

INSTRUCTOR'S SOLUTIONS MANUAL

PROBABILITY AND STATISTICAL INFERENCE

NINTH EDITION

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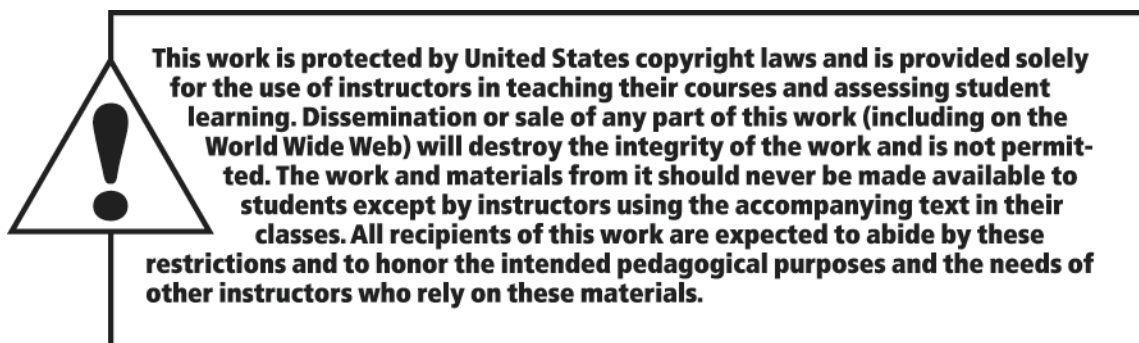
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ISBN-13: 978-0-321-91379-1
ISBN-10: 0-321-91379-5

www.pearsonhighered.com

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Contents

Preface	v
1 Probability	1
1.1 Properties of Probability	1
1.2 Methods of Enumeration	2
1.3 Conditional Probability	3
1.4 Independent Events	4
1.5 Bayes' Theorem	5
2 Discrete Distributions	7
2.1 Random Variables of the Discrete Type	7
2.2 Mathematical Expectation	10
2.3 Special Mathematical Expectations	11
2.4 The Binomial Distribution	14
2.5 The Negative Binomial Distribution	16
2.6 The Poisson Distribution	17
3 Continuous Distributions	19
3.1 Random Variables of the Continuous Type	19
3.2 The Exponential, Gamma, and Chi-Square Distributions	25
3.3 The Normal Distribution	28
3.4 Additional Models	29
4 Bivariate Distributions	33
4.1 Bivariate Distributions of the Discrete Type	33
4.2 The Correlation Coefficient	34
4.3 Conditional Distributions	35
4.4 Bivariate Distributions of the Continuous Type	36
4.5 The Bivariate Normal Distribution	41
5 Distributions of Functions of Random Variables	43
5.1 Functions of One Random Variable	43
5.2 Transformations of Two Random Variables	44
5.3 Several Random Variables	48
5.4 The Moment-Generating Function Technique	50
5.5 Random Functions Associated with Normal Distributions	52
5.6 The Central Limit Theorem	55
5.7 Approximations for Discrete Distributions	56
5.8 Chebyshev's Inequality and Convergence in Probability	58
5.9 Limiting Moment-Generating Functions	59

6	Point Estimation	61
6.1	Descriptive Statistics	61
6.2	Exploratory Data Analysis	63
6.3	Order Statistics	68
6.4	Maximum Likelihood Estimation	71
6.5	A Simple Regression Problem	74
6.6	Asymptotic Distributions of Maximum Likelihood Estimators	78
6.7	Sufficient Statistics	79
6.8	Bayesian Estimation	81
6.9	More Bayesian Concepts	83
7	Interval Estimation	85
7.1	Confidence Intervals for Means	85
7.2	Confidence Intervals for the Difference of Two Means	86
7.3	Confidence Intervals For Proportions	88
7.4	Sample Size	89
7.5	Distribution-Free Confidence Intervals for Percentiles	90
7.6	More Regression	92
7.7	Resampling Methods	98
8	Tests of Statistical Hypotheses	105
8.1	Tests About One Mean	105
8.2	Tests of the Equality of Two Means	107
8.3	Tests about Proportions	110
8.4	The Wilcoxon Tests	111
8.5	Power of a Statistical Test	115
8.6	Best Critical Regions	119
8.7	Likelihood Ratio Tests	121
9	More Tests	125
9.1	Chi-Square Goodness-of-Fit Tests	125
9.2	Contingency Tables	128
9.3	One-Factor Analysis of Variance	128
9.4	Two-Way Analysis of Variance	132
9.5	General Factorial and 2^k Factorial Designs	133
9.6	Tests Concerning Regression and Correlation	134
9.7	Statistical Quality Control	135

Preface

This solutions manual provides answers for the even-numbered exercises in *Probability and Statistical Inference*, 9th edition, by Robert V. Hogg, Elliot A. Tanis, and Dale L. Zimmerman. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these solutions and answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available for download at <http://www.math.hope.edu/tanis/>. Short descriptions of these procedures are provided on the “Maple Card.” Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8). You can download a copy of this manual at <http://www.math.hope.edu/tanis/MapleManual.pdf>.

Our hope is that this solutions manual will be helpful to each of you in your teaching.

If you find an error or wish to make a suggestion, send these to Elliot Tanis, tanis@hope.edu, and he will post corrections on his web page, <http://www.math.hope.edu/tanis/>.

R.V.H.
E.A.T.
D.L.Z.

Chapter 1

Probability

1.1 Properties of Probability

1.1-2 Sketch a figure and fill in the probabilities of each of the disjoint sets.

Let $A = \{\text{insure more than one car}\}$, $P(A) = 0.85$.

Let $B = \{\text{insure a sports car}\}$, $P(B) = 0.23$.

Let $C = \{\text{insure exactly one car}\}$, $P(C) = 0.15$.

It is also given that $P(A \cap B) = 0.17$. Since $A \cap C = \phi$, $P(A \cap C) = 0$. It follows that $P(A \cap B \cap C') = 0.17$. Thus $P(A' \cap B \cap C') = 0.06$ and $P(A' \cap B' \cap C) = 0.09$.

1.1-4 (a) $S = \{\text{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT}\}$;

(b) (i) $5/16$, (ii) 0 , (iii) $11/16$, (iv) $4/16$, (v) $4/16$, (vi) $9/16$, (vii) $4/16$.

1.1-6 (a) $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

$$\begin{aligned} \text{(b)} \quad A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B') &= 0.1; \end{aligned}$$

$$\text{(c)} \quad P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7.$$

1.1-8 Let $A = \{\text{lab work done}\}$, $B = \{\text{referral to a specialist}\}$,

$$P(A) = 0.41, P(B) = 0.53, P[(A \cup B)'] = 0.21.$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.79 &= 0.41 + 0.53 - P(A \cap B) \\ P(A \cap B) &= 0.41 + 0.53 - 0.79 = 0.15. \end{aligned}$$

$$\begin{aligned} \text{1.1-10} \quad A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

1.1-12 (a) $1/3$; (b) $2/3$; (c) 0 ; (d) $1/2$.

$$\mathbf{1.1-14} \quad P(A) = \frac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \frac{\sqrt{3}}{2}.$$

1.1-16 Note that the respective probabilities are p_0 , $p_1 = p_0/4$, $p_2 = p_0/4^2, \dots$.

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{p_0}{4^k} &= 1 \\ \frac{p_0}{1 - 1/4} &= 1 \\ p_0 &= \frac{3}{4} \\ 1 - p_0 - p_1 &= 1 - \frac{15}{16} = \frac{1}{16}. \end{aligned}$$

1.2 Methods of Enumeration

$$\mathbf{1.2-2} \quad (\mathbf{a}) \quad (4)(5)(2) = 40; \quad (\mathbf{b}) \quad (2)(2)(2) = 8.$$

$$\mathbf{1.2-4} \quad (\mathbf{a}) \quad 4 \binom{6}{3} = 80;$$

$$(\mathbf{b}) \quad 4(2^6) = 256;$$

$$(\mathbf{c}) \quad \frac{(4 - 1 + 3)!}{(4 - 1)!3!} = 20.$$

1.2-6 $S = \{ \text{DDD, DDFD, DFDD, FDDD, DDFD, DFDF, FDDF, DFFD, FDFD, FFDD, FFF, FFDF, FDFF, DFFF, FFDDF, FDFDF, DFFDF, FDDFF, DFDF, DDDFF} \}$ so there are 20 possibilities.

$$\mathbf{1.2-8} \quad 3 \cdot 3 \cdot 2^{12} = 36,864.$$

$$\begin{aligned} \mathbf{1.2-10} \quad \binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}. \end{aligned}$$

$$\mathbf{1.2-12} \quad 0 = (1-1)^n = \sum_{r=0}^n \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^n (-1)^r \binom{n}{r}.$$

$$2^n = (1+1)^n = \sum_{r=0}^n \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^n \binom{n}{r}.$$

$$\mathbf{1.2-14} \quad \binom{10-1+36}{36} = \frac{45!}{36!9!} = 886,163,135.$$

$$\mathbf{1.2-16} \quad (\mathbf{a}) \quad \frac{\binom{19}{3} \binom{52-19}{6}}{\binom{52}{9}} = \frac{102,486}{351,325} = 0.2917;$$

$$(\mathbf{b}) \quad \frac{\binom{19}{3} \binom{10}{2} \binom{7}{1} \binom{3}{0} \binom{5}{1} \binom{2}{0} \binom{6}{2}}{\binom{52}{9}} = \frac{7,695}{1,236,664} = 0.00622.$$

1.3 Conditional Probability

1.3-2 (a) $\frac{1041}{1456}$;

(b) $\frac{392}{633}$;

(c) $\frac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 (a) $P(\text{HH}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$;

(b) $P(\text{HC}) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$;

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$

$$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}.$$

1.3-6 Let $H = \{\text{died from heart disease}\}$; $P = \{\text{at least one parent had heart disease}\}$.

$$P(H | P') = \frac{N(H \cap P')}{N(P')} = \frac{110}{648}.$$

1.3-8 (a) $\frac{3}{20} \cdot \frac{2}{19} \cdot \frac{1}{18} = \frac{1}{1140}$;

(b) $\frac{\binom{3}{2} \binom{17}{1}}{\binom{20}{3}} \cdot \frac{1}{17} = \frac{1}{380}$;

(c) $\sum_{k=1}^9 \frac{\binom{3}{2} \binom{17}{2k-2}}{\binom{20}{2k}} \cdot \frac{1}{20-2k} = \frac{35}{76} = 0.4605$.

(d) Draw second. The probability of winning is $1 - 0.4605 = 0.5395$.

1.3-10 (a) $P(A) = \frac{52}{52} \cdot \frac{51}{52} \cdot \frac{50}{52} \cdot \frac{49}{52} \cdot \frac{48}{52} \cdot \frac{47}{52} = \frac{8,808,975}{11,881,376} = 0.74141$;

(b) $P(A') = 1 - P(A) = 0.25859$.

1.3-12 (a) It doesn't matter because $P(B_1) = \frac{1}{18}$, $P(B_5) = \frac{1}{18}$, $P(B_{18}) = \frac{1}{18}$;

(b) $P(B) = \frac{2}{18} = \frac{1}{9}$ on each draw.

1.3-14 (a) $P(A_1) = 30/100$;

(b) $P(A_3 \cap B_2) = 9/100$;

(c) $P(A_2 \cup B_3) = 41/100 + 28/100 - 9/100 = 60/100$;

$$(d) P(A_1 | B_2) = 11/41;$$

$$(e) P(B_1 | A_3) = 13/29.$$

$$\mathbf{1.3-16} \quad \frac{3}{5} \cdot \frac{5}{8} + \frac{2}{5} \cdot \frac{4}{8} = \frac{23}{40}.$$

1.4 Independent Events

$$\begin{aligned} \mathbf{1.4-2} \quad (a) \quad P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

$$(b) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.6} = 0.$$

$$\begin{aligned} \mathbf{1.4-4} \quad \text{Proof of (b):} \quad P(A' \cap B) &= P(B)P(A'|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A'). \end{aligned}$$

$$\begin{aligned} \text{Proof of (c):} \quad P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B'). \end{aligned}$$

$$\begin{aligned} \mathbf{1.4-6} \quad P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') | B] \\ &= P(B)[1 - P(A \cup C | B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C'). \end{aligned}$$

$$\begin{aligned} P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\ &\quad P(B)P(C) - P(A)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C'). \end{aligned}$$

$$\mathbf{1.4-8} \quad \frac{1}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} + \frac{5}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \frac{2}{9}.$$

$$\begin{aligned}
 \text{1.4-10 (a)} \quad & \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}; \\
 \text{(b)} \quad & \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} = \frac{9}{16}; \\
 \text{(c)} \quad & \frac{2}{4} \cdot \frac{1}{4} + \frac{2}{4} \cdot \frac{4}{4} = \frac{10}{16}.
 \end{aligned}$$

$$\begin{aligned}
 \text{1.4-12 (a)} \quad & \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2; \\
 \text{(b)} \quad & \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2; \\
 \text{(c)} \quad & \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2; \\
 \text{(d)} \quad & \frac{5!}{3!2!} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2.
 \end{aligned}$$

$$\begin{aligned}
 \text{1.4-14 (a)} \quad & 1 - (0.4)^3 = 1 - 0.064 = 0.936; \\
 \text{(b)} \quad & 1 - (0.4)^8 = 1 - 0.00065536 = 0.99934464.
 \end{aligned}$$

$$\begin{aligned}
 \text{1.4-16 (a)} \quad & \sum_{k=0}^{\infty} \frac{1}{5} \left(\frac{4}{5}\right)^{2k} = \frac{5}{9}; \\
 \text{(b)} \quad & \frac{1}{5} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{3}{5}.
 \end{aligned}$$

$$\text{1.4-18 (a) } 7; \text{ (b) } (1/2)^7; \text{ (c) } 63; \text{ (d) No! } (1/2)^{63} = 1/9,223,372,036,854,775,808.$$

1.4-20 No.

1.5 Bayes' Theorem

$$\begin{aligned}
 \text{1.5-2 (a)} \quad P(G) &= P(A \cap G) + P(B \cap G) \\
 &= P(A)P(G|A) + P(B)P(G|B) \\
 &= (0.40)(0.85) + (0.60)(0.75) = 0.79;
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad P(A|G) &= \frac{P(A \cap G)}{P(G)} \\
 &= \frac{(0.40)(0.85)}{0.79} = 0.43.
 \end{aligned}$$

1.5-4 Let event B denote an accident and let A_1 be the event that age of the driver is 16–25. Then

$$\begin{aligned}
 P(A_1|B) &= \frac{(0.1)(0.05)}{(0.1)(0.05) + (0.55)(0.02) + (0.20)(0.03) + (0.15)(0.04)} \\
 &= \frac{50}{50 + 110 + 60 + 60} = \frac{50}{280} = 0.179.
 \end{aligned}$$

1.5-6 Let B be the event that the policyholder dies. Let A_1, A_2, A_3 be the events that the deceased is standard, preferred and ultra-preferred, respectively. Then

$$\begin{aligned}
 P(A_1 | B) &= \frac{(0.60)(0.01)}{(0.60)(0.01) + (0.30)(0.008) + (0.10)(0.007)} \\
 &= \frac{60}{60 + 24 + 7} = \frac{60}{91} = 0.659; \\
 P(A_2 | B) &= \frac{24}{91} = 0.264; \\
 P(A_3 | B) &= \frac{7}{91} = 0.077.
 \end{aligned}$$

1.5-8 Let A be the event that the tablet is under warranty.

$$\begin{aligned}
 P(B_1 | A) &= \frac{(0.40)(0.10)}{(0.40)(0.10) + (0.30)(0.05) + (0.20)(0.03) + (0.10)(0.02)} \\
 &= \frac{40}{40 + 15 + 6 + 2} = \frac{40}{63} = 0.635; \\
 P(B_2 | A) &= \frac{15}{63} = 0.238; \\
 P(B_3 | A) &= \frac{6}{63} = 0.095; \\
 P(B_4 | A) &= \frac{2}{63} = 0.032.
 \end{aligned}$$

1.5-10 (a) $P(D^+) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674$;

(b) $P(A^- | D^+) = \frac{0.0490}{0.0674} = 0.727$; $P(A^+ | D^+) = \frac{0.0184}{0.0674} = 0.273$;

(c) $P(A^- | D^-) = \frac{(0.98)(0.95)}{(0.02)(0.08) + (0.98)(0.95)} = \frac{9310}{16 + 9310} = 0.998$;
 $P(A^+ | D^-) = 0.002$.

(d) Yes, particularly those in part (b).

1.5-12 Let $D = \{\text{has the disease}\}$, $DP = \{\text{detects presence of disease}\}$. Then

$$\begin{aligned}
 P(D | DP) &= \frac{P(D \cap DP)}{P(DP)} \\
 &= \frac{P(D) \cdot P(DP | D)}{P(D) \cdot P(DP | D) + P(D') \cdot P(DP | D')} \\
 &= \frac{(0.005)(0.90)}{(0.005)(0.90) + (0.995)(0.02)} \\
 &= \frac{0.0045}{0.0045 + 0.199} = \frac{0.0045}{0.2035} = 0.0221.
 \end{aligned}$$

1.5-14 Let $D = \{\text{defective roll}\}$. Then

$$\begin{aligned}
 P(I | D) &= \frac{P(I \cap D)}{P(D)} \\
 &= \frac{P(I) \cdot P(D | I)}{P(I) \cdot P(D | I) + P(II) \cdot P(D | II)} \\
 &= \frac{(0.60)(0.03)}{(0.60)(0.03) + (0.40)(0.01)} \\
 &= \frac{0.018}{0.018 + 0.004} = \frac{0.018}{0.022} = 0.818.
 \end{aligned}$$

Chapter 2

Discrete Distributions

2.1 Random Variables of the Discrete Type

2.1-2 (a)

$$f(x) = \begin{cases} 0.6, & x = 1, \\ 0.3, & x = 5, \\ 0.1, & x = 10, \end{cases}$$

(b)

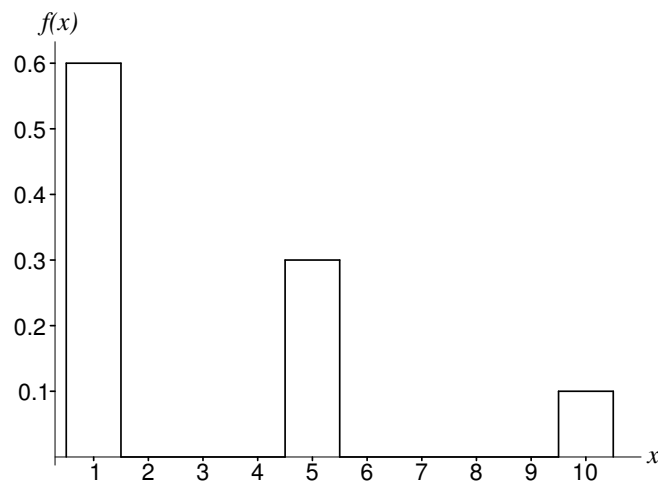


Figure 2.1-2: A probability histogram

2.1-4 (a) $f(x) = \frac{1}{10}, \quad x = 0, 1, 2, \dots, 9;$

(b) $\mathcal{N}(\{0\})/150 = 11/150 = 0.073; \quad \mathcal{N}(\{5\})/150 = 13/150 = 0.087;$
 $\mathcal{N}(\{1\})/150 = 14/150 = 0.093; \quad \mathcal{N}(\{6\})/150 = 22/150 = 0.147;$
 $\mathcal{N}(\{2\})/150 = 13/150 = 0.087; \quad \mathcal{N}(\{7\})/150 = 16/150 = 0.107;$
 $\mathcal{N}(\{3\})/150 = 12/150 = 0.080; \quad \mathcal{N}(\{8\})/150 = 18/150 = 0.120;$
 $\mathcal{N}(\{4\})/150 = 16/150 = 0.107; \quad \mathcal{N}(\{9\})/150 = 15/150 = 0.100.$

(c)

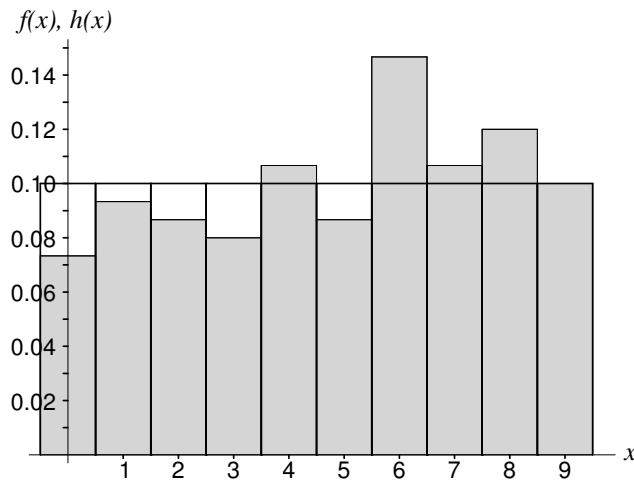


Figure 2.1-4: Michigan daily lottery digits

2.1-6 (a) $f(x) = \frac{6 - |7 - x|}{36}$, $x = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$.

(b)

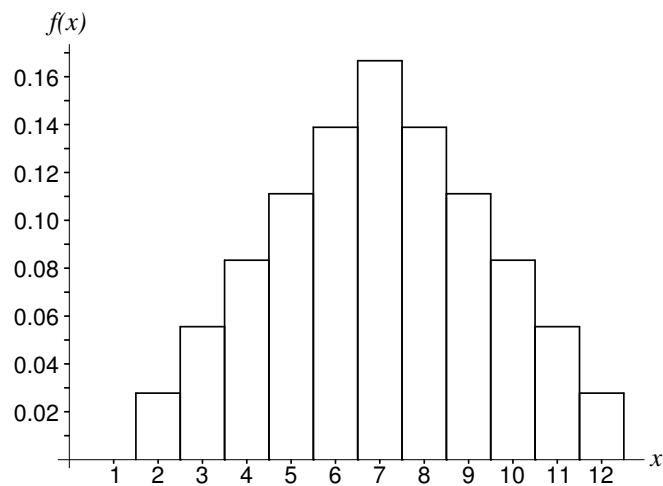


Figure 2.1-6: Probability histogram for the sum of a pair of dice

2.1-8 (a) The space of W is $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

$$P(W = 0) = P(X = 0, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}, \text{ assuming independence.}$$

$$P(W = 1) = P(X = 0, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},$$

$$P(W = 2) = P(X = 2, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},$$

$$P(W = 3) = P(X = 2, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},$$

$$P(W = 4) = P(X = 0, Y = 4) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},$$

$$P(W = 5) = P(X = 0, Y = 5) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},$$

$$P(W = 6) = P(X = 2, Y = 4) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},$$

$$P(W = 7) = P(X = 2, Y = 5) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}.$$

$$\text{That is, } f(w) = P(W = w) = \frac{1}{8}, \quad w \in S.$$

(b)

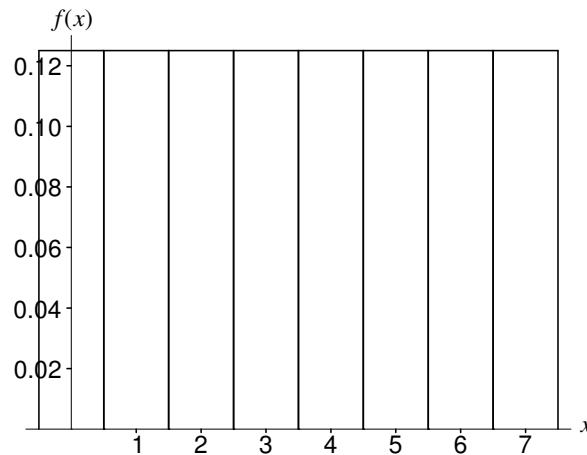


Figure 2.1-8: Probability histogram of sum of two special dice

$$\mathbf{2.1-10 \quad (a)} \quad \frac{\binom{3}{1} \binom{47}{9}}{\binom{50}{10}} = \frac{39}{98};$$

$$\mathbf{(b)} \quad \sum_{x=0}^1 \frac{\binom{3}{x} \binom{47}{10-x}}{\binom{50}{10}} = \frac{221}{245}.$$

$$\begin{aligned}
 \mathbf{2.1-12} \quad P(X \geq 4 | X \geq 1) &= \frac{P(X \geq 4)}{P(X \geq 1)} = \frac{1 - P(X \leq 3)}{1 - P(X = 0)} \\
 &= \frac{1 - [1 - 1/2 + 1/2 - 1/3 + 1/3 - 1/4 + 1/4 - 1/5]}{1 - [1 - 1/2]} = \frac{2}{5}.
 \end{aligned}$$

$$\mathbf{2.1-14} \quad P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\binom{3}{0}\binom{17}{5}}{\binom{20}{5}} = 1 - \frac{91}{228} = \frac{137}{228} = 0.60.$$

2.1-16 (a) $P(2, 1, 6, 10)$ means that 2 is in position 1 so 1 cannot be selected. Thus

$$\begin{aligned}
 P(2, 1, 6, 10) &= \frac{\binom{1}{0}\binom{1}{1}\binom{8}{5}}{\binom{10}{6}} = \frac{56}{210} = \frac{4}{15}; \\
 \mathbf{(b)} \quad P(i, r, k, n) &= \frac{\binom{i-1}{r-1}\binom{1}{1}\binom{n-i}{k-r}}{\binom{n}{k}}.
 \end{aligned}$$

2.2 Mathematical Expectation

$$\begin{aligned}
 \mathbf{2.2-2} \quad E(X) &= (-1)\left(\frac{4}{9}\right) + (0)\left(\frac{1}{9}\right) + (1)\left(\frac{4}{9}\right) = 0; \\
 E(X^2) &= (-1)^2\left(\frac{4}{9}\right) + (0)^2\left(\frac{1}{9}\right) + (1)^2\left(\frac{4}{9}\right) = \frac{8}{9}; \\
 E(3X^2 - 2X + 4) &= 3\left(\frac{8}{9}\right) - 2(0) + 4 = \frac{20}{3}.
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2.2-4} \quad 1 &= \sum_{x=0}^6 f(x) = \frac{9}{10} + c\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) \\
 c &= \frac{2}{49}; \\
 E(\text{Payment}) &= \frac{2}{49}\left(1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{1}{6}\right) = \frac{71}{490} \text{ units.}
 \end{aligned}$$

$$\mathbf{2.2-6} \quad \text{Note that } \sum_{x=1}^{\infty} \frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{6}{\pi^2} \frac{\pi^2}{6} = 1, \text{ so this is a pdf}$$

$$E(X) = \sum_{x=1}^{\infty} x \frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x} = +\infty$$

and it is well known that the sum of this harmonic series is not finite.

$$\mathbf{2.2-8} \quad E(|X - c|) = \frac{1}{7} \sum_{x \in S} |x - c|, \text{ where } S = \{1, 2, 3, 5, 15, 25, 50\}.$$

When $c = 5$,

$$E(|X - 5|) = \frac{1}{7} [(5 - 1) + (5 - 2) + (5 - 3) + (5 - 5) + (15 - 5) + (25 - 5) + (50 - 5)].$$

If c is either increased or decreased by 1, this expectation is increased by $1/7$. Thus $c = 5$, the median, minimizes this expectation while $b = E(X) = \mu$, the mean, minimizes $E[(X - b)^2]$. You could also let $h(c) = E(|X - c|)$ and show that $h'(c) = 0$ when $c = 5$.

$$\mathbf{2.2-10} \quad (1) \cdot \frac{15}{36} + (-1) \cdot \frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(1) \cdot \frac{15}{36} + (-1) \cdot \frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(4) \cdot \frac{6}{36} + (-1) \cdot \frac{30}{36} = \frac{-6}{36} = \frac{-1}{6}.$$

$$\mathbf{2.2-12} \quad (\mathbf{a}) \quad \text{The average class size is } \frac{(16)(25) + (3)(100) + (1)(300)}{20} = 50;$$

(b)

$$f(x) = \begin{cases} 0.4, & x = 25, \\ 0.3, & x = 100, \\ 0.3, & x = 300, \end{cases}$$

$$(\mathbf{c}) \quad E(X) = 25(0.4) + 100(0.3) + 300(0.3) = 130.$$

2.3 Special Mathematical Expectations

2.3-2 (a)

$$\begin{aligned} \mu &= E(X) \\ &= \sum_{x=1}^3 x \frac{3!}{x!(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\ &= 3 \left(\frac{1}{4}\right) \sum_{k=0}^2 \frac{2!}{k!(2-k)!} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{2-k} \\ &= 3 \left(\frac{1}{4}\right) \left(\frac{1}{4} + \frac{3}{4}\right)^2 = \frac{3}{4}; \\ E[X(X-1)] &= \sum_{x=2}^3 x(x-1) \frac{3!}{x!(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\ &= 2(3) \left(\frac{1}{4}\right)^2 \frac{3}{4} + 6 \left(\frac{1}{4}\right)^3 \\ &= 6 \left(\frac{1}{4}\right)^2 = 2 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right); \\ \sigma^2 &= E[X(X-1)] + E(X) - \mu^2 \\ &= (2) \left(\frac{3}{4}\right) \left(\frac{1}{4}\right) + \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 \\ &= (2) \left(\frac{3}{4}\right) \left(\frac{1}{4}\right) + \left(\frac{3}{4}\right) \left(\frac{1}{4}\right) = 3 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right); \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \mu &= E(X) \\
 &= \sum_{x=1}^4 x \frac{4!}{x!(4-x)!} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x} \\
 &= 4 \left(\frac{1}{2}\right) \sum_{k=0}^3 \frac{3!}{k!(3-k)!} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{3-k} \\
 &= 4 \left(\frac{1}{2}\right) \left(\frac{1}{2} + \frac{1}{2}\right)^3 = 2;
 \end{aligned}$$

$$\begin{aligned}
 E[X(X-1)] &= \sum_{x=2}^4 x(x-1) \frac{4!}{x!(4-x)!} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x} \\
 &= 2(6) \left(\frac{1}{2}\right)^4 + (6)(4) \left(\frac{1}{2}\right)^4 + (12) \left(\frac{1}{2}\right)^4 \\
 &= 48 \left(\frac{1}{2}\right)^4 = 12 \left(\frac{1}{2}\right)^2; \\
 \sigma^2 &= (12) \left(\frac{1}{2}\right)^2 + \frac{4}{2} - \left(\frac{4}{2}\right)^2 = 1.
 \end{aligned}$$

$$\mathbf{2.3-4} \quad E[(X - \mu)/\sigma] = (1/\sigma)[E(X) - \mu] = (1/\sigma)(\mu - \mu) = 0;$$

$$E\{[(X - \mu)/\sigma]^2\} = (1/\sigma^2)E[(X - \mu)^2] = (1/\sigma^2)(\sigma^2) = 1.$$

$$\mathbf{2.3-6} \quad f(1) = \frac{3}{8}, f(2) = \frac{2}{8}, f(3) = \frac{3}{8}$$

$$\mu = 1 \cdot \frac{3}{8} + 2 \cdot \frac{2}{8} + 3 \cdot \frac{3}{8} = 2,$$

$$\sigma^2 = 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{2}{8} + 3^2 \cdot \frac{3}{8} - 2^2 = \frac{3}{4}.$$

$$\begin{aligned}
 \mathbf{2.3-8} \quad E(X) &= \sum_{x=1}^4 x \cdot \frac{2x-1}{16} \\
 &= \frac{50}{16} = 3.125;
 \end{aligned}$$

$$\begin{aligned}
 E(X^2) &= \sum_{x=1}^4 x^2 \cdot \frac{2x-1}{16} \\
 &= \frac{85}{8};
 \end{aligned}$$

$$\text{Var}(X) = \frac{85}{8} - \left(\frac{25}{8}\right)^2 = \frac{55}{64} = 0.8594;$$

$$\sigma = \frac{\sqrt{55}}{8} = 0.9270.$$

2.3-10 We have $N = N_1 + N_2$. Thus

$$\begin{aligned}
 E[X(X-1)] &= \sum_{x=0}^n x(x-1)f(x) \\
 &= \frac{\sum_{x=2}^n x(x-1) \frac{N_1!}{x!(N_1-x)!} \cdot \frac{N_2!}{(n-x)!(N_2-n+x)!}}{\binom{N}{n}} \\
 &= N_1(N_1-1) \frac{\sum_{x=2}^n \frac{(N_1-2)!}{(x-2)!(N_1-x)!} \cdot \frac{N_2!}{(n-x)!(N_2-n+x)!}}{\binom{N}{n}}.
 \end{aligned}$$

In the summation, let $k = x - 2$, and in the denominator, note that

$$\binom{N}{n} = \frac{N!}{n!(N-n)!} = \frac{N(N-1)}{n(n-1)} \binom{N-2}{n-2}.$$

Thus

$$\begin{aligned}
 E[X(X-1)] &= \frac{N_1(N_1-1)}{\frac{N(N-1)}{n(n-1)}} \sum_{k=0}^{n-2} \frac{\binom{N_1-2}{k} \binom{N_2}{n-2-k}}{\binom{N-2}{n-2}} \\
 &= \frac{N_1(N_1-1)(n)(n-1)}{N(N-1)}.
 \end{aligned}$$

2.3-12 (a) $f(x) = \left(\frac{364}{365}\right)^{x-1} \left(\frac{1}{365}\right), \quad x = 1, 2, 3, \dots,$

(b) $\mu = \frac{1}{\frac{1}{365}} = 365,$

$$\sigma^2 = \frac{\frac{364}{365}}{\left(\frac{1}{365}\right)^2} = 132,860,$$

$$\sigma = 364.500;$$

(c) $P(X > 400) = \left(\frac{364}{365}\right)^{400} = 0.3337,$

$$P(X < 300) = 1 - \left(\frac{364}{365}\right)^{299} = 0.5597.$$

2.3-14 $P(X \geq 100) = P(X > 99) = (0.99)^{99} = 0.3697.$

2.3-16 (a) $f(x) = (1/2)^{x-1}, \quad x = 2, 3, 4, \dots;$

$$\begin{aligned}
 \text{(b)} \quad M(t) &= E[e^{tx}] = \sum_{x=2}^{\infty} e^{tx} (1/2)^{x-1} \\
 &= 2 \sum_{x=2}^{\infty} (e^t/2)^x \\
 &= \frac{2(e^t/2)^2}{1 - e^t/2} = \frac{e^{2t}}{2 - e^t}, \quad t < \ln 2;
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad M'(t) &= \frac{4e^{2t} - e^{3t}}{(2 - e^t)^2} \\
 \mu &= M'(0) = 3; \\
 M''(t) &= \frac{(2 - e^t)^2(8e^{2t} - 3e^{3t}) - (4e^{2t} - e^{3t})2 * (2 - e^t)(-e^t)}{(2 - e^t)^4} \\
 \sigma^2 &= M''(0) - \mu^2 = 11 - 9 = 2;
 \end{aligned}$$

$$\text{(d) (i)} \quad P(X \leq 3) = 3/4; \quad \text{(ii)} \quad P(X \geq 5) = 1/8; \quad \text{(iii)} \quad P(X = 3) = 1/4.$$

$$\begin{aligned}
 \text{2.3-18} \quad P(X > k + j | X > k) &= \frac{P(X > k + j)}{P(X > k)} \\
 &= \frac{q^{k+j}}{q^k} = q^j = P(X > j).
 \end{aligned}$$

2.4 The Binomial Distribution

$$\text{2.4-2} \quad f(-1) = \frac{11}{18}, \quad f(1) = \frac{7}{18};$$

$$\mu = (-1)\frac{11}{18} + (1)\frac{7}{18} = -\frac{4}{18};$$

$$\sigma^2 = \left(-1 + \frac{4}{18}\right)^2 \left(\frac{11}{18}\right) + \left(1 + \frac{4}{18}\right)^2 \left(\frac{7}{18}\right) = \frac{77}{81}.$$

$$\text{2.4-4 (a)} \quad X \text{ is } b(7, 0.15);$$

$$\text{(b) (i)} \quad P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.7166 = 0.2834;$$

$$\text{(ii)} \quad P(X = 1) = P(X \leq 1) - P(X \leq 0) = 0.7166 - 0.3206 = 0.3960;$$

$$\text{(iii)} \quad P(X \leq 3) = 0.9879.$$

$$\text{2.4-6 (a)} \quad X \text{ is } b(15, 0.75); \quad 15 - X \text{ is } b(15, 0.25);$$

$$\text{(b)} \quad P(X \geq 10) = P(15 - X \leq 5) = 0.8516;$$

$$\text{(c)} \quad P(X \leq 10) = P(15 - X \geq 5) = 1 - P(15 - X \leq 4) = 1 - 0.6865 = 0.3135;$$

$$\begin{aligned} \text{(d)} \quad P(X = 10) &= P(X \geq 10) - P(X \geq 11) \\ &= P(15 - X \leq 5) - P(15 - X \leq 4) = 0.8516 - 0.6865 = 0.1651; \end{aligned}$$

$$\text{(e)} \quad \mu = (15)(0.75) = 11.25, \quad \sigma^2 = (15)(0.75)(0.25) = 2.8125; \quad \sigma = \sqrt{2.8125} = 1.67705.$$

$$\text{2.4-8 (a)} \quad 1 - 0.01^4 = 0.99999999; \quad \text{(b)} \quad 0.99^4 = 0.960596.$$

$$\text{2.4-10 (a)} \quad X \text{ is } b(8, 0.90);$$

$$\text{(b) (i)} \quad P(X = 8) = P(8 - X = 0) = 0.4305;$$

$$\begin{aligned}
 \text{(ii)} \quad P(X \leq 6) &= P(8 - X \geq 2) \\
 &= 1 - P(8 - X \leq 1) = 1 - 0.8131 = 0.1869;
 \end{aligned}$$

$$\text{(iii)} \quad P(X \geq 6) = P(8 - X \leq 2) = 0.9619.$$

2.4-12 (a)

$$f(x) = \begin{cases} 125/216, & x = -1, \\ 75/216, & x = 1, \\ 15/216, & x = 2, \\ 1/216, & x = 3; \end{cases}$$

$$(b) \quad \mu = (-1) \cdot \frac{125}{216} + (1) \cdot \frac{75}{216} + (2) \cdot \frac{15}{216} + (3) \cdot \frac{1}{216} = -\frac{17}{216};$$

$$\sigma^2 = E(X^2) - \mu^2 = \frac{269}{216} - \left(-\frac{17}{216}\right)^2 = 1.2392;$$

$$\sigma = 1.11;$$

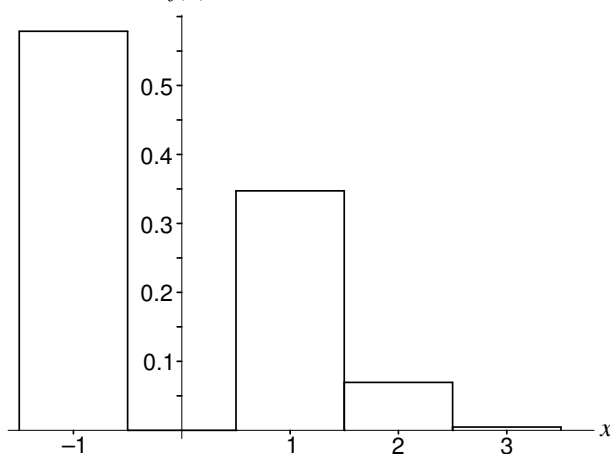
(c) See Figure 2.4-12. $f(x)$ 

Figure 2.4-12: Losses in chuck-a-luck

2.4-14 Let X equal the number of winning tickets when n tickets are purchased. Then

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \left(\frac{9}{10}\right)^n. \end{aligned}$$

$$(a) \quad 1 - (0.9)^n = 0.50$$

$$(0.9)^n = 0.50$$

$$n \ln 0.9 = \ln 0.5$$

$$n = \frac{\ln 0.5}{\ln 0.9} = 6.58$$

so $n = 7$.

$$(b) \quad 1 - (0.9)^n = 0.95$$

$$(0.9)^n = 0.05$$

$$n = \frac{\ln 0.05}{\ln 0.9} = 28.43$$

so $n = 29$.

2.4-16 It is given that X is $b(10, 0.10)$. We are to find M so that

$P(1000X \leq M) \geq 0.99$ or $P(X \leq M/1000) \geq 0.99$. From Appendix Table II,

$P(X \leq 4) = 0.9984 > 0.99$. Thus $M/1000 = 4$ or $M = 4000$ dollars.

2.4-18 X is $b(5, 0.05)$. The expected number of tests is

$$1P(X = 0) + 6P(X > 0) = 1(0.7738) + 6(1 - 0.7738) = 2.131.$$

2.4-20 (a) (i) $b(5, 0.7)$; (ii) $\mu = 3.5, \sigma^2 = 1.05$; (iii) 0.1607;

(b) (i) geometric, $p = 0.3$; (ii) $\mu = 10/3, \sigma^2 = 70/9$; (iii) 0.51;

(c) (i) Bernoulli, $p = 0.55$; (ii) $\mu = 0.55, \sigma^2 = 0.2475$; (iii) 0.55;

(d) (ii) $\mu = 2.1, \sigma^2 = 0.89$; (iii) 0.7;

(e) (i) discrete uniform on $1, 2, \dots, 10$; (ii) 5.5, 8.25; (iii) 0.2.

2.5 The Negative Binomial Distribution

$$\mathbf{2.5-2} \quad \binom{10-1}{5-1} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^5 = \frac{126}{1024} = \frac{63}{512}.$$

2.5-4 Let “being missed” be a success and let X equal the number of trials until the first success. Then $p = 0.01$.

$$P(X \leq 50) = 1 - 0.99^{50} = 1 - 0.605 = 0.395.$$

2.5-6 (a) $R(t) = \ln(1 - p + pe^t),$

$$R'(t) = \left[\frac{pe^t}{1 - p + pe^t} \right]_{t=0} = p,$$

$$R''(t) = \left[\frac{(1 - p + pe^t)(pe^t) - (pe^t)(pe^t)}{(1 - p + pe^t)^2} \right]_{t=0} = p(1 - p);$$

(b) $R(t) = n \ln(1 - p + pe^t),$

$$R'(t) = \left[\frac{npe^t}{1 - p + pe^t} \right]_{t=0} = np,$$

$$R''(t) = n \left[\frac{(1 - p + pe^t)(pe^t) - (pe^t)(pe^t)}{(1 - p + pe^t)^2} \right]_{t=0} = np(1 - p);$$

(c) $R(t) = \ln p + t - \ln[1 - (1 - p)e^t],$

$$R'(t) = \left[1 + \frac{(1 - p)e^t}{1 - (1 - p)e^t} \right]_{t=0} = 1 + \frac{1 - p}{p} = \frac{1}{p},$$

$$R''(t) = [(-1)\{1 - (1 - p)e^t\}^2\{-(1 - p)e^t\}]_{t=0} = \frac{1 - p}{p};$$

(d) $R(t) = r [\ln p + t - \ln\{1 - (1 - p)e^t\}],$

$$R'(t) = r \left[\frac{1}{1 - (1 - p)e^t} \right]_{t=0} = \frac{r}{p},$$

$$R''(t) = r [(-1)\{1 - (1 - p)e^t\}^{-2}\{-(1 - p)e^t\}]_{t=0} = \frac{r(1 - p)}{p^2}.$$

$$\mathbf{2.5-8} \quad (0.7)(0.7)(0.3) = 0.147.$$

2.5-10 (a) Let X equal the number of boxes that must be purchased. Then

$$E(X) = \sum_{x=1}^{12} \frac{1}{(13-x)/12} = \frac{86021}{2310} = 37.2385;$$

$$(b) \frac{100 \cdot E(X)}{365} \approx 10.2.$$

2.6 The Poisson Distribution

2.6-2 $\lambda = \mu = \sigma^2 = 3$ so $P(X = 2) = 0.423 - 0.199 = 0.224$.

$$\begin{aligned} \text{2.6-4} \quad 3 \frac{\lambda^1 e^{-\lambda}}{1!} &= \frac{\lambda^2 e^{-\lambda}}{2!} \\ e^{-\lambda} \lambda (\lambda - 6) &= 0 \\ \lambda &= 6 \end{aligned}$$

Thus $P(X = 4) = 0.285 - 0.151 = 0.134$.

2.6-6 $\lambda = (1)(50/100) = 0.5$, so $P(X = 0) = e^{-0.5}/0! = 0.607$.

2.6-8 $np = 1000(0.005) = 5$;

(a) $P(X \leq 1) \approx 0.040$;

(b) $P(X = 4, 5, 6) = P(X \leq 6) - P(X \leq 3) \approx 0.762 - 0.265 = 0.497$.

2.6-10 $\sigma = \sqrt{9} = 3$,

$$P(3 < X < 15) = P(X \leq 14) - P(X \leq 3) = 0.959 - 0.021 = 0.938.$$

2.6-12 Since $E(X) = 0.2$, the expected loss is $(0.02)(\$10,000) = \$2,000$.

Chapter 3

Continuous Distributions

3.1 Random Variables of the Continuous Type

3.1-2 $\mu = 0$, $\sigma^2 = (1 + 1)^2/12 = 1/3$.

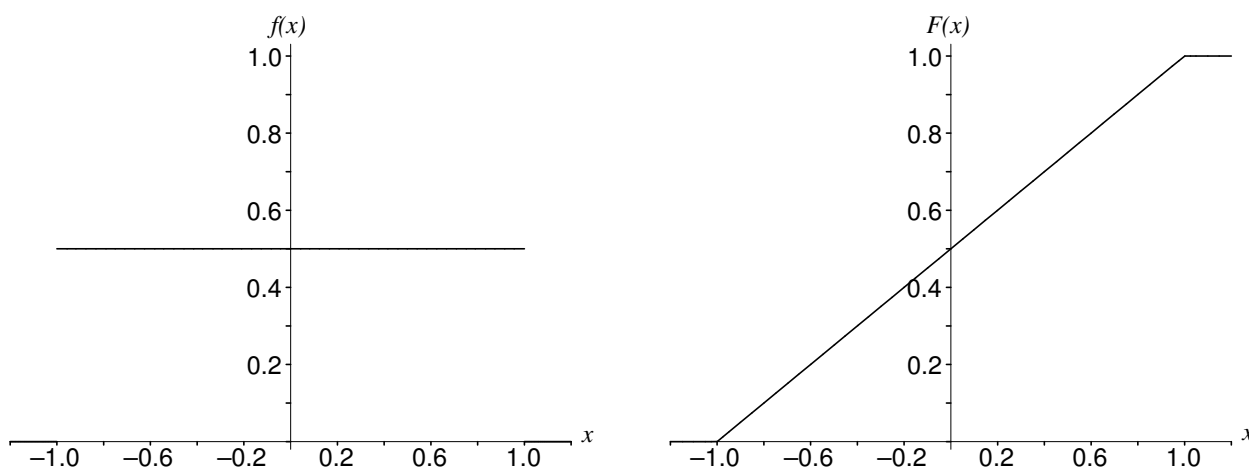


Figure 3.1-2: $f(x) = 1/2$ and $F(x) = (x + 1)/2$

3.1-4 X is $U(4, 5)$;

(a) $\mu = 9/2$; (b) $\sigma^2 = 1/12$; (c) 0.5.

$$\begin{aligned}
 \mathbf{3.1-6} \quad E(\text{profit}) &= \int_0^n [x - 0.5(n - x)] \frac{1}{200} dx + \int_n^{200} [n - 5(x - n)] \frac{1}{200} dx \\
 &= \frac{1}{200} \left[\frac{x^2}{2} + \frac{(n - x)^2}{4} \right]_0^n + \frac{1}{200} \left[6nx - \frac{5x^2}{2} \right]_n^{200} \\
 &= \frac{1}{200} [-3.25n^2 + 1200n - 100000] \\
 \text{derivative} &= \frac{1}{200} [-6.5n + 1200] = 0 \\
 n &= \frac{1200}{6.5} \approx 185.
 \end{aligned}$$

$$\mathbf{3.1-8} \quad (\mathbf{a}) \quad (\mathbf{i}) \quad \int_0^c x^3/4 \, dx = 1$$

$$c^4/16 = 1$$

$$c = 2;$$

$$(\mathbf{ii}) \quad F(x) = \int_{-\infty}^x f(t) \, dt$$

$$= \int_0^x t^3/4 \, dt$$

$$= x^4/16,$$

$$F(x) = \begin{cases} 0, & -\infty < x < 0, \\ x^4/16, & 0 \leq x < 2, \\ 1, & 2 \leq x < \infty. \end{cases}$$

(iii)

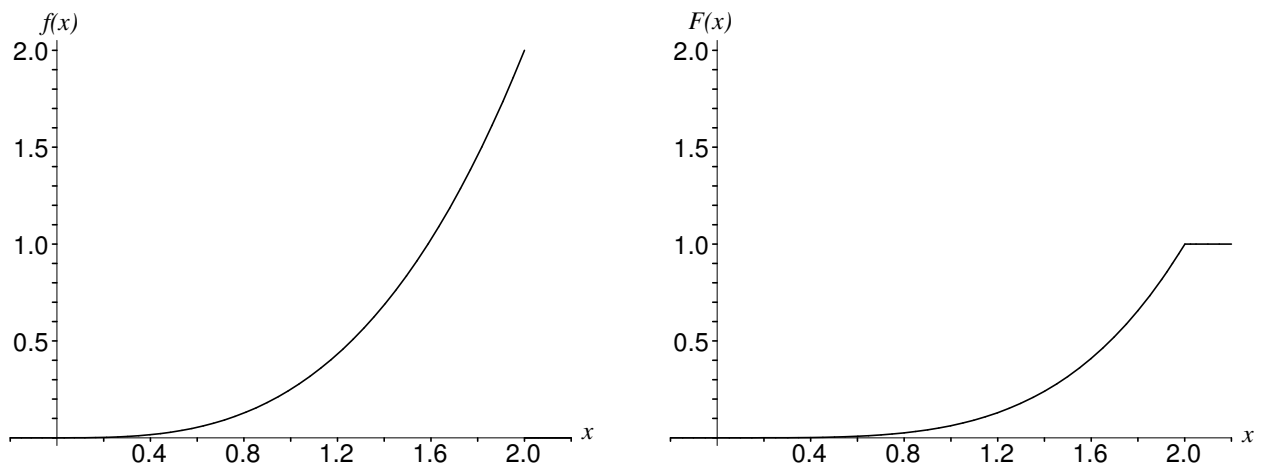


Figure 3.1-8: (a) Continuous distribution pdf and cdf

$$(\mathbf{iv}) \quad \mu = \int_0^2 (x)(x^3/4) \, dx = \frac{8}{5};$$

$$E(X^2) = \int_0^2 (x^2)(x^3/4) \, dx = \frac{8}{3};$$

$$\sigma^2 = \frac{8}{3} - \left(\frac{8}{5}\right)^2 = \frac{8}{75}.$$

$$\begin{aligned}
 \text{(b) (i)} \quad \int_{-c}^c (3/16)x^2 dx &= 1 \\
 c^3/8 &= 1 \\
 c &= 2;
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_{-2}^x (3/16)t^2 dt \\
 &= \left[\frac{t^3}{16} \right]_{-2}^x \\
 &= \frac{x^3}{16} + \frac{1}{2},
 \end{aligned}$$

$$F(x) = \begin{cases} 0, & -\infty < x < -2, \\ \frac{x^3}{16} + \frac{1}{2}, & -2 \leq x < 2, \\ 1, & 2 \leq x < \infty. \end{cases}$$

(iii)

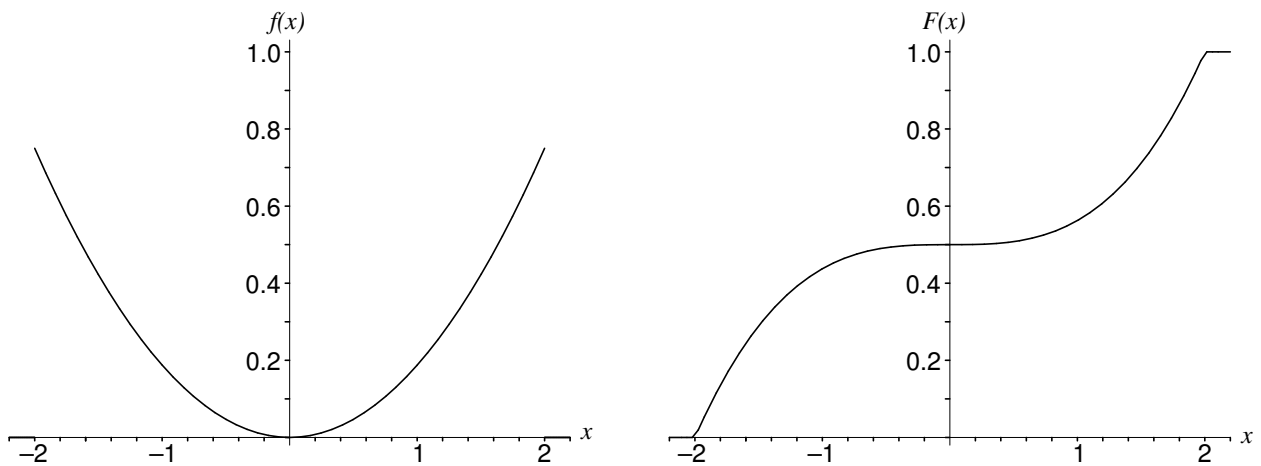


Figure 3.1-8: (b) Continuous distribution pdf and cdf

$$\begin{aligned}
 \text{(iv)} \quad \mu &= \int_{-2}^2 (x)(3/16)(x^2) dx = 0; \\
 \sigma^2 &= \int_{-2}^2 (x^2)(3/16)(x^2) dx = \frac{12}{5}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) (i)} \quad \int_0^1 \frac{c}{\sqrt{x}} dx &= 1 \\
 2c &= 1 \\
 c &= 1/2.
 \end{aligned}$$

The pdf in part (c) is unbounded.

$$\begin{aligned}
 \text{(ii)} \quad F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_0^x \frac{1}{2\sqrt{t}} dt \\
 &= \left[\sqrt{t} \right]_0^x = \sqrt{x}, \\
 F(x) &= \begin{cases} 0, & -\infty < x < 0, \\ \sqrt{x}, & 0 \leq x < 1, \\ 1, & 1 \leq x < \infty. \end{cases}
 \end{aligned}$$

(iii)

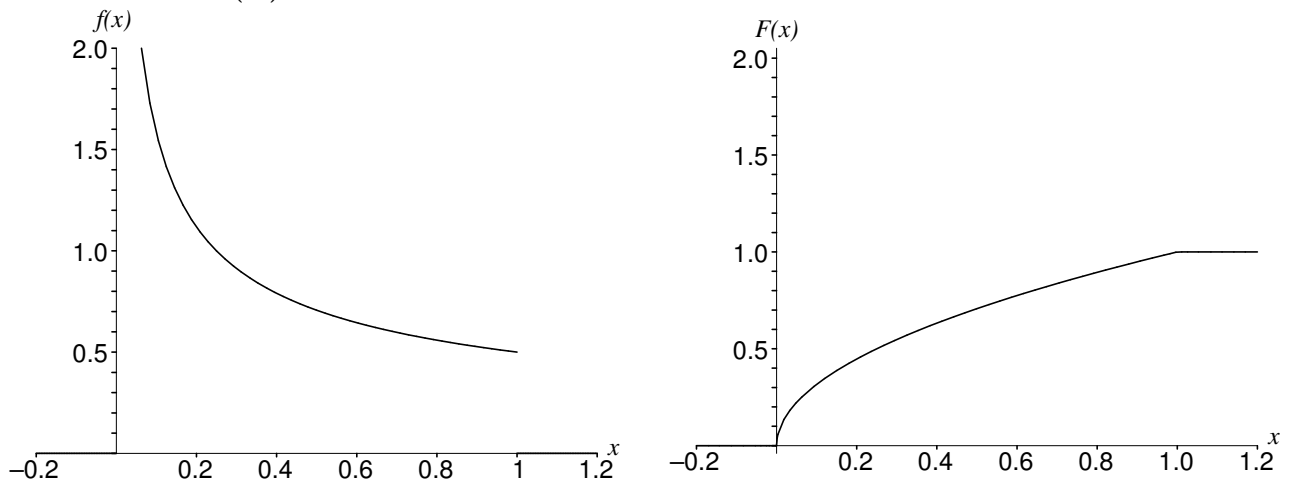


Figure 3.1-8: (c) Continuous distribution pdf and cdf

$$\begin{aligned}
 \text{(iv)} \quad \mu &= \int_0^1 (x)(1/2)/\sqrt{x} dx = \frac{1}{3}; \\
 E(X^2) &= \int_0^1 (x^2)(1/2)/\sqrt{x} dx = \frac{1}{5}; \\
 \sigma^2 &= \frac{1}{5} - \left(\frac{1}{3}\right)^2 = \frac{4}{45}.
 \end{aligned}$$

$$\text{3.1-10 (a)} \quad \int_1^{\infty} \frac{c}{x^2} dx = 1$$

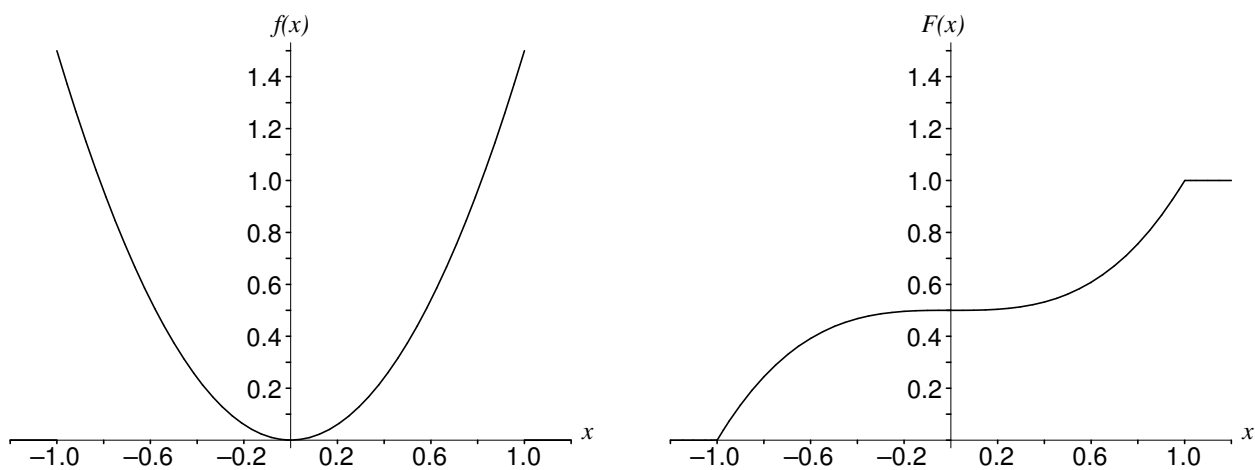
$$\left[\frac{-c}{x} \right]_1^{\infty} = 1$$

$$c = 1;$$

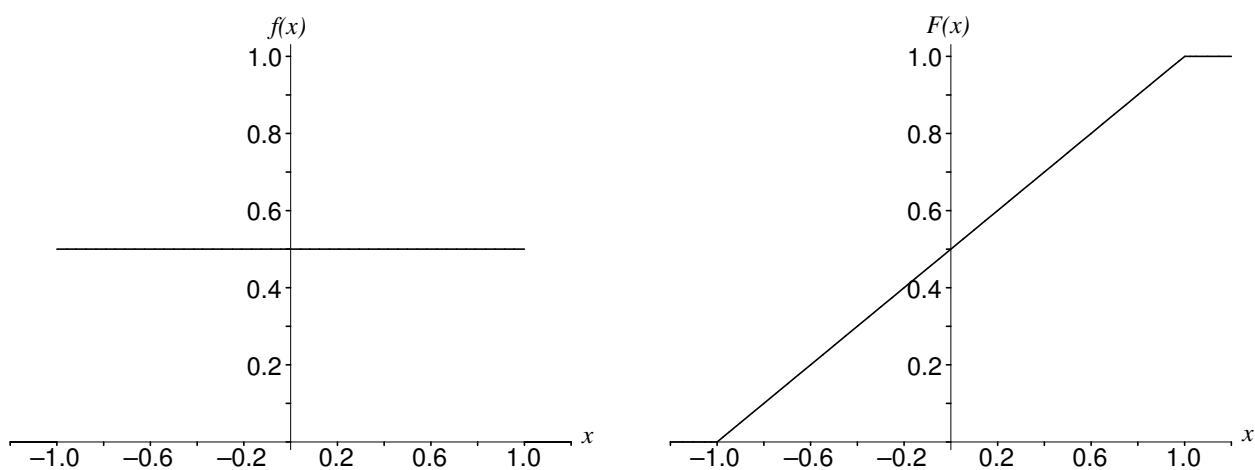
$$\text{(b)} \quad E(X) = \int_1^{\infty} \frac{x}{x^2} dx = [\ln x]_1^{\infty}, \text{ which is unbounded.}$$

3.1-12 (a)

$$F(x) = \begin{cases} 0, & -\infty < x < -1, \\ (x^3 + 1)/2, & -1 \leq x < 1, \\ 1, & 1 \leq x < \infty. \end{cases}$$

Figure 3.1-12: (a) $f(x) = (3/2)x^2$ and $F(x) = (x^3 + 1)/2$ **(b)**

$$F(x) = \begin{cases} 0, & -\infty < x < -1, \\ (x + 1)/2, & -1 \leq x < 1, \\ 1, & 1 \leq x < \infty. \end{cases}$$

Figure 3.1-12: (b) $f(x) = 1/2$ and $F(x) = (x + 1)/2$

$$(c) \quad F(x) = \begin{cases} 0, & -\infty < x < -1, \\ (x+1)^2/2, & -1 \leq x < 0, \\ 1 - (1-x)^2/2, & 0 \leq x < 1, \\ 1, & 1 \leq x < \infty. \end{cases}$$

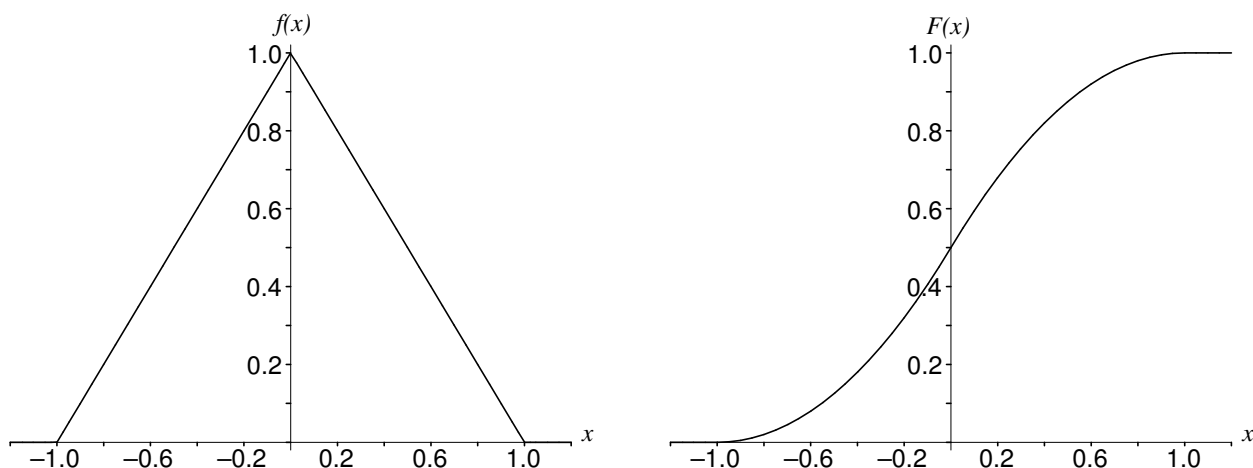


Figure 3.1-12: (c) $f(x)$ and $F(x)$ for Exercise 3.1-12(c)

3.1-14 (b)

$$F(x) = \begin{cases} 0, & -\infty < x \leq 0, \\ \frac{x}{2}, & 0 < x \leq 1, \\ \frac{1}{2}, & 1 < x \leq 2, \\ \frac{x}{2} - \frac{1}{2}, & 2 \leq x < 3, \\ 1, & 3 \leq x < \infty; \end{cases}$$

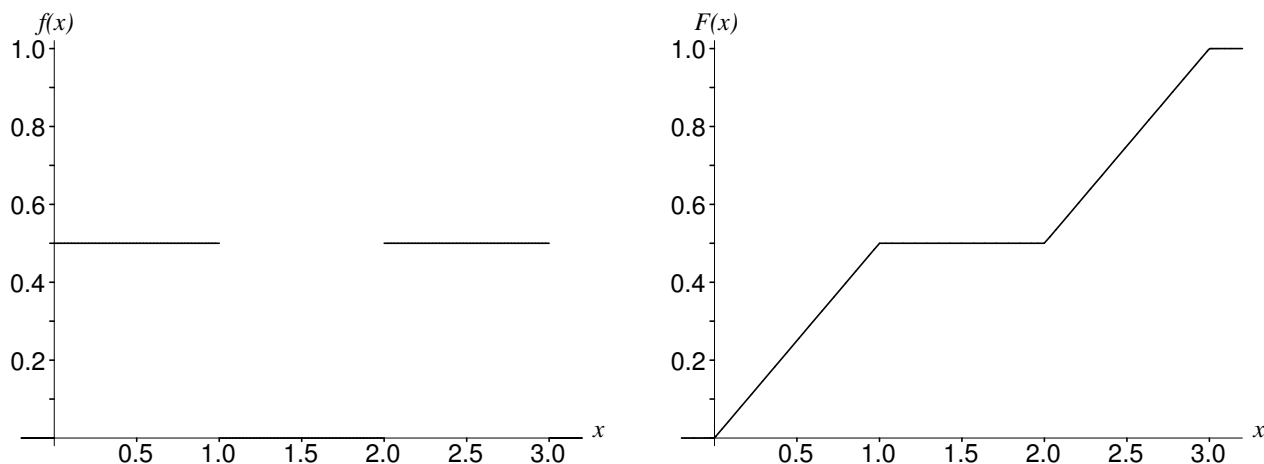


Figure 3.1-14: $f(x)$ and $F(x)$ for Exercise 3.1-14(a) and (b)