

CHAPTER 2: Describing Motion: Kinematics in One Dimension

Responses to Questions

1. A car speedometer measures only speed, since it gives no indication of the direction in which the car is traveling.
2. If the velocity of an object is constant, the speed must also be constant. (A constant velocity means that the speed and direction are both constant.) If the speed of an object is constant, the velocity CAN vary. For example, a car traveling around a curve at constant speed has a varying velocity, since the direction of the velocity vector is changing.
3. When an object moves with constant velocity, the average velocity and the instantaneous velocity are the same at all times.
4. No, if one object has a greater speed than a second object, it does not necessarily have a greater acceleration. For example, consider a speeding car, traveling at constant velocity, which passes a stopped police car. The police car will accelerate from rest to try to catch the speeder. The speeding car has a greater speed than the police car (at least initially!), but has zero acceleration. The police car will have an initial speed of zero, but a large acceleration.
5. The accelerations of the motorcycle and the bicycle are the same, assuming that both objects travel in a straight line. Acceleration is the change in velocity divided by the change in time. The magnitude of the change in velocity in each case is the same, 10 km/h, so over the same time interval the accelerations will be equal.
6. Yes, for example, a car that is traveling northward and slowing down has a northward velocity and a southward acceleration.
7. Yes. If the velocity and the acceleration have different signs (opposite directions), then the object is slowing down. For example, a ball thrown upward has a positive velocity and a negative acceleration while it is going up. A car traveling in the negative x -direction and braking has a negative velocity and a positive acceleration.
8. Both velocity and acceleration are negative in the case of a car traveling in the negative x -direction and speeding up. If the upward direction is chosen as $+y$, a falling object has negative velocity and negative acceleration.
9. Car A is going faster at this instant and is covering more distance per unit time, so car A is passing car B. (Car B is accelerating faster and will eventually overtake car A.)
10. Yes. Remember that acceleration is a *change in velocity* per unit time, or a *rate of change* in velocity. So, velocity can be increasing while the rate of increase goes down. For example, suppose a car is traveling at 40 km/h and a second later is going 50 km/h. One second after that, the car's speed is 55 km/h. The car's speed was increasing the entire time, but its acceleration in the second time interval was lower than in the first time interval.
11. If there were no air resistance, the ball's only acceleration during flight would be the acceleration due to gravity, so the ball would land in the catcher's mitt with the same speed it had when it left the bat, 120 km/h. The path of the ball as it rises and then falls would be symmetric.

12. (a) If air resistance is negligible, the acceleration of a freely falling object stays the same as the object falls toward the ground. (Note that the object's speed increases, but since it increases at a constant rate, the acceleration is constant.)
(b) In the presence of air resistance, the acceleration decreases. (Air resistance increases as speed increases. If the object falls far enough, the acceleration will go to zero and the velocity will become constant. See Section 5-6.)
13. Average speed is the displacement divided by the time. If the distances from A to B and from B to C are equal, then you spend more time traveling at 70 km/h than at 90 km/h, so your average speed should be less than 80 km/h. If the distance from A to B (or B to C) is x , then the total distance traveled is $2x$. The total time required to travel this distance is $x/70$ plus $x/90$. Then
- $$\bar{v} = \frac{d}{t} = \frac{2x}{x/70 + x/90} = \frac{2(90)(70)}{90 + 70} = 79 \text{ km/h.}$$
14. Yes. For example, a rock thrown straight up in the air has a constant, nonzero acceleration due to gravity for its entire flight. However, at the highest point it momentarily has a zero velocity. A car, at the moment it starts moving from rest, has zero velocity and nonzero acceleration.
15. Yes. Anytime the velocity is constant, the acceleration is zero. For example, a car traveling at a constant 90 km/h in a straight line has nonzero velocity and zero acceleration.
16. A rock falling from a cliff has a constant acceleration IF we neglect air resistance. An elevator moving from the second floor to the fifth floor making stops along the way does NOT have a constant acceleration. Its acceleration will change in magnitude and direction as the elevator starts and stops. The dish resting on a table has a constant acceleration (zero).
17. The time between clinks gets smaller and smaller. The bolts all start from rest and all have the same acceleration, so at any moment in time, they will all have the same speed. However, they have different distances to travel in reaching the floor and therefore will be falling for different lengths of time. The later a bolt hits, the longer it has been accelerating and therefore the faster it is moving. The time intervals between impacts decrease since the higher a bolt is on the string, the faster it is moving as it reaches the floor. In order for the clinks to occur at equal time intervals, the higher the bolt, the further it must be tied from its neighbor. Can you guess the ratio of lengths?
18. The slope of the position versus time curve is the velocity. The object starts at the origin with a constant velocity (and therefore zero acceleration), which it maintains for about 20 s. For the next 10 s, the positive curvature of the graph indicates the object has a positive acceleration; its speed is increasing. From 30 s to 45 s, the graph has a negative curvature; the object uniformly slows to a stop, changes direction, and then moves backwards with increasing speed. During this time interval its acceleration is negative, since the object is slowing down while traveling in the positive direction and then speeding up while traveling in the negative direction. For the final 5 s shown, the object continues moving in the negative direction but slows down, which gives it a positive acceleration. During the 50 s shown, the object travels from the origin to a point 20 m away, and then back 10 m to end up 10 m from the starting position.
19. The object begins with a speed of 14 m/s and increases in speed with constant positive acceleration from $t = 0$ until $t = 45$ s. The acceleration then begins to decrease, goes to zero at $t = 50$ s, and then goes negative. The object slows down from $t = 50$ s to $t = 90$ s, and is at rest from $t = 90$ s to $t = 108$ s. At that point the acceleration becomes positive again and the velocity increases from $t = 108$ s to $t = 130$ s.

Solutions to Problems

1. The distance of travel (displacement) can be found by rearranging Eq. 2-2 for the average velocity. Also note that the units of the velocity and the time are not the same, so the speed units will be converted.

$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow \Delta x = \bar{v} \Delta t = (110 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (2.0 \text{ s}) = 0.061 \text{ km} = \boxed{61 \text{ m}}$$

2. The average speed is given by Eq. 2-2.

$$\bar{v} = \Delta x / \Delta t = 235 \text{ km} / 3.25 \text{ h} = \boxed{72.3 \text{ km/h}}$$

3. The average velocity is given by Eq. 2.2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8.5 \text{ cm} - 4.3 \text{ cm}}{4.5 \text{ s} - (-2.0 \text{ s})} = \frac{4.2 \text{ cm}}{6.5 \text{ s}} = \boxed{0.65 \text{ cm/s}}$$

The average speed cannot be calculated. To calculate the average speed, we would need to know the actual distance traveled, and it is not given. We only have the displacement.

4. The average velocity is given by Eq. 2-2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-4.2 \text{ cm} - 3.4 \text{ cm}}{5.1 \text{ s} - 3.0 \text{ s}} = \frac{-7.6 \text{ cm}}{2.1 \text{ s}} = \boxed{-3.6 \text{ cm/s}}$$

The negative sign indicates the direction.

5. The speed of sound is intimated in the problem as 1 mile per 5 seconds. The speed is calculated as follows.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \left(\frac{1 \text{ mi}}{5 \text{ s}} \right) \left(\frac{1610 \text{ m}}{1 \text{ mi}} \right) = \boxed{300 \text{ m/s}}$$

The speed of 300 m/s would imply the sound traveling a distance of 900 meters (which is approximately 1 km) in 3 seconds. So the rule could be approximated as 1 km every 3 seconds.

6. The time for the first part of the trip is calculated from the initial speed and the first distance.

$$\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} \rightarrow \Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{130 \text{ km}}{95 \text{ km/h}} = 1.37 \text{ h} = 82 \text{ min}$$

The time for the second part of the trip is now calculated.

$$\Delta t_2 = \Delta t_{\text{total}} - \Delta t_1 = 3.33 \text{ h} - 1.37 \text{ h} = 1.96 \text{ h} = 118 \text{ min}$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} \rightarrow \Delta x_2 = \bar{v}_2 \Delta t_2 = (65 \text{ km/h})(1.96 \text{ h}) = 127.5 \text{ km} = 1.3 \times 10^2 \text{ km}$$

- (a) The total distance is then $\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = 130 \text{ km} + 127.5 \text{ km} = 257.5 \text{ km} \approx \boxed{2.6 \times 10^2 \text{ km}}$.
 (b) The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2-2.

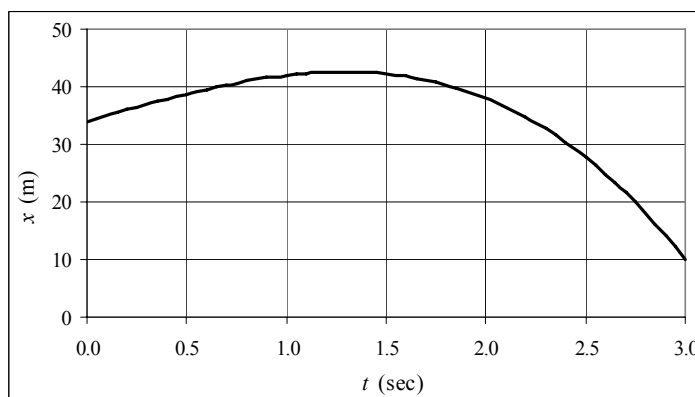
$$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{257.5 \text{ km}}{3.33 \text{ h}} = \boxed{77 \text{ km/h}}$$

7. The distance traveled is $116 \text{ km} + \frac{1}{2}(116 \text{ km}) = 174 \text{ km}$, and the displacement is $116 \text{ km} - \frac{1}{2}(116 \text{ km}) = 58 \text{ km}$. The total time is $14.0 \text{ s} + 4.8 \text{ s} = 18.8 \text{ s}$.

(a) Average speed = $\frac{\text{distance}}{\text{time elapsed}} = \frac{174 \text{ m}}{18.8 \text{ s}} = \boxed{9.26 \text{ m/s}}$

(b) Average velocity = $v_{\text{avg}} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{58 \text{ m}}{18.8 \text{ s}} = \boxed{3.1 \text{ m/s}}$

8. (a)



The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH02.XLS”, on tab “Problem 2.8a”.

- (b) The average velocity is the displacement divided by the elapsed time.

$$\bar{v} = \frac{x(3.0) - x(0.0)}{3.0 \text{ s} - 0.0 \text{ s}} = \frac{[34 + 10(3.0) - 2(3.0)^3] \text{ m} - (34 \text{ m})}{3.0 \text{ s}} = \boxed{-8.0 \text{ m/s}}$$

- (c) The instantaneous velocity is given by the derivative of the position function.

$$v = \frac{dx}{dt} = (10 - 6t^2) \text{ m/s} \quad 10 - 6t^2 = 0 \rightarrow t = \sqrt{\frac{5}{3}} \text{ s} = \boxed{1.3 \text{ s}}$$

This can be seen from the graph as the “highest” point on the graph.

9. Slightly different answers may be obtained since the data comes from reading the graph.

- (a) The instantaneous velocity is given by the slope of the tangent line to the curve. At $t = 10.0 \text{ s}$,

the slope is approximately $v(10) \approx \frac{3 \text{ m} - 0}{10.0 \text{ s} - 0} = \boxed{0.3 \text{ m/s}}$.

- (b) At $t = 30.0 \text{ s}$, the slope of the tangent line to the curve, and thus the instantaneous velocity, is

approximately $v(30) \approx \frac{22 \text{ m} - 10 \text{ m}}{35 \text{ s} - 25 \text{ s}} = \boxed{1.2 \text{ m/s}}$.

(c) The average velocity is given by $\bar{v} = \frac{x(5) - x(0)}{5.0 \text{ s} - 0 \text{ s}} = \frac{1.5 \text{ m} - 0}{5.0 \text{ s}} = \boxed{0.30 \text{ m/s}}$.

(d) The average velocity is given by $\bar{v} = \frac{x(30) - x(25)}{30.0 \text{ s} - 25.0 \text{ s}} = \frac{16 \text{ m} - 9 \text{ m}}{5.0 \text{ s}} = \boxed{1.4 \text{ m/s}}$.

(e) The average velocity is given by $\bar{v} = \frac{x(50) - x(40)}{50.0 \text{ s} - 40.0 \text{ s}} = \frac{10 \text{ m} - 19.5 \text{ m}}{10.0 \text{ s}} = \boxed{-0.95 \text{ m/s}}$.

10. (a) Multiply the reading rate times the bit density to find the bit reading rate.

$$N = \frac{1.2 \text{ m}}{1 \text{ s}} \times \frac{1 \text{ bit}}{0.28 \times 10^{-6} \text{ m}} = \boxed{4.3 \times 10^6 \text{ bits/s}}$$

- (b) The number of excess bits is $N - N_0$.

$$N - N_0 = 4.3 \times 10^6 \text{ bits/s} - 1.4 \times 10^6 \text{ bits/s} = 2.9 \times 10^6 \text{ bits/s}$$

$$\frac{N - N_0}{N} = \frac{2.9 \times 10^6 \text{ bits/s}}{4.3 \times 10^6 \text{ bits/s}} = 0.67 = \boxed{67\%}$$

11. Both objects will have the same time of travel. If the truck travels a distance Δx_{truck} , then the distance the car travels will be $\Delta x_{\text{car}} = \Delta x_{\text{truck}} + 110 \text{ m}$. Use Eq. 2-2 for average speed, $\bar{v} = \Delta x / \Delta t$, solve for time, and equate the two times.

$$\Delta t = \frac{\Delta x_{\text{truck}}}{\bar{v}_{\text{truck}}} = \frac{\Delta x_{\text{car}}}{\bar{v}_{\text{car}}} \quad \frac{\Delta x_{\text{truck}}}{75 \text{ km/h}} = \frac{\Delta x_{\text{truck}} + 110 \text{ m}}{95 \text{ km/h}}$$

$$\text{Solving for } \Delta x_{\text{truck}} \text{ gives } \Delta x_{\text{truck}} = (110 \text{ m}) \frac{(75 \text{ km/h})}{(95 \text{ km/h} - 75 \text{ km/h})} = 412.5 \text{ m}.$$

$$\text{The time of travel is } \Delta t = \frac{\Delta x_{\text{truck}}}{\bar{v}_{\text{truck}}} = \left(\frac{412.5 \text{ m}}{75000 \text{ m/h}} \right) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 0.33 \text{ min} = 19.8 \text{ s} = \boxed{2.0 \times 10^1 \text{ s}}.$$

$$\text{Also note that } \Delta t = \frac{\Delta x_{\text{car}}}{\bar{v}_{\text{car}}} = \left(\frac{412.5 \text{ m} + 110 \text{ m}}{95000 \text{ m/h}} \right) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 0.33 \text{ min} = 20 \text{ s}.$$

ALTERNATE SOLUTION:

The speed of the car relative to the truck is $95 \text{ km/h} - 75 \text{ km/h} = 20 \text{ km/h}$. In the reference frame of the truck, the car must travel 110 m to catch it.

$$\Delta t = \frac{0.11 \text{ km}}{20 \text{ km/h}} \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 19.8 \text{ s}$$

12. Since the locomotives have the same speed, they each travel half the distance, 4.25 km. Find the time of travel from the average speed.

$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow \Delta t = \frac{\Delta x}{\bar{v}} = \frac{4.25 \text{ km}}{95 \text{ km/h}} = 0.0447 \text{ h} \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 2.68 \text{ min} \approx \boxed{2.7 \text{ min}}$$

13. (a) The area between the concentric circles is equal to the length times the width of the spiral path.

$$\pi R_2^2 - \pi R_1^2 = w\ell \rightarrow$$

$$\ell = \frac{\pi(R_2^2 - R_1^2)}{w} = \frac{\pi[(0.058 \text{ m})^2 - (0.025 \text{ m})^2]}{1.6 \times 10^{-6} \text{ m}} = 5.378 \times 10^3 \text{ m} \approx \boxed{5400 \text{ m}}$$

$$(b) \quad 5.378 \times 10^3 \text{ m} \left(\frac{1 \text{ s}}{1.25 \text{ m}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = \boxed{72 \text{ min}}$$

14. The average speed for each segment of the trip is given by $\bar{v} = \frac{\Delta x}{\Delta t}$, so $\Delta t = \frac{\Delta x}{\bar{v}}$ for each

segment. For the first segment, $\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{3100 \text{ km}}{720 \text{ km/h}} = 4.306 \text{ h}$. For the second segment,

$$\Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{2800 \text{ km}}{990 \text{ km/h}} = 2.828 \text{ h}.$$

Thus the total time is $\Delta t_{\text{tot}} = \Delta t_1 + \Delta t_2 = 4.306 \text{ h} + 2.828 \text{ h} = 7.134 \text{ h} \approx \boxed{7.1 \text{ h}}$.

The average speed of the plane for the entire trip is $\bar{v} = \frac{\Delta x_{\text{tot}}}{\Delta t_{\text{tot}}} = \frac{3100 \text{ km} + 2800 \text{ km}}{7.134 \text{ h}} = 827 \text{ km/h}$
 $\approx \boxed{830 \text{ km/h}}$.

15. The distance traveled is 500 km (250 km outgoing, 250 km return, keep 2 significant figures). The displacement (Δx) is 0 because the ending point is the same as the starting point.

(a) To find the average speed, we need the distance traveled (500 km) and the total time elapsed.

During the outgoing portion, $\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1}$ and so $\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{250 \text{ km}}{95 \text{ km/h}} = 2.632 \text{ h}$. During the

return portion, $\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2}$, and so $\Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{250 \text{ km}}{55 \text{ km/h}} = 4.545 \text{ h}$. Thus the total time,

including lunch, is $\Delta t_{\text{total}} = \Delta t_1 + \Delta t_{\text{lunch}} + \Delta t_2 = 8.177 \text{ h}$.

$$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{500 \text{ km}}{8.177 \text{ h}} = \boxed{61 \text{ km/h}}$$

(b) Average velocity = $\boxed{\bar{v} = \Delta x / \Delta t = 0}$

16. We are given that $x(t) = 2.0 \text{ m} - (3.6 \text{ m/s})t + (1.1 \text{ m/s}^2)t^2$.

$$(a) \quad x(1.0 \text{ s}) = 2.0 \text{ m} - (3.6 \text{ m/s})(1.0 \text{ s}) + (1.1 \text{ m/s}^2)(1.0 \text{ s})^2 = \boxed{-0.5 \text{ m}}$$

$$x(2.0 \text{ s}) = 2.0 \text{ m} - (3.6 \text{ m/s})(2.0 \text{ s}) + (1.1 \text{ m/s}^2)(2.0 \text{ s})^2 = \boxed{-0.8 \text{ m}}$$

$$x(3.0 \text{ s}) = 2.0 \text{ m} - (3.6 \text{ m/s})(3.0 \text{ s}) + (1.1 \text{ m/s}^2)(3.0 \text{ s})^2 = \boxed{1.1 \text{ m}}$$

$$(b) \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{1.1 \text{ m} - (-0.5 \text{ m})}{2.0 \text{ s}} = \boxed{0.80 \text{ m/s}}$$

(c) The instantaneous velocity is given by $v(t) = \frac{dx(t)}{dt} = -3.6 \text{ m/s} + (2.2 \text{ m/s}^2)t$.

$$v(2.0 \text{ s}) = -3.6 \text{ m/s} + (2.2 \text{ m/s}^2)(2.0 \text{ s}) = \boxed{0.8 \text{ m/s}}$$

$$v(3.0 \text{ s}) = -3.6 \text{ m/s} + (2.2 \text{ m/s}^2)(3.0 \text{ s}) = \boxed{3.0 \text{ m/s}}$$

17. The distance traveled is $120 \text{ m} + \frac{1}{2}(120 \text{ m}) = 180 \text{ m}$, and the displacement is $120 \text{ m} - \frac{1}{2}(120 \text{ m}) = 60 \text{ m}$. The total time is $8.4 \text{ s} + \frac{1}{3}(8.4 \text{ s}) = 11.2 \text{ s}$.

$$(a) \text{ Average speed} = \frac{\text{distance}}{\text{time elapsed}} = \frac{180 \text{ m}}{11.2 \text{ s}} = \boxed{16 \text{ m/s}}$$

$$(b) \text{ Average velocity} = v_{\text{avg}} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{60 \text{ m}}{11.2 \text{ s}} = \boxed{+5 \text{ m/s}} \text{ (in original direction) (1 sig fig)}$$

18. For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either $d_{\text{car}} = v_{\text{car}}t = (95 \text{ km/h})t$ or $d_{\text{car}} = \ell_{\text{train}} + v_{\text{train}}t = 1.10 \text{ km} + (75 \text{ km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95 \text{ km/h})t = 1.10 \text{ km} + (75 \text{ km/h})t \rightarrow t = \frac{1.10 \text{ km}}{20 \text{ km/h}} = 0.055 \text{ h} = \boxed{3.3 \text{ min}}$$

The distance the car travels during this time is $d = (95 \text{ km/h})(0.055 \text{ h}) = 5.225 \text{ km} \approx \boxed{5.2 \text{ km}}$.

If the train is traveling the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either $d_{\text{car}} = (95 \text{ km/h})t$ or $d_{\text{car}} = 1.10 \text{ km} - (75 \text{ km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95 \text{ km/h})t = 1.10 \text{ km} - (75 \text{ km/h})t \rightarrow t = \frac{1.10 \text{ km}}{170 \text{ km/h}} = 6.47 \times 10^{-3} \text{ h} = \boxed{23.3 \text{ s}}$$

The distance the car travels during this time is $d = (95 \text{ km/h})(6.47 \times 10^{-3} \text{ h}) = \boxed{0.61 \text{ km}}$.

- [19] The average speed of sound is given by $v_{\text{sound}} = \Delta x / \Delta t$, and so the time for the sound to travel from the end of the lane back to the bowler is $\Delta t_{\text{sound}} = \frac{\Delta x}{v_{\text{sound}}} = \frac{16.5 \text{ m}}{340 \text{ m/s}} = 4.85 \times 10^{-2} \text{ s}$. Thus the time for the ball to travel from the bowler to the end of the lane is given by $\Delta t_{\text{ball}} = \Delta t_{\text{total}} - \Delta t_{\text{sound}} = 2.50 \text{ s} - 4.85 \times 10^{-2} \text{ s} = 2.4515 \text{ s}$. And so the speed of the ball is as follows.

$$v_{\text{ball}} = \frac{\Delta x}{\Delta t_{\text{ball}}} = \frac{16.5 \text{ m}}{2.4515 \text{ s}} = \boxed{6.73 \text{ m/s}}.$$

20. The average acceleration is found from Eq. 2-5.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{95 \text{ km/h} - 0 \text{ km/h}}{4.5 \text{ s}} = \frac{(95 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)}{4.5 \text{ s}} = \boxed{5.9 \text{ m/s}^2}$$

21. The time can be found from the average acceleration, $\bar{a} = \Delta v / \Delta t$.

$$\Delta t = \frac{\Delta v}{\bar{a}} = \frac{110 \text{ km/h} - 80 \text{ km/h}}{1.8 \text{ m/s}^2} = \frac{(30 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)}{1.8 \text{ m/s}^2} = 4.630 \text{ s} \approx \boxed{5 \text{ s}}$$

22. (a) The average acceleration of the sprinter is $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{9.00 \text{ m/s} - 0.00 \text{ m/s}}{1.28 \text{ s}} = \boxed{7.03 \text{ m/s}^2}$.

(b) $\bar{a} = (7.03 \text{ m/s}^2) \left(\frac{1 \text{ km}}{1000 \text{ m}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right)^2 = \boxed{9.11 \times 10^4 \text{ km/h}^2}$

23. Slightly different answers may be obtained since the data comes from reading the graph.

(a) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t \approx 48 \text{ s}}$.

(b) The indication of a constant velocity on a velocity–time graph is a slope of 0, which occurs from $\boxed{t = 90 \text{ s to } t \approx 108 \text{ s}}$.

(c) The indication of a constant acceleration on a velocity–time graph is a constant slope, which occurs from $\boxed{t = 0 \text{ s to } t \approx 42 \text{ s}}$, again from $\boxed{t \approx 65 \text{ s to } t \approx 83 \text{ s}}$, and again from $\boxed{t = 90 \text{ s to } t \approx 108 \text{ s}}$.

(d) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t \approx 65 \text{ s to } t \approx 83 \text{ s}}$.

24. The initial velocity of the car is the average speed of the car before it accelerates.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{110 \text{ m}}{5.0 \text{ s}} = 22 \text{ m/s} = v_0$$

The final speed is $v = 0$, and the time to stop is 4.0 s. Use Eq. 2-12a to find the acceleration.

$$v = v_0 + at \rightarrow a = \frac{v - v_0}{t} = \frac{0 - 22 \text{ m/s}}{4.0 \text{ s}} = -5.5 \text{ m/s}^2$$

Thus the magnitude of the acceleration is $\boxed{5.5 \text{ m/s}^2}$, or $(5.5 \text{ m/s}^2) \left(\frac{1 \text{ g}}{9.80 \text{ m/s}^2} \right) = \boxed{0.56 \text{ g's}}$.

25. (a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{385 \text{ m} - 25 \text{ m}}{20.0 \text{ s} - 3.0 \text{ s}} = \boxed{21.2 \text{ m/s}}$

(b) $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{45.0 \text{ m/s} - 11.0 \text{ m/s}}{20.0 \text{ s} - 3.0 \text{ s}} = \boxed{2.00 \text{ m/s}^2}$

26. Slightly different answers may be obtained since the data comes from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being “in” a certain gear.

(a) The average acceleration in 2nd gear is given by $\bar{a}_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{24 \text{ m/s} - 14 \text{ m/s}}{8 \text{ s} - 4 \text{ s}} = \boxed{2.5 \text{ m/s}^2}$.

(b) The average acceleration in 4th gear is given by $\bar{a}_4 = \frac{\Delta v_4}{\Delta t_4} = \frac{44 \text{ m/s} - 37 \text{ m/s}}{27 \text{ s} - 16 \text{ s}} = \boxed{0.6 \text{ m/s}^2}$.

(c) The average acceleration through the first four gears is given by $\bar{a} = \frac{\Delta v}{\Delta t} =$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{44 \text{ m/s} - 0 \text{ m/s}}{27 \text{ s} - 0 \text{ s}} = \boxed{1.6 \text{ m/s}^2}.$$

27. The acceleration is the second derivative of the position function.

$$x = 6.8t + 8.5t^2 \rightarrow v = \frac{dx}{dt} = 6.8 + 17.0t \rightarrow a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \boxed{17.0 \text{ m/s}^2}$$

28. To estimate the velocity, find the average velocity over each time interval, and assume that the car had that velocity at the midpoint of the time interval. To estimate the acceleration, find the average acceleration over each time interval, and assume that the car had that acceleration at the midpoint of the time interval. A sample of each calculation is shown.

From 2.00 s to 2.50 s, for average velocity:

$$t_{\text{mid}} = \frac{2.50 \text{ s} + 2.00 \text{ s}}{2} = 2.25 \text{ s}$$

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{13.79 \text{ m} - 8.55 \text{ m}}{2.50 \text{ s} - 2.00 \text{ s}} = \frac{5.24 \text{ m}}{0.50 \text{ s}} = 10.48 \text{ m/s}$$

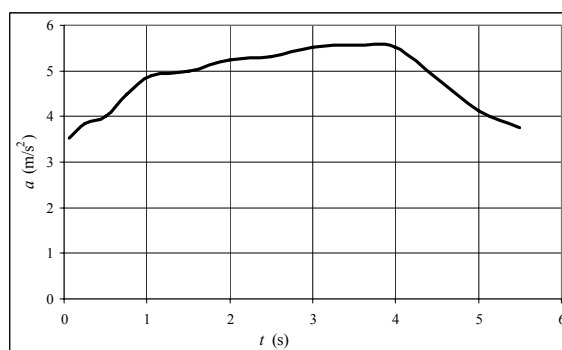
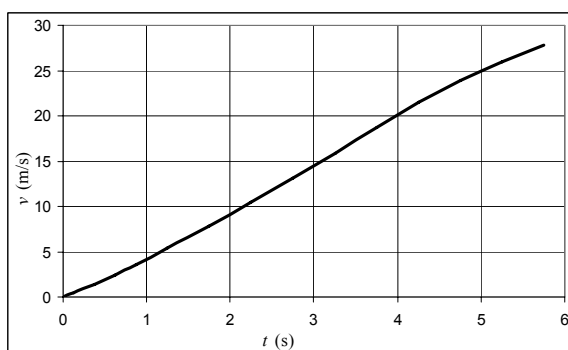
From 2.25 s to 2.75 s, for average acceleration:

$$t_{\text{mid}} = \frac{2.25 \text{ s} + 2.75 \text{ s}}{2} = 2.50 \text{ s}$$

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{13.14 \text{ m/s} - 10.48 \text{ m/s}}{2.75 \text{ s} - 2.25 \text{ s}} = \frac{2.66 \text{ m/s}}{0.50 \text{ s}} = 5.32 \text{ m/s}^2$$

Table of Calculations

t (s)	x (m)	t (s)	v (m/s)	t (s)	a (m/s ²)
0.00	0.00	0.00	0.00	0.063	3.52
		0.125	0.44		
0.25	0.11			0.25	3.84
		0.375	1.40		
0.50	0.46			0.50	4.00
		0.625	2.40		
0.75	1.06			0.75	4.48
		0.875	3.52		
1.00	1.94			1.06	4.91
		1.25	5.36		
1.50	4.62			1.50	5.00
		1.75	7.86		
2.00	8.55			2.00	5.24
		2.25	10.48		
2.50	13.79			2.50	5.32
		2.75	13.14		
3.00	20.36			3.00	5.52
		3.25	15.90		
3.50	28.31			3.50	5.56
		3.75	18.68		
4.00	37.65			4.00	5.52
		4.25	21.44		
4.50	48.37			4.50	4.84
		4.75	23.86		
5.00	60.30			5.00	4.12
		5.25	25.92		
5.50	73.26			5.50	3.76
		5.75	27.80		
6.00	87.16				



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.28."

29. (a) Since the units of A times the units of t must equal meters, the units of A must be $\boxed{\text{m/s}}$.

Since the units of B times the units of t^2 must equal meters, the units of B must be

$$\boxed{\text{m/s}^2}.$$

(b) The acceleration is the second derivative of the position function.

$$x = At + Bt^2 \rightarrow v = \frac{dx}{dt} = A + 2Bt \rightarrow a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \boxed{2B \text{ m/s}^2}$$

(c) $v = A + 2Bt \rightarrow v(5) = \boxed{(A + 10B) \text{ m/s}} \quad a = \boxed{2B \text{ m/s}^2}$

(d) The velocity is the derivative of the position function.

$$x = At + Bt^{-3} \rightarrow v = \frac{dx}{dt} = \boxed{A - 3Bt^{-4}}$$

30. The acceleration can be found from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (25 \text{ m/s})^2}{2(85 \text{ m})} = \boxed{-3.7 \text{ m/s}^2}$$

31. By definition, the acceleration is $a = \frac{v - v_0}{t} = \frac{21 \text{ m/s} - 12 \text{ m/s}}{6.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$.

The distance of travel can be found from Eq. 2-12b.

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 = (12 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2} (1.5 \text{ m/s}^2)(6.0 \text{ s})^2 = \boxed{99 \text{ m}}$$

32. Assume that the plane starts from rest. The runway distance is found by solving Eq. 2-12c for $x - x_0$.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{(32 \text{ m/s})^2 - 0}{2(3.0 \text{ m/s}^2)} = \boxed{1.7 \times 10^2 \text{ m}}$$

33. For the baseball, $v_0 = 0$, $x - x_0 = 3.5 \text{ m}$, and the final speed of the baseball (during the throwing motion) is $v = 41 \text{ m/s}$. The acceleration is found from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(41 \text{ m/s})^2 - 0}{2(3.5 \text{ m})} = \boxed{240 \text{ m/s}^2}$$

34. The average velocity is defined by Eq. 2-2, $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$. Compare this expression to Eq. 2-12d, $\bar{v} = \frac{1}{2}(v + v_0)$. A relation for the velocity is found by integrating the expression for the acceleration, since the acceleration is the derivative of the velocity. Assume the velocity is v_0 at time $t = 0$.

$$a = A + Bt = \frac{dv}{dt} \rightarrow dv = (A + Bt) dt \rightarrow \int_{v_0}^v dv = \int_0^t (A + Bt) dt \rightarrow v = v_0 + At + \frac{1}{2} Bt^2$$

Find an expression for the position by integrating the velocity, assuming that $x = x_0$ at time $t = 0$.

$$v = v_0 + At + \frac{1}{2} Bt^2 = \frac{dx}{dt} \rightarrow dx = \left(v_0 + At + \frac{1}{2} Bt^2 \right) dt \rightarrow$$

$$\int_{x_0}^x dx = \int_0^t \left(v_0 + At + \frac{1}{2} Bt^2 \right) dt \rightarrow x - x_0 = v_0 t + \frac{1}{2} At^2 + \frac{1}{6} Bt^3$$

Compare $\frac{x - x_0}{t}$ to $\frac{1}{2}(v + v_0)$.

$$\bar{v} = \frac{x - x_0}{t} = \frac{v_0 t + \frac{1}{2} A t^2 + \frac{1}{6} B t^3}{t} = v_0 + \frac{1}{2} A t + \frac{1}{6} B t^2$$

$$\frac{1}{2}(v + v_0) = \frac{v_0 + v_0 + A t + \frac{1}{2} B t^2}{2} = v_0 + \frac{1}{2} A t + \frac{1}{4} B t^2$$

They are different, so $\boxed{\bar{v} \neq \frac{1}{2}(v + v_0)}$.

35. The sprinter starts from rest. The average acceleration is found from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(11.5 \text{ m/s})^2 - 0}{2(15.0 \text{ m})} = 4.408 \text{ m/s}^2 \approx \boxed{4.41 \text{ m/s}^2}$$

Her elapsed time is found by solving Eq. 2-12a for time.

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{11.5 \text{ m/s} - 0}{4.408 \text{ m/s}^2} = \boxed{2.61 \text{ s}}$$

36. Calculate the distance that the car travels during the reaction time and the deceleration.

$$\Delta x_1 = v_0 \Delta t = (18.0 \text{ m/s})(0.200 \text{ s}) = 3.6 \text{ m}$$

$$v^2 = v_0^2 + 2a\Delta x_2 \rightarrow \Delta x_2 = \frac{v^2 - v_0^2}{2a} = \frac{0 - (18.0 \text{ m/s})^2}{2(-3.65 \text{ m/s}^2)} = 44.4 \text{ m}$$

$$\Delta x = 3.6 \text{ m} + 44.4 \text{ m} = 48.0 \text{ m}$$

He will NOT be able to stop in time.

- 37.** The words “slows down uniformly” implies that the car has a constant acceleration. The distance of travel is found from combining Eqs. 2-2 and 2-9.

$$x - x_0 = \frac{v_0 + v}{2} t = \left(\frac{18.0 \text{ m/s} + 0 \text{ m/s}}{2} \right) (5.00 \text{ sec}) = \boxed{45.0 \text{ m}}$$

38. The final velocity of the car is zero. The initial velocity is found from Eq. 2-12c with $v = 0$ and solving for v_0 . Note that the acceleration is negative.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{0 - 2(-4.00 \text{ m/s}^2)(85 \text{ m})} = \boxed{26 \text{ m/s}}$$

39. (a) The final velocity of the car is 0. The distance is found from Eq. 2-12c with an acceleration of $a = -0.50 \text{ m/s}^2$ and an initial velocity of 85 km/h.

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left[(85 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2}{2(-0.50 \text{ m/s}^2)} = 557 \text{ m} \approx \boxed{560 \text{ m}}$$

- (b) The time to stop is found from Eq. 2-12a.

$$t = \frac{v - v_0}{a} = \frac{0 - \left[(85 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]}{(-0.50 \text{ m/s}^2)} = 47.22 \text{ s} \approx \boxed{47 \text{ s}}$$

- (c) Take $x_0 = x(t = 0) = 0$ m. Use Eq. 2-12b, with $a = -0.50 \text{ m/s}^2$ and an initial velocity of 85 km/h . The first second is from $t = 0$ s to $t = 1$ s, and the fifth second is from $t = 4$ s to $t = 5$ s.

$$x(0) = 0 ; x(1) = 0 + (85 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (1 \text{ s}) + \frac{1}{2} (-0.50 \text{ m/s}^2) (1 \text{ s})^2 = 23.36 \text{ m} \rightarrow$$

$$x(1) - x(0) = \boxed{23 \text{ m}}$$

$$x(4) = 0 + (85 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (4 \text{ s}) + \frac{1}{2} (-0.50 \text{ m/s}^2) (4 \text{ s})^2 = 90.44 \text{ m}$$

$$x(5) = 0 + (85 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (5 \text{ s}) + \frac{1}{2} (-0.50 \text{ m/s}^2) (5 \text{ s})^2 = 111.81 \text{ m}$$

$$x(5) - x(4) = 111.81 \text{ m} - 90.44 \text{ m} = 21.37 \text{ m} \approx \boxed{21 \text{ m}}$$

40. The final velocity of the driver is zero. The acceleration is found from Eq. 2-12c with $v = 0$ and solving for a .

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - \left[(105 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2}{2(0.80 \text{ m})} = -531.7 \text{ m/s}^2 \approx \boxed{-5.3 \times 10^2 \text{ m/s}^2}$$

Converting to “g’s”: $a = \frac{-531.7 \text{ m/s}^2}{(9.80 \text{ m/s}^2)/g} = \boxed{-54 \text{ g's}}$

41. The origin is the location of the car at the beginning of the reaction time. The initial speed of the car is $(95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s}$. The location where the brakes are applied is found from

the equation for motion at constant velocity: $x_0 = v_0 t_R = (26.39 \text{ m/s})(1.0 \text{ s}) = 26.39 \text{ m}$. This is now the starting location for the application of the brakes. In each case, the final speed is 0.

- (a) Solve Eq. 2-12c for the final location.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x = x_0 + \frac{v^2 - v_0^2}{2a} = 26.39 \text{ m} + \frac{0 - (26.39 \text{ m/s})^2}{2(-5.0 \text{ m/s}^2)} = \boxed{96 \text{ m}}$$

- (b) Solve Eq. 2-12c for the final location with the second acceleration.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 26.39 \text{ m} + \frac{0 - (26.39 \text{ m/s})^2}{2(-7.0 \text{ m/s}^2)} = \boxed{76 \text{ m}}$$

42. Calculate the acceleration from the velocity–time data using Eq. 2-12a, and then use Eq. 2-12b to calculate the displacement at $t = 2.0$ s and $t = 6.0$ s. The initial velocity is $v_0 = 65 \text{ m/s}$.

$$a = \frac{v - v_0}{t} = \frac{162 \text{ m/s} - 65 \text{ m/s}}{10.0 \text{ s}} = 9.7 \text{ m/s}^2 \quad x = x_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow$$

$$x(6.0 \text{ s}) - x(2.0 \text{ s}) = \left[x_0 + v_0 (6.0 \text{ s}) + \frac{1}{2} a (6.0 \text{ s})^2 \right] - \left[x_0 + v_0 (2.0 \text{ s}) + \frac{1}{2} a (2.0 \text{ s})^2 \right]$$

$$\begin{aligned}
 &= v_0 (6.0 \text{ s} - 2.0 \text{ s}) + \frac{1}{2} a \left[(6.0 \text{ s})^2 - (2.0 \text{ s})^2 \right] = (65 \text{ m/s})(4.0 \text{ s}) + \frac{1}{2} (9.7 \text{ m/s}^2)(32 \text{ s}^2) \\
 &= 415 \text{ m} \approx \boxed{4.2 \times 10^2 \text{ m}}
 \end{aligned}$$

43. Use the information for the first 180 m to find the acceleration, and the information for the full motion to find the final velocity. For the first segment, the train has $v_0 = 0 \text{ m/s}$, $v_1 = 23 \text{ m/s}$, and a displacement of $x_1 - x_0 = 180 \text{ m}$. Find the acceleration from Eq. 2-12c.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \rightarrow a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)} = \frac{(23 \text{ m/s})^2 - 0}{2(180 \text{ m})} = 1.469 \text{ m/s}^2$$

Find the speed of the train after it has traveled the total distance (total displacement of $x_2 - x_0 = 255 \text{ m}$) using Eq. 2-12c.

$$v_2^2 = v_0^2 + 2a(x_2 - x_0) \rightarrow v_2 = \sqrt{v_0^2 + 2a(x_2 - x_0)} = \sqrt{2(1.469 \text{ m/s}^2)(255 \text{ m})} = \boxed{27 \text{ m/s}}$$

44. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car, and find another time that both cars have the same displacement from the origin.

For the speeder, traveling with a constant speed, the displacement is given by the following.

$$\Delta x_s = v_s t = (135 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (t) = (37.5 t) \text{ m}$$

For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1 second of reaction time.

$$\Delta x_{p1} = v_{p1} (1.00 \text{ s}) = (95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (1.00 \text{ s}) = 26.39 \text{ m}$$

The second part of the police car displacement is that during the accelerated motion, which lasts for $(t - 1.00) \text{ s}$. So this second part of the police car displacement, using Eq. 2-12b, is given as follows.

$$\Delta x_{p2} = v_{p1} (t - 1.00) + \frac{1}{2} a_p (t - 1.00)^2 = \left[(26.39 \text{ m/s})(t - 1.00) + \frac{1}{2} (2.00 \text{ m/s}^2)(t - 1.00)^2 \right] \text{ m}$$

So the total police car displacement is $\Delta x_p = \Delta x_{p1} + \Delta x_{p2} = (26.39 + 26.39(t - 1.00) + (t - 1.00)^2) \text{ m}$.

Now set the two displacements equal, and solve for the time.

$$26.39 + 26.39(t - 1.00) + (t - 1.00)^2 = 37.5 t \rightarrow t^2 - 13.11t + 1.00 = 0$$

$$t = \frac{13.11 \pm \sqrt{(13.11)^2 - 4.00}}{2} = 7.67 \times 10^{-2} \text{ s}, \boxed{13.0 \text{ s}}$$

The answer that is approximately 0 s corresponds to the fact that both vehicles had the same displacement of zero when the time was 0. The reason it is not exactly zero is rounding of previous values. The answer of 13.0 s is the time for the police car to overtake the speeder.

As a check on the answer, the speeder travels $\Delta x_s = (37.5 \text{ m/s})(13.0 \text{ s}) = 488 \text{ m}$, and the police car travels $\Delta x_p = [26.39 + 26.39(12.0) + (12.0)^2] \text{ m} = 487 \text{ m}$. The difference is due to rounding.

45. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car. Both cars have the same displacement 8.00 s after the initial passing by the speeder.

For the speeder, traveling with a constant speed, the displacement is given by $\Delta x_s = v_s t = (8.00 v_s) \text{ m}$.

For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1.00 s of reaction time.

$$\Delta x_{p1} = v_{p1} (1.00 \text{ s}) = (95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (1.00 \text{ s}) = 26.39 \text{ m}$$

The second part of the police car displacement is that during the accelerated motion, which lasts for 7.00 s. So this second part of the police car displacement, using Eq. 2-12b, is given by the following.

$$\Delta x_{p2} = v_{p1} (7.00 \text{ s}) + \frac{1}{2} a_p (7.00 \text{ s})^2 = (26.39 \text{ m/s}) (7.00 \text{ s}) + \frac{1}{2} (2.00 \text{ m/s}^2) (7.00 \text{ s})^2 = 233.73 \text{ m}$$

Thus the total police car displacement is $\Delta x_p = \Delta x_{p1} + \Delta x_{p2} = (26.39 + 233.73) \text{ m} = 260.12 \text{ m}$.

Now set the two displacements equal, and solve for the speeder's velocity.

$$(8.00 v_s) \text{ m} = 260.12 \text{ m} \rightarrow v_s = (32.5 \text{ m/s}) \left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}} \right) = \boxed{117 \text{ km/h}}$$

46. During the final part of the race, the runner must have a displacement of 1100 m in a time of 180 s (3.0 min). Assume that the starting speed for the final part is the same as the average speed thus far.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8900 \text{ m}}{(27 \times 60) \text{ s}} = 5.494 \text{ m/s} = v_0$$

The runner will accomplish this by accelerating from speed v_0 to speed v for t seconds, covering a distance d_1 , and then running at a constant speed of v for $(180 - t)$ seconds, covering a distance d_2 .

We have these relationships from Eq. 2-12a and Eq. 2-12b.

$$v = v_0 + at \quad d_1 = v_0 t + \frac{1}{2} at^2 \quad d_2 = v(180 - t) = (v_0 + at)(180 - t)$$

$$1100 \text{ m} = d_1 + d_2 = v_0 t + \frac{1}{2} at^2 + (v_0 + at)(180 - t) \rightarrow 1100 \text{ m} = 180 v_0 + 180 at - \frac{1}{2} at^2 \rightarrow$$

$$1100 \text{ m} = (180 \text{ s})(5.494 \text{ m/s}) + (180 \text{ s})(0.2 \text{ m/s}^2)t - \frac{1}{2}(0.2 \text{ m/s}^2)t^2$$

$$0.1t^2 - 36t + 111 = 0 \quad t = 357 \text{ s}, 3.11 \text{ s}$$

Since we must have $t < 180 \text{ s}$, the solution is $\boxed{t = 3.1 \text{ s}}$.

47. For the runners to cross the finish line side-by-side means they must both reach the finish line in the same amount of time from their current positions. Take Mary's current location as the origin. Use Eq. 2-12b.

$$\text{For Sally: } 22 = 5 + 5t + \frac{1}{2}(-.5)t^2 \rightarrow t^2 - 20t + 68 = 0 \rightarrow$$

$$t = \frac{20 \pm \sqrt{20^2 - 4(68)}}{2} = 4.343 \text{ s}, 15.66 \text{ s}$$

The first time is the time she first crosses the finish line, and so is the time to be used for the problem. Now find Mary's acceleration so that she crosses the finish line in that same amount of time.

$$\text{For Mary: } 22 = 0 + 4t + \frac{1}{2}at^2 \rightarrow a = \frac{22 - 4t}{\frac{1}{2}t^2} = \frac{22 - 4(4.343)}{\frac{1}{2}(4.343)^2} = \boxed{0.49 \text{ m/s}^2}$$

48. Choose downward to be the positive direction, and take $y_0 = 0$ at the top of the cliff. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80 \text{ m/s}^2$. The displacement is found from Eq. 2-12b, with x replaced by y .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow y - 0 = 0 + \frac{1}{2} (9.80 \text{ m/s}^2) (3.75 \text{ s})^2 \rightarrow y = \boxed{68.9 \text{ m}}$$

49. Choose downward to be the positive direction. The initial velocity is $v_0 = 0$, the final velocity is $v = (55 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 15.28 \text{ m/s}$, and the acceleration is $a = 9.80 \text{ m/s}^2$. The time can be found by solving Eq. 2-12a for the time.

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{15.28 \text{ m/s} - 0}{9.80 \text{ m/s}^2} = \boxed{1.6 \text{ s}}$$

50. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the top of the Empire State Building. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80 \text{ m/s}^2$.

- (a) The elapsed time can be found from Eq. 2-12b, with x replaced by y .

$$y - y_0 = v_0 t + \frac{1}{2} a t^2 \rightarrow t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(380 \text{ m})}{9.80 \text{ m/s}^2}} = 8.806 \text{ s} \approx \boxed{8.8 \text{ s}}$$

- (b) The final velocity can be found from Eq. 2-12a.

$$v = v_0 + at = 0 + (9.80 \text{ m/s}^2) (8.806 \text{ s}) = \boxed{86 \text{ m/s}}$$

51. Choose upward to be the positive direction, and take $y_0 = 0$ to be at the height where the ball was hit. For the upward path, $v_0 = 20 \text{ m/s}$, $v = 0$ at the top of the path, and $a = -9.80 \text{ m/s}^2$.

- (a) The displacement can be found from Eq. 2-12c, with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (20 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{20 \text{ m}}$$

- (b) The time of flight can be found from Eq. 2-12b, with x replaced by y , using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow t(v_0 + \frac{1}{2} a t) = 0 \rightarrow t = 0, t = \frac{2v_0}{-a} = \frac{2(20 \text{ m/s})}{-9.80 \text{ m/s}^2} = \boxed{4 \text{ s}}$$

The result of $t = 0 \text{ s}$ is the time for the original displacement of zero (when the ball was hit), and the result of $t = 4 \text{ s}$ is the time to return to the original displacement. Thus the answer is $t = 4 \text{ s}$.

52. Choose upward to be the positive direction, and take $y_0 = 0$ to be the height from which the ball was thrown. The acceleration is $a = -9.80 \text{ m/s}^2$. The displacement upon catching the ball is 0, assuming it was caught at the same height from which it was thrown. The starting speed can be found from Eq. 2-12b, with x replaced by y .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow$$

$$v_0 = \frac{y - y_0 - \frac{1}{2} a t^2}{t} = -\frac{1}{2} a t = -\frac{1}{2} (-9.80 \text{ m/s}^2) (3.2 \text{ s}) = 15.68 \text{ m/s} \approx \boxed{16 \text{ m/s}}$$

The height can be calculated from Eq. 2-12c, with a final velocity of $v = 0$ at the top of the path.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (15.68 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 12.54 \text{ m} \approx \boxed{13 \text{ m}}$$

53. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the maximum height of the kangaroo. Consider just the downward motion of the kangaroo. Then the displacement is $y = 1.65 \text{ m}$, the acceleration is $a = 9.80 \text{ m/s}^2$, and the initial velocity is $v_0 = 0 \text{ m/s}$. Use Eq. 2-12b to calculate the time for the kangaroo to fall back to the ground. The total time is then twice the falling time.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow y = \frac{1}{2} a t^2 \rightarrow t_{\text{fall}} = \sqrt{\frac{2y}{a}} \rightarrow$$

$$t_{\text{total}} = 2\sqrt{\frac{2y}{a}} = 2\sqrt{\frac{2(1.65 \text{ m})}{(9.80 \text{ m/s}^2)}} = \boxed{1.16 \text{ s}}$$

54. Choose upward to be the positive direction, and take $y_0 = 0$ to be at the floor level, where the jump starts. For the upward path, $y = 1.2 \text{ m}$, $v = 0$ at the top of the path, and $a = -9.80 \text{ m/s}^2$.

(a) The initial speed can be found from Eq. 2-12c, with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a(y - y_0)} = \sqrt{-2a y} = \sqrt{-2(-9.80 \text{ m/s}^2)(1.2 \text{ m})} = 4.8497 \text{ m/s} \approx \boxed{4.8 \text{ m/s}}$$

(b) The time of flight can be found from Eq. 2-12b, with x replaced by y , using a displacement of 0 for the displacement of the jumper returning to the original height.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow t(v_0 + \frac{1}{2} a t) = 0 \rightarrow$$

$$t = 0, t = \frac{2v_0}{-a} = \frac{2(4.897 \text{ m/s})}{9.80 \text{ m/s}^2} = \boxed{0.99 \text{ s}}$$

The result of $t = 0 \text{ s}$ is the time for the original displacement of zero (when the jumper started to jump), and the result of $t = 0.99 \text{ s}$ is the time to return to the original displacement. Thus the answer is $t = 0.99 \text{ seconds}$.

55. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height where the object was released. The initial velocity is $v_0 = -5.10 \text{ m/s}$, the acceleration is $a = 9.80 \text{ m/s}^2$, and the displacement of the package will be $y = 105 \text{ m}$. The time to reach the ground can be found from Eq. 2-12b, with x replaced by y .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow t^2 + \frac{2v_0}{a} t - \frac{2y}{a} = 0 \rightarrow t^2 + \frac{2(-5.10 \text{ m/s})}{9.80 \text{ m/s}^2} t - \frac{2(105 \text{ m})}{9.80 \text{ m/s}^2} = 0 \rightarrow$$

$$t = 5.18 \text{ s}, -4.14 \text{ s}$$

The correct time is the positive answer, $\boxed{t = 5.18 \text{ s}}$.

56. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height from which the object is released. The initial velocity is $v_0 = 0$, and the acceleration is $a = g$. Then we can calculate the position as a function of time from Eq. 2-12b, with x replaced by y , as $y(t) = \frac{1}{2}gt^2$. At the end of each second, the position would be as follows.

$$y(0) = 0 ; \quad y(1) = \frac{1}{2}g ; \quad y(2) = \frac{1}{2}g(2)^2 = 4y(1) ; \quad y(3) = \frac{1}{2}g(3)^2 = 9y(1)$$

The distance traveled during each second can be found by subtracting two adjacent position values from the above list.

$$d(1) = y(1) - y(0) = y(1) ; \quad d(2) = y(2) - y(1) = 3y(1) ; \quad d(3) = y(3) - y(2) = 5y(1)$$

We could do this in general.

$$y(n) = \frac{1}{2}gn^2 \quad y(n+1) = \frac{1}{2}g(n+1)^2$$

$$d(n+1) = y(n+1) - y(n) = \frac{1}{2}g(n+1)^2 - \frac{1}{2}gn^2 = \frac{1}{2}g((n+1)^2 - n^2)$$

$$= \frac{1}{2}g(n^2 + 2n + 1 - n^2) = \frac{1}{2}g(2n + 1)$$

The value of $(2n + 1)$ is always odd, in the sequence $[1, 3, 5, 7, \dots]$.

57. Choose upward to be the positive direction, and $y_0 = 0$ to be the level from which the ball was thrown. The initial velocity is v_0 , the instantaneous velocity is $v = 14 \text{ m/s}$, the acceleration is $a = -9.80 \text{ m/s}^2$, and the location of the window is $y = 23 \text{ m}$.

- (a) Using Eq. 2-12c and substituting y for x , we have

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v_0 = \pm \sqrt{v^2 - 2a(y - y_0)} = \pm \sqrt{(14 \text{ m/s})^2 - 2(-9.8 \text{ m/s}^2)(23 \text{ m})} = 25.43 \text{ m/s} \approx \boxed{25 \text{ m/s}}$$

Choose the positive value because the initial direction is upward.

- (b) At the top of its path, the velocity will be 0, and so we can use the initial velocity as found above, along with Eq. 2-12c.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (25.43 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{33 \text{ m}}$$

- (c) We want the time elapsed from throwing (speed $v_0 = 25.43 \text{ m/s}$) to reaching the window (speed $v = 14 \text{ m/s}$). Using Eq. 2-12a, we have the following.

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{14 \text{ m/s} - 25.43 \text{ m/s}}{-9.80 \text{ m/s}^2} = 1.166 \text{ s} \approx \boxed{1.2 \text{ s}}$$

- (d) We want the time elapsed from the window (speed $v_0 = 14 \text{ m/s}$) to reaching the street (speed $v = -25.43 \text{ m/s}$). Using Eq. 2-12a, we have the following.

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{-25.43 \text{ m/s} - 14 \text{ m/s}}{-9.80 \text{ m/s}^2} = 4.0 \text{ s}$$

This is the elapsed time after passing the window. The total time of flight of the baseball from passing the window to reaching the street is $4.0 \text{ s} + 1.2 \text{ s} = \boxed{5.2 \text{ s}}$.

58. (a) Choose upward to be the positive direction, and $y_0 = 0$ at the ground. The rocket has $v_0 = 0$, $a = 3.2 \text{ m/s}^2$, and $y = 950 \text{ m}$ when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq 2-12c, with x replaced by y .

$$v_{950 \text{ m}}^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v_{950 \text{ m}} = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \pm \sqrt{0 + 2(3.2 \text{ m/s}^2)(950 \text{ m})} = 77.97 \text{ m/s} \approx \boxed{78 \text{ m/s}}$$

The positive root is chosen since the rocket is moving upwards when it runs out of fuel.

- (b) The time to reach the 950 m location can be found from Eq. 2-12a.

$$v_{950 \text{ m}} = v_0 + at_{950 \text{ m}} \rightarrow t_{950 \text{ m}} = \frac{v_{950 \text{ m}} - v_0}{a} = \frac{77.97 \text{ m/s} - 0}{3.2 \text{ m/s}^2} = 24.37 \text{ s} \approx \boxed{24 \text{ s}}$$

- (c) For this part of the problem, the rocket will have an initial velocity $v_0 = 77.97 \text{ m/s}$, an acceleration of $a = -9.80 \text{ m/s}^2$, and a final velocity of $v = 0$ at its maximum altitude. The altitude reached from the out-of-fuel point can be found from Eq. 2-12c.

$$v^2 = v_{950 \text{ m}}^2 + 2a(y - 950 \text{ m}) \rightarrow$$

$$y_{\text{max}} = 950 \text{ m} + \frac{0 - v_{950 \text{ m}}^2}{2a} = 950 \text{ m} + \frac{-(77.97 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 950 \text{ m} + 310 \text{ m} = \boxed{1260 \text{ m}}$$

- (d) The time for the “coasting” portion of the flight can be found from Eq. 2-12a.

$$v = v_{950 \text{ m}} + at_{\text{coast}} \rightarrow t_{\text{coast}} = \frac{v - v_0}{a} = \frac{0 - 77.97 \text{ m/s}}{-9.80 \text{ m/s}^2} = 7.96 \text{ s}$$

Thus the total time to reach the maximum altitude is $t = 24.37 \text{ s} + 7.96 \text{ s} = 32.33 \text{ s} \approx \boxed{32 \text{ s}}$.

- (e) For the falling motion of the rocket, $v_0 = 0 \text{ m/s}$, $a = -9.80 \text{ m/s}^2$, and the displacement is -1260 m (it falls from a height of 1260 m to the ground). Find the velocity upon reaching the Earth from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \pm \sqrt{0 + 2(-9.80 \text{ m/s}^2)(-1260 \text{ m})} = -157 \text{ m/s} \approx \boxed{-160 \text{ m/s}}$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.

- (f) The time for the rocket to fall back to the Earth is found from Eq. 2-12a.

$$v = v_0 + at \rightarrow t_{\text{fall}} = \frac{v - v_0}{a} = \frac{-157 \text{ m/s} - 0}{-9.80 \text{ m/s}^2} = 16.0 \text{ s}$$

Thus the total time for the entire flight is $t = 32.33 \text{ s} + 16.0 \text{ s} = 48.33 \text{ s} \approx \boxed{48 \text{ s}}$.

59. (a) Choose $y = 0$ to be the ground level, and positive to be upward. Then $y = 0 \text{ m}$, $y_0 = 15 \text{ m}$, $a = -g$, and $t = 0.83 \text{ s}$ describe the motion of the balloon. Use Eq. 2-12b.

$$y = y_0 + v_0 t + \frac{1}{2} at^2 \rightarrow$$

$$v_0 = \frac{y - y_0 - \frac{1}{2} at^2}{t} = \frac{0 - 15 \text{ m} - \frac{1}{2}(-9.80 \text{ m/s}^2)(0.83 \text{ s})^2}{(0.83 \text{ s})} = -14 \text{ m/s}$$

So the speed is $\boxed{14 \text{ m/s}}$.

- (b) Consider the change in velocity from being released to being at Roger's room, using Eq. 2-12c.

$$v^2 = v_0^2 + 2a\Delta y \rightarrow \Delta y = \frac{v^2 - v_0^2}{2a} = \frac{-(-14 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 10 \text{ m}$$

Thus the balloons are coming from 2 floors above Roger, and so the fifth floor.

60. Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 24.0 \text{ m/s}$, $a = -9.80 \text{ m/s}^2$, and $y - y_0 = 13.0 \text{ m}$.

- (a) The velocity can be found from Eq. 2-12c, with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) = 0 \rightarrow$$

$$v = \pm \sqrt{v_0^2 + 2ay} = \pm \sqrt{(24.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(13.0 \text{ m})} = \pm 17.9 \text{ m/s}$$

Thus the speed is $|v| = 17.9 \text{ m/s}$.

- (b) The time to reach that height can be found from Eq. 2-12b.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow t^2 + \frac{2(24.0 \text{ m/s})}{-9.80 \text{ m/s}^2} t + \frac{2(-13.0 \text{ m})}{-9.80 \text{ m/s}^2} = 0 \rightarrow$$

$$t^2 - 4.898 t + 2.653 = 0 \rightarrow \boxed{t = 4.28 \text{ s}, 0.620 \text{ s}}$$

- (c) There are two times at which the object reaches that height – once on the way up ($t = 0.620 \text{ s}$), and once on the way down ($t = 4.28 \text{ s}$).

61. Choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is dropped. Call the location of the top of the window y_w , and the time for the stone to fall from release to the top of the window is t_w . Since the stone is dropped from rest, using Eq. 2-12b with y substituting for x , we have $y_w = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} g t_w^2$. The location of the bottom of the window is $y_w + 2.2 \text{ m}$, and the time for the stone to fall from release to the bottom of the window is $t_w + 0.33 \text{ s}$. Since the stone is dropped from rest, using Eq. 2-12b, we have the following:

$y_w + 2.2 \text{ m} = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} g (t_w + 0.33 \text{ s})^2$. Substitute the first expression for y_w into the second expression.

$$\frac{1}{2} g t_w^2 + 2.2 \text{ m} = \frac{1}{2} g (t_w + 0.33 \text{ s})^2 \rightarrow t_w = 0.515 \text{ s}$$

Use this time in the first equation to get the height above the top of the window from which the stone fell.

$$y_w = \frac{1}{2} g t_w^2 = \frac{1}{2} (9.80 \text{ m/s}^2) (0.515 \text{ s})^2 = \boxed{1.3 \text{ m}}$$

62. Choose upward to be the positive direction, and $y_0 = 0$ to be the location of the nozzle. The initial velocity is v_0 , the acceleration is $a = -9.80 \text{ m/s}^2$, the final location is $y = -1.5 \text{ m}$, and the time of flight is $t = 2.0 \text{ s}$. Using Eq. 2-12b and substituting y for x gives the following.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow v_0 = \frac{y - \frac{1}{2} a t^2}{t} = \frac{-1.5 \text{ m} - \frac{1}{2} (-9.80 \text{ m/s}^2) (2.0 \text{ s})^2}{2.0 \text{ s}} = \boxed{9.1 \text{ m/s}}$$

63. Choose up to be the positive direction, so $a = -g$. Let the ground be the $y = 0$ location. As an intermediate result, the velocity at the bottom of the window can be found from the data given. Assume the rocket is at the bottom of the window at $t = 0$, and use Eq. 2-12b.

$$y_{\text{top of window}} = y_{\text{bottom of window}} + v_{\text{bottom of window}} t_{\text{pass window}} + \frac{1}{2} a t_{\text{pass window}}^2 \rightarrow$$

$$10.0 \text{ m} = 8.0 \text{ m} + v_{\text{bottom of window}} (0.15 \text{ s}) + \frac{1}{2} (-9.80 \text{ m/s}^2) (0.15 \text{ s})^2 \rightarrow v_{\text{bottom of window}} = 14.07 \text{ m/s}$$

Now use the velocity at the bottom of the window with Eq. 2-12c to find the launch velocity, assuming the launch velocity was achieved at the ground level.

$$v_{\text{bottom of window}}^2 = v_{\text{launch}}^2 + 2a(y - y_0) \rightarrow$$

$$v_{\text{launch}} = \sqrt{v_{\text{bottom of window}}^2 - 2a(y - y_0)} = \sqrt{(14.07 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(8.0 \text{ m})} = 18.84 \text{ m/s}$$

$$\approx \boxed{18.8 \text{ m/s}}$$

The maximum height can also be found from Eq. 2-12c, using the launch velocity and a velocity of 0 at the maximum height.

$$v_{\text{maximum height}}^2 = v_{\text{launch}}^2 + 2a(y_{\text{max}} - y_0) \rightarrow$$

$$y_{\text{max}} = y_0 + \frac{v_{\text{maximum height}}^2 - v_{\text{launch}}^2}{2a} = \frac{-(18.84 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{18.1 \text{ m}}$$

64. Choose up to be the positive direction. Let the bottom of the cliff be the $y = 0$ location. The equation of motion for the dropped ball is $y_{\text{ball}} = y_0 + v_0 t + \frac{1}{2} a t^2 = 50.0 \text{ m} + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2$. The equation of motion for the thrown stone is $y_{\text{stone}} = y_0 + v_0 t + \frac{1}{2} a t^2 = (24.0 \text{ m/s}) t + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2$. Set the two equations equal and solve for the time of the collision. Then use that time to find the location of either object.

$$y_{\text{ball}} = y_{\text{stone}} \rightarrow 50.0 \text{ m} + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2 = (24.0 \text{ m/s}) t + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2 \rightarrow$$

$$50.0 \text{ m} = (24.0 \text{ m/s}) t \rightarrow t = \frac{50.0 \text{ m}}{24.0 \text{ m/s}} = 2.083 \text{ s}$$

$$y_{\text{ball}} = y_0 + v_0 t + \frac{1}{2} a t^2 = 50.0 \text{ m} + \frac{1}{2} (-9.80 \text{ m/s}^2) (2.083 \text{ s})^2 = \boxed{28.7 \text{ m}}$$

65. For the falling rock, choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is dropped. The initial velocity is $v_0 = 0 \text{ m/s}$, the acceleration is $a = g$, the displacement is $y = H$, and the time of fall is t_1 . Using Eq. 2-12b with y substituting for x , we have $H = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} g t_1^2$. For the sound wave, use the constant speed equation that $v_s = \frac{\Delta x}{\Delta t} = \frac{H}{T - t_1}$, which can be rearranged to give $t_1 = T - \frac{H}{v_s}$, where $T = 3.4 \text{ s}$ is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for t_1 into the equation for H from the stone, and solve for H .

$$H = \frac{1}{2}g \left(T - \frac{H}{v_s} \right)^2 \rightarrow \frac{g}{2v_s^2} H^2 - \left(\frac{gT}{v_s} + 1 \right) H + \frac{1}{2}gT^2 = 0 \rightarrow$$

$$4.239 \times 10^{-5} H^2 - 1.098 H + 56.64 = 0 \rightarrow H = 51.7 \text{ m}, 2.59 \times 10^4 \text{ m}$$

If the larger answer is used in $t_1 = T - \frac{H}{v_s}$, a negative time of fall results, and so the physically

correct answer is $\boxed{H = 52 \text{ m}}$.

66. (a) Choose up to be the positive direction. Let the throwing height of both objects be the $y = 0$ location, and so $y_0 = 0$ for both objects. The acceleration of both objects is $a = -g$. The equation of motion for the rock, using Eq. 2-12b, is $y_{\text{rock}} = y_0 + v_{0 \text{ rock}} t + \frac{1}{2} a t^2 = v_{0 \text{ rock}} t - \frac{1}{2} g t^2$, where t is the time elapsed from the throwing of the rock. The equation of motion for the ball, being thrown 1.00 s later, is $y_{\text{ball}} = y_0 + v_{0 \text{ ball}} (t - 1.00 \text{ s}) + \frac{1}{2} a (t - 1.00 \text{ s})^2 = v_{0 \text{ ball}} (t - 1.00 \text{ s}) - \frac{1}{2} g (t - 1.00 \text{ s})^2$. Set the two equations equal (meaning the two objects are at the same place) and solve for the time of the collision.

$$\begin{aligned} y_{\text{rock}} = y_{\text{ball}} &\rightarrow v_{0 \text{ rock}} t - \frac{1}{2} g t^2 = v_{0 \text{ ball}} (t - 1.00 \text{ s}) - \frac{1}{2} g (t - 1.00 \text{ s})^2 \rightarrow \\ (12.0 \text{ m/s}) t - \frac{1}{2} (9.80 \text{ m/s}^2) t^2 &= (18.0 \text{ m/s}) (t - 1.00 \text{ s}) - \frac{1}{2} (9.80 \text{ m/s}^2) (t - 1.00 \text{ s})^2 \rightarrow \\ (15.8 \text{ m/s}) t &= (22.9 \text{ m}) \rightarrow t = \boxed{1.45 \text{ s}} \end{aligned}$$

- (b) Use the time for the collision to find the position of either object.

$$y_{\text{rock}} = v_{0 \text{ rock}} t - \frac{1}{2} g t^2 = (12.0 \text{ m/s}) (1.45 \text{ s}) - \frac{1}{2} (9.80 \text{ m/s}^2) (1.45 \text{ s})^2 = \boxed{7.10 \text{ m}}$$

- (c) Now the ball is thrown first, and so $y_{\text{ball}} = v_{0 \text{ ball}} t - \frac{1}{2} g t^2$ and

$y_{\text{rock}} = v_{0 \text{ rock}} (t - 1.00 \text{ s}) - \frac{1}{2} g (t - 1.00 \text{ s})^2$. Again set the two equations equal to find the time of collision.

$$\begin{aligned} y_{\text{ball}} = y_{\text{rock}} &\rightarrow v_{0 \text{ ball}} t - \frac{1}{2} g t^2 = v_{0 \text{ rock}} (t - 1.00 \text{ s}) - \frac{1}{2} g (t - 1.00 \text{ s})^2 \rightarrow \\ (18.0 \text{ m/s}) t - \frac{1}{2} (9.80 \text{ m/s}^2) t^2 &= (12.0 \text{ m/s}) (t - 1.00 \text{ s}) - \frac{1}{2} (9.80 \text{ m/s}^2) (t - 1.00 \text{ s})^2 \rightarrow \\ (3.80 \text{ m/s}) t &= 16.9 \text{ m} \rightarrow t = 4.45 \text{ s} \end{aligned}$$

But this answer can be deceptive. Where do the objects collide?

$$y_{\text{ball}} = v_{0 \text{ ball}} t - \frac{1}{2} g t^2 = (18.0 \text{ m/s}) (4.45 \text{ s}) - \frac{1}{2} (9.80 \text{ m/s}^2) (4.45 \text{ s})^2 = -16.9 \text{ m}$$

Thus, assuming they were thrown from ground level, they collide below ground level, which cannot happen. Thus $\boxed{\text{they never collide}}$.

67. The displacement is found from the integral of the velocity, over the given time interval.

$$\begin{aligned} \Delta x &= \int_{t_1}^{t_2} v dt = \int_{t=1.5 \text{ s}}^{t=3.1 \text{ s}} (25 + 18t) dt = (25t + 9t^2) \Big|_{t=1.5 \text{ s}}^{t=3.1 \text{ s}} = [25(3.1) + 9(3.1)^2] - [25(1.5) + 9(1.5)^2] \\ &= \boxed{106 \text{ m}} \end{aligned}$$

68. (a) The speed is the integral of the acceleration.

$$a = \frac{dv}{dt} \rightarrow dv = a dt \rightarrow dv = A\sqrt{t} dt \rightarrow \int_{v_0}^v dv = A \int_0^t \sqrt{t} dt \rightarrow$$

$$v - v_0 = \frac{2}{3} At^{3/2} \rightarrow v = v_0 + \frac{2}{3} At^{3/2} \rightarrow \boxed{v = 7.5 \text{ m/s} + \frac{2}{3} (2.0 \text{ m/s}^{5/2}) t^{3/2}}$$

- (b) The displacement is the integral of the velocity.

$$v = \frac{dx}{dt} \rightarrow dx = v dt \rightarrow dx = \left(v_0 + \frac{2}{3} At^{3/2} \right) dt \rightarrow$$

$$\int_{0 \text{ m}}^x dx = \int_0^t \left(v_0 + \frac{2}{3} At^{3/2} \right) dt \rightarrow x = v_0 t + \frac{2}{3} \cdot \frac{2}{5} At^{5/2} = \boxed{(7.5 \text{ m/s})t + \frac{4}{15} (2.0 \text{ m/s}^{5/2}) t^{5/2}}$$

(c) $a(t = 5.0 \text{ s}) = (2.0 \text{ m/s}^{5/2})\sqrt{5.0 \text{ s}} = \boxed{4.5 \text{ m/s}^2}$

$$v(t = 5.0 \text{ s}) = 7.5 \text{ m/s} + \frac{2}{3} (2.0 \text{ m/s}^{5/2}) (5.0 \text{ s})^{3/2} = 22.41 \text{ m/s} \approx \boxed{22 \text{ m/s}}$$

$$x(t = 5.0 \text{ s}) = (7.5 \text{ m/s})(5.0 \text{ s}) + \frac{4}{15} (2.0 \text{ m/s}^{5/2}) (5.0 \text{ s})^{5/2} = 67.31 \text{ m} \approx \boxed{67 \text{ m}}$$

69. (a) The velocity is found by integrating the acceleration with respect to time. Note that with the substitution given in the hint, the initial value of u is $u_0 = g - kv_0 = g$.

$$a = \frac{dv}{dt} \rightarrow dv = a dt \rightarrow dv = (g - kv) dt \rightarrow \frac{dv}{g - kv} = dt$$

Now make the substitution that $u \equiv g - kv$.

$$u \equiv g - kv \rightarrow dv = -\frac{du}{k} \quad \frac{dv}{g - kv} = dt \rightarrow -\frac{du}{k} \frac{1}{u} = dt \rightarrow \frac{du}{u} = -k dt$$

$$\int_g^u \frac{du}{u} = -k \int_0^t dt \rightarrow \ln u \Big|_g^u = -kt \rightarrow \ln \frac{u}{g} = -kt \rightarrow u = g e^{-kt} = g - kv \rightarrow$$

$$\boxed{v = \frac{g}{k} (1 - e^{-kt})}$$

- (b) As t goes to infinity, the value of the velocity is $v_{\text{term}} = \lim_{t \rightarrow \infty} \frac{g}{k} (1 - e^{-kt}) = \boxed{\frac{g}{k}}$. We also note that

if the acceleration is zero (which happens at terminal velocity), then $a = g - kv = 0 \rightarrow$

$$v_{\text{term}} = \frac{g}{k}.$$

70. (a) The train's constant speed is $v_{\text{train}} = 5.0 \text{ m/s}$, and the location of the empty box car as a function of time is given by $x_{\text{train}} = v_{\text{train}} t = (5.0 \text{ m/s})t$. The fugitive has $v_0 = 0 \text{ m/s}$ and $a = 1.2 \text{ m/s}^2$ until his final speed is 6.0 m/s . The elapsed time during the acceleration is $t_{\text{acc}} = \frac{v - v_0}{a} = \frac{6.0 \text{ m/s}}{1.2 \text{ m/s}^2} = 5.0 \text{ s}$. Let the origin be the location of the fugitive when he starts to run. The first possibility to consider is, "Can the fugitive catch the empty box car before he reaches his maximum speed?" During the fugitive's acceleration, his location as a function of time is given by Eq. 2-12b, $x_{\text{fugitive}} = x_0 + v_0 t + \frac{1}{2} at^2 = 0 + 0 + \frac{1}{2} (1.2 \text{ m/s}^2) t^2$. For him to catch

the train, we must have $x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0 \text{ m/s})t = \frac{1}{2}(1.2 \text{ m/s}^2)t^2$. The solutions of this are $t = 0 \text{ s}, 8.3 \text{ s}$. Thus the fugitive cannot catch the car during his 5.0 s of acceleration.

Now the equation of motion of the fugitive changes. After the 5.0 s of acceleration, he runs with a constant speed of 6.0 m/s. Thus his location is now given (for times $t > 5 \text{ s}$) by the following.

$$x_{\text{fugitive}} = \frac{1}{2}(1.2 \text{ m/s}^2)(5.0 \text{ s})^2 + (6.0 \text{ m/s})(t - 5.0 \text{ s}) = (6.0 \text{ m/s})t - 15.0 \text{ m}$$

So now, for the fugitive to catch the train, we again set the locations equal.

$$x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0 \text{ m/s})t = (6.0 \text{ m/s})t - 15.0 \text{ m} \rightarrow t = \boxed{15.0 \text{ s}}$$

(b) The distance traveled to reach the box car is given by the following.

$$x_{\text{fugitive}}(t = 15.0 \text{ s}) = (6.0 \text{ m/s})(15.0 \text{ s}) - 15.0 \text{ m} = \boxed{75 \text{ m}}$$

71. Choose the upward direction to be positive, and $y_0 = 0$ to be the level from which the object was thrown. The initial velocity is v_0 and the velocity at the top of the path is $v = 0 \text{ m/s}$. The height at the top of the path can be found from Eq. 2-12c with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y - y_0 = \frac{-v_0^2}{2a}$$

From this we see that the displacement is inversely proportional to the acceleration, and so if the acceleration is reduced by a factor of 6 by going to the Moon, and the initial velocity is unchanged, the displacement increases by a factor of 6.

72. (a) For the free-falling part of the motion, choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the person jumped. The initial velocity is $v_0 = 0$, acceleration is $a = 9.80 \text{ m/s}^2$, and the location of the net is $y = 15.0 \text{ m}$. Find the speed upon reaching the net from Eq. 2-12c with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow v = \pm \sqrt{0 + 2a(y - 0)} = \pm \sqrt{2(9.80 \text{ m/s}^2)(15.0 \text{ m})} = 17.1 \text{ m/s}$$

The positive root is selected since the person is moving downward.

For the net-stretching part of the motion, choose downward to be the positive direction, and $y_0 = 15.0 \text{ m}$ to be the height at which the person first contacts the net. The initial velocity is $v_0 = 17.1 \text{ m/s}$, the final velocity is $v = 0$, and the location at the stretched position is $y = 16.0 \text{ m}$. Find the acceleration from Eq. 2-12c with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0^2 - (17.1 \text{ m/s})^2}{2(1.0 \text{ m})} = \boxed{-150 \text{ m/s}^2}$$

- (b) For the acceleration to be smaller, in the above equation we see that the displacement should be larger. This means that the net should be “loosened”.

73. The initial velocity of the car is $v_0 = (100 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 27.8 \text{ m/s}$. Choose $x_0 = 0$ to be the

location at which the deceleration begins. We have $v = 0 \text{ m/s}$ and $a = -30g = -294 \text{ m/s}^2$. Find the displacement from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x = x_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (27.8 \text{ m/s})^2}{2(-2.94 \times 10^2 \text{ m/s}^2)} = 1.31 \text{ m} \approx \boxed{1.3 \text{ m}}$$

74. Choose downward to be the positive direction, and $y_0 = 0$ to be at the start of the pelican's dive.

The pelican has an initial velocity is $v_0 = 0$, an acceleration of $a = g$, and a final location of $y = 16.0 \text{ m}$. Find the total time of the pelican's dive from Eq. 2-12b, with x replaced by y .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow y = 0 + 0 + \frac{1}{2} a t^2 \rightarrow t_{\text{dive}} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(16.0 \text{ m})}{9.80 \text{ m/s}^2}} = 1.81 \text{ s}.$$

The fish can take evasive action if he sees the pelican at a time of $1.81 \text{ s} - 0.20 \text{ s} = 1.61 \text{ s}$ into the dive. Find the location of the pelican at that time from Eq. 2-12b.

$$y = y_0 + v_0 t + \frac{1}{2} a t = 0 + 0 + \frac{1}{2} (9.80 \text{ m/s}^2) (1.61 \text{ s})^2 = 12.7 \text{ m}$$

Thus the fish must spot the pelican at a minimum height from the surface of the water of $16.0 \text{ m} - 12.7 \text{ m} = \boxed{3.3 \text{ m}}$.

75. (a) Choose downward to be the positive direction, and $y_0 = 0$ to be the level from which the car was dropped. The initial velocity is $v_0 = 0$, the final location is $y = H$, and the acceleration is $a = g$. Find the final velocity from Eq. 2-12c, replacing x with y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow v = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \pm \sqrt{2gH}.$$

The speed is the magnitude of the velocity, $\boxed{v = \sqrt{2gH}}$.

- (b) Solving the above equation for the height, we have that $H = \frac{v^2}{2g}$. Thus for a collision of

$$v = (50 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 13.89 \text{ m/s}, \text{ the corresponding height is as follows.}$$

$$H = \frac{v^2}{2g} = \frac{(13.89 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 9.84 \text{ m} \approx \boxed{10 \text{ m}}$$

- (c) For a collision of $v = (100 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 27.78 \text{ m/s}$, the corresponding height is as follow.

$$H = \frac{v^2}{2g} = \frac{(27.78 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 39.37 \text{ m} \approx \boxed{40 \text{ m}}$$

76. Choose downward to be the positive direction, and $y_0 = 0$ to be at the roof from which the stones are dropped. The first stone has an initial velocity of $v_0 = 0$ and an acceleration of $a = g$. Eqs. 2-12a and 2-12b (with x replaced by y) give the velocity and location, respectively, of the first stone as a function of time.

$$v = v_0 + at \rightarrow v_1 = gt_1 \quad y = y_0 + v_0 t + \frac{1}{2} at^2 \rightarrow y_1 = \frac{1}{2} gt_1^2$$

The second stone has the same initial conditions, but its elapsed time $t - 1.50 \text{ s}$, and so has velocity and location equations as follows.

$$v_2 = g(t_1 - 1.50 \text{ s}) \quad y_2 = \frac{1}{2}g(t_1 - 1.50 \text{ s})^2$$

The second stone reaches a speed of $v_2 = 12.0 \text{ m/s}$ at a time given by the following.

$$t_1 = 1.50 \text{ s} + \frac{v_2}{g} = 1.50 \text{ s} + \frac{12.0 \text{ m/s}}{9.80 \text{ m/s}^2} = 2.72 \text{ s}$$

The location of the first stone at that time is $y_1 = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(2.72 \text{ s})^2 = 36.4 \text{ m}$.

The location of the second stone at that time is $y_2 = \frac{1}{2}g(t_1 - 1.50 \text{ s})^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(2.72 - 1.50 \text{ s})^2 = 7.35 \text{ m}$. Thus the distance between the two stones is

$$y_1 - y_2 = 36.4 \text{ m} - 7.35 \text{ m} = \boxed{29.0 \text{ m}}.$$

77. The initial velocity is $v_0 = (15 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 4.17 \text{ m/s}$. The final velocity is

$v_0 = (75 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 20.83 \text{ m/s}$. The displacement is $x - x_0 = 4.0 \text{ km} = 4000 \text{ m}$. Find the average acceleration from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(20.83 \text{ m/s})^2 - (4.17 \text{ m/s})^2}{2(4000 \text{ m})} = \boxed{5.2 \times 10^{-2} \text{ m/s}^2}$$

78. The speed limit is $50 \text{ km/h}\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 13.89 \text{ m/s}$.

(a) For your motion, you would need to travel $(10 + 15 + 50 + 15 + 70 + 15) \text{ m} = 175 \text{ m}$ to get the front of the car all the way through the third intersection. The time to travel the 175 m is found using the distance and the constant speed.

$$\Delta x = \bar{v}\Delta t \rightarrow \Delta t = \frac{\Delta x}{\bar{v}} = \frac{175 \text{ m}}{13.89 \text{ m/s}} = 12.60 \text{ s}$$

Yes, you can make it through all three lights without stopping.

(b) The second car needs to travel 165 m before the third light turns red. This car accelerates from $v_0 = 0 \text{ m/s}$ to a maximum of $v = 13.89 \text{ m/s}$ with $a = 2.0 \text{ m/s}^2$. Use Eq. 2-12a to determine the duration of that acceleration.

$$v = v_0 + at \rightarrow t_{\text{acc}} = \frac{v - v_0}{a} = \frac{13.89 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ m/s}^2} = 6.94 \text{ s}$$

The distance traveled during that time is found from Eq. 2-12b.

$$(x - x_0)_{\text{acc}} = v_0 t_{\text{acc}} + \frac{1}{2}at_{\text{acc}}^2 = 0 + \frac{1}{2}(2.0 \text{ m/s}^2)(6.94 \text{ s})^2 = 48.2 \text{ m}$$

Since 6.94 s have elapsed, there are $13 - 6.94 = 6.06 \text{ s}$ remaining to clear the intersection. The car travels another 6.06 s at a speed of 13.89 m/s, covering a distance of $\Delta x_{\text{constant speed}} = v_{\text{avg}} t =$

$(13.89 \text{ m/s})(6.06 \text{ s}) = 84.2 \text{ m}$. Thus the total distance is $48.2 \text{ m} + 84.2 \text{ m} = 132.4 \text{ m}$. No, the car cannot make it through all three lights without stopping.

The car has to travel another 32.6 m to clear the third intersection, and is traveling at a speed of 13.89 m/s. Thus the care would enter the intersection a time $t = \frac{\Delta x}{v} = \frac{32.6 \text{ m}}{13.89 \text{ m/s}} = \boxed{2.3 \text{ s}}$ after the light turns red.

79. First consider the “uphill lie,” in which the ball is being putted down the hill. Choose $x_0 = 0$ to be the ball’s original location, and the direction of the ball’s travel as the positive direction. The final velocity of the ball is $v = 0 \text{ m/s}$, the acceleration of the ball is $a = -1.8 \text{ m/s}^2$, and the displacement of the ball will be $x - x_0 = 6.0 \text{ m}$ for the first case and $x - x_0 = 8.0 \text{ m}$ for the second case. Find the initial velocity of the ball from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-1.8 \text{ m/s}^2)(6.0 \text{ m})} = 4.6 \text{ m/s} \\ \sqrt{0 - 2(-1.8 \text{ m/s}^2)(8.0 \text{ m})} = 5.4 \text{ m/s} \end{cases}$$

The range of acceptable velocities for the uphill lie is $\boxed{4.6 \text{ m/s to } 5.4 \text{ m/s}}$, a spread of 0.8 m/s.

Now consider the “downhill lie,” in which the ball is being putted up the hill. Use a very similar set-up for the problem, with the basic difference being that the acceleration of the ball is now $a = -2.8 \text{ m/s}^2$. Find the initial velocity of the ball from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-2.8 \text{ m/s}^2)(6.0 \text{ m})} = 5.8 \text{ m/s} \\ \sqrt{0 - 2(-2.8 \text{ m/s}^2)(8.0 \text{ m})} = 6.7 \text{ m/s} \end{cases}$$

The range of acceptable velocities for the downhill lie is $\boxed{5.8 \text{ m/s to } 6.7 \text{ m/s}}$, a spread of 0.9 m/s.

Because the range of acceptable velocities is smaller for putting down the hill, more control in putting is necessary, and so putting the ball downhill (the “uphill lie”) is more difficult.

80. To find the distance, we divide the motion of the robot into three segments. First, the initial acceleration from rest; second, motion at constant speed; and third, deceleration back to rest.
- $$d_1 = v_0 t + \frac{1}{2} a_1 t^2 = 0 + \frac{1}{2} (0.20 \text{ m/s}^2) (5.0 \text{ s})^2 = 2.5 \text{ m} \quad v_1 = a_1 t_1 = (0.20 \text{ m/s}^2) (5.0 \text{ s}) = 1.0 \text{ m/s}$$
- $$d_2 = v_1 t_2 = (1.0 \text{ m/s}) (68 \text{ s}) = 68 \text{ m} \quad v_2 = v_1 = 1.0 \text{ m/s}$$
- $$d_3 = v_2 t_3 + \frac{1}{2} a_1 t_1^2 = (1.0 \text{ m/s}) (2.5 \text{ s}) + \frac{1}{2} (-0.40 \text{ m/s}^2) (2.5 \text{ s})^2 = 1.25 \text{ m}$$
- $$d = d_1 + d_2 + d_3 = 2.5 \text{ m} + 68 \text{ m} + 1.25 \text{ m} = 71.75 \text{ m} \approx \boxed{72 \text{ m}}$$

81. Choose downward to be the positive direction, and $y_0 = 0$ to be at the top of the cliff. The initial velocity is $v_0 = -12.5 \text{ m/s}$, the acceleration is $a = 9.80 \text{ m/s}^2$, and the final location is $y = 75.0 \text{ m}$.

(a) Using Eq. 2-12b and substituting y for x , we have the following.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow (4.9 \text{ m/s}^2) t^2 - (12.5 \text{ m/s}) t - 75.0 \text{ m} = 0 \rightarrow t = -2.839 \text{ s}, 5.390 \text{ s}$$

The positive answer is the physical answer: $\boxed{t = 5.39 \text{ s}}$.

- (b) Using Eq. 2-12a, we have $v = v_0 + at = -12.5 \text{ m/s} + (9.80 \text{ m/s}^2) (5.390 \text{ s}) = \boxed{40.3 \text{ m/s}}$.

- (c) The total distance traveled will be the distance up plus the distance down. The distance down will be 75.0 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Using Eq. 2-12c we have the following.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (-12.5 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = -7.97 \text{ m}$$

Thus the distance up is 7.97 m, the distance down is 82.97 m, and the total distance traveled is 90.9 m.

82. (a) In the interval from A to B, it is moving in the negative direction, because its displacement is negative.
- (b) In the interval from A to B, it is speeding up, because the magnitude of its slope is increasing (changing from less steep to more steep).
- (c) In the interval from A to B, the acceleration is negative, because the graph is concave down, indicating that the slope is getting more negative, and thus the acceleration is negative.
- (d) In the interval from D to E, it is moving in the positive direction, because the displacement is positive.
- (e) In the interval from D to E, it is speeding up, because the magnitude of its slope is increasing (changing from less steep to more steep).
- (f) In the interval from D to E, the acceleration is positive, because the graph is concave upward, indicating the slope is getting more positive, and thus the acceleration is positive.
- (g) In the interval from C to D, the object is not moving in either direction.
- The velocity and acceleration are both 0.

83. This problem can be analyzed as a series of three one-dimensional motions: the acceleration phase, the constant speed phase, and the deceleration phase. The maximum speed of the train is as follows.

$$(95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s}$$

In the acceleration phase, the initial velocity is $v_0 = 0 \text{ m/s}$, the acceleration is $a = 1.1 \text{ m/s}^2$, and the final velocity is $v = 26.39 \text{ m/s}$. Find the elapsed time for the acceleration phase from Eq. 2-12a.

$$v = v_0 + at \rightarrow t_{\text{acc}} = \frac{v - v_0}{a} = \frac{26.39 \text{ m/s} - 0}{1.1 \text{ m/s}^2} = 23.99 \text{ s}$$

Find the displacement during the acceleration phase from Eq. 2-12b.

$$(x - x_0)_{\text{acc}} = v_0 t + \frac{1}{2} at^2 = 0 + \frac{1}{2} (1.1 \text{ m/s}^2) (23.99 \text{ s})^2 = 316.5 \text{ m}$$

In the deceleration phase, the initial velocity is $v_0 = 26.39 \text{ m/s}$, the acceleration is $a = -2.0 \text{ m/s}^2$, and the final velocity is $v = 0 \text{ m/s}$. Find the elapsed time for the deceleration phase from Eq. 2-12a.

$$v = v_0 + at \rightarrow t_{\text{dec}} = \frac{v - v_0}{a} = \frac{0 - 26.39 \text{ m/s}}{-2.0 \text{ m/s}^2} = 13.20 \text{ s}$$

Find the distance traveled during the deceleration phase from Eq. 2-12b.

$$(x - x_0)_{\text{dec}} = v_0 t + \frac{1}{2} at^2 = (26.39 \text{ m/s})(13.20 \text{ s}) + \frac{1}{2} (-2.0 \text{ m/s}^2) (13.20 \text{ s})^2 = 174.1 \text{ m}$$

The total elapsed time and distance traveled for the acceleration / deceleration phases are:

$$t_{\text{acc}} + t_{\text{dec}} = 23.99 \text{ s} + 13.20 \text{ s} = 37.19 \text{ s}$$

$$(x - x_0)_{\text{acc}} + (x - x_0)_{\text{dec}} = 316.5 \text{ m} + 174.1 \text{ m} = 491 \text{ m}$$

- (a) If the stations are spaced $1.80 \text{ km} = 1800 \text{ m}$ apart, then there is a total of $\frac{9000 \text{ m}}{1800 \text{ m}} = 5$ inter-station segments. A train making the entire trip would thus have a total of 5 inter-station segments and 4 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $1800 \text{ m} - 491 \text{ m} = 1309 \text{ m}$ of each segment is traveled at an average speed of $\bar{v} = 26.39 \text{ m/s}$. The time for that 1309 m is given by $\Delta x = \bar{v} \Delta t \rightarrow$
- $$\Delta t_{\text{constant speed}} = \frac{\Delta x}{\bar{v}} = \frac{1309 \text{ m}}{26.39 \text{ m/s}} = 49.60 \text{ s.}$$
- Thus a total inter-station segment will take $37.19 \text{ s} + 49.60 \text{ s} = 86.79 \text{ s}$. With 5 inter-station segments of 86.79 s each, and 4 stops of 22 s each, the total time is given by $t_{0.8 \text{ km}} = 5(86.79 \text{ s}) + 4(22 \text{ s}) = 522 \text{ s} = \boxed{8.7 \text{ min}}$.

- (b) If the stations are spaced $3.0 \text{ km} = 3000 \text{ m}$ apart, then there is a total of $\frac{9000 \text{ m}}{3000 \text{ m}} = 3$ inter-station segments. A train making the entire trip would thus have a total of 3 inter-station segments and 2 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $3000 \text{ m} - 491 \text{ m} = 2509 \text{ m}$ of each segment is traveled at an average speed of $\bar{v} = 26.39 \text{ m/s}$. The time for that 2509 m is given by $d = \bar{v} t \rightarrow$
- $$t = \frac{d}{\bar{v}} = \frac{2509 \text{ m}}{26.39 \text{ m/s}} = 95.07 \text{ s.}$$
- Thus a total inter-station segment will take $37.19 \text{ s} + 95.07 \text{ s} = 132.3 \text{ s}$. With 3 inter-station segments of 132.3 s each, and 2 stops of 22 s each, the total time is $t_{3.0 \text{ km}} = 3(132.3 \text{ s}) + 2(22 \text{ s}) = 441 \text{ s} = \boxed{7.3 \text{ min}}$.

84. For the motion in the air, choose downward to be the positive direction, and $y_0 = 0$ to be at the height of the diving board. The diver has $v_0 = 0$ (assuming the diver does not jump upward or downward), $a = g = 9.80 \text{ m/s}^2$, and $y = 4.0 \text{ m}$ when reaching the surface of the water. Find the diver's speed at the water's surface from Eq. 2-12c, with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \sqrt{0 + 2(9.80 \text{ m/s}^2)(4.0 \text{ m})} = 8.85 \text{ m/s}$$

For the motion in the water, again choose down to be positive, but redefine $y_0 = 0$ to be at the surface of the water. For this motion, $v_0 = 8.85 \text{ m/s}$, $v = 0$, and $y - y_0 = 2.0 \text{ m}$. Find the acceleration from Eq. 2-12c, with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0 - (8.85 \text{ m/s})^2}{2(2.0 \text{ m})} = -19.6 \text{ m/s}^2 \approx \boxed{-20 \text{ m/s}^2}$$

The negative sign indicates that the acceleration is directed upwards.

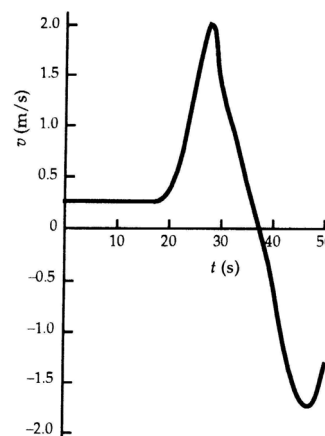
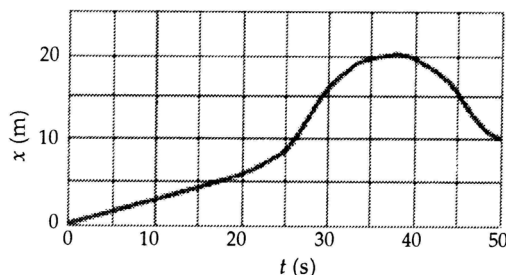
- 85.** Choose upward to be the positive direction, and the origin to be at the level where the ball was thrown. The velocity at the top of the ball's path will be $v = 0$, and the ball will have an acceleration of $a = -g$. If the maximum height that the ball reaches is $y = H$, then the relationship

between the initial velocity and the maximum height can be found from Eq. 2-12c, with x replaced by y .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow 0 = v_0^2 + 2(-g)H \rightarrow H = v_0^2/2g$$

It is given that $v_{0 \text{ Bill}} = 1.5v_{0 \text{ Joe}}$, so $\frac{H_{\text{Bill}}}{H_{\text{Joe}}} = \frac{(v_{0 \text{ Bill}})^2/2g}{(v_{0 \text{ Joe}})^2/2g} = \frac{(v_{0 \text{ Bill}})^2}{(v_{0 \text{ Joe}})^2} = 1.5^2 = 2.25 \approx \boxed{2.3}$.

86. The v vs. t graph is found by taking the slope of the x vs. t graph. Both graphs are shown here.



87. The car's initial speed is $v_o = (45 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 12.5 \text{ m/s}$.

Case I: trying to stop. The constraint is, with the braking deceleration of the car ($a = -5.8 \text{ m/s}^2$), can the car stop in a 28 m displacement? The 2.0 seconds has no relation to this part of the problem. Using Eq. 2-12c, the distance traveled during braking is as follows.

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12.5 \text{ m/s})^2}{2(-5.8 \text{ m/s}^2)} = 13.5 \text{ m} \rightarrow \boxed{\text{She can stop the car in time.}}$$

Case II: crossing the intersection. The constraint is, with the given acceleration of the car

$$\left[a = \left(\frac{65 \text{ km/h} - 45 \text{ km/h}}{6.0 \text{ s}} \right) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 0.9259 \text{ m/s}^2 \right], \text{ can she get through the intersection}$$

(travel 43 meters) in the 2.0 seconds before the light turns red? Using Eq. 2-12b, the distance traveled during the 2.0 sec is as follows.

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2 = (12.5 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (0.927 \text{ m/s}^2)(2.0 \text{ s})^2 = 26.9 \text{ m}$$

She should stop.

88. The critical condition is that the total distance covered by the passing car and the approaching car must be less than 400 m so that they do not collide. The passing car has a total displacement composed of several individual parts. These are: i) the 10 m of clear room at the rear of the truck, ii) the 20 m length of the truck, iii) the 10 m of clear room at the front of the truck, and iv) the distance the truck travels. Since the truck travels at a speed of $\bar{v} = 25 \text{ m/s}$, the truck will have a displacement of $\Delta x_{\text{truck}} = (25 \text{ m/s})t$. Thus the total displacement of the car during passing is $\Delta x_{\text{passing car}} = 40 \text{ m} + (25 \text{ m/s})t$.

To express the motion of the car, we choose the origin to be at the location of the passing car when the decision to pass is made. For the passing car, we have an initial velocity of $v_0 = 25 \text{ m/s}$ and an acceleration of $a = 1.0 \text{ m/s}^2$. Find $\Delta x_{\text{passing car}}$ from Eq. 2-12b.

$$\Delta x_{\text{passing car}} = x_c - x_0 = v_0 t + \frac{1}{2} a t^2 = (25 \text{ m/s}) t + \frac{1}{2} (1.0 \text{ m/s}^2) t^2$$

Set the two expressions for $\Delta x_{\text{passing car}}$ equal to each other in order to find the time required to pass.

$$40 \text{ m} + (25 \text{ m/s}) t_{\text{pass}} = (25 \text{ m/s}) t_{\text{pass}} + \frac{1}{2} (1.0 \text{ m/s}^2) t_{\text{pass}}^2 \rightarrow 40 \text{ m} = \frac{1}{2} (1.0 \text{ m/s}^2) t_{\text{pass}}^2 \rightarrow$$

$$t_{\text{pass}} = \sqrt{80 \text{ s}^2} = 8.94 \text{ s}$$

Calculate the displacements of the two cars during this time.

$$\Delta x_{\text{passing car}} = 40 \text{ m} + (25 \text{ m/s})(8.94 \text{ s}) = 264 \text{ m}$$

$$\Delta x_{\text{approaching car}} = v_{\text{approaching car}} t = (25 \text{ m/s})(8.94 \text{ s}) = 224 \text{ m}$$

Thus the two cars together have covered a total distance of 488 m, which is more than allowed.

The car should not pass.

89. Choose downward to be the positive direction, and $y_0 = 0$ to be at the height of the bridge. Agent Bond has an initial velocity of $v_0 = 0$, an acceleration of $a = g$, and will have a displacement of $y = 13 \text{ m} - 1.5 \text{ m} = 11.5 \text{ m}$. Find the time of fall from Eq. 2-12b with x replaced by y .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(11.5 \text{ m})}{9.80 \text{ m/s}^2}} = 1.532 \text{ s}$$

If the truck is approaching with $v = 25 \text{ m/s}$, then he needs to jump when the truck is a distance away given by $d = vt = (25 \text{ m/s})(1.532 \text{ s}) = 38.3 \text{ m}$. Convert this distance into “poles.”

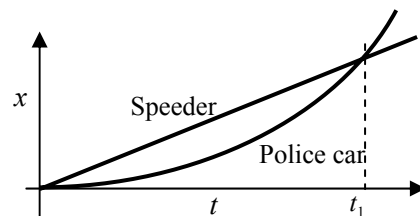
$$d = (38.3 \text{ m})(1 \text{ pole}/25 \text{ m}) = 1.53 \text{ poles}$$

So he should jump when the truck is about 1.5 poles away from the bridge.

90. Take the origin to be the location where the speeder passes the police car. The speeder's constant speed is $v_{\text{speeder}} = (130 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 36.1 \text{ m/s}$, and the location of the speeder as a function of time is given by $x_{\text{speeder}} = v_{\text{speeder}} t_{\text{speeder}} = (36.1 \text{ m/s}) t_{\text{speeder}}$. The police car has an initial velocity of $v_0 = 0 \text{ m/s}$ and a constant acceleration of a_{police} . The location of the police car as a function of time is given by Eq. 2-12b: $x_{\text{police}} = v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} a_{\text{police}} t_{\text{police}}^2$.

(a) The position vs. time graphs would qualitatively look like the graph shown here.

(b) The time to overtake the speeder occurs when the speeder has gone a distance of 750 m. The time is found using the speeder's equation from above.



$$750 \text{ m} = (36.1 \text{ m/s}) t_{\text{speeder}} \rightarrow t_{\text{speeder}} = \frac{750 \text{ m}}{36.1 \text{ m/s}} = 20.8 \text{ s} \approx \boxed{21 \text{ s}}$$

- (c) The police car's acceleration can be calculated knowing that the police car also had gone a distance of 750 m in a time of 22.5 s.

$$750 \text{ m} = \frac{1}{2} a_p (20.8 \text{ s})^2 \rightarrow a_p = \frac{2(750 \text{ m})}{(20.8 \text{ s})^2} = 3.47 \text{ m/s}^2 \approx \boxed{3.5 \text{ m/s}^2}$$

- (d) The speed of the police car at the overtaking point can be found from Eq. 2-12a.

$$v = v_0 + at = 0 + (3.47 \text{ m/s}^2)(20.8 \text{ s}) = 72.2 \text{ m/s} \approx \boxed{72 \text{ m/s}}$$

Note that this is exactly twice the speed of the speeder.

91. The speed of the conveyor belt is given by $d = \bar{v} \Delta t \rightarrow \bar{v} = \frac{d}{\Delta t} = \frac{1.1 \text{ m}}{2.5 \text{ min}} = \boxed{0.44 \text{ m/min}}$. The rate of burger production, assuming the spacing given is center to center, can be found as follows.

$$\left(\frac{1 \text{ burger}}{0.15 \text{ m}} \right) \left(\frac{0.44 \text{ m}}{1 \text{ min}} \right) = \boxed{2.9 \frac{\text{burgers}}{\text{min}}}$$

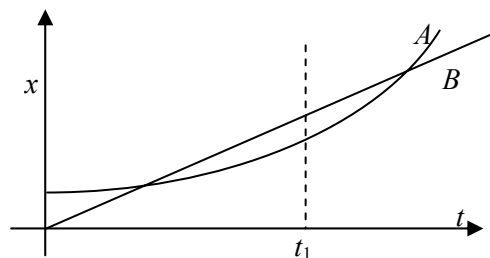
92. Choose downward to be the positive direction, and the origin to be at the top of the building. The barometer has $y_0 = 0$, $v_0 = 0$, and $a = g = 9.8 \text{ m/s}^2$. Use Eq. 2-12b to find the height of the building, with x replaced by y .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} (9.8 \text{ m/s}^2) t^2$$

$$y_{t=2.0} = \frac{1}{2} (9.8 \text{ m/s}^2) (2.0 \text{ s})^2 = 20 \text{ m} \quad y_{t=2.3} = \frac{1}{2} (9.8 \text{ m/s}^2) (2.3 \text{ s})^2 = 26 \text{ m}$$

The difference in the estimates is 6 m. If we assume the height of the building is the average of the two measurements, then the % difference in the two values is $\frac{6 \text{ m}}{23 \text{ m}} \times 100 = \boxed{26\%}$.

93. (a) The two bicycles will have the same velocity at any time when the instantaneous slopes of their x vs. t graphs are the same. That occurs near the time t_1 as marked on the graph.
 (b) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.



- (c) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, and so is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle B.
 (d) Bicycle B has the highest instantaneous velocity at all times until the time t_1 , where both graphs have the same slope. For all times after t_1 , bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.
 (e) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.

94. In this problem, note that $a < 0$ and $x > 0$. Take your starting position as 0. Then your position is given by Eq. 2-12b, $x_1 = v_M t + \frac{1}{2} a t^2$, and the other car's position is given by $x_2 = x + v_A t$. Set the two positions equal to each other and solve for the time of collision. If this time is negative or imaginary, then there will be no collision.

$$x_1 = x_2 \rightarrow v_M t + \frac{1}{2} a t^2 = x + v_A t \rightarrow \frac{1}{2} a t^2 + (v_M - v_A) t - x = 0$$

$$t = \frac{(v_A - v_M) \pm \sqrt{(v_M - v_A)^2 - 4 \frac{1}{2} a (-x)}}{2 \frac{1}{2} a}$$

$$\text{No collision: } (v_M - v_A)^2 - 4 \frac{1}{2} a (-x) < 0 \rightarrow x > \frac{(v_M - v_A)^2}{-2a}$$

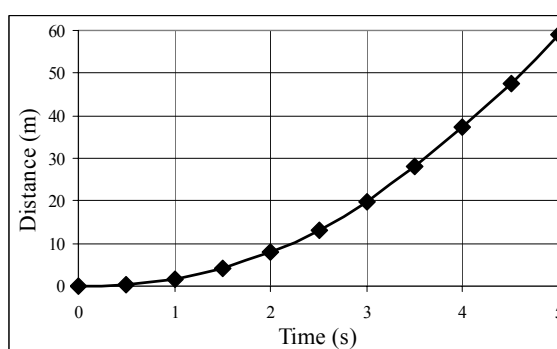
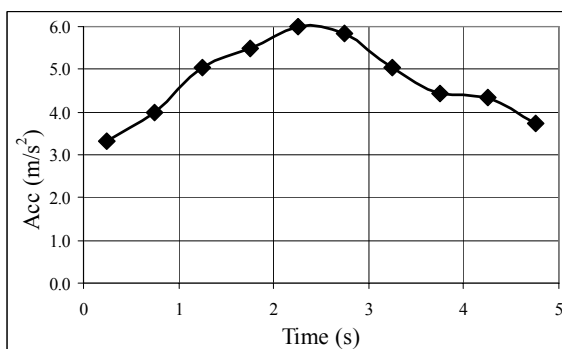
95. The velocities were changed from km/h to m/s by multiplying the conversion factor that 1 km/hr = 1/3.6 m/s.

(a) The average acceleration for each interval is calculated by $a = \Delta v / \Delta t$, and taken to be the acceleration at the midpoint of the time interval. In the spreadsheet, $a_{n+\frac{1}{2}} = \frac{v_{n+1} - v_n}{t_{n+1} - t_n}$. The accelerations are shown in the table below.

(b) The position at the end of each interval is calculated by $x_{n+1} = x_n + \frac{1}{2} (v_n + v_{n+1}) (t_{n+1} - t_n)$. This can also be represented as $x = x_0 + \bar{v} \Delta t$. These are shown in the table below.

t (s)	v (km/h)	v (m/s)	t (s)	a (m/s ²)	t (s)	x (m)
0.0	0.0	0.0	0.25	3.33	0.0	0.00
0.5	6.0	1.7	0.75	4.00	0.5	0.42
1.0	13.2	3.7	1.25	5.06	1.0	1.75
1.5	22.3	6.2	1.75	5.50	1.5	4.22
2.0	32.2	8.9	2.25	6.00	2.0	8.00
2.5	43.0	11.9	2.75	5.83	2.5	13.22
3.0	53.5	14.9	3.25	5.06	3.0	19.92
3.5	62.6	17.4	3.75	4.44	3.5	27.99
4.0	70.6	19.6	4.25	4.33	4.0	37.24
4.5	78.4	21.8	4.75	3.72	4.5	47.58
5.0	85.1	23.6			5.0	58.94

(c) The graphs are shown below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.95c."



96. For this problem, a spreadsheet was designed. The columns of the spreadsheet are time, acceleration, velocity, and displacement. The time starts at 0 and with each interval is incremented by 1.00 s. The acceleration at each time is from the data given in the problem. The velocity at each time is found by multiplying the average of the accelerations at the current time and the previous time, by the time interval, and then adding that to the previous velocity. Thus

$v_{n+1} = v_n + \frac{1}{2}(a_n + a_{n+1})(t_{n+1} - t_n)$. The displacement from the starting position at each time interval is calculated by a constant acceleration model, where the acceleration is as given above. Thus the positions is calculated as follows.

$$x_{n+1} = x_n + v_n(t_{n+1} - t_n) + \frac{1}{2}[\frac{1}{2}(a_n + a_{n+1})](t_{n+1} - t_n)^2$$

The table of values is reproduced here.

(a) $v(17.00) = 30.3 \text{ m/s}$

(b) $x(17.00) = 305 \text{ m}$

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH02.XLS,” on tab “Problem 2.96.”

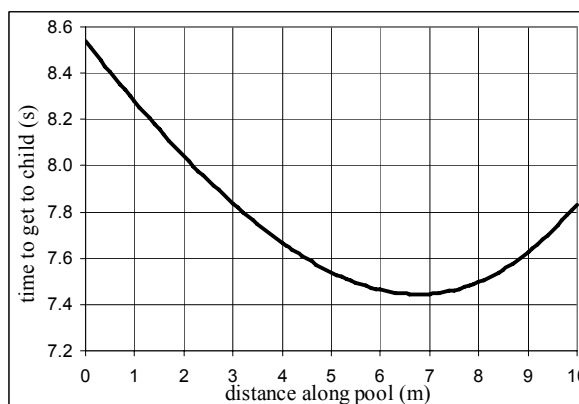
t (s)	a (m/s ²)	v (m/s)	x (m)
0.0	1.25	0.0	0
1.0	1.58	1.4	1
2.0	1.96	3.2	3
3.0	2.40	5.4	7
4.0	2.66	7.9	14
5.0	2.70	10.6	23
6.0	2.74	13.3	35
7.0	2.72	16.0	50
8.0	2.60	18.7	67
9.0	2.30	21.1	87
10.0	2.04	23.3	109
11.0	1.76	25.2	133
12.0	1.41	26.8	159
13.0	1.09	28.0	187
14.0	0.86	29.0	215
15.0	0.51	29.7	245
16.0	0.28	30.1	275
17.0	0.10	30.3	305

97. (a) For each segment of the path, the time is given by the distance divided by the speed.

$$t = t_{\text{land}} + t_{\text{pool}} = \frac{d_{\text{land}}}{v_{\text{land}}} + \frac{d_{\text{pool}}}{v_{\text{pool}}}$$

$$= \frac{x}{v_R} + \frac{\sqrt{D^2 + (d - x)^2}}{v_S}$$

- (b) The graph is shown here. The minimum time occurs at a distance along the pool of about $x = 6.8 \text{ m}$.



An analytic differentiation to solve for the minimum point gives $x = 6.76 \text{ m}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH02.XLS,” on tab “Problem 2.97b.”