SOLUTIONS MANUAL

SYSTEM DYNAMICS

Fourth Edition

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Preface

This Solutions Manual presents solutions to all unsolved B-problems. For some problems, solutions include more materials than are required in problem statements to aid the user of the book.

The text may be used in a few different ways depending on the course objective and the time allocated to the course.

Sample course coverages are listed below.

If this book is used as a text for a quarter-length course (with approximately 30 lecture hours and 18 recitation hours), Chapters 1 through 7 may be covered.

If the book is used as a text for a semester-length course (with approximately 40 lecture hours and 26 recitation hours), then the first nine chapters may be covered or, alternatively, the first seven chapters plus Chapters 10 and 11 may be covered.

If the course devotes 50 to 60 hours to lectures, then the entire book may be covered in a semester.

The instructor will always have an option to omit certain subjects depending on the course objective.

Katsuhiko Ogata

Solutions to B Problems

CHAPTER 2

B-2-1.

$$f(t) = 0 \qquad t < 0$$
$$= t\, e^{-2t} \qquad t \geq 0$$

Note that

$$\mathcal{L}[t] = \frac{1}{s^2}$$

Referring to Equation (2-2), we obtain

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[t\, e^{-2t}] = \frac{1}{(s+2)^2}$$

B-2-2.

(a)

$$f_1(t) = 0 \qquad t < 0$$
$$= 3 \sin(5t + 45^\circ) \qquad t \geq 0$$

Note that

$$3 \sin(5t + 45^\circ) = 3 \sin 5t \cos 45^\circ + 3 \cos 5t \sin 45^\circ$$
$$= \frac{3}{\sqrt{2}} \sin 5t + \frac{3}{\sqrt{2}} \cos 5t$$

So we have

$$F_1(s) = \mathcal{L}[f_1(t)] = \frac{3}{\sqrt{2}} \frac{5}{s^2 + 5^2} + \frac{3}{\sqrt{2}} \frac{s}{s^2 + 5^2}$$
$$= \frac{3}{\sqrt{2}} \frac{s + 5}{s^2 + 25}$$

(b)

$$f_2(t) = 0 \qquad t < 0$$
$$= 0.03(1 - \cos 2t) \qquad t \geq 0$$

$$F_2(s) = \mathcal{L}[f_2(t)] = 0.03 \frac{1}{s} - 0.03 \frac{s}{s^2 + 2^2} = \frac{0.12}{s(s^2 + 4)}$$

B-2-3.

$$f(t) = 0 \qquad t < 0$$
$$= t^2 e^{-at} \qquad t \geq 0$$

Note that

$$\mathcal{L}[t^2] = \frac{2}{s^3}$$

Referring to Equation (2-2), we obtain

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[t^2 e^{-at}] = \frac{2}{(s+a)^3}$$

B-2-4.

$$f(t) = 0 \qquad t < 0$$
$$= \cos 2\omega t \cos 3\omega t \qquad t \geq 0$$

Noting that

$$\cos 2\omega t \cos 3\omega t = \tfrac{1}{2}(\cos 5\omega t + \cos \omega t)$$

we have

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[\tfrac{1}{2}(\cos 5\omega t + \cos \omega t)]$$
$$= \frac{1}{2}\left(\frac{s}{s^2 + 25\omega^2} + \frac{s}{s^2 + \omega^2}\right) = \frac{(s^2 + 13\omega^2)s}{(s^2 + 25\omega^2)(s^2 + \omega^2)}$$

B-2-5. The function f(t) can be written as

$$f(t) = (t - a)\, 1(t - a)$$

The Laplace transform of f(t) is

$$F(s) = \mathcal{L}[f(t)] = \mathcal{L}[(t - a)\, 1(t - a)] = \frac{e^{-as}}{s^2}$$

B-2-6.

$$f(t) = c\, 1(t - a) - c\, 1(t - b)$$

The Laplace transform of f(t) is

$$F(s) = c\,\frac{e^{-as}}{s} - c\,\frac{e^{-bs}}{s} = \frac{c}{s}(e^{-as} - e^{-bs})$$

B-2-7. The function f(t) can be written as

$$f(t) = \frac{10}{a^2} - \frac{12.5}{a^2} 1(t - \frac{a}{5}) + \frac{2.5}{a^2} 1(t - a)$$

So the Laplace transform of f(t) becomes

$$F(s) = \mathcal{L}[f(t)] = \frac{10}{a^2}\frac{1}{s} - \frac{12.5}{a^2}\frac{1}{s} e^{-(a/5)s} + \frac{2.5}{a^2}\frac{1}{s} e^{-as}$$

$$= \frac{1}{a^2 s}(10 - 12.5\, e^{-(a/5)s} + 2.5\, e^{-as})$$

As a approaches zero, the limiting value of F(s) becomes as follows:

$$\lim_{a\to 0} F(s) = \lim_{a\to 0} \frac{10 - 12.5\, e^{-(a/5)s} + 2.5\, e^{-as}}{a^2 s}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da}(10 - 12.5\, e^{-(a/5)s} + 2.5\, e^{-as})}{\frac{d}{da} a^2 s}$$

$$= \lim_{a\to 0} \frac{2.5\, s\, e^{-(a/5)s} - 2.5\, s\, e^{-as}}{2as}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da}(2.5\, e^{-(a/5)s} - 2.5\, e^{-as})}{\frac{d}{da} 2a}$$

$$= \lim_{a\to 0} \frac{-0.5\, s\, e^{-(a/5)s} + 2.5\, s\, e^{-as}}{2}$$

$$= \frac{-0.5\, s + 2.5\, s}{2} = \frac{2s}{2} = s$$

B-2-8. The function f(t) can be written as

$$f(t) = \frac{24}{a^3} t - \frac{24}{a^2} 1(t - \frac{a}{2}) - \frac{24}{a^3}(t - a)\, 1(t - a)$$

So the Laplace transform of f(t) becomes

$$F(s) = \frac{24}{a^3}\frac{1}{s^2} - \frac{24}{a^2}\frac{1}{s} e^{-\frac{1}{2}as} - \frac{24}{a^3}\frac{e^{-as}}{s^2}$$

$$= \frac{24}{a^3}\left(\frac{1}{s^2} - \frac{a}{s} e^{-\frac{1}{2}as} - \frac{e^{-as}}{s^2}\right)$$

The limiting value of F(s) as a approaches zero is

$$\lim_{a\to 0} F(s) = \lim_{a\to 0} \frac{24(1 - as\, e^{-\frac{1}{2}as} - e^{-as})}{a^3 s^2}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da} 24(1 - as\, e^{-\frac{1}{2}as} - e^{-as})}{\frac{d}{da} a^3 s^2}$$

$$= \lim_{a\to 0} \frac{24(-s\, e^{-\frac{1}{2}as} + \frac{as^2}{2} e^{-\frac{1}{2}as} + s\, e^{-as})}{3a^2 s^2}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da} 8(- e^{-\frac{1}{2}as} + \frac{as}{2} e^{-\frac{1}{2}as} + e^{-as})}{\frac{d}{da} a^2 s}$$

$$= \lim_{a\to 0} \frac{8\left[\frac{s}{2} e^{-\frac{1}{2}as} + \frac{s}{2} e^{-\frac{1}{2}as} + \frac{as}{2}\left(\frac{-s}{2}\right) e^{-\frac{1}{2}as} - se^{-as}\right]}{2as}$$

$$= \lim_{a\to 0} \frac{\frac{d}{da} (4\, e^{-\frac{1}{2}as} - as\, e^{-\frac{1}{2}as} - 4\, e^{-as})}{\frac{d}{da} a}$$

$$= \lim_{a\to 0} \frac{-2s\, e^{-\frac{1}{2}as} - s\, e^{-\frac{1}{2}as} + as \frac{s}{2} e^{-\frac{1}{2}as} + 4s\, e^{-as}}{1}$$

$$= -2s - s + 4s = s$$

B-2-9.

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} \frac{s\, 5(s + 2)}{s(s + 1)} = \frac{5 \times 2}{1} = 10$$

B-2-10.

$$f(0+) = \lim_{t\to 0+} f(t) = \lim_{s\to\infty} \frac{s\, 2(s + 2)}{s(s + 1)(s + 3)} = 0$$

B-2-11. Define

$$y = \dot{x}$$

Then

$$y(0+) = \dot{x}(0+)$$

The initial value of y can be obtained by use of the initial value theorem as follows:

$$y(0+) = \lim_{s \to \infty} sY(s)$$

Since

$$Y(s) = \mathcal{L}_+[y(t)] = \mathcal{L}_+[\dot{x}(t)] = sX(s) - x(0+)$$

we obtain

$$y(0+) = \lim_{s \to \infty} sY(s) = \lim_{s \to \infty} s[sX(s) - x(0+)]$$

$$= \lim_{s \to \infty} [s^2X(s) - sx(0+)]$$

B-2-12. Note that

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0)$$

$$\mathcal{L}\left[\frac{d^2}{dt^2} f(t)\right] = s^2F(s) - sf(0) - \dot{f}(0)$$

Define

$$g(t) = \frac{d^2}{dt^2} f(t)$$

Then

$$\mathcal{L}\left[\frac{d^3}{dt^3} f(t)\right] = \mathcal{L}\left[\frac{d}{dt} g(t)\right] = sG(s) - g(0)$$

$$= s[s^2F(s) - sf(0) - \dot{f}(0)] - \ddot{f}(0)$$

$$= s^3F(s) - s^2f(0) - s\dot{f}(0) - \ddot{f}(0)$$

B-2-13.

(a)

$$F_1(s) = \frac{s + 5}{(s + 1)(s + 3)} = \frac{a_1}{s + 1} + \frac{a_2}{s + 3}$$

where

$$a_1 = \left.\frac{s + 5}{s + 3}\right|_{s = -1} = \frac{4}{2} = 2$$

$$a_2 = \left.\frac{s + 5}{s + 1}\right|_{s = -3} = \frac{2}{-2} = -1$$

$F_1(s)$ can thus be written as

$$F_1(s) = \frac{2}{s+1} - \frac{1}{s+3}$$

and the inverse Laplace transform of $F_1(s)$ is

$$f_1(t) = 2\,e^{-t} - e^{-3t}$$

(b)

$$F_2(s) = \frac{3(s+4)}{s(s+1)(s+2)} = \frac{a_1}{s} + \frac{a_2}{s+1} + \frac{a_3}{s+2}$$

where

$$a_1 = \left.\frac{3(s+4)}{(s+1)(s+2)}\right|_{s=0} = \frac{3 \times 4}{2} = 6$$

$$a_2 = \left.\frac{3(s+4)}{s(s+2)}\right|_{s=-1} = \frac{3 \times 3}{(-1) \times 1} = -9$$

$$a_3 = \left.\frac{3(s+4)}{s(s+1)}\right|_{s=-2} = \frac{3 \times 2}{(-2)(-1)} = 3$$

$F_2(s)$ can thus be written as

$$F_2(s) = \frac{6}{s} - \frac{9}{s+1} + \frac{3}{s+2}$$

and the inverse Laplace transform of $F_2(s)$ is

$$f_2(t) = 6 - 9\,e^{-t} + 3\,e^{-2t}$$

B-2-14.

(a)

$$F_1(s) = \frac{6s+3}{s^2} = \frac{6}{s} + \frac{3}{s^2}$$

The inverse Laplace transform of $F_1(s)$ is

$$f_1(t) = 6 + 3t$$

(b)

$$F_2(s) = \frac{5s + 2}{(s + 1)(s + 2)^2} = \frac{a}{s + 1} + \frac{b_2}{(s + 2)^2} + \frac{b_1}{s + 2}$$

where

$$a = \frac{5s + 2}{(s + 2)^2}\bigg|_{s = -1} = \frac{-5 + 2}{1^2} = -3$$

$$b_2 = \frac{5s + 2}{s + 1}\bigg|_{s = -2} = \frac{-10 + 2}{-2 + 1} = 8$$

$$b_1 = \frac{d}{ds}\left(\frac{5s + 2}{s + 1}\right)\bigg|_{s = -2} = \frac{5(s + 1) - (5s + 2)}{(s + 1)^2}\bigg|_{s = -2}$$

$$= \frac{5(-1) - (-10 + 2)}{1^2} = 3$$

$F_2(s)$ can thus be written as

$$F_2(s) = \frac{-3}{s + 1} + \frac{8}{(s + 2)^2} + \frac{3}{s + 2}$$

and the inverse Laplace transform of $F_2(s)$ is

$$f_2(t) = -3\, e^{-t} + 8t\, e^{-2t} + 3\, e^{-2t}$$

B-2-15.

$$F(s) = \frac{2s^2 + 4s + 5}{s(s + 1)} = 2 + \frac{2}{s + 1} + \frac{5}{s(s + 1)}$$

$$= 2 + \frac{2}{s + 1} + \frac{5}{s} - \frac{5}{s + 1} = 2 - \frac{3}{s + 1} + \frac{5}{s}$$

The inverse Laplace transform of $F(s)$ is

$$f(t) = 2\,\delta(t) - 3\, e^{-t} + 5$$

B-2-16.

$$F(s) = \frac{s^2 + 2s + 4}{s^2} = 1 + \frac{2}{s} + \frac{4}{s^2}$$

The inverse Laplace transform of F(s) is

$$f(t) = \delta(t) + 2 + 4t$$

B-2-17.

$$F(s) = \frac{s}{s^2 + 2s + 10} = \frac{s + 1 - 1}{(s + 1)^2 + 3^2}$$

$$= \frac{s + 1}{(s + 1)^2 + 3^2} - \frac{3}{(s + 1)^2 + 3^2} \frac{1}{3}$$

Hence

$$f(t) = e^{-t} \cos 3t - \frac{1}{3} e^{-t} \sin 3t$$

B-2-18.

$$F(s) = \frac{s^2 + 2s + 5}{s^2(s + 1)} = \frac{a}{s^2} + \frac{b}{s} + \frac{c}{s + 1}$$

where

$$a = \left. \frac{s^2 + 2s + 5}{s + 1} \right|_{s = 0} = 5$$

$$b = \left. \frac{(2s + 2)(s + 1) - (s^2 + 2s + 5)}{(s + 1)^2} \right|_{s = 0} = \frac{2 - 5}{1} = -3$$

$$c = \left. \frac{s^2 + 2s + 5}{s^2} \right|_{s = -1} = \frac{1 - 2 + 5}{1} = 4$$

Hence

$$F(s) = \frac{5}{s^2} + \frac{-3}{s} + \frac{4}{s + 1}$$

The inverse Laplace transform of F(s) is

$$f(t) = 5t - 3 + 4 e^{-t}$$

B-2-19.

$$F(s) = \frac{2s + 10}{(s + 1)^2(s + 4)} = \frac{a}{(s + 1)^2} + \frac{b}{s + 1} + \frac{c}{s + 4}$$

where

$$a = \left.\frac{2s + 10}{s + 4}\right|_{s = -1} = \frac{-2 + 10}{3} = \frac{8}{3}$$

$$b = \left.\frac{2(s + 4) - (2s + 10)}{(s + 4)^2}\right|_{s = -1} = \frac{6 - 8}{3^2} = \frac{-2}{9}$$

$$c = \left.\frac{2s + 10}{(s + 1)^2}\right|_{s = -4} = \frac{-8 + 10}{9} = \frac{2}{9}$$

Hence

$$F(s) = \frac{8}{3}\frac{1}{(s + 1)^2} - \frac{2}{9}\frac{1}{s + 1} + \frac{2}{9}\frac{1}{s + 4}$$

The inverse Laplace transform of F(s) is

$$f(t) = \frac{8}{3} te^{-t} - \frac{2}{9} e^{-t} + \frac{2}{9} e^{-4t}$$

B-2-20.

$$F(s) = \frac{1}{s^2(s^2 + \omega^2)} = \left(\frac{1}{s^2} - \frac{1}{s^2 + \omega^2}\right)\frac{1}{\omega^2}$$

The inverse Laplace transform of F(s) is

$$f(t) = \frac{1}{\omega^2}(t - \frac{1}{\omega} \sin \omega t)$$

B-2-21.

$$F(s) = \frac{c}{s^2}(1 - e^{-as}) - \frac{b}{s} e^{-as} \qquad a > 0$$

The inverse Laplace transform of F(s) is

$$f(t) = ct - c(t - a)1(t - a) - b\,1(t - a)$$
