1

FOUNDATIONS

Questions and Problems

1.1. The statement makes it clear the instruments were lost. This means that one cannot device experiments to verify the claim. That makes the statement a non-valid hypothesis.

1.2. You assume that the competing product contains non-zero fat, and that the serving sizes of the two are equal. Say the two foods contain the same amount of fat per ounce. The maker of the product being advertised could print his label showing a *recommended serving size* 50 percent smaller than the recommended serving size of the competing product. This makes the claim of 50 percent less fat *per serving* true but of course misleading.

1.3. You assume the sequence is linear, meaning each entry is larger than the previous one by a constant amount. As an alternative, the sequence could be formed by starting with 1, 2 and then setting the nth term c_n equal to the sum of the previous two terms: $c_n = c_{n-1} + c_{n-2}$. This would work for $c_3 = c_2 + c_1 = 2 + 1 = 3,$ and would yield 5 as the next number in the sequence.

1.4. If you assume that the coins are currently circulated U.S. currency, you would not be able to find a solution. If, however, you consider that the word "cents" is also used to refer to hundredths of other currencies, then all that would be required is that increments of 10 and 20 centers exist in some currency. As an example, "cents" may refer to hundredths of a Euro. You would say the coins must be worth 10 and 20 Euro cents, respectively (which do exist).

1.5. There are 12 ways:

4|3|2|1	4|3|2|1	4|3|2|1	4|3|2|1
1|2|3|4	1|2|3|4	1|2|3|4	1|2|3|4
3|1|4|2	2|1|4|3	2|4|1|3	3|4|1|2
2|4|1|3	3|4|1|2	3|1|4|2	2|1|4|3

4|3|1|2	4|3|1|2	4|3|1|2	4|3|1|2
1|2|3|4	1|2|3|4	1|2|4|3	1|2|4|3
2|1|4|3	3|4|1|2	2|1|3|4	2|4|3|1
3|4|2|1	2|1|4|3	3|4|2|1	3|1|2|4

4|3|1|2	
1|2|4|3	
3|1|2|4	
2|4|3|1	

4|3|1|2
1|2|4|3
3|4|2|1
2|1|3|4

| |
|---|
| 4\|3\|2\|1 |
| 1\|2\|4\|3 |
| 2\|1\|3\|4 |
| 3\|4\|1\|2 |

4|3|2|1
1|2|4|3
3|4|1|2
2|1|3|4

1.6. There is one axis of reflection symmetry. It is marked by the dashed line:
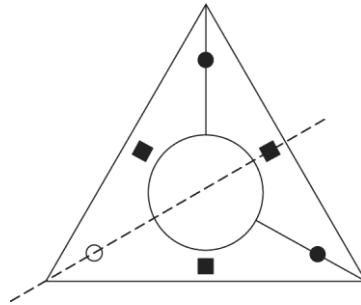
1.7. One. An axis of rotational symmetry is an axis about which the object can be rotated (through some angle other than a multiple of 360 degrees), that results in an indistinguishable appearance compared to the original orientation of the object. For a cone the axis passing through the center of the circular face and through the vertex (point) of the cone is the only axis of rotational symmetry.

**1.8.** One unit down and left from the upper right corner. This way the coins form a square:
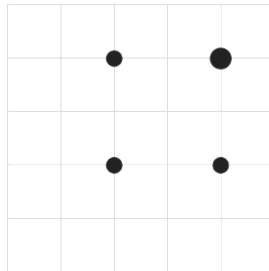
1.9. "T" and "A" are reflection symmetric across a vertical line passing through the center of the letter. "E" and "B" are reflection symmetric across a horizontal line passing through the center of the letter. "L" and "S" have no reflection symmetry.

1.10. Reflection and rotation symmetry. A sphere is reflection symmetric across any plane that passes through its center, and rotationally symmetric around any axis passing through its center.

1.11. A cube has 9 planes of reflection symmetry and 13 axes of rotational symmetry. 3 planes each bisect four sides of the cube. The remaining six planes pass through edges that are diagonally opposite each other. 3 of the axes of rotational symmetry pass orthogonally through the centers of two square faces. 4 of the axes pass through corners that are opposite each other along the body diagonal. 6 axes pass through the centers of edges that are opposite each other along the body diagonal.

1.12. (*a*) The two possible axes of reflective symmetry are drawn.
(*b*) Rotation about an axis through the center that is perpendicular to the page by $180°$ will also bring the disk onto itself.
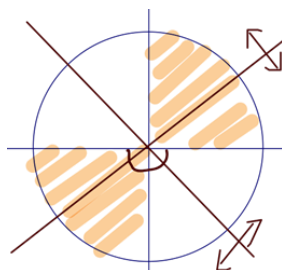
1.13. The maximum number of axes of reflection symmetry is two. There are two sides about which we have no information. Let us assume the side opposite the visible blue side is also blue, and the side opposite the visible red side is also red. In that case, the object would be reflection symmetric about a vertical plane that bisects both blue sides, and a about a vertical plane that bisects both red sides.

1.14. We use $v_x = \Delta x/\Delta t$ for constant speed in the *x* direction to write:

$$\Delta x = v_x \Delta t = \frac{299{,}792{,}458 \text{ m}}{1\text{s}} \times \frac{3600 \text{ s}}{1 \text{ h}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{365.25 \text{ days}}{1 \text{ yr}} \times 78 \text{ yrs} = 7.4 \times 10^{17} \text{ m}$$

1.15. (*a*) 8.33 min = 8.33 × 60 seconds = 499.8 seconds.
The distance light travels during this time is (499.80 sec) × (299792458 m/s) = 149836270508.40 m $\approx$ 149 million kilometers.
(*b*) 149836270508.40 m /696000000 m = 215.4. This means you can comfortably fit 214 Suns!

1.16. The blue whale and a human have densities that are the same orders of magnitude, and contain similar concentrations of different atom types. Hence it is reasonable to say that the ratio of the numbers of atoms in these two species should be roughly equal to the ratio of volumes of these two species: $\frac{V_{bw}}{V_{human}} = \frac{10^{32}}{10^{29}} = 10^3$. Since whales and people are three dimensional, this corresponds to blue whales being roughly one order of magnitude larger in each of three dimensions. Hence, the blue whale is approximately 10 times longer than a human is tall.

1.17. Because the gastrotrich lifetime is given as three days, this should technically be treated as being on the order of one day. However, this is an approximate value of a lifetime, and if it were slightly higher, it would be treated as being on the order of ten days. Either of these is defensible in order of magnitude treatments. One tortoise lifetime can be related to gastrotrich lifetimes using the following order of magnitude conversions:

$$10^2 \text{ yr} \times \frac{10^3 \text{ days}}{1 \text{ yr}} \times \frac{1 \text{ gastrotrich lifetime}}{1 \text{ day}} = 10^5 \text{ gastrotrich lifetimes}$$

If we had used 10 days as the order of magnitude for a gastrotrich lifetime we would have obtained 10^4 gastrotrich lifetimes. Hence there are 10,000 to 100,000 gastrotrich lifetimes in one tortoise lifetime. Either of these is acceptable.

1.18. Take 60 kg as an average adult mass and assume that the density of the body is similar to that of water, approximately 1000kg/m^3. This gives an average volume of $V = 60/1000 = 0.06$ m^3. The volume allocated for RBCs is $0.35 \times 0.08 \times 0.06 = 0.0016$ m^3. The volume of one RBC is approximately $(10^{-5}$ m$)^3 = 10^{-15}$ m^3. This means one will have about 1.6×10^{12} RBCs or about a trillion!

1.19. Answers may vary by an order of magnitude since some textbooks may be somewhat thicker than 3.0 cm, and others may be thinner. My textbook has a thickness that is on the order of 10^{-1} m. The distance to the moon is

3.84×10^8 m, meaning the distance is of order 10^9 m. Dividing the distance by the thickness of one textbook yields

$$\text{Number of books} = \frac{\text{Distance to Moon}}{\text{Thickness of book}} = \frac{10^9}{10^{-1}} = 10^{10}.$$ Hence 10^{10} copies of my physics textbook could fit between Earth and the Moon.

1.20. We proceed by finding the total mass of water in the pool, and dividing this by the mass of a single molecule of water:

$N = \dfrac{m_{pool}}{m_{molecule}} = \dfrac{\rho_{water} V_{pool}}{M / N_A}$ where $\rho_{water} = 1000 \text{ kg/m}^3$ is the density of water, $V_{pool} = (15 \times 8.5 \times 1.5) \text{ m}^3$ is the volume

of the swimming pool, $M = 0.018 \text{kg}$ is the molar mass of water, and $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ is Avogadro's number. Using this information we find

$$N = \frac{(1.0 \times 10^3 \text{ kg/m}^3)(191 \text{ m}^3)}{(0.018 \text{ kg/mol})} (6.02 \times 10^{23} \text{ mol}^{-1}) = 6.4 \times 10^{30} \text{ molecules in the pool.}$$

1.21. (*a*) Since numbers are not given, it might be natural to cancel lengths and be left with a factor of 2^3. In that case the volume of the cube would increase by one order of magnitude. But if one were asked to use an order of magnitude estimate to first express ℓ_2 in terms of ℓ_1, one might find that they have the same order of magnitude and that the volume therefore does not increase. Either of these answers (one order of magnitude, or zero orders of magnitude) is acceptable. (*b*) Yes, because of the rules of rounding numerical values. For example, if $V_1 = 3.5 \text{ m}^3$, that value would round to an order of magnitude of 10 m^3. Then $V_2 = 8V_1 = 28 \text{ m}^3$, which also rounds to an order of magnitude of 10 m^3.

1.22. The speed of light is of order 10^8 m/s. The length of Earth's trip around the Sun is of order 10^{12} m. Hence the order of magnitude of the time light would need to make the same trip around the sun is

$\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{10^{12} \text{ m}}{10^8 \text{ m/s}} = 10^4$ s. Hence light would take approximately 10^4 s to complete this trip.

1.23. The surface area of the roughly spherical distribution of leaves is $4\pi r^2$, where r is the radius of the tree's sphere of leaves. The surface area of an individual leaf is just its length ℓ times its width w. We can find the number

of leaves by dividing the total surface area by the area of one leaf: $N = \dfrac{4\pi r^2}{\ell w} = \dfrac{10(10 \text{ m})^2}{(0.1 \text{m})(0.1 \text{ m})} = 10^5$ leaves.

1.24. 70 years $\approx$ 70 year $\times$ (365.25 day/year) $\times$ (24 hour/day) $\times$ (60 min/hour) $\approx 3.68 \times 10^7$ minutes. The heart will then beat about $3.68 \times 10^7 \times 72 = 2.65 \times 10^9$ times in a lifetime.

1.25. Not reasonable. Because light travels much faster than sound, any thunder peal is delayed compared to the light signal caused by the lightning bolt event. From the principle of causality, the lightning you see after you hear the peal cannot have caused the peal. The peal you heard must have come from a previous lightning strike.

1.26. Call the period of time required for one such oscillation T. Then the second is defined such that

$1.0 \text{ s} = (9.19 \times 10^9)T$ or $T = \dfrac{1.0 \text{ s}}{(9.19 \times 10^9)} = 1.09 \times 10^{-10}$ s.

1.27. That the barrier lowers time after time 30 s before a train passes is consistent with a causal relationship between the two events. The single negative result, however, tells you that the lowering of the barrier cannot be the *direct* cause of the passing of the train. More likely, the lowering is triggered when the train passes a sensor quite a distance up the tracks from the barrier and the sensor sends an electrical signal to the lowering mechanism. A malfunction in either the sensor, the electrical connections, or the lowering mechanism would account for the one negative result you observed.

1.28.

$$\lambda = \frac{299792458}{9.19 \times 10^9} = .0326 \, m$$

In fact this indicates that this radiation (related to the hyperfine splitting of cesium 133) is about 3.26 mm in wavelength, so one millimetre is less than 1/3 of one wavelength.

**1.29.** Symbols: λ wavelength, f frequency, c speed of light.
The statement can be translated into: $\lambda = \chi/\phi$.

1.30. Rectangle, parallelogram, and two equilateral triangles meeting at a vertex forming an hourglass shape.

1.31. The problem states that two adjacent sides must make an angle of $30°$. This most likely means the interior angle between actual sides. This angle is labeled α in the figure below. But one might also describe the exterior angle in this way. This angle is labeled β below. If one accepts this interpretation, some of the $30°$ angles could be interior and some could be exterior.
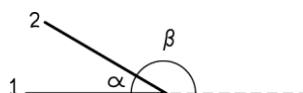

Let us first address the case in which the $30°$ angle refers to the interior angle. Remembering that the segments cannot cross, there is only one possible arrangement of segments that fits the description:
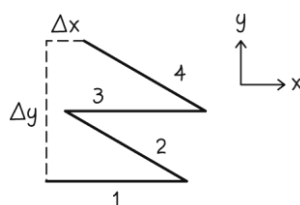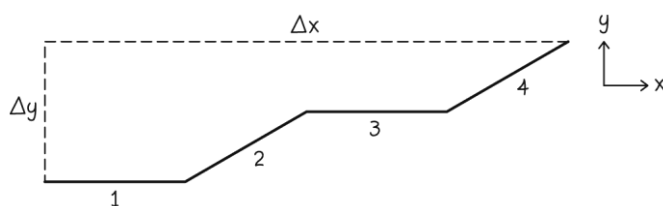

Clearly $\Delta y = 2\ell \sin(\theta)$ and $\Delta x = 2\ell(1 - \cos(\theta))$. The Pythagorean Theorem tells us that the distance between the two

unconnected points must be $d = \sqrt{\Delta x^2 + \Delta y^2} = 2\ell\left((1 - \cos(30°))^2 + \sin^2(30°)\right)^{1/2} = 1.0\ell$.

So the distance between unconnected ends is ℓ.

If the $30°$ angle refers to the exterior angle, we can obtain several possible shapes. The two shapes that give the shortest and longest distances are shown below:
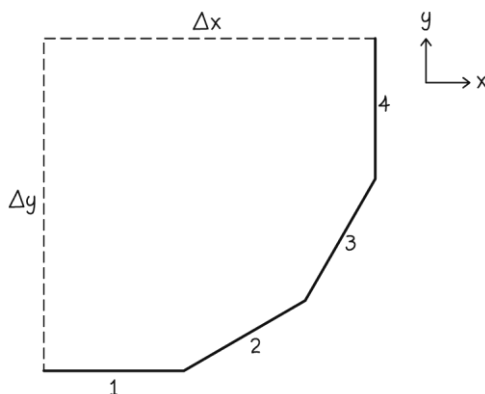
We can see in the top figure (that yields the shortest distance) $\Delta x = \ell(1 + \cos(30°) + \cos(60°) + 0)$, and since the x and y distances are equal based on symmetry, we find $d = \sqrt{2}\ell(1 + \cos(30°) + \cos(60°)) = 3.3\ell$.

In the second figure (that yields the greatest distance) we find $\Delta x = 2\ell(1 + \cos(30°))$ and $\Delta x = 2\ell(\sin(30°))$. Again using the Pythagorean Theorem yields a distance of $d = \left[(2\ell(1 + \cos(30°)))^2 + (2\ell\sin(30°))^2 \right]^{1/2} = 3.9\ell$.

If a combination of interior and exterior angles is used, there are even more possibilities. The shortest of these distances is 0 (parallelogram). It can be shown that other possible distances include 2.0ℓ, 2.2ℓ, 2.4ℓ.

1.32. Uncle, cousin, grandmother, aunt, grandfather, brother.

1.33. Consider the diagram below.
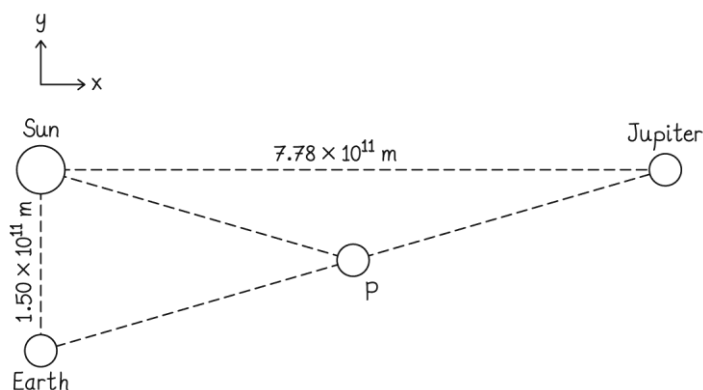
The distance from the Sun to point P is
$$\sqrt{\Delta x^2 + \Delta y^2} = (1/2)\sqrt{(1.50 \times 10^{11} \text{ m})^2 + (7.78 \times 10^{11} \text{ m})^2} = 3.96 \times 10^{11} \text{ m}$$
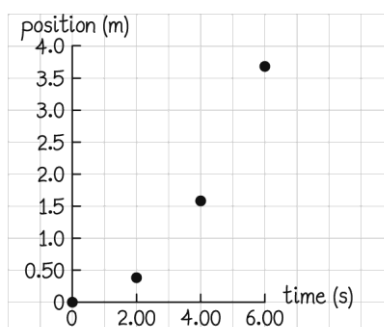The time that light would require to cross this distance can be found using
$$\Delta t = d/v = (3.96 \times 10^{11} \text{ m})/(3.00 \times 10^8 \text{ m/s}) = 1.32 \times 10^3 \text{ s}$$
which is equivalent to about 22.0 minutes.

1.34.

Time (s)	Position (m)
0.00	0.00
2.00	0.40
4.00	1.59
6.00	3.64

1.35. (*a*) The position decreases linearly as a function of time, from an initial position of 4.0 m to a final position of zero at a time of 8.0 s, with a slope of -0.5 m/s. (*b*) $x(t) = mt + b$ where $m = -0.5$ m/s, and $b = 4.0$ m.

1.36. Forming a regular tetrahedron from these triangles will automatically satisfy all conditions.

1.37.

$$v = \frac{100 \text{ m}}{9.58 \text{ s}} = 10.44 \frac{\text{m}}{\text{s}}$$

$$1 \frac{\text{m}}{\text{s}} = \frac{\frac{1}{1000} \text{ km}}{\frac{1}{3600} \text{ hr}} = 3.6 \frac{\text{km}}{\text{hr}} = \frac{3.6}{1.609} \frac{\text{mile}}{\text{hr}}$$

This means $v = 37.57$ m/s = 23.36 mile/hr.

1.38. We change units using known conversion factors:

$$35{,}000 \text{ ft} \times \frac{1 \text{ mile}}{5280 \text{ ft}} = 6.629 \text{ miles}$$

$$35{,}000 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{25.4 \text{ mm}}{1 \text{ in}} \times \frac{1 \text{ m}}{10^3 \text{ mm}} \times \frac{1 \text{ km}}{10^3 \text{ m}} = 10.7 \text{ km}$$

1.39. (*a*) The density will be the same. (*b*) The density will be the same.

1.40.

$$1 \frac{\text{m}}{\text{s}} = \frac{\frac{1}{1000} \text{ km}}{\frac{1}{3600} \text{ hr}} = 3.6 \frac{\text{km}}{\text{hr}} = \frac{3.6}{1.609} \frac{\text{mile}}{\text{hr}}$$

$$1 \frac{\text{m}}{\text{s}} = \frac{39.37 \text{in}}{10^9 \text{ nanoseconds}} = \frac{3.6}{1.609} \frac{\text{mile}}{3600 \text{ second}} = 6.2 \times 10^{-4} \frac{\text{mile}}{\text{second}}$$

This means $v = 1234 \frac{\text{km}}{\text{hr}} = 1.35 \times 10^5 \frac{\text{in}}{\text{s}} = 0.213$ mile/sec.

1.41. If the two stones are made from the same material, they should have roughly the same density. We calculate the density of each stone and compare them:

$$\rho_1 = \frac{m_1}{V_1} = \frac{2.9 \times 10^{-2} \text{ kg}}{10.0 \text{ cm}^3} = 2.9 \times 10^{-3} \text{ kg/cm}^3$$

$$\rho_2 = \frac{m_2}{V_2} = \frac{2.5 \times 10^{-2} \text{ kg}}{7.50 \text{ cm}^3} = 3.3 \times 10^{-3} \text{ kg/cm}^3$$

No, it is not likely. Stone 2 has considerably higher density.

1.42. This length can be expressed as $1.000 \text{ mi} + 440 \text{ yd} \times \dfrac{3 \text{ ft}}{1 \text{ yd}} \times \dfrac{1 \text{ mi}}{5,280 \text{ ft}} = 1.250 \text{ mi}.$ We now convert this entirely

to feet: $1.250 \text{ mi} \times \dfrac{5280 \text{ ft}}{1 \text{ mi}} = 6{,}600 \text{ ft}.$

1.43. Manipulating the expression algebraically, we get $T^2 = 4\pi^2 \dfrac{l}{g}$

The units of g are then $\dfrac{\text{units of } l}{\text{units of } T^2} = \dfrac{\text{m}}{\text{s}^2}$.

1.44. Mass density is the ratio of mass per unit volume. While one could have any mass of a given substance, or any volume of that substance, the density tends to be a constant value for a given material (under certain conditions).

1.45. For all cases we find the order of magnitude of the mass of Earth using $m = \rho V = \rho \left(\dfrac{4}{3} \pi R_{\text{E}}^3 \right).$

(*a*) $m = \rho_{\text{air}} \left(\dfrac{4}{3} \pi R_{\text{E}}^3 \right) = (10^0 \text{ kg/m}^3)(1 \times 10 \times (10^7 \text{ m})^3) = 10^{22} \text{ kg}$

(*b*) $m = \rho_{5515} \left(\dfrac{4}{3} \pi R_{\text{E}}^3 \right) = (10^4 \text{ kg/m}^3)(1 \times 10 \times (10^7 \text{ m})^3) = 10^{26} \text{ kg}$

(*c*) $m = \rho_{\text{nucleus}} \left(\dfrac{4}{3} \pi R_{\text{E}}^3 \right) = (10^{18} \text{ kg/m}^3)(1 \times 10 \times (10^7 \text{ m})^3) = 10^{40} \text{ kg}$

1.46. We rearrange the given expression to solve for *y* and then write all units in terms of SI base units and powers of ten:

$$y = \left(\frac{x}{a} \right)^{2/3} = \left(\frac{61.7 \text{ Eg} \cdot \text{fm}^2/\text{ms}^3}{7.81 \text{ }\mu\text{g/Tm}} \right)^{2/3}$$

$$y = \left(\frac{61.7 \, (10^{18} \text{ g}) \cdot (10^{-15} \text{ m})^2/(10^{-3} \text{ s})^3}{7.81 \, (10^{-6} \text{ g})/(10^{12} \text{ m})} \right)^{2/3}$$

$$y = 3.97 \times 10^{10} \text{ m}^2/\text{s}^2$$

1.47. (*a*) 3.00×10^8 m/s (*b*) 8.99×10^{16} m²/s² (*c*) No. There is a small difference because the answer to (*a*) was rounded before squaring. The answer to (*b*) was obtained using more digits of the speed of light, and only the result was rounded to three significant digits.

1.48. The given distance can also be written as 1.25 miles. We now convert to kilometers using known the known conversion factor:

$$1.25 \text{ mi} \times \frac{1.609 \text{ km}}{1 \text{ mi}} = 2.012 \text{ km}$$

1.49. Your answer has four significant digits. When dividing a quantity by an integer, the number of significant digits should not change.

1.50. Yes, there is a difference in the precision. You will calculate your gas mileage by dividing the number of miles you travel by the gallons of gasoline used. Since the gas pump gives you thousandths (and most vehicles take 10 gal or more) you know the fuel used to five significant digits. Neither distance given has this many, making the precision

of the distance value the limiting factor. Thus you can calculate mileage to three significant digits when you use 40.0 mi for distance and to four significant digits when you use 400.0 mi.

1.51. The additional distance you travel is $20 \times 725 = 14500$ m $= 14.5$ km.

Following the notation in this textbook, we assign an infinite number of significant digits to 20 since it is a counting number. When multiplying we take the significance of 725 which is 3 digits.

The total distance is $23456 + 14.5 = 23470.5$ keeping one decimal point (when adding) in keeping with the notation used in this book.

1.52. We convert the given amount of ingested caffeine using known conversion factors:

$$\frac{34 \text{ mg}}{\text{serving}} \times \frac{2 \text{ servings}}{\text{day}} \times \frac{365.25 \text{ days}}{1 \text{ yr}} \times \frac{1\text{g}}{10^3 \text{ mg}} \times \frac{1 \text{ mol}}{194.19 \text{ g}} \times \frac{6.02 \times 10^{23} \text{ molecules}}{1\text{mol}} = 7.7 \times 10^{22} \text{ molecules/yr}$$

1.53. The molarity of the solution must be known to a precision of 1 part in 15. Since we can measure volume to arbitrary precision and we have the molar mass to very high precision, the limiting factor on the precision of our molarity is the precision of the mass we use. Hence the precision of the mass we use must be at least as good as the required precision of the molarity. We can only measure tenths of a gram, so we must have at least 1.5 grams in order to know the precision of our mass to 1 part in 15 or better. Mathematically:

$$\frac{m}{MV} \pm \frac{\Delta m}{MV} = (0.15 \pm 0.01) \text{ mol/L}$$
$$0.1 \text{ g } \leq \Delta m = 0.01 MV$$
$$V \geq \frac{0.1 \text{ g}}{(0.01 \text{ mol/L})(58.44 \text{ g/mol})}$$
$$V \geq 0.17 \text{ L}$$
$$m = 0.15(MV) = 1.5 \text{ g}$$

As can be seen from the above calculation, the answer could also be given in terms of the minimum volume of solution prepared. That minimum volume is 0.17 L.

1.54. The mass density is given by $\rho = m/V$. The volume is given in milliliters, which are equivalent to cubic centimeters. Hence

$$\rho = \frac{25.403 \text{ g}}{23.42 \text{ cm}^3} \times \frac{10^{-3} \text{ kg}}{1 \text{ g}} \times \frac{10^6 \text{ cm}^3}{1 \text{ m}^3} = 1.085 \times 10^3 \text{ kg/m}^3$$

1.55. The mass evaporated each second is given by

$$\frac{|\Delta m|}{\Delta t} = \frac{m_{\text{dish}} + m_{\text{liquid}} - m_{\text{final}}}{\Delta t}$$
$$= \frac{(145.67 \text{ g}) + (0.335 \text{ g}) - (145.82 \text{ g})}{(25.01 \text{ s})}$$
$$= 7.4 \times 10^{-3} \text{ g/s}$$

Note that we have only two significant digits. Because two of the masses were only known to the hundredths place, we only had two digits of precision after taking the sum (and difference) of masses.

1.56. There are too many significant digits. The volume is only given to two significant digits. The answer could have at most two significant digits.

1.57. Let the time be t seconds. The distance travelled by the hour meter is $2\pi \times .01 \times t$ m and by the second meter $2\pi \times 0.012 \times t \times 60$, where t is in hours.

$$107.01 \text{ m} = 2\pi \times 0.012 \times t \times 60 - 2\pi \times .01 \times t$$

$$t = \frac{100.0}{2\pi[60 \times 0.012 - 0.01]} = 22.4 \text{ hrs} = 22 \text{ hr } 25 \text{ min.}$$

1.58. The path lengths are exactly the same along the straight sections of track. We need only consider the two curved ends. Along those sections, the runner in lane 1 moves through a circle of radius $R_1 = 36.80$ m. A runner in lane 8 moves through a circle of radius $R_8 = 36.80 \text{ m} + 7(1.22 \text{ m}) = 45.34 \text{ m}.$ The total path length difference will then be given by the difference in the circumferences of these two circles: $\Delta x = 2\pi(R_8 - R_1) = 2\pi(8.54 \text{ m}) = 53.7 \text{ m}.$

1.59. Place two coins on the balance, and hold the third in your hand. If they are balanced, the one coin in your hand is the counterfeit. If the two coins are not balanced, then one must be the counterfeit. Swap the lighter of the coins on the balance for the one in your hand. If the two coins now have equal mass, then the counterfeit coin is the one you just removed, and it is lighter than real coins. If they are still unbalanced, then the counterfeit is the one that left in place on the scale, and it is heavier than real coins.

1.60. (*a*) We proceed by assuming nearly perfect packing of the grains of rice (no unoccupied volume in the cup). This would almost certainly not be exactly true, but since the rice is not rigid after cooking, deformations could allow the rice to be tightly packed almost to this limit. With that assumption, we find the number of grains by dividing the total volume by the volume of a single grain:
$$N = \frac{V_{\text{cup}}}{V_{\text{grain}}} = \frac{V_{\text{cup}}}{1 \, \pi R^2} = \frac{250 \text{ cm}^3}{(0.6 \text{ cm})\pi(0.1 \text{ cm})^2} = 1 \times 10^4 \text{ grains}$$

(*b*) We simply divide the given energy by the number of grains we found in part (*a*): $\dfrac{785 \text{ Cal}}{1.3 \times 10^4 \text{ grains}} = 0.06 \text{ Cal/grain.}$

(*c*) The total food calories needed would be 8,000. Dividing this by the 785 food calories in one cup, we find that 10.2 cups of such rice would be required.

1.61. From *Principles* Figure 1.9 we see that there are approximately 10^{80} atoms in the universe. We also know that the number of atoms in a mole is of order 10^{24} atoms/Mole. Hence the number of Moles is given by
$$\frac{10^{80} \text{ atoms}}{10^{24} \text{ atoms/Mole}} = 10^{56} \text{ Moles}$$

1.62. We divide the volume of the bread by the number of raisins in the bread and find:
$$\frac{V_{\text{bread}}}{N_{\text{raisin}}} = \frac{(1)(10 \text{ in})^3}{10^2 \text{ raisins}} = 10 \text{ in}^3/\text{raisin}$$
This volume could be treated as a spherical or cubic region of bread surrounding each raisin. The side length of such a cubic region would be of order 1 inch. This also means that 1 inch would be the order of magnitude of the distance between raisins.

1.63. We could divide the volume of the tree by the volume of the board to obtain the number of boards. But if a half-integer number of boards fits along the height of the tree, this does not help us and we have to round down. So, let us first find how many boards fit along the height.
$$h_{\text{tree}} = 32 \text{ m} \times \frac{10^3 \text{ mm}}{1 \text{ m}} \times \frac{1 \text{ in}}{25.4 \text{ mm}} \times \frac{1 \text{ ft}}{12 \text{ in}} = 105 \text{ ft or } 17.5 \text{ boards}$$
So we can only fit 17 entire boards along the height of the tree. Now if we look at a cross-section of the tree, we can calculate how many boards can fit across the tree. We will treat the tree as though it has a circular cross section.
$$\frac{A_{\text{tree}}}{A_{\text{board}}} = \frac{\pi(0.40 \text{ m})^2}{(2 \text{ in})^2} \times \left(\frac{10^3 \text{ mm}^3}{1 \text{ m}}\right)^2 \times \left(\frac{1 \text{ in}}{25.4 \text{ mm}}\right)^2 = 190 \text{ boards}$$
Clearly, this overestimates the number of boards that can actually be cut, because of the curvature of the tree trunk. We might estimate how many boards will be affected by the curvature by determining how many boards would lie

along the circumference of the trunk. We do this by dividing the circumference of the tree by the width of one board. Some might lie with an edge parallel to the trunk, whereas most will lie with their edges at some angle to the trunk. Still, as this is only an estimate, let us approximate the width of a single board as 2 in. Then the number of boards that would lie along the edge is

$$N = \frac{C_{\text{tree}}}{w_{\text{board}}} = \frac{2\pi r_{\text{tree}}}{w_{\text{board}}} = \frac{2\pi(0.40 \text{ m})}{2 \text{ in}} \times \frac{10^3 \text{ mm}}{1 \text{ m}} \times \frac{1 \text{ in}}{25.4 \text{ mm}} = 49 \text{ boards}$$

So we estimate that we can fit a cross section of 140 complete boards in the cross section of the tree.
Finally, we have a total number of boards given by the number of boards that fit length-wise along the height of the tree times the number of complete boards in a cross section:

$$N_{\text{total}} = N_{\text{high}} N_{\text{across}} = 17 \times 140 = 2 \times 10^3$$

In the last step we have rounded to one significant figure. In addition to being given only one significant figure in the width of the board, we have also ignored secondary corrections such as wood that is lost to the sawing process. Hence our best estimate is 2×10^3 boards.

1.64. Estimates will vary, as there are many methods of estimating. One method of estimation would be to count the number of times it occurs on a few lines to obtain an average number of occurrences per line of text. One could then multiply by the number of lines in a book. One could also estimate the number of letters in the book and assume that "d" occurs just as often as all other letters (not strictly true, but correct to an order of magnitude). The former method is crude, but requires no prior knowledge of the frequency of letters' occurrence in the English language. For this reason, we proceed with the former method. After counting the number of times "d" occurs on a few lines, I estimate that it occurs approximately twice on each line (of order 1). Each page of this book contains a number of lines that is of order 10^2. The book has a number of pages that is of order 10^3. Multiplying yields:

$$\frac{1 \text{ "d"}}{1 \text{ line}} \times \frac{10^2 \text{ lines}}{1 \text{ page}} \times \frac{10^3 \text{ pages}}{1 \text{ book}} = 10^5 \text{ "d"/book}$$

One could obtain a much more reliable estimate if one happens to know that the letter "d" accounts for approximately 4% of the letters in a typical English text. But this method agrees with our crude method to the nearest order of magnitude.

1.65. One human hair is about 100μm in diameter (Thick!). Assume the hair forms a hemispherical shell of inner radius of 100 mm and outer radius of 105 mm (for the 5 mm long hair).

The total hair volume is $V_h = 0.5 \times \frac{4\pi}{3}[0.105^3 - 0.100^3] = 3.3 \times 10^{-4} \text{ m}^3$

The volume of one hair is $V_{1h} = \pi(10^{-4}/2)^2 \times 0.005 = 3.92 \times 10^{-11} \text{ m}^3$, assuming a cylindrical shape.

The number of hairs $= \frac{3.3 \times 10^{-4}}{3.92 \times 10^{-11}} = 8.4 \times 10^6$

Total hair length $= 8.4 \times 10^6 * 0.005 = 42 \times 10^3 \text{m}!$ More than 40 kilometres!

1.66. Not feasible during a four year stay at university, but perhaps possible by starting at age 5. $200K of university cost and 5 cents per can means the student must collect 4 million cans. Doing this during a 4-year university career would require almost 3000 cans every day. Because these must be pulled out of waste bins, even at one can every 10 seconds that uses more than 8 hours each and every day. This is not feasible. But if the student started collecting at age 5 and spent 12 full years at the job, at about 3 hours per day, by the time she reached university she would have her tuition money. Not a fun childhood, but perhaps feasible.

1.67. Call S the storage capacity and A the area required. We wish to obtain an order of magnitude by which the fraction S/A has increased. Hence we estimate

$$\frac{S_{\text{f}}/A_{\text{f}}}{S_{\text{i}}/A_{\text{i}}} = \frac{(10^{12} \text{ bytes})/\left((10^1 \text{ plates})(10^{-2} \text{ m}^2/\text{plates})\right)}{(10^7 \text{ bytes})/\left((10^2 \text{ plates})(10^0 \text{ m}^2/\text{plates})\right)} = 10^8$$

Hence storage per unit area has increased by a factor of approximately 10^8.

1.68. 829.8 m = 828.9 × 10^9 nm = 8.3 × 10^{11} nm.

No. of atoms needed = $8.3 \times \frac{10^{11}}{0.1} = 8.3 \times 10^{12}$ atoms stacked on end!

From table 1.9 (Principles) you can get that number from one human cell!

1.69. We first convert from inches to millimeters:

$$2.75 \text{ in} \times \frac{25.4 \text{ mm}}{1 \text{ in}} = 69.9 \text{ mm}$$

We know this thickness is from 200 sheets, meaning $200t = 69.9$ mm or $t = \frac{69.9 \text{ mm}}{200 \text{ sheets}} = 0.349$ mm/sheet. Hence

the thickness of each sheet is 0.349 mm.

1.70. (*a*) 1 litre = 10^{-3} m^3. So, the tank's capacity is 100 m^3.
(*b*) The mass of water = $\rho V = 1000 \frac{\text{kg}}{\text{m}^3} \times 100 \text{m}^3 = 10^5 \text{kg} = 10^{11} \mu\text{g}$.
(*c*) $\frac{10^5}{0.5 \times 10} = 20000$ days = 54.75 years!

Hence the water would last about 30 years. Answers up to 45 years would also be plausible given different assumptions about the glass volume.

1.71. No, the model will not fit. Atoms are typically 10^5 times larger than nuclei. In order for the nucleus to be that large, the atom itself would need to be about 50 km across.

1.72. We calculate both sides of the equation and check for agreement.
(*a*) Initially keeping all the significant digits provided:
$$gR_E^2 = GM_E$$
$$(9.80665 \text{ m/s}^2)(6.378140 \times 10^6 \text{ m})^2 = (6.6738 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2))(5.9736 \times 10^{24} \text{ kg})$$
$$3.98941 \times 10^{14} \text{ m}^3/\text{s}^2 = 3.98666 \times 10^{14} \text{ m}^3/\text{s}^2$$
The results agree to three significant digits.
(*b*) Proceeding in exactly the same way as in part (*a*) one can round each quantity to various numbers of significant digits and calculate the two sides of the equation. We do this and look for a point at which the two answers no longer agree to three significant digits. One obtains:
$$\text{4 sig. dig. : } 3.98938 \times 10^{14} \text{ m}^3/\text{s}^2 = 3.98705 \times 10^{14} \text{ m}^3/\text{s}^2$$
$$\text{3 sig. dig. : } 3.99310 \times 10^{14} \text{ m}^3/\text{s}^2 = 3.98199 \times 10^{14} \text{ m}^3/\text{s}^2$$
$$\text{2 sig. dig. : } 4.01408 \times 10^{14} \text{ m}^3/\text{s}^2 = 4.02000 \times 10^{14} \text{ m}^3/\text{s}^2$$
$$\text{1 sig. dig. : } 3.6 \times 10^{14} \text{ m}^3/\text{s}^2 = 4.2 \times 10^{14} \text{ m}^3/\text{s}^2$$
Clearly, the two sides of the equation agree to three significant digits unless the physical values are all rounded to only three significant digits or fewer.

1.73.
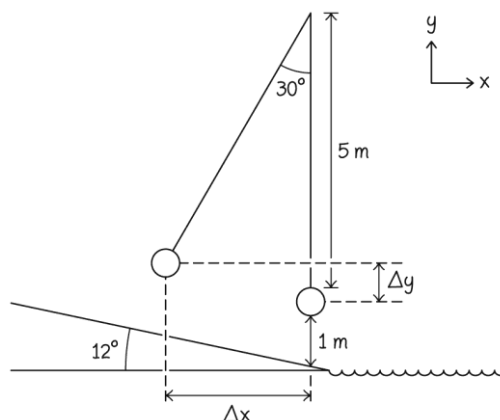
When the tire swing moves through an angle θ from vertical, it will have risen a vertical distance $L(1-\cos(\theta))$, and it will have moved over a horizontal distance $L\sin(\theta)$. Call the angle that the ground makes with the horizontal ϕ. Then $\tan(\phi) = \Delta y_{\text{ground}} / \Delta x_{\text{ground}}$. Hence we can write the amount by which the ground has risen, when the tire swing moves through a certain angle: $\Delta y_{\text{ground}} = \Delta x_{\text{ground}} \tan(\phi) = L\sin(\theta)\tan(\phi)$. The total vertical distance from the ground to the tire swing is then

$$y_{\text{i}} + \Delta y_{\text{swing}} - \Delta y_{\text{ground}} = (1.0 \text{ m}) + L(1-\cos(\theta)) - L\sin(\theta)\tan(\phi)$$

Not surprisingly, the maximum of this height occurs when the swing is at its maximum angle. This height is only about 1.1 m, which is a perfectly safe height. Of course, swinging out over the water (where the ground does not rise up to meet the swing) will result in a greater height above the water. But this is also only about 1.7 m, which is also not a dangerous height for falling into water. You might also have a concern that the tire swing may drag a child across the ground. We can find the minimum height of the tire swing above the ground by differentiating with respect to θ and requiring that the derivative be equal to zero. This yields the condition $\tan(\theta) = \tan(\phi)$, or $\theta = \phi$. This tells us that the minimum height of the tire swing above the ground is about 0.89 m. This is high enough that it is not likely that a child would unexpectedly strike the ground. There is no need for concern. The swing is perfectly safe.

1.74. We begin by calculating how much oxygen is actually needed per breath. Since oxygen is only 20.95% of the air we breathe, the volume of oxygen taken in during each breath is only 0.943 L. Since each breath only absorbs 25% of the oxygen present, the oxygen that actually gets used in each breath is only 0.236 L. This corresponds to approximately 2.36×10^{-4} kg of oxygen per breath. Finally we calculate how much oxygen is needed for an entire year using simple conversion factors:

$$\frac{2.36 \times 10^{-4} \text{ kg}}{\text{breath}} \times \frac{15 \text{ breaths}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ yr}} = 1900 \text{ kg}$$

The mass of oxygen required is around 1900 kg. This is reasonable. However, at atmospheric pressure and room temperature this air would occupy 1900 cubic meters, or a cubic room of side length 12 m. In an environment in which space is a precious commodity, this is probably not feasible. Alternatives would be to use highly compressed, cooled air to decrease storage requirements, or rely on plant life in the ship to replenish oxygen.

1.75. (*a*) The density is given by $\dfrac{m}{V} = \dfrac{10^{30} \text{ kg}}{(1)(10)(10^{4} \text{ m})^{3}} = 10^{17} \text{ kg/m}^{3}$

(*b*) To the nearest order of magnitude, the density of Earth is 10^{4} kg/m^{3}, and the density of water is 10^{3} kg/m^{3}. Hence the neutron star is 13 orders of magnitude greater than the density of Earth and 14 orders of magnitude greater than the density of water. (*c*) The mass of a liter of water is 1.0 kg. Since the neutron star is 14 orders of magnitude denser than water, the mass contained in a 2-L container would be of the order 10^{14} kg.