}\dot{Q}_H;$$

$$\Rightarrow \left(1-\frac{T_C}{T_I}\right) - \left(1-\frac{T_I}{T_H}\right) = \left(1-\frac{T_C}{T_I}\right)\left(1-\frac{T_I}{T_H}\right);$$

$$\Rightarrow -\frac{T_C}{T_I} + \frac{T_I}{T_H} = 1 - \frac{T_C}{T_I} - \frac{T_I}{T_H} + \frac{T_C}{T_H}; \qquad \Rightarrow 2\frac{T_I}{T_H} = 1 + \frac{T_C}{T_H};$$

$$\Rightarrow T_I = \frac{T_H}{2} + \frac{T_C}{2}; \qquad \Rightarrow T_I = \frac{2500+300}{2}; \qquad \Rightarrow T_I = 1400 \text{ K}$$

2-5-24 [GA] A reversible heat engine operates in outer space. The only way heat can be rejected is by radiation, which is proportional to the fourth power of the temperature and the area of the radiating surface. Show that for a given power output and a given source temperature (T_1), the area of the radiator is minimized when the radiating surface temperature is $T_2 = 0.75\, T_1$.

SOLUTION

$$\dot{W}_{\text{net}} = \dot{Q}_{\text{H}} - \dot{Q}_{\text{C}};$$

For the reversible engine the entropy balance equation produces:

$$\cancel{\frac{dS}{dt}}^{0} = \cancel{\dot{S}_{\text{net}}}^{0} + \frac{\dot{Q}_H}{T_1} - \frac{\dot{Q}_C}{T_2} + \cancel{\dot{S}_{\text{gen,univ}}}^{0}; \qquad \Rightarrow \dot{Q}_{\text{H}} = \frac{T_1}{T_2}\dot{Q}_{\text{C}};$$

Therefore,

$$\dot{W}_{\text{net}} = \frac{T_1}{T_2}\dot{Q}_{\text{C}} - \dot{Q}_{\text{C}}; \qquad \Rightarrow \dot{W}_{\text{net}} = \left(\frac{T_1}{T_2} - 1\right)\dot{Q}_{\text{C}};$$

$$\Rightarrow \dot{W}_{\text{ext}} = \left(\frac{T_1}{T_2} - 1\right) kAT_2^4;$$

For minimum area:

$$\frac{dA}{dT_2} = 0;$$

$$A = \frac{\dot{W}_{\text{ext}}}{(T_1 - T_2)kT_2^3};$$

$$\frac{dA}{dT_2} = -\left(\frac{\dot{W}_{\text{ext}}}{(T_1 - T_2)kT_2^3}\right)\frac{d\left[(T_1 - T_2)kT_2^3\right]}{dT_2}; \qquad \Rightarrow \frac{dA}{dT_2} = 0;$$

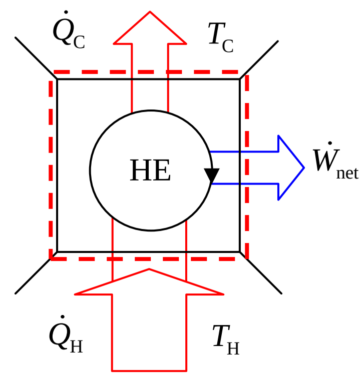
$$\Rightarrow (T_1 - T_2)\frac{d\left(kT_2^3\right)}{dT_2} + \left(kT_2^3\right)\frac{d\left(T_1 - T_2\right)}{dT_2} = 0;$$

$$\Rightarrow (T_1 - T_2)\left(3kT_2^2\right) + \left(kT_2^3\right)(-1) = 0;$$

$$\Rightarrow 3(T_1 - T_2) = T_2; \qquad \Rightarrow T_2 = 0.75T_1$$

2-5-25 [GH] A heat engine receives heat at a rate of 3000 kJ/min from a reservoir at 1000 K and rejects the waste heat to the atmosphere at 300 K. If the engine produces 20 kW of power, determine (a) the thermal efficiency and (b) the entropy generated in the engine's universe.

SOLUTION



(a) $\dot{Q}_H = \left(3000\frac{\text{kJ}}{\text{min}}\right)\left(\frac{1}{60}\frac{\text{min}}{\text{s}}\right); \quad \Rightarrow \dot{Q}_H = 50\ \frac{\text{kJ}}{\text{s}};$

$\Rightarrow \dot{Q}_H = 50\text{ kW};$

$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}; \quad \Rightarrow \eta_{th} = \frac{20}{50}; \quad \Rightarrow \eta_{th} = 0.4;$

$\Rightarrow \eta_{th} = 40\%$

(b) From the energy balance equation we obtain:

$$\frac{dE}{dt}^{0} = \dot{J}_{net}^{0} + \dot{Q} - \dot{W}_{ext} = \left(\dot{Q}_H - \dot{Q}_C\right) - \dot{W}_{net};$$

$$\Rightarrow \dot{Q}_C = \dot{Q}_H - \dot{W}_{net}; \quad \Rightarrow \dot{Q}_C = 50 - 20; \quad \Rightarrow \dot{Q}_C = 30\text{ kW};$$

The entropy balance equation yields:

$$\frac{dS}{dt}^{0} = \dot{S}_{net}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \quad \Rightarrow \frac{dS}{dt}^{0} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{gen,univ};$$

$$\dot{S}_{gen,univ} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{30}{300} - \frac{50}{1000}; \quad \Rightarrow \dot{S}_{gen,univ} = 0.05\ \frac{\text{kW}}{\text{K}}$$

2-5-26 [GN] A household freezer operates in a kitchen at 25°C. Heat must be transferred from the cold space at a rate of 2.5 kW to maintain its temperature at -25°C. What is the smallest (power) motor required to operate the freezer.

SOLUTION

The minimum power input requirement occurs when the COP of this freezer is equal to the Carnot COP.

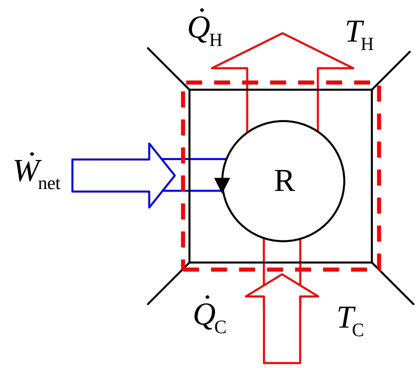
$$\mathrm{COP_R} = \mathrm{COP_{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\mathrm{net}}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = \frac{(2.5)\left(25-(-25)\right)}{(273-25)};$$

$$\Rightarrow \dot{W}_{\mathrm{net}} = 0.504 \text{ kW}$$



TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-27 [GE] To keep a refrigerator in steady state at 2°C, heat has to be removed from it at a rate of 200 kJ/min. If the surrounding air is at 27°C, determine (a) the minimum power input to the refrigerator and (b) the maximum COP.

SOLUTION

(a) The minimum power input requirement occurs when the COP of this refrigerator is equal to the Carnot COP.

$$COP_R = COP_{R,Carnot};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C (T_H - T_C)}{T_C}; \qquad \Rightarrow \dot{W}_{net} = \frac{(200)(27-2)}{(273+2)};$$

$$\Rightarrow \dot{W}_{net} = 18\frac{\text{kJ}}{\text{min}}; \qquad \Rightarrow \dot{W}_{net} = 0.3 \text{ kW}$$

(b) The maximum COP can be determined by evaluating the Carnot Cop.

$$COP_{R,Carnot} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow COP_{R,Carnot} = \frac{(273+2)}{(27-2)}; \qquad \Rightarrow COP_{R,Carnot} = 11$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-28 [GI] A Carnot refrigerator consumes 2 kW of power while operating in a room at 20°C. If the food compartment of the refrigerator is to be maintained at 3°C, determine the rate of heat removal in kJ/min from the compartment.

SOLUTION

The COP of the refrigerator is the Carnot COP.

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_\text{net}}; \quad \Rightarrow \text{COP}_\text{R} = \text{COP}_\text{R,Carnot}; \quad \Rightarrow \text{COP}_\text{R} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{Q}_C = \frac{\dot{W}_\text{net} T_C}{T_H - T_C}; \quad \Rightarrow \dot{Q}_C = \frac{(2)(273+3)}{20-3}; \quad \Rightarrow \dot{Q}_C = 32.47 \text{ kW};$$

$$\Rightarrow \dot{Q}_C = 1948.2 \ \frac{\text{kJ}}{\text{min}}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-29 [BEQ] An actual refrigerator operates with a COP that is half the Carnot COP. It removes 10 kW of heat from a cold reservoir at 250 K and dumps the waste heat into the atmosphere at 300 K. (a) Determine the net work consumed by the refrigerator. (b) What-if Scenario: How would the answer change if the cold storage were to be maintained at 200 K without altering the rate of heat transfer?

SOLUTION

(a) The COP of the refrigerator is half that of the Carnot COP. Therefore,

$$\text{COP}_\text{R} = \frac{\dot{Q}_C}{\dot{W}_\text{net}}; \quad \Rightarrow \text{COP}_\text{R} = \frac{1}{2}\text{COP}_\text{R,Carnot}; \quad \Rightarrow \text{COP}_\text{R} = \frac{1}{2}\frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_\text{net} = \frac{2\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \quad \Rightarrow \dot{W}_\text{net} = \frac{(2)(10)(300-250)}{250};$$

$$\Rightarrow \dot{W}_\text{net} = 4 \text{ kW}$$

(b) For $T_C = 200$ K:

$$\Rightarrow \dot{W}_\text{net} = \frac{2\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \quad \Rightarrow \dot{W}_\text{net} = \frac{(2)(10)(300-200)}{200};$$

$$\Rightarrow \dot{W}_\text{net} = 10 \text{ kW}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-30 [GL] An inventor claims to have developed a refrigerator with a COP of 10 that maintains a cold space at -10^{o}C, while operating in a 25^{o}C kitchen. Is this claim plausible? (1:Yes; 0:No)

SOLUTION

This claim can be evaluated by comparing the claimed COP with the Carnot COP of this refrigerator.

$$\text{COP}_{\text{R,Carnot}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow \text{COP}_{\text{R,Carnot}} = \frac{(273-10)}{\{25-(-10)\}}; \qquad \Rightarrow \text{COP}_{\text{R,Carnot}} = 7.51;$$

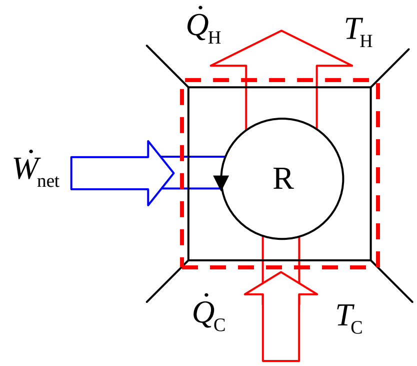
The maximum COP or Carnot COP is found to be less than the claimed COP. Therefore, this claim is not plausible.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-31 [GG] A refrigeration cycle removes heat at a rate of 250 kJ/min from a cold space maintained at -10°C while rejecting heat to the atmosphere at 25°C. If the power consumption rate is 0.75 kW, determine if the cycle is (1: reversible; 2: irreversible; 3: impossible).

SOLUTION



The energy balance equation yields:

$$\frac{dE}{dt}^{0} = \dot{J}_{net}^{0} + \dot{Q} - \dot{W}_{ext}; \quad \Rightarrow \frac{dE}{dt}^{0} = \left(\dot{Q}_C - \dot{Q}_H\right) - \left(-\dot{W}_{net}\right);$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_C + \dot{W}_{net}; \quad \Rightarrow \dot{Q}_H = \frac{250}{60} + 0.75;$$

$$\Rightarrow \dot{Q}_H = 4.917 \text{ kW};$$

The entropy balance equation yields:

$$\frac{dS}{dt}^{0} = \dot{S}_{net}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \quad \Rightarrow \frac{dS}{dt}^{0} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen,univ};$$

$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{4.917}{273+25} - \frac{250}{60(273-10)};$$

$$\Rightarrow \dot{S}_{gen,univ} = 6.57 \times 10^{4} \frac{\text{kW}}{\text{K}};$$

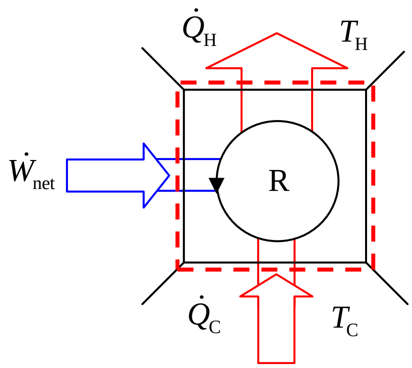
The cycle is 2: irreversible.

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-32 [GZ] A refrigeration cycle removes heat at a rate of 250 kJ/min from a cold space maintained at -10^{o}C while rejecting heat to the atmosphere at 25^{o}C. If the power consumption rate is 1.5 kW, (a) do a first-law analysis to determine the rate of heat rejection to the atmosphere in kW. (b) Do a second-law analysis to determine the entropy generation rate in the refrigerator's universe.

SOLUTION



(a) $$\frac{dE}{dt}^{0} = \dot{J}_{net}^{0} + \dot{Q} - \dot{W}_{ext};$$

$$\Rightarrow \frac{dE}{dt}^{0} = \left(\dot{Q}_C - \dot{Q}_H\right) - \left(-\dot{W}_{net}\right);$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_C + \dot{W}_{ext}; \qquad \Rightarrow \dot{Q}_H = \frac{250}{60} + 1.5;$$

$$\Rightarrow \dot{Q}_H = 5.67\text{ kW}$$

(b) $$\frac{dS}{dt}^{0} = \dot{S}_{net}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \qquad \Rightarrow \frac{dS}{dt}^{0} = \dot{S}_{net}^{0} + \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen,univ};$$

$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C}; \qquad \Rightarrow \dot{S}_{gen,univ} = \frac{5.67}{298} - \frac{4.167}{263}; \qquad \Rightarrow \dot{S}_{gen,univ} = 0.00318\ \frac{\text{kW}}{\text{K}}$$

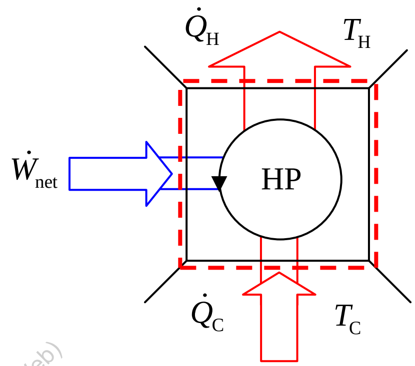
TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-33 [GK] In a cryogenic experiment a container is maintained at -120°C, although it gains 200 W due to heat transfer from the surroundings. What is the minimum power of a motor that is needed for a heat pump to absorb heat from the container and reject heat to the room at 25°C?

SOLUTION

The heat pump is working as a refrigerator to withdraw $\dot{Q}_C = 200$ W of heat from the container to maintain the container at steady state.



For minimum power consumption,

$COP_R = COP_{R,Carnot}$;

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{(200)(25-(-120))}{(273-120)};$$

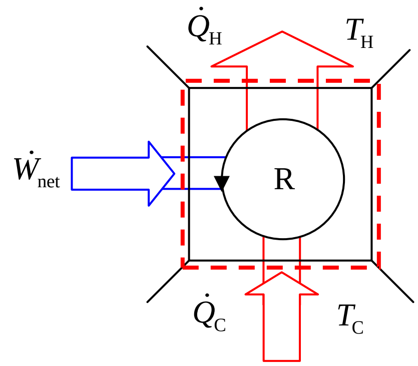
$$\Rightarrow \dot{W}_{net} = 189.542 \text{ W}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-34 [GP] An air-conditioning system maintains a house at a temperature of 20°C while the outside temperature is 40°C. If the cooling load on this house is 10 tons, determine (a) the minimum power requirement. (b) What-if Scenario: What would the minimum power requirement be if the interior were 5 degrees warmer?

SOLUTION



(a) From the problem statement,

$$\dot{Q}_C = 10 \text{ tons} = 35.1667 \text{ kW};$$

The minimum power input requirement occurs when the COP of this refrigerator is equal to the Carnot COP.

$$\text{COP}_\text{R} = \text{COP}_\text{R,Carnot};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_\text{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \dot{W}_\text{net} = \frac{\dot{Q}_C (T_H - T_C)}{T_C}; \quad \Rightarrow \dot{W}_\text{net} = \frac{(35.1667)(40-20)}{(273+20)};$$

$$\Rightarrow \dot{W}_\text{net} = 2.4 \text{ kW}$$

(b) $$\dot{W}_\text{net} = \frac{\dot{Q}_C (T_H - T_C)}{T_C}; \quad \Rightarrow \dot{W}_\text{net} = \frac{(35.1667)(40-(20+5))}{(273+(20+5))}; \quad \Rightarrow \dot{W}_\text{net} = 1.77 \text{ kW}$$

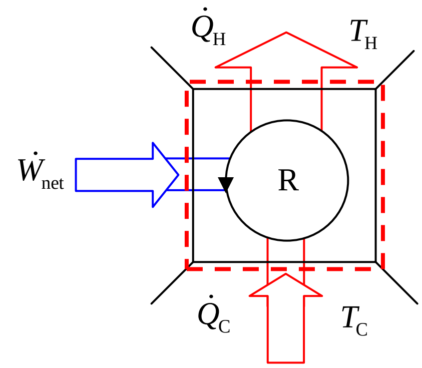
TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-35 [GU] An air-conditioning system is required to transfer heat from a house at a rate of 800 kJ/min to maintain its temperature at 20°C. (a) If the COP of the system is 3.7, determine the power required for air conditioning the house. (b) If the outdoor temperature is 35°C, determine the minimum possible power required.

SOLUTION

(a) From the definition of the COP of a refrigerator we obtain:



$$\mathrm{COP_R} = \frac{\dot{Q}_C}{\dot{W}_{net}};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C}{\mathrm{COP_R}}; \qquad \Rightarrow \dot{W}_{net} = \frac{800}{3.7};$$

$$\Rightarrow \dot{W}_{net} = 216.2\frac{\text{kJ}}{\text{min}}; \qquad \Rightarrow \dot{W}_{net} = 3.6 \text{ kW}$$

(b) The minimum power input requirement occurs when the COP of this refrigerator is equal to the Carnot COP.

$$\mathrm{COP_R} = \mathrm{COP_{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C}; \qquad \Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\left(\frac{800}{60}\right)(35-20)}{(273+20)}; \qquad \Rightarrow \dot{W}_{net} = 0.68 \text{ kW}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-36 [GX] A solar-powered refrigeration system receives heat from a solar collector at T_H, rejects heat to the atmosphere at T_0 and extracts heat from a cold space at T_C. The three heat transfer rates are $\dot{Q}_H$, $\dot{Q}_0$ and $\dot{Q}_C$, respectively. (a) Do an energy and entropy analysis of the system to derive an expression for the maximum possible COP, defined as the ratio $\dot{Q}_C/\dot{Q}_H$. (b) Determine the COP for T_H= 400°C, T_0 = 30°C, and T_C = -20 °C.

SOLUTION

(a) For maximum possible COP, the system should be reversible. An energy and entropy analysis yields:

$$\frac{dE}{dt}^{0} = \dot{J}_{\text{net}}^{0} + \dot{Q} - \dot{W}_{\text{ext}}^{0}; \quad \Rightarrow \frac{dE}{dt}^{0} = \left(\dot{Q}_C + \dot{Q}_H - \dot{Q}_0\right);$$

$$\Rightarrow \dot{Q}_0 = \dot{Q}_H + \dot{Q}_C;$$

$$\frac{dS}{dt}^{0} = \dot{S}_{\text{net}}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{\text{gen,univ}}^{0}; \quad \Rightarrow \frac{dS}{dt}^{0} = \dot{S}_{\text{net}}^{0} + \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_0}{T_0};$$

$$\Rightarrow \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_H}{T_H} = \frac{\dot{Q}_C + \dot{Q}_H}{T_0};$$

$$\Rightarrow \dot{Q}_H\left(\frac{1}{T_H} - \frac{1}{T_0}\right) = \dot{Q}_C\left(\frac{1}{T_0} - \frac{1}{T_C}\right);$$

$$\Rightarrow \text{COP}_{\max} = \frac{\dot{Q}_C}{\dot{Q}_H}; \quad \Rightarrow \text{COP}_{\max} = \frac{T_C\left(T_H - T_0\right)}{T_H\left(T_0 - T_C\right)}$$

(b) Substituting the given values of temperature (in Kelvin), we obtain:

$$\text{COP}_{\max} = \frac{\dot{Q}_C}{\dot{Q}_H}; \quad \Rightarrow \text{COP}_{\max} = \frac{T_C\left(T_H - T_0\right)}{T_H\left(T_0 - T_C\right)}; \quad \Rightarrow \text{COP}_{\max} = \frac{(273-20)(400-30)}{(273+400)(30-(-20))};$$

$$\Rightarrow \text{COP}_{\max} = 2.78$$

2-5-37 [GC] Assume T_H = 425 K, T_0 = 298 K, T_C = 250 K and $\dot{Q}_C$ = 20 kW in the above system of problem 2-5-36[GX]. (a) Determine the maximum COP of the system. (b) If the collector captures 0.2 kW/m^2, determine the minimum collector area required.

SOLUTION

(a) Using the results from the previous problem:

$$\mathrm{COP}_{\max} = \frac{\dot{Q}_C}{\dot{Q}_H}; \qquad \Rightarrow \mathrm{COP}_{\max} = \frac{T_C\left(T_H - T_0\right)}{T_H\left(T_0 - T_C\right)};$$

$$\Rightarrow \mathrm{COP}_{\max} = \frac{250(425-298)}{425(298-250)}; \qquad \Rightarrow \mathrm{COP}_{\max} = 1.556$$

(b) To solve for the minimum collector area required to extract 20 kW of heat

$$0.2A = \dot{Q}_H;$$

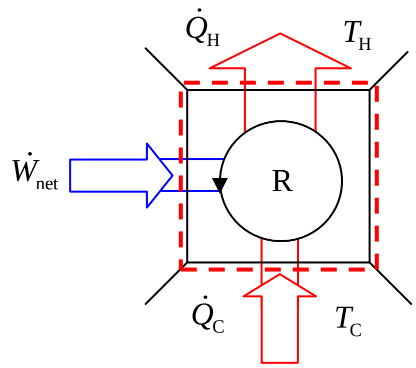
$$\Rightarrow A = \frac{\dot{Q}_H}{0.2}; \qquad \Rightarrow A = \frac{\dot{Q}_H}{\dot{Q}_C}\frac{\dot{Q}_C}{0.2}; \qquad \Rightarrow A = \frac{\dot{Q}_C}{(0.2)\mathrm{COP}_{\max}};$$

$$\Rightarrow A = \frac{20}{(0.2)(1.556)}; \qquad \Rightarrow A = 64.3 \text{ m}^2$$

2-5-38 [GV] A refrigerator with a COP of 2.0 extracts heat from a cold chamber at 0°C at a rate of 400 kJ/min. If the atmospheric temperature is 20°C, determine (a) the power drawn by the refrigerator and (b) the rate of entropy generation in the refrigerator's universe.

SOLUTION

(a) The power drawn by this refrigerator can be found using the COP equation.



$$\text{COP}_R = \frac{\dot{Q}_C}{\dot{W}_{net}}; \quad \Rightarrow \dot{W}_{net} = \frac{\dot{Q}_C}{\text{COP}_R};$$

$$\Rightarrow \dot{W}_{net} = \frac{400}{2}; \quad \Rightarrow \dot{W}_{net} = 200\frac{\text{kJ}}{\text{min}};$$

$$\Rightarrow \dot{W}_{net} = 3.33 \text{ kW}$$

(b) The energy balance equation produces:

$$\frac{dE}{dt}^{0} = \dot{J}_{net}^{0} + \dot{Q} - \dot{W}_{ext}; \quad \Rightarrow \frac{dE}{dt}^{0} = \left(\dot{Q}_H - \dot{Q}_C\right) - \dot{W}_{net};$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_C - \dot{W}_{net}; \quad \Rightarrow \dot{Q}_H = \frac{400}{60} - 3.33; \quad \Rightarrow \dot{Q}_H = 3.33 \text{ kW};$$

An entropy analysis of the refrigerator's universe produces:

$$\frac{dS}{dt}^{0} = \dot{S}_{net}^{0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen,univ}; \quad \Rightarrow \frac{dS}{dt}^{0} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{S}_{gen,univ};$$

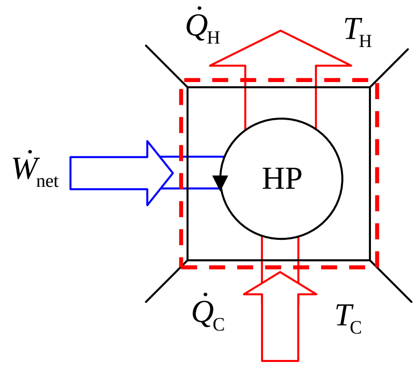
$$\Rightarrow \dot{S}_{gen,univ} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}; \quad \Rightarrow \dot{S}_{gen,univ} = \frac{6.67}{273} - \frac{3.33}{(273+20)}; \quad \Rightarrow \dot{S}_{gen,univ} = 0.0131 \frac{\text{kW}}{\text{K}}$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the refrigerator radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-39 [GQ] On a cold night a house is losing heat at a rate of 15 kW. A reversible heat pump maintains the house at 20°C while the outside temperature is 0°C. (a) Determine the heating cost for the night (8 hours). (b) Also determine the heating cost if resistance heating were used instead. Assume the price of electricity to be 15 cents/kWh.

SOLUTION



(a) Since this heat pump is reversible, it's COP is equal to the Carnot COP.

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{(15)(20-0)}{(273+20)};$$

$$\Rightarrow \dot{W}_{\text{net}} = 1.023 \text{ kW};$$

Knowing the power input, the cost per night can be calculated as:

$$\frac{\text{cost}}{\text{night}} = (1.022 \text{ kW})(8\text{hrs})\left(0.15\frac{\$}{\text{kWh}}\right); \quad \Rightarrow \frac{\text{cost}}{\text{night}} = \$1.23$$

(b) With resistance heating (the energy equation can be used to show that, at steady state, the entire amount of electric work is converted to heat).

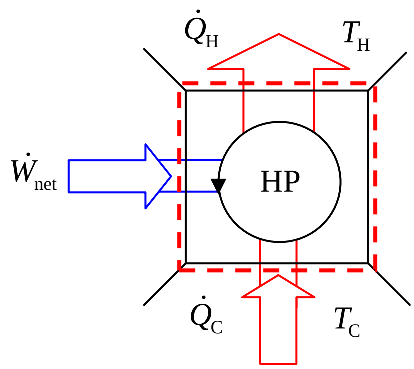
$$\frac{\text{cost}}{\text{night}} = (15 \text{ kW})(8\text{hrs})\left(0.15\frac{\$}{\text{kWh}}\right); \quad \Rightarrow \frac{\text{cost}}{\text{night}} = \$18$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat pump radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-40 [GT] On a cold night a house is losing heat at a rate of 80,000 Btu/h. A reversible heat pump maintains the house at 70°F, while the outside temperature is 30°F. Determine (a) the heating cost for the night (8 hours) assuming the price of 10 cents/kWh for electricity. Also determine (b) the heating cost if resistance heating were used instead.

SOLUTION



(a) Using the Engineering Converter,

$$\dot{Q}_H = 80,000\ \frac{\text{Btu}}{\text{h}} = 23.44\ \text{kW};$$

Since this heat pump is reversible,

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H\left(T_H - T_C\right)}{T_H};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{(23.44)(294.26-272)}{294.26}; \qquad \Rightarrow \dot{W}_{\text{net}} = 1.77\ \text{kW};$$

Knowing the power input, the cost per night can be calculated as:

$$\frac{\text{cost}}{\text{night}} = (1.77\ \text{kW})(8\text{hrs})\left(0.1\frac{\$}{\text{kWh}}\right); \qquad \Rightarrow \frac{\text{cost}}{\text{night}} = \$1.42$$

(b) With resistance heating (the energy equation can be used to show that, at steady state, the entire amount of electric work is converted to heat).

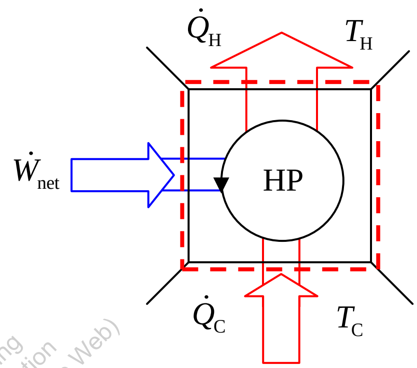
$$\frac{\text{cost}}{\text{night}} = (23.44\ \text{kW})(8\text{hrs})\left(0.1\frac{\$}{\text{kWh}}\right); \qquad \Rightarrow \frac{\text{cost}}{\text{night}} = \$18.75$$

TEST Solution:

Launch the closed-steady cycle TESTcalc. Select the heat pump radio-button. Enter the known information about the device to calculate some of the desired unknowns. Secondary variables can be calculated in the I/O panel.

2-5-41 [GY] A house is maintained at a temperature of 20°C by a heat pump pumping heat from the atmosphere. Heat transfer rate through the wall and roof is estimated at 0.6 kW per unit temperature difference between inside and outside. (a) If the atmospheric temperature is -10°C, what is the minimum power required to drive the pump? (b) It is proposed to use the same pump to cool the house in the summer. For the same room temperature, the same heat transfer rate, and the same power input to the pump, determine the maximum permissible atmospheric temperature. (c) What-if Scenario: What would the answer in part (b) be if the heat transfer rate between the house and outside were estimated at 0.7 kW per unit temperature difference?

SOLUTION



(a) From the problem statement,

$$\dot{Q}_H = 0.6\Delta T; \quad \Rightarrow \dot{Q}_H = 0.6(T_H - T_C);$$

$$\Rightarrow \dot{Q}_H = 0.6\left(20-(-10)\right); \quad \Rightarrow \dot{Q}_H = 18 \text{ kW};$$

The minimum power requirement will occur when the pump's COP is equal to the Carnot COP.

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{T_H}{T_H - T_C}; \quad \Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H (T_H - T_C)}{T_H}; \quad \Rightarrow \dot{W}_{\text{net}} = 1.84 \text{ kW}$$

(b) As a refrigerator,

$$\dot{Q}_C = 0.6\Delta T; \quad \Rightarrow \dot{Q}_C = 0.6(T_H - T_C); \quad \Rightarrow \dot{Q}_C = 0.6(T_H - 293);$$

The maximum outdoor temperature that the refrigerator can handle will occur when it's COP equals the Carnot COP.

$$\text{COP}_{\text{R}} = \text{COP}_{\text{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C}; \quad \Rightarrow \frac{0.6(T_H - 293)}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

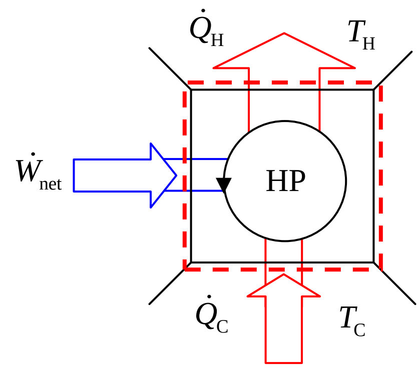
$$\Rightarrow T_H = 323 \text{ K}; \quad \Rightarrow T_H = 50^{\circ}\text{C}$$

(c) $\dot{Q}_C = 0.7\Delta T; \quad \Rightarrow \dot{Q}_C = 0.7(T_H - T_C); \quad \Rightarrow \dot{Q}_C = 0.7(T_H - 293);$

$$\frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C}; \quad \Rightarrow \frac{0.7(T_H - 293)}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C}; \quad \Rightarrow T_H = 307 \text{ K}; \quad \Rightarrow T_H = 34^{\circ}\text{C}$$

2-5-42 [GF] A house is maintained at a temperature T_H by a heat pump that is powered by an electric motor. The outside air at T_C is used as the low-temperature TER. Heat loss from the house to the surroundings is directly proportional to the temperature difference and is given by $\dot{Q}_{loss} = U(T_H - T_C)$. (a) Determine the minimum electric power to drive the heat pump as a function of the given variables. (b) The electric power consumption is calculated to be 10 kW for T_H = 20 deg-C and $T_{H=}$ 10 deg-C. Determine the power consumption if the outside temperature drops to 0 deg-C.

SOLUTION



(a) The minimum power requirement will occur when this heat pump runs as a reversible device.

$$\mathrm{COP_{HP}} = \mathrm{COP_{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{net}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{net} = \frac{U(T_H - T_C)(T_H - T_C)}{T_H};$$

$$\Rightarrow \dot{W}_{net} = \frac{U(T_H - T_C)^2}{T_H}$$

(b) The expression derived in part (a) shows that the power consumption is proportional to the square of temperature difference. Therefore, as the temperature difference doubles,

$$\dot{W}_{net,new} = \frac{U(T_H - T_{C,new})^2}{T_H}; \quad \Rightarrow \dot{W}_{net,new} = \frac{U(T_H - T_{C,old})^2}{T_H} \frac{(T_H - T_{C,new})^2}{(T_H - T_{C,old})^2};$$

$$\Rightarrow \dot{W}_{net,new} = (10)\frac{(20-0)^2}{(20-10)^2}; \quad \Rightarrow \dot{W}_{net,new} = 40 \text{ kW}$$

2-5-43 [GD] A house is maintained at steady state (closed system) at 300 K while the outside temperature is 275 K. The heat loss ($Q^{\cdot}_{C}$) is measured at 2 kW. Two approaches are being considered: (A) electrical heating at 100% efficiency and (B) an ideal heat pump that operates with Carnot COP. The price of electricity is $0.2/kWh. (a) Determine the cost of option A and (b) option B over a 10 hour operating period.

SOLUTION

(a) $\dot{W}_{\text{el,in}} = \dot{Q}_{\text{loss}} = 2 \text{ kW};$

$$\Rightarrow W_{\text{el}} = \dot{W}_{\text{el}} \Delta t = 20 \text{ kWh};$$

$$\text{Cost of Option A} = (0.2)(10)(2); \qquad \Rightarrow \text{Cost} = \$4$$

(b) $\text{COP}_{\text{HP,Carnot}} = \dfrac{T_H}{T_H - T_c}; \qquad \Rightarrow \text{COP}_{\text{HP,Carnot}} = \dfrac{300}{300-275}; \qquad \Rightarrow \text{COP}_{\text{HP,Carnot}} = 12;$

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = 12;$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{2}{12};$$

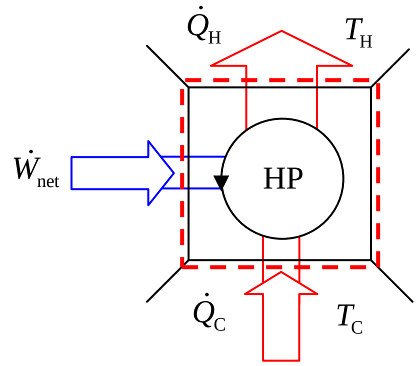
$$\Rightarrow \dot{W}_{\text{net}} = 1.84 \text{ kW};$$

$$\text{Cost of Option B} = (0.2)\dot{W}_{\text{el}} \Delta t; \qquad \Rightarrow \text{Cost} = (0.2)\left(\frac{1}{6}\right)(10); \qquad \Rightarrow \text{Cost} = \$0.33$$

2-5-44 [GM] A house is maintained at a temperature of 25°C by a reversible heat pump powered by an electric motor. The outside air at 10°C is used as the low-temperature TER. Determine the percent saving in electrical power consumption if the house is kept at 20°C instead. Assume that the heat loss from the house to the surroundings is directly proportional to the temperature difference.

SOLUTION

The percent savings can be evaluated by examining the differences in power inputs as this heat pump runs as a reversible device.



$$\mathrm{COP_{HP}} = \mathrm{COP_{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{net}} = \frac{T_H}{T_H - T_C};$$

$$\Rightarrow \dot{W}_{net} = \frac{\dot{Q}_H (T_H - T_C)}{T_H};$$

And $\dot{Q}_H = A(T_H - T_C);$

$$\Rightarrow \dot{W}_{net} = \frac{A(T_H - T_C)^2}{T_H};$$

$$\dot{W}_{net,25} = \frac{A(T_H - T_C)^2}{T_H}; \quad \Rightarrow \dot{W}_{net,25} = \frac{A(25-10)^2}{273+25}; \quad \Rightarrow \dot{W}_{net,25} = 0.755A \text{ kW};$$

$$\dot{W}_{net,20} = \frac{A(T_H - T_C)^2}{T_H}; \quad \Rightarrow \dot{W}_{net,20} = \frac{A(20-10)^2}{273+20}; \quad \Rightarrow \dot{W}_{net,20} = 0.341A \text{ kW};$$

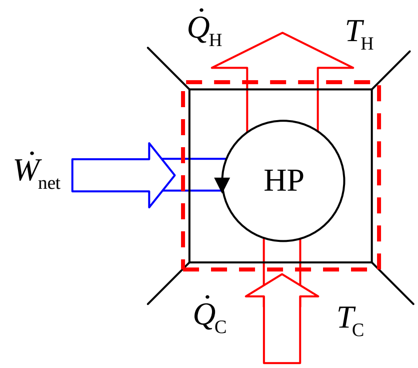
$$\%\text{Savings} = 1 - \frac{\dot{W}_{net,20}}{\dot{W}_{net,25}}; \quad \Rightarrow \%\text{Savings} = 1 - \frac{0.341\cancel{A}}{0.755\cancel{A}}; \quad \Rightarrow \%\text{Savings} = 0.548;$$

$$\Rightarrow \%\text{Savings} = 54.8\%$$

2-5-45 [GJ] A house is heated and maintained at 25°C by a heat pump. Determine the maximum possible COP if heat is extracted from the outside atmosphere at (a) 10°C, (b) 0°C, (c) -10°C and (d) -40°C. (e) Based on these results, would you recommend heat pumps at locations with a severe climate?

SOLUTION

The maximum possible COP occurs when the COP of the heat pump is equal to the Carnot COP.



$$COP_{HP,max} = COP_{HP,Carnot}; \quad \Rightarrow COP_{HP,max} = \frac{T_H}{T_H - T_C};$$

(a) $$COP_{10^\circ C} = \frac{273+25}{25-10}; \quad \Rightarrow COP_{10^\circ C} = 19.9$$

(b) $$COP_{0^\circ C} = \frac{273+25}{25-0}; \quad \Rightarrow COP_{0^\circ C} = 11.92$$

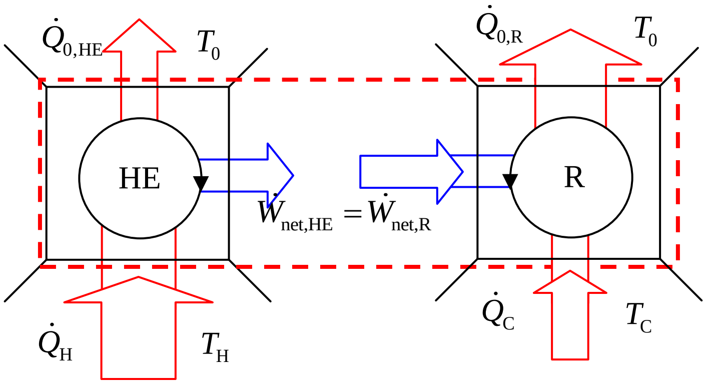
(c) $$COP_{-10^\circ C} = \frac{273+25}{25-(-10)}; \quad \Rightarrow COP_{-10^\circ C} = 8.51$$

(d) $$COP_{-40^\circ C} = \frac{273+25}{25-(-40)}; \quad \Rightarrow COP_{-40^\circ C} = 4.58$$

(e) Given that a real heat pump is significantly less efficient that a Carnot heat pump, the advantage of a heat pump over electric heating start diminishing rapidly when the outside temperature becomes really low. As the COP decreases, so does the heating power. Heat pump is ideal for milder climate.

2-5-46 [GW] A Carnot heat engine receives heat at 800 K and rejects the waste heat to the surroundings at 300 K. The output from the heat engine is used to drive a Carnot refrigerator that removes heat from the cooled space at -20°C at a rate of 400 kJ/min and rejects it to the same surroundings at 300 K. Determine (a) the rate of heat supplied to the heat engine and (b) the total rate of heat rejection to the surroundings. What-if Scenario: (c) What would the rate of heat supplied be if the temperature of the cooled space were -30°C?

SOLUTION



(a) The work required by the refrigerator is equal to the work produced by the heat engine.

$$\mathrm{COP}_{\mathrm{R,Carnot}} = \frac{\dot{Q}_C}{\dot{W}_{\mathrm{net,R}}}; \qquad \Rightarrow \mathrm{COP}_{\mathrm{R,Carnot}} = \frac{T_C}{T_H - T_C};$$

$$\dot{W}_{\mathrm{net,R}} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \qquad \Rightarrow \dot{W}_{\mathrm{net,R}} = \frac{\dot{Q}_C\left(T_0 - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{\mathrm{net,R}} = \frac{400\left[300-(273-20)\right]}{273-20}; \qquad \Rightarrow \dot{W}_{\mathrm{net,R}} = 74.3\ \frac{\mathrm{kJ}}{\mathrm{min}};$$

$$\eta_{th,\mathrm{Carnot}} = \frac{\dot{W}_{\mathrm{net,HE}}}{\dot{Q}_{\mathrm{H}}}; \qquad \Rightarrow \eta_{th,\mathrm{Carnot}} = 1 - \frac{T_C}{T_H};$$

$$\dot{Q}_{\mathrm{H}} = \frac{\dot{W}_{\mathrm{net,HE}}}{\left(1-\frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{\mathrm{H}} = \frac{\dot{W}_{\mathrm{net,R}}}{\left(1-\frac{T_C}{T_H}\right)}; \qquad \Rightarrow \dot{Q}_{\mathrm{H}} = \frac{74.3}{\left(1-\frac{300}{800}\right)}; \qquad \Rightarrow \dot{Q}_{\mathrm{H}} = 118.88\ \frac{\mathrm{kJ}}{\mathrm{min}}$$

(b) An energy balance on the combined system produces:

$$\frac{dE}{dt}^{0} = \dot{J}_{\mathrm{net}}^{0} + \dot{Q} - \dot{W}_{\mathrm{ext}}^{0}; \qquad \Rightarrow \frac{dE}{dt}^{0} = \dot{Q}_{\mathrm{H}} + \dot{Q}_{\mathrm{C}} - \dot{Q}_{0,\mathrm{HE}} - \dot{Q}_{0,\mathrm{R}}; \qquad \Rightarrow \frac{dE}{dt} = 118.88 + 400 - \dot{Q}_0;$$

$$\Rightarrow \dot{Q}_0 = \dot{Q}_{0,\mathrm{HE}} + \dot{Q}_{0,\mathrm{R}}; \qquad \Rightarrow \dot{Q}_0 = \dot{Q}_{\mathrm{H}} + \dot{Q}_{\mathrm{C}};$$

$$\Rightarrow \dot{Q}_0 = 118.88 + 400; \qquad \Rightarrow \dot{Q}_0 = 518.88\ \frac{\mathrm{kJ}}{\mathrm{min}}$$

(c) If the temperature of the cooled space were -30°C, $\mathrm{COP}_{\mathrm{R,Carnot}}$ would change.

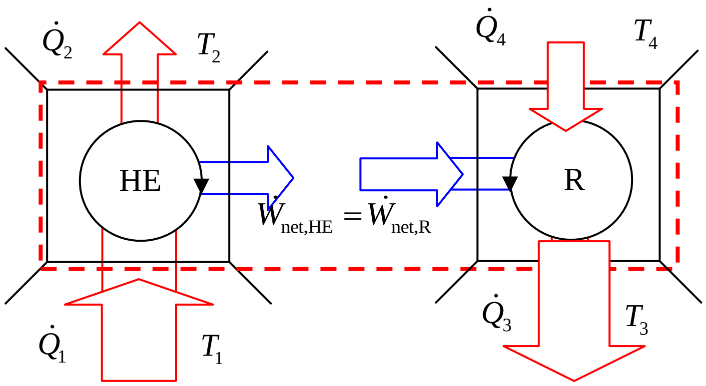
$$\dot{W}_{\text{net,R}} = \frac{\dot{Q}_C}{\text{COP}_{\text{R,Carnot}}}; \quad \Rightarrow \dot{W}_{\text{net,R}} = \frac{\dot{Q}_C\left(T_H - T_C\right)}{T_C}; \quad \Rightarrow \dot{W}_{\text{net,R}} = \frac{\dot{Q}_C\left(T_0 - T_C\right)}{T_C};$$

$$\Rightarrow \dot{W}_{\text{net,R}} = \frac{400\left\lfloor 300 - \left(273 - 30\right)\right\rfloor}{273 - 30}; \quad \Rightarrow \dot{W}_{\text{net,R}} = 93.8\ \frac{\text{kJ}}{\text{min}};$$

$$\dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net,HE}}}{\left(1 - \frac{T_C}{T_H}\right)}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{\dot{W}_{\text{net,R}}}{\left(1 - \frac{T_C}{T_H}\right)}; \quad \Rightarrow \dot{Q}_{\text{H}} = \frac{93.8}{\left(1 - \frac{300}{800}\right)}; \quad \Rightarrow \dot{Q}_{\text{H}} = 150.08\ \frac{\text{kJ}}{\text{min}}$$

2-5-47 [ZR] A reversible heat engine is used to drive a reversible heat pump. The power cycle takes in Q_1 heat units at T_1 and rejects Q_2 heat units at T_2. The heat pump extracts Q_4 from a heat sink at T_4 and discharges Q_3 at T_3. (a) Develop an expression for Q_4/Q_1 in terms of the four given temperatures. (b) Evaluate the expression for T_1 = 500 K, T_2 = 300 K, T_3 = 400 K, and T_4 = 300 K.

SOLUTION



(a) An energy analysis of this total system produces:

$$\frac{dE^{\,0}}{dt} = \dot{J}_{\text{net}}^{\,0} + \dot{Q} - \dot{W}_{\text{ext}}^{\,0}; \quad \Rightarrow \frac{dE^{\,0}}{dt} = \dot{Q}_1 - \dot{Q}_2 - \dot{Q}_3 + \dot{Q}_4; \quad \Rightarrow \dot{Q}_1 - \dot{Q}_2 + \dot{Q}_4 = \dot{Q}_3;$$

The entropy equation for the engine and the heat pump yields:

$$\frac{dS^{\,0}}{dt} = \dot{S}_{net}^{\,0} + \frac{\dot{Q}}{T_B} + \dot{S}_{gen}^{\,0}; \quad \Rightarrow \frac{\dot{Q}}{T_B} = 0;$$

Heat engine: $\dfrac{\dot{Q}_1}{T_1} - \dfrac{\dot{Q}_2}{T_2} = 0; \quad \Rightarrow \dot{Q}_2 = \dfrac{T_2}{T_1}\dot{Q}_1;$

Heat pump: $\dfrac{\dot{Q}_4}{T_4} - \dfrac{\dot{Q}_3}{T_3} = 0; \quad \Rightarrow \dot{Q}_3 = \dfrac{T_3}{T_4}\dot{Q}_4;$

By substituting these equations into the result of the energy balance equation:

$$\Rightarrow \dot{Q}_1 - \frac{T_2}{T_1}\dot{Q}_1 + \dot{Q}_4 = \frac{T_3}{T_4}\dot{Q}_4; \quad \Rightarrow \dot{Q}_1\left(1 - \frac{T_2}{T_1}\right) = \dot{Q}_4\left(\frac{T_3}{T_4} - 1\right);$$

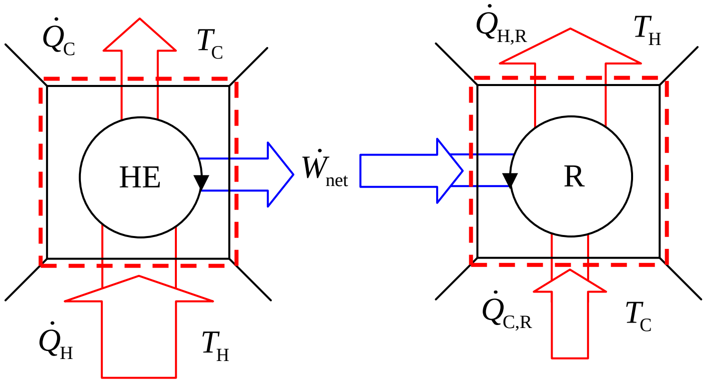
$$\Rightarrow \frac{\dot{Q}_4}{\dot{Q}_1} = \frac{\left(1 - \dfrac{T_2}{T_1}\right)}{\left(\dfrac{T_3}{T_4} - 1\right)}; \quad \Rightarrow \frac{\dot{Q}_4}{\dot{Q}_1} = \frac{T_4\left(T_1 - T_2\right)}{T_1\left(T_3 - T_4\right)}$$

(b) Substituting the given values:

$$\frac{\dot{Q}_1}{\dot{Q}_4} = \frac{T_4\left(T_1 - T_2\right)}{T_1\left(T_3 - T_4\right)}; \quad \Rightarrow \frac{\dot{Q}_1}{\dot{Q}_4} = \frac{(300)(500-300)}{(500)(400-300)}; \quad \Rightarrow \frac{\dot{Q}_1}{\dot{Q}_4} = 1.2$$

2-5-48 [ZO] A heat engine with a thermal efficiency (η_{th}) of 35% is used to drive a refrigerator having a COP of 4. (a) What is the heat input to the engine for each MJ removed from the cold region by the refrigerator? (b) If the system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input into the engine?

SOLUTION



(a) $$\frac{\dot{Q}_H}{\dot{Q}_{C,R}} = \frac{\dot{Q}_H}{\dot{W}_{net,HE}}\frac{\dot{W}_{net,HE}}{\dot{Q}_{C,R}} = \frac{\dot{Q}_H}{\dot{W}_{net,HE}}\frac{\dot{W}_{net,R}}{\dot{Q}_{C,R}} = \frac{1}{\eta_{th}}\frac{1}{COP_R};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{Q}_{C,R}} = \frac{1}{4(0.35)} = 0.714;$$

$$\Rightarrow \dot{Q}_H = (0.714)\dot{Q}_{C,R} = (0.714)(1) = 0.714 \text{ MJ}$$

(b)

$$COP_{HP} = \frac{\dot{Q}_{H,R}}{\dot{W}_{net,R}} = \frac{\dot{W}_{net,R} + \dot{Q}_{C,R}}{\dot{W}_{net,R}} = 1 + COP_R = 5$$

$$\frac{\dot{Q}_H}{\dot{Q}_{H,R}} = \frac{\dot{Q}_H}{\dot{W}_{net,HE}}\frac{\dot{W}_{net,HE}}{\dot{Q}_{H,R}} = \frac{\dot{Q}_H}{\dot{W}_{net,HE}}\frac{\dot{W}_{net,R}}{\dot{Q}_{H,R}} = \frac{1}{\eta_{th}}\frac{1}{COP_{HP}} = \frac{1}{5(0.35)} = 0.571;$$

$$\Rightarrow \dot{Q}_{H,R} = \frac{\dot{Q}_H}{0.571} = \frac{1}{0.571} = 1.75 \text{ MJ}$$

2-5-49 [ZB] A heat engine operates between two TERs at 1000°C and 20°C respectively. Two-thirds of the work output is used to drive a heat pump that removes heat from the cold surroundings at 0°C and transfers it to a house kept at 20°C. If the house is losing heat at a rate of 60,000 kJ/h, determine (a) the minimum rate of heat supply to the heat engine. (b) What-if Scenario: What would the minimum heat supply be if the outside temperature dropped to -10°C?

SOLUTION

(a) For minimum heat supply, both the heat engine and heat pump must be reversible.

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_{H,HP}}{\dot{W}_{\text{net},HP}} = \frac{T_H}{T_H - T_C};$$

$$\dot{W}_{\text{net,HP}} = \frac{2}{3}\dot{W}_{\text{net,HE}}; \quad \Rightarrow \dot{W}_{\text{net,HE}} = \frac{\dot{Q}_{H,\text{HP}}(T_H - T_C)}{T_H}\left(\frac{3}{2}\right); \quad \Rightarrow \dot{W}_{\text{net,HE}} = \frac{60,000(20-0)}{273+20}\left(\frac{3}{2}\right);$$

$$\Rightarrow \dot{W}_{\text{net,HE}} = 6143.3\ \frac{\text{kJ}}{\text{h}}; \quad \Rightarrow \dot{W}_{\text{net,HE}} = 1.706\ \text{kW};$$

The minimum heat amount of heat can be supplied when the engine is reversible.

$$\eta_{\text{th}} = \eta_{\text{th, Carnot}};$$

$$\Rightarrow \frac{\dot{W}_{\text{net,HE}}}{\dot{Q}_{H,\text{HE}}} = 1 - \frac{T_C}{T_H}; \quad \Rightarrow \dot{Q}_{H,\text{HE}} = \frac{\dot{W}_{\text{net,HE}}}{\left(1 - \frac{T_C}{T_H}\right)}; \quad \Rightarrow \dot{Q}_{H,\text{HE}} = \frac{1.706}{\left(1 - \frac{273+20}{273+1000}\right)};$$

$$\Rightarrow \dot{Q}_{H,\text{HE}} = 2.22\ \text{kW}$$

(b) With a temperature drop of 10 degrees,

$$\dot{W}_{\text{net,HE}} = \frac{\dot{Q}_{H,HP}(T_H - T_C)}{T_H}\left(\frac{3}{2}\right); \quad \Rightarrow \dot{W}_{\text{net,HE}} = \frac{60,000(20-(-10))}{273+20}\left(\frac{3}{2}\right);$$

$$\Rightarrow \dot{W}_{\text{net,HE}} = 9215.02\ \frac{\text{kJ}}{\text{hr}}; \quad \Rightarrow \dot{W}_{\text{net,HE}} = 2.56\ \text{kW};$$

$$\Rightarrow \dot{Q}_{H,\text{HE}} = \frac{\dot{W}_{\text{net,HE}}}{\left(1 - \frac{T_C}{T_H}\right)}; \quad \Rightarrow \dot{Q}_{H,\text{HE}} = \frac{2.56}{\left(1 - \frac{273+20}{273+1000}\right)}; \quad \Rightarrow \dot{Q}_{H,\text{HE}} = 3.33\ \text{kW}$$

2-5-50 [ZS] A heat engine is used to drive a heat pump. The waste heat from the heat engine and the heat transfer from the heat pump are used to heat the water circulating through the radiator of a building. The thermal efficiency of the heat engine is 30% and the COP of the heat pump is 4.2. Evaluate the COP of the combined system, defined as the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.

SOLUTION

The efficiency of a heat engine is defined as:

$$\eta_{\text{th,Engine}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H,Engine}}}; \quad \Rightarrow \eta_{\text{th,Engine}} = \frac{\dot{Q}_{\text{H,Engine}} - \dot{Q}_{\text{C,Engine}}}{\dot{Q}_{\text{H,Engine}}}; \quad \Rightarrow \eta_{\text{th,Engine}} = 1 - \frac{\dot{Q}_{\text{C,Engine}}}{\dot{Q}_{\text{H,Engine}}};$$

$$\Rightarrow \dot{Q}_{\text{H,Engine}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}}; \quad \Rightarrow \dot{Q}_{\text{H,Engine}} = \frac{\dot{W}_{\text{net}}}{0.3};$$

And $\dot{Q}_{\text{C,Engine}} = \dot{Q}_{\text{H,Engine}} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{C,Engine}} = \frac{\dot{W}_{\text{net}}}{0.3} - \dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{C,Engine}} = \frac{7}{3}\dot{W}_{\text{net}};$

The heat pump uses the net work produced by the engine. The COP of the heat pump is defined as:

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_{\text{H,HP}}}{\dot{W}_{\text{net}}};$$

$$\dot{Q}_{\text{H,HP}} = \text{COP}_{\text{HP}}\dot{W}_{\text{net}}; \quad \Rightarrow \dot{Q}_{\text{H,HP}} = 4.2\dot{W}_{\text{net}};$$

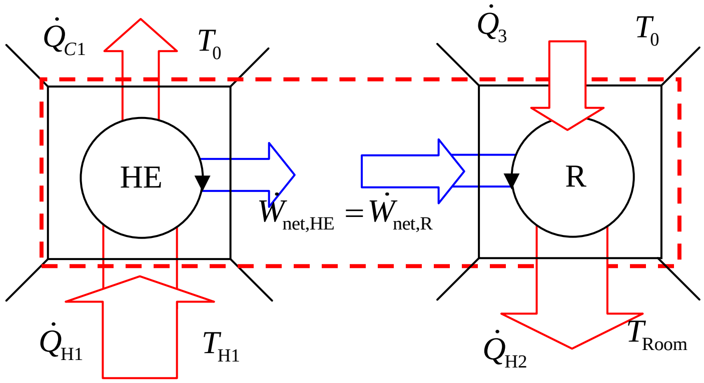
The COP of the combined system is calculated as:

$$\text{COP}_{\text{SYSTEM}} = \frac{\dot{Q}_{\text{C,Engine}} + \dot{Q}_{\text{H,HP}}}{\dot{Q}_{\text{H,Engine}}}; \quad \Rightarrow \text{COP}_{\text{SYSTEM}} = \frac{2.33\cancel{\dot{W}_{\text{net}}} + 4.2\cancel{\dot{W}_{\text{net}}}}{\left(\frac{\cancel{\dot{W}_{\text{net}}}}{0.3}\right)};$$

$$\Rightarrow \text{COP}_{\text{SYSTEM}} = 1.96$$

2-5-51 [ZA] A furnace delivers heat at a rate of $\dot{Q}_{H1}$ at T_{H1}. Instead of directly using this for room heating, it is used to drive a heat engine that rejects the waste heat to atmosphere at T_0. The heat engine drives a heat pump that delivers $\dot{Q}_{H2}$ at T_{room} using the atmosphere as the cold reservoir. Find the ratio $\dot{Q}_{H2}/\dot{Q}_{H1}$, the energetic efficiency of the system as a function of the given temperatures. Why is this a better set-up than direct room heating from the furnace?

SOLUTION



(a) For the heat engine:

$$\eta_{th} = \eta_{th,\text{Carnot}}; \qquad \Rightarrow \eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{H1}}}; \qquad \Rightarrow \eta_{th} = 1 - \frac{T_0}{T_{\text{H1}}};$$

$$\Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H1}}\left(1 - \frac{T_0}{T_{\text{H1}}}\right);$$

For the heat pump:

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_{\text{H2}}}{\dot{W}_{\text{net}}} = \frac{T_{\text{Room}}}{T_{\text{Room}} - T_0}; \qquad \Rightarrow \dot{W}_{\text{net}} = \dot{Q}_{\text{H2}}\left(\frac{T_{\text{Room}} - T_0}{T_{\text{Room}}}\right);$$

Equating the net work produced by the engine with the work consumed by the heat pump, we obtain:

$$\Rightarrow \frac{\dot{Q}_{\text{H2}}}{\dot{Q}_{\text{H1}}} = \frac{T_{\text{Room}}\left(T_{\text{H1}} - T_0\right)}{T_{\text{H1}}\left(T_{\text{Room}} - T_0\right)};$$

(b) Substituting the given temperatures, values: T_{H1}=2000K, T_{room}=300K and T_0=270K, the energetic efficiency of this system would be:

$$\frac{\dot{Q}_{\text{H2}}}{\dot{Q}_{\text{H1}}} = \frac{300(2000-270)}{2000(300-270)}; \qquad \Rightarrow \frac{\dot{Q}_{\text{H2}}}{\dot{Q}_{\text{H1}}} = 8.65$$

2-5-52 [ZH] A heat pump is used for heating a house in the winter and cooling it in the summer by reversing the flow of the refrigerant. The interior temperature should be 20°C in the winter and 25°C in the summer. Heat transfer through the walls and ceilings is estimated to be 2500 kJ per hour per °C temperature difference between the inside and outside. (a) If the winter outside temperature is 0°C, what is the minimum power required to drive the heat pump? (b) For the same power input as in part (a), what is the maximum outside summer temperature for which the house can be maintained at 25°C?

SOLUTION

(a) $\dot{Q}_H = \dot{Q}_{\text{winter}}; \quad \Rightarrow \dot{Q}_H = \frac{2500}{3600}(20-0); \quad \Rightarrow \dot{Q}_H = 13.89 \text{ kW};$

As a heat pump:

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{HP,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{W}_{net}} = \frac{T_H}{T_H - T_C}; \quad \Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_H}{\left(\frac{T_H}{T_H - T_C}\right)};$$

$$\Rightarrow \dot{W}_{\text{net}} = \frac{\dot{Q}_{\text{winter}}}{\left(\frac{T_H}{T_H - T_C}\right)}; \quad \Rightarrow \dot{W}_{\text{net}} = \frac{13.89}{\left(\frac{273+20}{20-0}\right)}; \Rightarrow \dot{W}_{\text{net}} = 0.948 \text{ kW}$$

(b) $\dot{Q}_{\text{summer}} = \frac{2500}{3600}\Delta T; \quad \Rightarrow \dot{Q}_{\text{summer}} = \frac{2500}{3600}(T_H - T_C);$

As a refrigerator:

$$\text{COP}_{\text{R}} = \text{COP}_{\text{R,Carnot}};$$

$$\Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_{\text{summer}}}{\dot{W}_{\text{net}}}; \quad \Rightarrow \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow \frac{\frac{2500}{3600}(T_H - T_C)}{\dot{W}_{net}} = \frac{T_C}{T_H - T_C};$$

$$\Rightarrow T_H = \sqrt{T_C \dot{W}_{\text{net}} \frac{3600}{2500}} + 25; \quad \Rightarrow T_H = \sqrt{(273+25)(0.948)\frac{3600}{2500}} + 25;$$

$$\Rightarrow T_H = 45.16^\circ \text{C}$$