

## Chapter 2

1. For a particle Newton's second law says  $\vec{F} = m\vec{a} = m \left[ \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k} \right]$ .

Take the second derivative of each of the expressions in Equation (2.1):

$$\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2} \quad \frac{d^2y'}{dt^2} = \frac{d^2y}{dt^2} \quad \frac{d^2z'}{dt^2} = \frac{d^2z}{dt^2}. \text{ Substitution into the previous equation gives}$$

$$\vec{F} = m\vec{a} = m \left[ \frac{d^2x'}{dt^2} \hat{i} + \frac{d^2y'}{dt^2} \hat{j} + \frac{d^2z'}{dt^2} \hat{k} \right] = \vec{F}'.$$

2. From Equation (2.1)  $\vec{p} = m \left[ \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} \right]$ .

$$\text{In a Galilean transformation } \frac{dx'}{dt} = \frac{dx}{dt} - v \quad \frac{dy'}{dt} = \frac{dy}{dt} \quad \frac{dz'}{dt} = \frac{dz}{dt}.$$

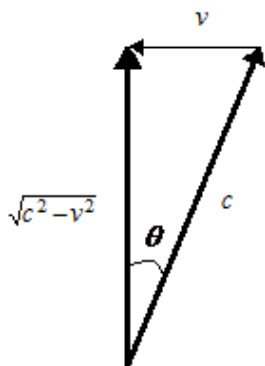
$$\text{Substitution into Equation (2.1) gives } \vec{p} = m \left[ \frac{dx'}{dt} + v \right] \hat{i} + \frac{dy'}{dt} \hat{j} + \frac{dz'}{dt} \hat{k} \neq \vec{p}'.$$

However, because  $\vec{p}' = m \left[ \frac{dx'}{dt} \hat{i} + \frac{dy'}{dt} \hat{j} + \frac{dz'}{dt} \hat{k} \right]$  the same form is clearly retained, given

$$\text{the velocity transformation } \frac{dx'}{dt} = \frac{dx}{dt} - v.$$

3. Using the vector triangle shown, the speed of light coming toward the mirror is  $\sqrt{c^2 - v^2}$  and the same on the return trip. Therefore the total time is  $t_2 = \frac{\text{distance}}{\text{speed}} = \frac{2\ell_2}{\sqrt{c^2 - v^2}}.$

Notice that  $\sin \theta = \frac{v}{c}$ , so  $\theta = \sin^{-1} \left( \frac{v}{c} \right).$



4. As in Problem 3,  $\sin \theta = v_1 / v_2$ , so  $\theta = \sin^{-1}(v_1 / v_2) = \sin^{-1}\left[\frac{0.350 \text{ m/s}}{1.25 \text{ m/s}}\right] = 16.3^\circ$  and

$$v = \sqrt{v_2^2 - v_1^2} = \sqrt{(1.25 \text{ m/s})^2 - (0.35 \text{ m/s})^2} = 1.20 \text{ m/s}.$$

5. When the apparatus is rotated by  $90^\circ$ , the situation is equivalent, except that we have effectively interchanged  $\ell_1$  and  $\ell_2$ . Interchanging  $\ell_1$  and  $\ell_2$  in Equation (2.3) leads to Equation (2.4).
6. Let  $n$  = the number of fringes shifted; then  $n = \frac{\Delta d}{\lambda}$ . Because  $\Delta d = c(\Delta t' - \Delta t)$ , we have

$$n = \frac{c(\Delta t' - \Delta t)}{\lambda} = \frac{v^2(\ell_1 + \ell_2)}{c^2 \lambda}. \text{ Solving for } v \text{ and noting that } \ell_1 + \ell_2 = 22 \text{ m,}$$

$$v = c \sqrt{\frac{n\lambda}{\ell_1 + \ell_2}} = (3.00 \times 10^8 \text{ m/s}) \sqrt{\frac{(0.005)(589 \times 10^{-9} \text{ m})}{22 \text{ m}}} = 3.47 \text{ km/s}.$$

7. Letting  $\ell_1 \rightarrow \ell_1 \sqrt{1 - \beta^2}$  (where  $\beta = v/c$ ) the text equation (not currently numbered) for  $t_1$  becomes

$$t_1 = \frac{2\ell_1 \sqrt{1 - \beta^2}}{c(1 - \beta)} = \frac{2\ell_1}{c\sqrt{1 - \beta^2}}$$

which is identical to  $t_2$  when  $\ell_1 = \ell_2$  so  $\Delta t = 0$  as required.

8. Since the Lorentz transformations depend on  $c$  (and the fact that  $c$  is the same constant for all inertial frames), different values of  $c$  would necessarily lead two observers to different conclusions about the order or positions of two spacetime events, in violation of postulate 1.

9. Let an observer in  $K$  send a light signal along the  $+x$ -axis with speed  $c$ . According to the Galilean transformations, an observer in  $K'$  measures the speed of the signal to be

$$\frac{dx'}{dt} = \frac{dx}{dt} - v = c - v. \text{ Therefore the speed of light cannot be constant under the Galilean transformations.}$$

10. From the Principle of Relativity, we know the correct transformation must be of the form (assuming  $y = y'$  and  $z = z'$ ):

$$x = ax' + bt'; \quad x' = ax - bt.$$

The spherical wave front equations (2.9a) and (2.9b) give us:

$$ct = (ac + b)t'; \quad ct' = (ac - b)t.$$

Solve the second wave front equation for  $t'$  and substitute into the first:

$$ct = \left[ \frac{(ac+b)(ac-b)t}{c} \right] \text{ or } c^2 = (ac+b)(ac-b) = a^2c^2 - b^2.$$

Now  $v$  is the speed of the origin of the  $x'$ -axis. We can find that speed by setting  $x' = 0$  which gives  $0 = ax - bt$ , or  $v = x/t = b/a$ , or equivalently  $b = av$ . Substituting this into the equation above for  $c^2$  yields  $c^2 = a^2c^2 - a^2v^2 = a^2(c^2 - v^2)$ . Solving for  $a$ :

$$a = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma.$$

This expression, along with  $b = av$ , can be substituted into the original expressions for  $x$  and  $x'$  to obtain:

$$x = \gamma(x' + vt'); \quad x' = \gamma(x - vt)$$

which in turn can be solved for  $t$  and  $t'$  to complete the transformation.

11. When  $v \ll c$  we find  $1 - \beta^2 \rightarrow 1$ , so:

$$x' = \frac{x - \beta ct}{\sqrt{1 - \beta^2}} \rightarrow x - \beta ct = x - vt;$$

$$t' = \frac{t - \beta x/c}{\sqrt{1 - \beta^2}} \rightarrow t - \beta x/c \approx t;$$

$$x = \frac{x' + \beta ct'}{\sqrt{1 - \beta^2}} \rightarrow x' + \beta ct' = x' + vt';$$

$$t = \frac{t' + \beta x'/c}{\sqrt{1 - \beta^2}} \rightarrow t' + \beta x'/c \approx t'.$$

12. (a) First we convert to SI units:  $95 \text{ km/h} = 26.39 \text{ m/s}$ , so

$$\beta = v/c = (26.39 \text{ m/s}) / (3.00 \times 10^8 \text{ m/s}) = 8.8 \times 10^{-8}$$

$$(b) \beta = v/c = (240 \text{ m/s}) / (3.00 \times 10^8 \text{ m/s}) = 8.0 \times 10^{-7}$$

$$(c) v = 2.3 v_{\text{sound}} = (2.3 \times 330 \text{ m/s}) \text{ so}$$

$$\beta = v/c = (2.3 \times 330 \text{ m/s}) / (3.00 \times 10^8 \text{ m/s}) = 2.5 \times 10^{-6}$$

(d) Converting to SI units,  $27,000 \text{ km/h} = 7500 \text{ m/s}$ , so

$$\beta = v/c = (7500 \text{ m/s}) / (3.00 \times 10^8 \text{ m/s}) = 2.5 \times 10^{-5}$$

$$(e) (25 \text{ cm}) / (2 \text{ ns}) = 1.25 \times 10^8 \text{ m/s so } \beta = v/c = (1.25 \times 10^8 \text{ m/s}) / (3.00 \times 10^8 \text{ m/s}) = 0.42$$

$$(f) (1 \times 10^{-14} \text{ m}) / (0.35 \times 10^{-22} \text{ s}) = 2.857 \times 10^8 \text{ m/s , so}$$

$$\beta = v/c = (2.857 \times 10^8 \text{ m/s}) / (3.00 \times 10^8 \text{ m/s}) = 0.95$$

13. From the Lorentz transformations  $\Delta t' = \gamma [\Delta t - v\Delta x / c^2]$ . But  $\Delta t' = 0$  in this case, so solving for  $v$  we find  $v = c^2 \Delta t / \Delta x$ . Inserting the values  $\Delta t = t_2 - t_1 = -a / 2c$  and  $\Delta x = x_2 - x_1 = a$ , we find  $v = \frac{c^2 (-a / 2c)}{a} = -c / 2$ . We conclude that the frame  $K'$  travels at a speed  $c/2$  in the  $-x$ -direction. Note that there is no motion in the transverse direction.

14. Try setting  $\Delta x' = 0 = \gamma (\Delta x - v\Delta t)$ . Thus  $0 = \Delta x - v\Delta t = a + va / 2c$ . Solving for  $v$  we find  $v = -2c$ , which is impossible. There is no such frame  $K'$ .

15. For the smaller values of  $\beta$  we use the binomial expansion  $\gamma = (1 - \beta^2)^{-1/2} \approx 1 + \beta^2 / 2$ .

(a)  $\gamma \approx 1 + \beta^2 / 2 = 1 + 3.87 \times 10^{-15}$

(b)  $\gamma \approx 1 + \beta^2 / 2 = 1 + 3.2 \times 10^{-13}$

(c)  $\gamma \approx 1 + \beta^2 / 2 = 1 + 3.1 \times 10^{-12}$

(d)  $\gamma \approx 1 + \beta^2 / 2 = 1 + 3.1 \times 10^{-10}$

(e)  $\gamma = (1 - \beta^2)^{-1/2} = (1 - 0.42^2)^{-1/2} = 1.10$

(f)  $\gamma = (1 - \beta^2)^{-1/2} = (1 - 0.95^2)^{-1/2} = 3.20$

16. There is no motion in the transverse direction, so  $y = z = 3.5$  m.

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - 0.8^2}} = 5/3$$

$$x = \gamma (x' + vt') = \frac{5}{3} (2\text{ m} + 0.8c(0)) = 10/3 \text{ m}$$

$$t = \gamma (t' + vx' / c^2) = \frac{5}{3} (0 + (0.8c)(2 \text{ m}) / c^2) = 8.9 \times 10^{-9} \text{ s}$$

17. (a)  $t = \frac{\sqrt{x^2 + y^2 + z^2}}{c} = \frac{\sqrt{(3 \text{ m})^2 + (5 \text{ m})^2 + (10 \text{ m})^2}}{3.00 \times 10^8 \text{ m/s}} = 3.86 \times 10^{-8} \text{ s}$

(b) With  $\beta = 0.8$  we find  $\gamma = 5/3$ . Then  $y' = y = 5$  m,  $z' = z = 10$  m,

$$x' = \gamma (x - vt) = \frac{5}{3} [3 \text{ m} - (2.40 \times 10^8 \text{ m/s})(3.86 \times 10^{-8} \text{ s})] = -10.4 \text{ m}$$

$$t' = \gamma (t - vx / c^2) = \frac{5}{3} [(3.86 \times 10^{-8} \text{ s}) - (2.40 \times 10^8 \text{ m/s})(3 \text{ m}) / (3.00 \times 10^8 \text{ m/s})^2] = 51.0 \text{ ns}$$

(c)  $\frac{\sqrt{x'^2 + y'^2 + z'^2}}{t'} = \frac{\sqrt{(-10.4 \text{ m})^2 + (5 \text{ m})^2 + (10 \text{ m})^2}}{51.0 \times 10^{-9} \text{ s}} = 2.994 \times 10^8 \text{ m/s}$  which equals  $c$  to within rounding errors.

18. At the point of reflection the light has traveled a distance  $L + v\Delta t_1 = c\Delta t_1$ . On the return trip it travels  $L - v\Delta t_2 = c\Delta t_2$ . Then the total time is  $\Delta t = \Delta t_1 + \Delta t_2 = \frac{2Lc}{c^2 - v^2} = \frac{2L/c}{1 - v^2/c^2}$ .

But from time dilation we know (with  $\Delta t' = \text{proper time} = 2L_0/c$ ) that

$$\Delta t = \gamma \Delta t' = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}}. \text{ Comparing these two results for } \Delta t \text{ we get } \frac{2L/c}{1 - v^2/c^2} = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}}$$

which reduces to  $L = L_0 \sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma}$ . This is Equation (2.21).

19. (a) With a contraction of 1%,  $L/L_0 = 0.99 = \sqrt{1 - v^2/c^2}$ . Thus  $1 - \beta^2 = (0.99)^2 = 0.9801$ .

Solving for  $\beta$ , we find  $\beta = 0.14$  or  $v = 0.14c$ .

(b) The time for the trip in the Earth-based frame is

$$\Delta t = \frac{d}{v} = \frac{5.00 \times 10^6 \text{ m}}{0.14 \times 3.00 \times 10^8 \text{ m/s}} = 1.19 \times 10^{-1} \text{ s}. \text{ With the relativistic factor } \gamma = 1.01$$

(corresponding to a 1% shortening of the ship's length), the elapsed time on the rocket ship is 1% less than the Earth-based time, or a difference of

$$(0.01)1.2 \times 10^{-1} \text{ s} = 1.2 \times 10^{-3} \text{ s}.$$

20. The round-trip distance is  $d = 40 \text{ ly}$ . Assume the same constant speed  $v = \beta c$  for the entire round trip. In the rocket's reference frame the distance is only  $d' = d\sqrt{1 - \beta^2}$ . Then

$$\text{in the rocket's frame of reference } v = \frac{\text{distance}}{\text{time}} = \frac{d'}{40 \text{ y}} = \frac{40 \text{ ly } \sqrt{1 - \beta^2}}{40 \text{ y}} = c\sqrt{1 - \beta^2}.$$

$$\text{Rearranging } \beta = \frac{v}{c} = \sqrt{1 - \beta^2}. \text{ Solving for } \beta \text{ we find } \beta = \sqrt{0.5}, \text{ or } v = \sqrt{0.5}c \approx 0.71c.$$

$$\text{To find the elapsed time } t \text{ on Earth, we know } t' = 40 \text{ y, so } t = \gamma t' = \frac{1}{\sqrt{1 - \beta^2}} 40 \text{ y} = 56.6 \text{ y}.$$

21. In the muon's frame  $T_0 = 2.2 \text{ } \mu\text{s}$ . In the lab frame the time is longer; see Equation (2.19):

$$T' = \gamma T_0. \text{ In the lab the distance traveled is } 9.5 \text{ cm} = vT' = v\gamma T_0 = \beta c\gamma T_0, \text{ since } v = \beta c.$$

Therefore  $\beta = \frac{9.5 \text{ cm}(\sqrt{1-\beta^2})}{cT_0}$ , so  $\beta = \frac{v}{c} = \frac{9.5 \text{ cm}(\sqrt{1-\beta^2})}{c(2.2\mu\text{s})}$ . Now all quantities are known except  $\beta$ . Solving for  $\beta$  we find  $\beta = 1.4 \times 10^{-4}$  or  $v = 1.4 \times 10^{-4} c$ .

22. Converting the speed to m/s we find  $25,000 \text{ mi/h} = 11,176 \text{ m/s}$ . From tables the distance is  $3.84 \times 10^8 \text{ m}$ . In the earth's frame of reference the time is the distance divided by

speed, or  $t = \frac{d}{v} = \frac{3.84 \times 10^8 \text{ m}}{11,176 \text{ m/s}} = 34,359 \text{ s}$ . In the astronauts' frame the time elapsed is

$$t' = t/\gamma = t\sqrt{1-\beta^2}. \text{ The time difference is } \Delta t = t - t' = t - t\sqrt{1-\beta^2} = t[1 - \sqrt{1-\beta^2}].$$

Evaluating numerically  $\Delta t = 34,359 \text{ s} \left[ 1 - \sqrt{1 - \left( \frac{11,176 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2} \right] = 2.4 \times 10^{-5} \text{ s}$ .

23.  $T' = \gamma T_0$ , so we know that  $\gamma = 5/3 = \frac{1}{\sqrt{1-v^2/c^2}}$ . Solving for  $v$  we find  $v = 4c/5$ .

24.  $L = L_0/\gamma$  so clearly  $\gamma = 2$  in this case. Thus  $2 = \frac{1}{\sqrt{1-v^2/c^2}}$  and solving for  $v$  we find

$$v = \frac{\sqrt{3}c}{2}.$$

25. The clocks' rates differ by a factor of  $\gamma = 1/\sqrt{1-v^2/c^2}$ . Because  $\beta$  is very small we will use the binomial theorem approximation  $\gamma \approx 1 + \beta^2/2$ . Then the time difference is  $\Delta t = t - t' = t - \gamma t = t(\gamma - 1)$ . Using  $\gamma - 1 \approx \beta^2/2$  and the fact that the time for the trip equals distance divided by speed,

$$\Delta t = t(\beta^2/2) = \frac{8 \times 10^6 \text{ m}}{375 \text{ m/s}} \frac{\left( \frac{375 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2}{2}$$

$$\Delta t = 1.67 \times 10^{-8} \text{ s} = 16.7 \text{ ns}.$$

26. (a)  $L' = L/\gamma = L\sqrt{1-v^2/c^2} = (3.58 \times 10^4 \text{ km})\sqrt{1-0.94^2} = 1.22 \times 10^4 \text{ km}$

$$(b) \text{ Earth's frame: } t = L/v = \frac{3.58 \times 10^7 \text{ m}}{(0.94)(3.00 \times 10^8 \text{ m/s})} = 0.127 \text{ s}$$

$$\text{Golf ball's frame: } t' = t/\gamma = 0.127 \text{ s} \sqrt{1-0.94^2} = 0.0433 \text{ s}$$

27. Spacetime invariant (see Section 2.9):  $c^2 \Delta t^2 - \Delta x^2 = c^2 \Delta t'^2 - \Delta x'^2$ . We know  $\Delta x = 4$  km,

$$\Delta t = 0, \text{ and } \Delta x' = 5 \text{ km. Thus } \Delta t'^2 = \frac{\Delta x'^2 - \Delta x^2}{c^2} = \frac{(5000 \text{ m})^2 - (4000 \text{ m})^2}{(3.00 \times 10^8 \text{ m/s})^2} = 1.0 \times 10^{-10} \text{ s}^2$$

and  $\Delta t' = 1.0 \times 10^{-5} \text{ s}$ .

28. (a) Converting  $v = 120 \text{ km/h} = 33.3 \text{ m/s}$ . Now with  $c = 100 \text{ m/s}$ , we have

$$\beta = v/c = 0.333 \text{ and } \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.333^2}} = 1.061. \text{ We conclude that the moving}$$

person ages 6.1% slower.

$$(b) L' = L/\gamma = (1 \text{ m})/(1.061) = 0.942 \text{ m.}$$

29. Converting  $v = 300 \text{ km/h} = 83.3 \text{ m/s}$ . Now with  $c = 100 \text{ m/s}$ , we have  $\beta = v/c = 0.833$

$$\text{and } \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.833^2}} = 1.81. \text{ So the length is } L = L_0/\gamma = 40/1.81 = 22.1 \text{ m.}$$

30. Let subscript 1 refer to firing and subscript 2 to striking the target. Therefore we can see that  $x_1 = 1 \text{ m}$ ,  $x_2 = 121 \text{ m}$ , and  $t_1 = 3 \text{ ns}$ .

$$t_2 = t_1 + \frac{\text{distance}}{\text{speed}} = 3 \text{ ns} + \frac{120 \text{ m}}{0.98c} = 3 \text{ ns} + 408 \text{ ns} = 411 \text{ ns.}$$

To find the four primed quantities we can use the Lorentz transformations with the

known values of  $x_1$ ,  $x_2$ ,  $t_1$ , and  $t_2$ . Note that with  $v = 0.8c$ ,  $\gamma = \sqrt{1-v^2/c^2} = 5/3$ .

$$t'_1 = \gamma(t_1 - vx_1/c^2) = 0.56 \text{ ns}$$

$$t'_2 = \gamma(t_2 - vx_2/c^2) = 147 \text{ ns}$$

$$x'_1 = \gamma(x_1 - vt_1) = 0.47 \text{ m}$$

$$x'_2 = \gamma(x_2 - vt_2) = 37.3 \text{ m}$$

31. Start from the formula for velocity addition, Equation (2.23a):  $u_x = \frac{u'_x + v}{1 + vu'_x/c^2}$ .

$$(a) u_x = \frac{0.62c + 0.84c}{1 + (0.62c)(0.84c)/c^2} = \frac{1.46c}{1.52} = 0.96c$$

$$(b) u_x = \frac{-0.62c + 0.84c}{1 + (-0.62c)(0.84c)/c^2} = \frac{0.22c}{0.48} = 0.46c$$

32. Velocity addition, Equation (2.24):  $u'_x = \frac{u_x - v}{1 - vu_x/c^2}$  with  $v = -0.8c$  and  $u_x = 0.8c$ .

$$u'_x = \frac{0.8c - (-0.8c)}{1 - (-0.8c)(0.8c)/c^2} = \frac{1.6c}{1.64} = 0.976c$$

33. Conversion:  $110 \text{ km/h} = 30.556 \text{ m/s}$  and  $140 \text{ km/h} = 38.889 \text{ m/s}$ . Let  $u_x = 30.556 \text{ m/s}$  and  $v = -38.889 \text{ m/s}$ . Our premise is that  $c = 100 \text{ m/s}$ . Then by velocity addition,

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} = \frac{30.556 \text{ m/s} - (-38.889 \text{ m/s})}{1 - (-38.889 \text{ m/s})(30.556 \text{ m/s})/(100 \text{ m/s})^2} = 62.1 \text{ m/s.}$$

By symmetry each observer sees the other one traveling at the same speed.

34. From Example 2.5 we have  $u = \frac{c}{n} \left[ \frac{1 + nv/c}{1 + v/nc} \right]$ . For light traveling in opposite directions

$$\Delta u = \frac{c}{n} \left[ \frac{1 + nv/c}{1 + v/nc} - \frac{1 - nv/c}{1 - v/nc} \right].$$

Because  $v/c$  is very small, use the binomial expansion:  $\frac{1 + nv/c}{1 + v/nc} = (1 + nv/c)(1 + v/nc)^{-1} \approx (1 + nv/c)(1 - v/nc) \approx 1 + nv/c - v/nc$ , where we

have dropped terms of order  $v^2/c^2$ . Similarly  $\frac{1 - nv/c}{1 - v/nc} \approx 1 - nv/c + v/nc$ . Thus

$$\Delta u \approx \frac{c}{n} \left[ (1 + nv/c - v/nc) - (1 - nv/c + v/nc) \right] = \frac{2v}{n} (1 - 1/n^2) = 2v \left( 1 - \frac{1}{n^2} \right).$$

Evaluating numerically we find  $\Delta u \approx 2(5 \text{ m/s}) \left( 1 - \frac{1}{1.33^2} \right) = 4.35 \text{ m/s}$ .

35. Clearly the speed of B is just  $0.60c$ . To find the speed of C use  $u_x = 0.60c$  and

$$v = -0.60c: \quad u'_x = \frac{u_x - v}{1 - vu_x/c^2} = \frac{0.60c - (-0.60c)}{1 - (-0.60c)(0.60c)/c^2} = 0.88c.$$

36. We can ignore the 400 km, which is small compared with the Earth-to-moon distance  $3.84 \times 10^8 \text{ m}$ . The rotation rate is  $\omega = 2\pi \text{ rad} \times 100 \text{ s}^{-1} = 2\pi \times 10^2 \text{ rad/s}$ . Then the speed across the moon's surface is  $v = \omega R = (2\pi \times 10^2 \text{ rad/s})(3.84 \times 10^8 \text{ m}) = 2.41 \times 10^{11} \text{ m/s}$ .

37. Classical:  $t = \frac{4205 \text{ m}}{0.98c} = 1.43 \times 10^{-5} \text{ s}$ . Then  $N = N_0 \exp \left[ \frac{-(\ln 2)t}{t_{1/2}} \right] = 14.6$

or about 15 muons.

$$\text{Relativistic: } t' = t/\gamma = \frac{1.43 \times 10^{-5} \text{ s}}{5} = 2.86 \times 10^{-6} \text{ s so}$$

$$N = N_0 \exp \left[ \frac{-(\ln 2)t}{t_{1/2}} \right] = 2710 \text{ muons.}$$

Because of the exponential nature of the decay curve, a factor of five (shorter) in time results in many more muons surviving.

38. The circumference of the fixed point's rotational path is  $2\pi R_E \cos(39^\circ)$ , where  $R_E = \text{Earth's radius} = 6378 \text{ km}$ . Thus the circumference of the path is 31,143 km. The



rotational speed of that point is  $v = (31,143 \text{ km}) / 24 \text{ h} = 1298 \text{ km/h} = 360.5 \text{ m/s}$ . The

observatory clock runs slow by a factor of  $\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 1 + \beta^2 / 2 = 1 + 7.22 \times 10^{-13}$ . In

41.2 h the observatory clock is slow by  $(41.2 \text{ h})(7.22 \times 10^{-13}) = 2.9746 \times 10^{-11} \text{ h} = 107 \text{ ns}$ .

In 48.6 h it is slow by  $(48.6 \text{ h})(7.22 \times 10^{-13}) = 3.5089 \times 10^{-11} \text{ h} = 126 \text{ ns}$ . The Eastward-moving clock has a ground speed of  $31,143 \text{ km} / 41.2 \text{ h} = 755.9 \text{ km/h} = 210.0 \text{ m/s}$  and thus has a net speed of  $210.0 \text{ m/s} + 360.5 \text{ m/s} = 570.5 \text{ m/s}$ . For this clock

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 1 + \beta^2 / 2 = 1 + 1.81 \times 10^{-12}$  and in 41.2 hours it runs slow by

$(41.2 \text{ h})(1.81 \times 10^{-12}) = 7.4572 \times 10^{-11} \text{ h} = 268 \text{ ns}$ . The Westward-moving clock has a ground speed of  $31,143 \text{ km} / 48.6 \text{ h} = 640.8 \text{ km/h} = 178.0 \text{ m/s}$  and thus has a net speed of  $360.5 \text{ m/s} - 178.0 \text{ m/s} = 182.5 \text{ m/s}$ . For this clock

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 1 + \beta^2 / 2 = 1 + 1.85 \times 10^{-13}$  and in 48.6 hours it runs slow by

$(48.6 \text{ h})(1.85 \times 10^{-13}) = 8.991 \times 10^{-12} \text{ h} = 32 \text{ ns}$ . So our prediction is that the Eastward-moving clock is off by  $107 \text{ ns} - 269 \text{ ns} = -162 \text{ ns}$ , while the Westward-moving clock is off by  $126 \text{ ns} - 32 \text{ ns} = 94 \text{ ns}$ . These results are correct for special relativity but do not reconcile with those in the table in the text, because general relativistic effects are of the same order of magnitude.

39. The derivations of Equations (2.31) and (2.32) in the beginning of Section 2.10 will suffice. Mary receives signals at a rate  $f'$  for  $t'_1$  and a rate  $f''$  for  $t'_2$ . Frank receives signals at a rate  $f'$  for  $t_1$  and a rate  $f''$  for  $t_2$ .

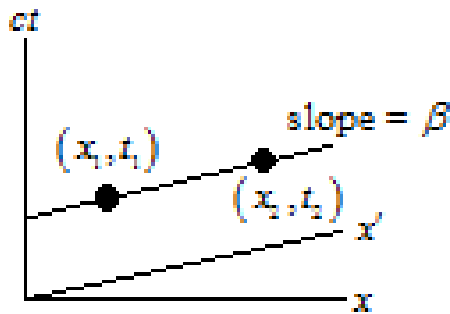
40.  $T = t_1 + t_2 = \frac{L}{v} + \frac{L}{c} + \frac{L}{v} - \frac{L}{c} = \frac{2L}{v}$ ; Frank sends signals at rate  $f$ , so Mary receives  $f T = 2f L / v$  signals.

$T' = t'_1 + t'_2 = \frac{2L}{\gamma v}$ ; Mary sends signals at rate  $f$ , so Frank receives  $f T' = 2f L / \gamma v$  signals.

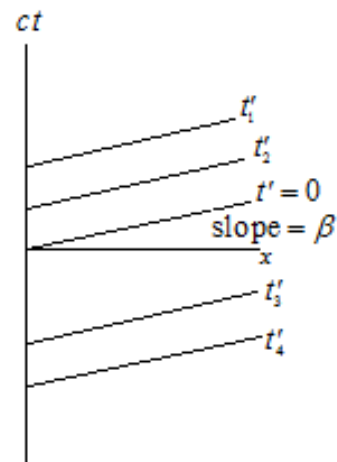
41.  $s^2 = x^2 + y^2 + z^2 - c^2 t^2$ ; Using the Lorentz transformation

$$\begin{aligned} s^2 &= \gamma^2 (x' + vt')^2 + y'^2 + z'^2 - c^2 \gamma^2 (t' + vx' / c^2)^2 \\ &= x'^2 \gamma^2 (1 - v^2 / c^2) + y'^2 + z'^2 - c^2 t'^2 \gamma^2 (1 - v^2 / c^2) \\ &= x'^2 + y'^2 + z'^2 - c^2 t'^2 = s'^2 \end{aligned}$$

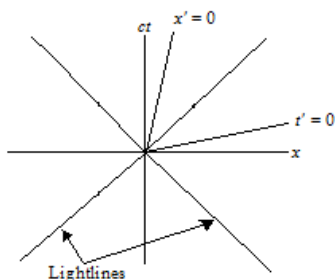
42. For a timelike interval  $\Delta s^2 < 0$  so  $\Delta x^2 < c^2 \Delta t^2$ . We will prove by contradiction. Suppose that there is a frame  $K'$  in which the two events were simultaneous, so that  $\Delta t' = 0$ . Then by the spacetime invariant  $\Delta x^2 - c^2 \Delta t^2 = \Delta x'^2 - c^2 \Delta t'^2 = \Delta x'^2$ . But because  $\Delta x^2 < c^2 \Delta t^2$ , this implies  $\Delta x'^2 < 0$  which is impossible because  $\Delta x'$  is real.
43. As in Problem 42, we know that for a spacelike interval  $\Delta s^2 > 0$  so  $\Delta x^2 > c^2 \Delta t^2$ . Then in a frame  $K'$  in which the two events occur in the same place,  $\Delta x' = 0$  and  $\Delta x^2 - c^2 \Delta t^2 = \Delta x'^2 - c^2 \Delta t'^2 = -c^2 \Delta t'^2$ . But because  $\Delta x^2 > c^2 \Delta t^2$  we have  $c^2 \Delta t'^2 < 0$ , which is impossible because  $\Delta t'$  is real.
44. In order for two events to be simultaneous in  $K'$ , the two events must lie along the  $x'$  axis, or along a line parallel to the  $x'$  axis. The slope of the  $x'$  axis is  $\beta = v/c$ , so  $v/c = \text{slope} = \frac{c \Delta t}{\Delta x}$ . Solving for  $v$ , we find  $v = c^2 \Delta t / \Delta x$ . Since the slope of the  $x'$  axis must be less than one, we see that  $\Delta x > c \Delta t$  so  $s^2 = \Delta x^2 - c^2 \Delta t^2 > 0$  is required.



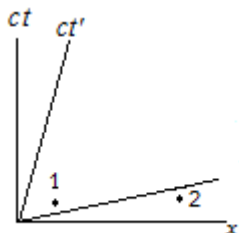
45. parts (a) and (b) To find the equation of the line use the Lorentz transformation. With  $t' = 0$  we have  $t' = 0 = \gamma(t - vx/c^2)$  or, rearranging,  $ct = vx/c = \beta x$ . Thus the graph of  $ct$  vs.  $x$  is a straight line with a slope  $\beta$ .
- (c) Now with  $t'$  constant, the Lorentz transformation gives  $t' = \gamma(t - vx/c^2)$ . Again we solve for  $ct$ :  $ct = \beta x + ct'/\gamma = \beta x + \text{constant}$ . This line is parallel to the  $t' = 0$  line we found earlier but shifted by the constant.
- (d) Here both the  $x'$  and  $ct'$  axes are shifted from their normal  $(x, ct)$  orientation and they are not perpendicular.



46. The diagram is shown here. Note that there is only one worldline for light, and it bisects both the  $x, ct$  axes and the  $x', ct'$  axes. The  $x'$  and  $ct'$  axes are not perpendicular. This can be seen as a result of the Lorentz transformations, since  $x' = 0$  defines the  $ct'$  axis and  $t' = 0$  defines the  $x'$  axis.



47. The diagram shows that the events A and B that occur at the same time in K occur at different times in K'.



48. The Doppler shift gives  $\lambda = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}}$ . With numerical values  $\lambda_0 = 650 \text{ nm}$  and

$\lambda = 540 \text{ nm}$ , solving this equation for  $\beta$  gives  $\beta = 0.183$ . The astronaut's speed is  $v = \beta c = 5.50 \times 10^7 \text{ m/s}$ . In addition to a red light violation, the astronaut gets a speeding ticket.

49. According to the fixed source (K) the signal and receiver move at speeds  $c$  and  $v$ , respectively, in opposite directions, so their relative speed is  $c + v$ . The time interval between receipt of signals is  $\Delta t = \lambda / (c + v) = 1 / f_0$ . By time dilation  $\Delta t' = \frac{\Delta t}{\gamma} = \frac{\lambda}{\gamma(c + v)}$ .

Using  $\lambda = c / \nu_0$  and  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$  we find  $\Delta t' = \frac{c\sqrt{1-v^2/c^2}}{f_0(c + v)} = \frac{\sqrt{1-\beta^2}}{f_0(1+\beta)}$  and

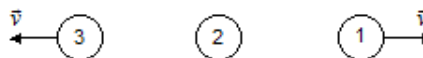
$$f' = \frac{1}{\Delta t'} = \frac{f_0(1+\beta)}{\sqrt{1-\beta^2}} = f_0 \sqrt{\frac{1+\beta}{1-\beta}}.$$

50. For a fixed source and moving receiver, the length of the wave train is  $cT + vT$ . Since  $n$  waves are emitted during time  $T$ ,  $\lambda = \frac{cT + vT}{n}$  and the frequency  $f = c / \lambda$  is  $f = \frac{cn}{cT + vT}$ .

As in the text  $n = f_0 T'_0$  and  $T'_0 = T / \gamma$ . Therefore  $f = \frac{cf_0 T / \gamma}{cT + vT} = \frac{f_0 \sqrt{1 - \beta^2}}{1 + \beta} = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}}$ .

51.  $f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}} = (1400 \text{ kHz}) \sqrt{\frac{1 - 0.95}{1 + 0.95}} = 224 \text{ kHz}$

52. The Doppler shift function  $f' = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}}$

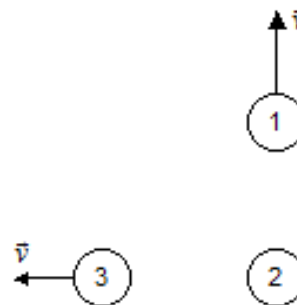


is the rate at which #1 and #2 receive signals from each other and the rate at which #2 and #3 receive signals from each other. But for signals between #1 and #3 the rate is

$$f'' = f' \sqrt{\frac{1 - \beta}{1 + \beta}} = f_0 \frac{1 - \beta}{1 + \beta}.$$

53. The Doppler shift function  $f' = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}}$

is the rate at which #1 and #2 receive signals from each other and the rate at which #2 and #3 receive signals from each other. As for #1 and #3 we will assume that these plumbing vans are non-relativistic ( $v \ll c$ ). Otherwise it would be necessary to use the velocity addition law and apply the transverse Doppler shift. From the figure we see



that  $f' = \frac{1}{t_0 + (t_2 - t_1)}$ . Now  $f_0 = 1 / t_0$  and

$$t_2 - t_1 = \frac{2x}{c} = \frac{2vt_0 \cos \theta}{c}. \text{ With an angle of } 45^\circ, \cos(45) = 1/\sqrt{2} \text{ and}$$

$$f' = \frac{1}{1/f_0 + (2v \cos \theta)/cf_0} = \frac{f_0}{1 + (2v \cos \theta)/c} = \frac{f_0}{1 + \sqrt{2}v/c}.$$

54. The Doppler shift to higher wavelengths is (with  $\lambda_0 = 589 \text{ nm}$ )  $\lambda = 700 \text{ nm} = \lambda_0 \sqrt{\frac{1 + \beta}{1 - \beta}}$ .

Solving for  $\beta$  we find  $\beta = 0.171$ . Then  $t = \frac{v}{a} = \frac{(0.171)(3.00 \times 10^8 \text{ m/s})}{29.4 \text{ m/s}^2} = 1.75 \times 10^6 \text{ s}$

which is 20.25 days. One problem with this analysis is that we have only computed the

time as measured by Earth. We are not prepared to handle the non-inertial frame of the spaceship.

55. Let the instantaneous momentum be in the  $x$ -direction and the force be in the  $y$ -direction.

Then  $d\vec{p} = \vec{F} dt = \gamma m d\vec{v}$  and  $d\vec{v}$  is also in the  $y$ -direction. So we have  $\vec{F} = \gamma m \frac{d\vec{v}}{dt} = \gamma m \vec{a}$ .

56. The magnitude of the centripetal force is  $\gamma m a = \gamma m \frac{v^2}{r}$  for circular motion. For a charged

particle  $F = qvB$ , so  $qvB = \gamma m \frac{v^2}{r}$  or, rearranging  $qBr = \gamma mv = p$ . Therefore

$$r = \frac{p}{qB}.$$

When the speed increases the momentum increases, and thus for a given value of  $B$  the radius must increase.

57.  $\vec{p} = \gamma m \vec{v} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$  and  $\vec{F} = \frac{d\vec{p}}{dt}$ . The momentum is the product of two factors that contain the velocity, so we apply the product rule for derivatives:

$$\begin{aligned} \vec{F} &= m \frac{d}{dt} \left[ \frac{m\vec{v}}{\sqrt{1-v^2/c^2}} \right] \\ &= m \left[ \frac{d\vec{v}/dt}{\sqrt{1-v^2/c^2}} + \vec{v} \frac{d}{dt} \left( \frac{1}{\sqrt{1-v^2/c^2}} \right) \right] \\ &= \gamma m \vec{a} + m\vec{v} \left( -\frac{1}{2} \right) \left( -\frac{2v}{c^2} \right) \gamma^3 \frac{dv}{dt} \\ &= \gamma m \vec{a} + \gamma^3 m \vec{a} \left( \frac{v^2}{c^2} \right) \\ &= \gamma^3 m \vec{a} \left[ 1 - \frac{v^2}{c^2} + \frac{v^2}{c^2} \right] \\ &= \gamma^3 m \vec{a} \end{aligned}$$

58. From the preceding problem  $F = \gamma^3 m a$ . We have  $a = 10^{19} \text{ m/s}^2$  and  $m = 1.67 \times 10^{-27} \text{ kg}$ .

$$(a) \quad \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.01^2}} = 1.00005$$

$$F = (1.00005)^3 (1.67 \times 10^{-27} \text{ kg}) (10^{19} \text{ m/s}^2) = 1.67 \times 10^{-8} \text{ N}$$

(b) As in (a)  $\gamma = 1.005$  and  $F = 1.70 \times 10^{-8} \text{ N}$

(c) As in (a)  $\gamma = 2.294$  and  $F = 2.02 \times 10^{-7} \text{ N}$

(d) As in (a)  $\gamma = 7.0888$  and  $F = 5.95 \times 10^{-6} \text{ N}$

$$59. p = \gamma mv \text{ with } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.92^2}} = 2.5516;$$

$$m = \frac{p}{\gamma v} = \frac{10^{-16} \text{ kg} \cdot \text{m/s}}{(2.5516)(0.92)(3.00 \times 10^8 \text{ m/s})} = 1.42 \times 10^{-25} \text{ kg}$$

$$60. \text{ The initial momentum is } p_0 = \gamma mv = \frac{1}{\sqrt{1 - (0.5)^2}} m(0.5c) = 0.57735mc.$$

$$(a) p/p_0 = 1.01$$

$$1.01 = \frac{\gamma mv}{0.57735mc}$$

$$\gamma v = (1.01)(0.57735c) = 0.58312c$$

$$\text{Substituting for } \gamma \text{ and solving for } v, v = \left[ \frac{1}{(.58312c)^2} + \frac{1}{c^2} \right]^{-1/2} = 0.504c.$$

$$(b) \text{ Similarly } v = \left[ \frac{1}{(.63509c)^2} + \frac{1}{c^2} \right]^{-1/2} = 0.536c$$

$$(c) \text{ Similarly } v = \left[ \frac{1}{(1.1547c)^2} + \frac{1}{c^2} \right]^{-1/2} = 0.756c$$

61. 6.3 GeV protons have  $K = 6.3 \times 10^3 \text{ MeV}$  and  $E = K + E_0 = 7238 \text{ MeV}$ . Then

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = 7177 \text{ MeV}/c. \text{ Converting to SI units}$$

$$p = 7177 \text{ MeV}/c \left( \frac{1.60 \times 10^{-13} \text{ J}}{\text{MeV}} \right) \left( \frac{c}{3.00 \times 10^8 \text{ m/s}} \right) = 3.83 \times 10^{-18} \text{ kg} \cdot \text{m/s}$$

$$\text{From Problem 56 we have } B = \frac{p}{qr} = \frac{3.83 \times 10^{-18} \text{ kg} \cdot \text{m/s}}{(1.60 \times 10^{-19} \text{ C})(15.2 \text{ m})} = 1.57 \text{ T}.$$

62. Initially Mary throws her ball with velocity (primes showing the measurements are in Mary's frame):  $u'_{M_x} = 0$   $u'_{M_y} = -u_0$ . After the elastic collision, the signs on the above expressions are reversed, so the change in momentum as measured by Mary is

$$\Delta p'_M = \frac{mu_0}{\sqrt{1 - u_0^2/c^2}} - \frac{-mu_0}{\sqrt{1 - u_0^2/c^2}} = \frac{2mu_0}{\sqrt{1 - u_0^2/c^2}}.$$

Now for Frank's ball, we know  $u_{F_x} = 0$  and  $u_{F_y} = u_0$ . The velocity transformations give

$$\text{for Frank's ball as measured by Mary: } u'_{F_x} = -v \quad u'_{F_y} = u_0 \sqrt{1 - v^2 / c^2}.$$

To find  $\gamma$  for Frank's ball, note that  $(u'_{F_x})^2 + (u'_{F_y})^2 = v^2 + u_0^2 (1 - v^2 / c^2)$ . Then

$$\gamma = \frac{1}{\sqrt{1 - u'^2_F / c^2}} = \frac{1}{\sqrt{1 - v^2 / c^2 - u_0^2 (1 - v^2 / c^2) / c^2}} = \frac{1}{\sqrt{(1 - u_0^2 / c^2)(1 - v^2 / c^2)}}. \text{ Using}$$

$p' = \gamma mu'$  along with the reversal of velocities in an elastic collision, we find

$$\begin{aligned} \Delta p'_F &= \gamma m(-u_0) \sqrt{1 - v^2 / c^2} - \gamma mu_0 \sqrt{1 - v^2 / c^2} = -2\gamma mu_0 \sqrt{1 - v^2 / c^2} \\ &= \frac{-2mu_0 \sqrt{1 - v^2 / c^2}}{\sqrt{(1 - u_0^2 / c^2)(1 - v^2 / c^2)}} = \frac{-2mu_0}{\sqrt{(1 - u_0^2 / c^2)}} \end{aligned}$$

$$\text{Finally } \Delta p' = \Delta p'_F + \Delta p'_M = \frac{-2mu_0 + 2mu_0}{\sqrt{(1 - u_0^2 / c^2)}} = 0 \text{ as required.}$$

63. To prove by contradiction, suppose that  $K = \frac{1}{2} \gamma mv^2$ . Then

$$K = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 = \frac{1}{2} \gamma mv^2. \text{ This implies } \gamma - 1 = v^2 / 2c^2, \text{ or}$$

$$\gamma = 1 + v^2 / 2c^2, \text{ which is clearly false.}$$

64. The source of the energy is the internal energy associated with the change of state, commonly called that latent heat of fusion  $L_f$ . Let  $m$  be the mass equivalent of 2 grams

$$\text{and } M \text{ be the mass of ice required. } m = \frac{E}{c^2} = \frac{L_f M}{c^2} \text{ Rearranging}$$

$$M = \frac{mc^2}{L_f} = \frac{(0.002 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2}{334 \times 10^3 \text{ J/kg}} = 5.39 \times 10^8 \text{ kg}$$

65. In general  $K = (\gamma - 1)mc^2$ , so  $\gamma = 1 + \frac{K}{mc^2}$ . For 9 GeV electrons:

$$\gamma = 1 + \frac{9000 \text{ MeV}}{0.511 \text{ MeV}} = 1.76 \times 10^4 \text{ Then from the definition of } \gamma \text{ we have}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.76 \times 10^4)^2}} = 1 - 1.6 \times 10^{-9}. \text{ Thus}$$

$$v = (1 - 1.6 \times 10^{-9})c = 0.9999999984c.$$

For 3.1 GeV positrons:  $\gamma = 1 + \frac{3100 \text{ MeV}}{0.511 \text{ MeV}} = 6068$  and  $\beta = \sqrt{1 - \frac{1}{(6068)^2}} = 1 - 1.4 \times 10^{-8}$ .

Thus  $v = (1 - 1.4 \times 10^{-8})c = 0.999999986c$ .

66. Note that the proton's mass is  $938 \text{ MeV}/c^2$ . In general  $K = (\gamma - 1)mc^2$ , so  $\gamma = 1 + \frac{K}{mc^2}$ .

Then from the definition of  $\gamma$  we have  $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$ . For the first section  $K = 0.750$

MeV, and  $\gamma = 1 + \frac{0.750 \text{ MeV}}{938 \text{ MeV}} = 1.00080$  with  $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \beta = \sqrt{1 - \frac{1}{(1.00080)^2}} = 0.040$ .

Thus  $v = 0.04c$  at the end of the first stage. For the other stages the computations are similar, and we tabulate the results:

$K \text{ (GeV)}$	$\gamma$	$\beta$
0.400	1.43	0.71
8	9.53	0.994
150	160.9	0.99998
1000	1067	0.9999996

$$67. (a) \quad p = \gamma mu = \frac{(511 \text{ keV}/c^2)(0.020c)}{\sqrt{1 - 0.020^2}} = 10.22 \text{ keV}/c;$$

$$E = \gamma mc^2 = \frac{(511 \text{ keV}/c^2)(c^2)}{\sqrt{1 - 0.02^2}} = 511.102 \text{ keV};$$

$$K = E - E_0 = 511.102 \text{ keV} - 511.00 \text{ keV} = 102 \text{ eV}$$

The results for (b) and (c) follow with similar computations and are tabulated:

$\beta$	$p \text{ (keV}/c)$	$E \text{ (keV)}$	$K \text{ (keV)}$
0.20	104.3	521.5	10.5
0.90	1055	1172	661

$$68. \quad E = 2E_0 = \gamma E_0 \text{ so } \gamma = 2. \text{ Then } \beta = \sqrt{1 - \frac{1}{\gamma^2}} = \frac{\sqrt{3}}{2} \text{ and } v = \frac{\sqrt{3}c}{2}.$$



69. For a constant force, work = change in kinetic energy =  $Fd = mc^2/4$ , because

$$25\% = 1/4. \quad d = \frac{mc^2}{4F} = \frac{(80 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2}{4(8 \text{ N})} = 2.25 \times 10^{17} \text{ m} = 23.8 \text{ ly}.$$

70.  $E = K + E_0 = 2E_0 + E_0 = 3E_0 = \gamma E_0$  so  $\gamma = 3$ . Then

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \beta = \sqrt{1 - \frac{1}{3^2}} = \frac{2\sqrt{2}}{3} = 0.943. \quad \text{Thus } v = 0.943c.$$

71. (a)  $E = K + E_0 = 0.1E_0 + E_0 = 1.1E_0 = \gamma E_0$ , so  $\gamma = 1.1$ . Then

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \beta = \sqrt{1 - \frac{1}{1.1^2}} = 0.417 \quad \text{and } v = 0.417c.$$

(b) As in (a)  $\gamma = 2$  and  $v = \sqrt{3}c/2$ .

(c) As in (a)  $\gamma = 11$  and  $v = 0.996c$ .

72.  $K = E - E_0 = \frac{E_0}{\sqrt{1 - \beta^2}} - E_0$  so  $K + E_0 = \frac{E_0}{\sqrt{1 - \beta^2}}$ . Rearranging  $1 - \beta^2 = \left(\frac{E_0}{K + E_0}\right)^2$  which

$$\text{can be written as } \beta^2 = 1 - \left(\frac{E_0}{K + E_0}\right)^2 \text{ or } \beta = \sqrt{1 - \left(\frac{E_0}{K + E_0}\right)^2}.$$

73. Using  $E = \gamma mc^2$  along with  $p = \gamma mv$  we see that  $\gamma = E/mc^2 = p/mv$ . Solving for  $v/c$  we find  $\beta = v/c = pc/E$ .

74. The speed is the same for protons, electrons, or any particle.

$$K = (\gamma - 1)mc^2 = 1.01 \left( \frac{1}{2}mv^2 \right) = 0.505mc^2\beta^2 \text{ so } \gamma - 1 = \frac{1}{\sqrt{1 - \beta^2}} - 1 = 0.505\beta^2.$$

Rearranging and solving for  $\beta$ , we find  $\beta = 0.114$  or  $v = 0.114c$ .

75. Converting  $0.1 \text{ ounce} = 2.835 \times 10^{-3} \text{ kg}$ .

$$E = mc^2 = (2.835 \times 10^{-3} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 2.55 \times 10^{14} \text{ J}.$$

Eating 10 ounces results in a factor of 100 greater mass-energy increase, or  $2.55 \times 10^{16} \text{ J}$ . This is a small increase compared with your original mass-energy, but it will tend to increase your weight; depending on how they are prepared, peanuts generally contain about 100 kcal of food energy per ounce.

76. The energy needed equals the kinetic energy of the spaceship.

$$K = (\gamma - 1)mc^2 = \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) mc^2$$

$$= \left( \frac{1}{\sqrt{1 - 0.3^2}} - 1 \right) (10^4 \text{ kg}) (3.00 \times 10^8 \text{ m/s})^2 = 4.35 \times 10^{19} \text{ J}$$

or 4.35% of  $10^{21} \text{ J}$ .

77. Up to Equation (2.57) the derivation in the text is complete. Then using the integration by parts formula,  $\int x dy = xy - \int y dx$  and noting that in this case  $x = u$  and  $y = \gamma u$ , we have  $\int u d(\gamma u) = \gamma u^2 - \int \gamma u du$ . Thus

$$K = m \int_0^{\gamma u} u d(\gamma u) = \gamma m u^2 - m \int \gamma u du$$

$$= \gamma m u^2 - m \int \frac{u}{\sqrt{1 - u^2 / c^2}} du$$

Using integral tables or simple substitution:

$$K = \gamma m u^2 + m c^2 \sqrt{1 - u^2 / c^2} \Big|_0^u$$

$$= \gamma m u^2 + m c^2 \sqrt{1 - u^2 / c^2} - m c^2$$

$$= \frac{m c^2 + m c^2 (1 - u^2 / c^2)}{\sqrt{1 - u^2 / c^2}} - m c^2$$

$$= \gamma m c^2 - m c^2 = m c^2 (\gamma - 1)$$

78. Converting  $0.11 \text{ cal} \cdot \text{g}^{-1} \cdot ^\circ \text{C}^{-1} = 460 \text{ J} \cdot \text{kg}^{-1} \cdot ^\circ \text{C}^{-1} = c_v$

(specific heat at constant volume). From thermodynamics the energy  $\Delta E$  used to change the temperature by  $\Delta T$  is  $mc_v \Delta T$ . Thus

$$\Delta E = (1000 \text{ kg}) \left[ (460 \text{ J}/(\text{kg} \cdot ^\circ \text{C})) (0.5^\circ \text{C}) \right] = 2.30 \times 10^5 \text{ J} \text{ and}$$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{2.30 \times 10^5 \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 2.56 \times 10^{-12} \text{ kg} . \text{ The source of this energy is the internal energy of the arrangement of atoms and molecules prior to the collision.}$$

79.

$$\begin{aligned}
 E_b &= [2m_p + 2m_n - m(\text{He})]c^2 \\
 &= [2(1.007276 \text{ u}) + 2(1.008665 \text{ u}) - 4.001505 \text{ u}]c^2 \left( \frac{931.494 \text{ MeV}}{c^2 \cdot \text{u}} \right) = 28.3 \text{ MeV}
 \end{aligned}$$

80.

$$\begin{aligned}
 \Delta E &= [m_n - m_p - m_e]c^2 \\
 &= [1.008665 \text{ u} - 1.007276 \text{ u} - 0.000549 \text{ u}]c^2 \left( \frac{931.494 \text{ MeV}}{c^2 \cdot \text{u}} \right) = 0.782 \text{ MeV}
 \end{aligned}$$

81.  $E = K + E_0 = 1 \text{ TeV} + 938 \text{ MeV} \approx 1 \text{ TeV}$ ;

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(1 \text{ TeV} + 938 \text{ MeV})^2 - (938 \text{ MeV})^2}}{c} = 1.000938 \text{ TeV}/c$$

$$\gamma = \frac{E + E_0}{E_0} = \frac{1.000938 \text{ TeV}}{0.000938 \text{ TeV}} = 1067; \quad \beta^2 = 1 - \frac{1}{\gamma^2} = 1 - 8.78 \times 10^{-7}$$

$$\beta = \sqrt{1 - 8.78 \times 10^{-7}} \approx 1 - 4.39 \times 10^{-7}; \text{ and } v = \beta c \approx 0.999999561c$$

82. (a)  $E = \sqrt{p^2 c^2 + E_0^2} = \sqrt{(40 \text{ GeV})^2 + (511 \text{ keV})^2} \approx 40.0 \text{ GeV}$ 

$$K = E - E_0 = 40.0 \text{ GeV}$$

$$(b) \quad E = \sqrt{p^2 c^2 + E_0^2} = \sqrt{(40 \text{ GeV})^2 + (0.938 \text{ GeV})^2} = 40.011 \text{ GeV}$$

$$K = E - E_0 = 40.011 \text{ GeV} - 0.938 \text{ GeV} = 39.07 \text{ GeV}$$

83.  $E = K + E_0 = 200 \text{ MeV} + 106 \text{ MeV} = 306 \text{ MeV}$ 

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(306 \text{ MeV})^2 - (106 \text{ MeV})^2}}{c} = 287.05 \text{ MeV}/c$$

$$\gamma = \frac{E}{E_0} = \frac{306 \text{ MeV}}{106 \text{ MeV}} = 2.887$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.938$$

$$\text{so } v = 0.938c$$

84. (a) The mass-energy imbalance occurs because the helium-3 ( $^3\text{He}$ ) nucleus is more tightly bound than the two separate deuterium nuclei ( $^2\text{H}$ ). (Masses from Appendix 8.)

$$\begin{aligned}\Delta E &= ([2m(^2\text{H})] - [m_n + m(^3\text{He})])c^2 \\ &= [2(2.014102 \text{ u}) - (1.008665 \text{ u} + 3.016029 \text{ u})]c^2 \left( \frac{931.494 \text{ MeV}}{c^2 \cdot \text{u}} \right) = 3.27 \text{ MeV}\end{aligned}$$

(b) The initial rest energy is  $2m(^2\text{H}) = 2(2.014102 \text{ u}) \left( \frac{931.494 \text{ MeV}}{c^2 \cdot \text{u}} \right) = 3752 \text{ MeV}$ .

Thus the answer in (a) is about 0.09% of the initial rest energy.

85. (a) The mass-energy imbalance occurs because the helium-4 ( $^4\text{He}$ ) is more tightly bound than the deuterium ( $^2\text{H}$ ) and tritium nuclei ( $^3\text{H}$ ).

$$\begin{aligned}\Delta E &= ([m(^2\text{H}) + m(^3\text{H})] - [m_n + m(^4\text{He})])c^2 \\ &= [(2.014102 \text{ u} + 3.016029 \text{ u}) - (1.008665 \text{ u} + 4.002603 \text{ u})]c^2 \left( \frac{931.494 \text{ MeV}}{c^2 \cdot \text{u}} \right) \\ &= 17.6 \text{ MeV}\end{aligned}$$

(b) The initial rest energy is  $[m(^2\text{H}) + m(^3\text{H})] \cdot c^2 = [(5.030131 \text{ u})] \cdot c^2 \left( \frac{931.494 \text{ MeV}}{c^2 \cdot \text{u}} \right) = 4686 \text{ MeV}$ . Thus the answer in (a) is about 0.37% of the initial rest energy.

86. (a) In the inertial frame moving with the negative charges in wire 1, the negative charges in wire 2 are stationary, but the positive charges are moving. The density of the positive charges in wire 2 is thus greater than the density of negative charges, and there is a net attraction between the wires.

(b) By the same reasoning as in (a), note that the positive charges in wire 2 will be stationary and have a normal density, but the negative charges are moving and have an increased density, causing a net attraction between the wires.

(c) There are two facts to be considered. First, (a) and (b) are consistent with the physical result being independent of inertial frame. Second, we know from classical physics that two parallel wires carrying current in the same direction attract each other. That is, the same result is achieved in the “lab” frame.

87. As in the solution to Problem 21 we have  $\beta = \frac{v}{c} = \frac{d\sqrt{1-\beta^2}}{ct'}$  where  $d$  is the length of the particle track and  $t'$  the particle's lifetime in its rest frame. In this problem  $t' = 8.2 \times 10^{-11} \text{ s}$

and  $d = 24$  mm. Solving the above equation we find  $\beta = 0.698$ . Then

$$E = \frac{E_0}{\sqrt{1-\beta^2}} = \frac{1672 \text{ MeV}}{\sqrt{1-0.698^2}} = 2330 \text{ MeV}$$

88.

$$\begin{aligned} F &= \frac{dp}{dt} = \frac{d}{dt}(\gamma mv) = \frac{d}{dt} \left[ mv \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \right] \\ &= m\gamma \frac{dv}{dt} - \frac{mv}{2} \gamma^3 \left( -\frac{2v}{c^2} \right) \frac{dv}{dt} \\ &= m\gamma \frac{dv}{dt} \left[ 1 + \frac{v^2 \gamma^2}{c^2} \right] = m\gamma \frac{dv}{dt} \left[ 1 + \frac{v^2 / c^2}{1 - v^2 / c^2} \right] \\ &= m\gamma \frac{dv}{dt} \left[ \frac{1}{1 - v^2 / c^2} \right] = m\gamma^3 \frac{dv}{dt} = m \frac{dv}{dt} \frac{1}{(1 - v^2 / c^2)^{3/2}} \end{aligned}$$

89. (a) The number  $n$  received by Frank at  $f'$  is half the number sent by Mary at that rate, or  $fL / \gamma v$ . The detected time of turnaround is

$$t = \frac{n}{f'} = \frac{fL / \gamma v}{v \sqrt{(1-\beta)/(1+\beta)}} = \frac{L \sqrt{1+\beta}}{\gamma v \sqrt{1-\beta}} = \frac{L(1+\beta)}{v} = \frac{L}{v} + \frac{L}{c}.$$

(b) Similarly, the number  $n'$  received by Mary at  $f'$  is

$$n' = f' \frac{T'}{2} = f \sqrt{\frac{1-\beta}{1+\beta}} \frac{L}{\gamma v} = \frac{f L (1-\beta)}{v}. \quad \text{Her turnaround time is } T' / 2 = L / \gamma v.$$

(c) For Frank, the time  $t_2$  for the remainder of the trip is  $t_2 = T - t_1 = L/v - L/c$ .

$$\text{Number of signals} = f'' t_2 = f \sqrt{\frac{1+\beta}{1-\beta}} (L/v - L/c) = \frac{fL}{\gamma v}.$$

$$\text{Total number received} = \frac{fL}{\gamma v} + \frac{fL}{\gamma v} = \frac{2fL}{\gamma v}.$$

$$\text{Mary's age} = \frac{\text{Total number received}}{f} = \frac{2L}{\gamma v}.$$

(d) For Mary,  $t'_2 = T' - t'_1 = L / \gamma v$ .

$$\text{Number of signals} = f'' t'_2 = f \sqrt{\frac{1+\beta}{1-\beta}} \frac{L}{\gamma v} = \frac{fL}{v} (1+\beta).$$

$$\text{Total number received} = \frac{fL}{v} (1-\beta + 1+\beta) = \frac{2fL}{v}.$$

$$\text{Frank's age} = \frac{\text{Total number received}}{f} = \frac{2L}{v}.$$

90. In the fixed frame, the distance is  $\Delta x = 8 \text{ ly}$  and the elapsed time is  $\Delta t = 10 \text{ y}$ , so the interval is  $s^2 = \Delta x^2 - c^2 \Delta t^2 = 64 \text{ ly}^2 - 100 \text{ ly}^2 = -36 \text{ ly}^2$ . In the moving frame, Mary's clock is at rest, so  $\Delta x' = 0$ , and the time interval is  $\Delta t' = 6 \text{ y}$ . Thus the interval is  $s'^2 = \Delta x'^2 - c^2 \Delta t'^2 = 0 \text{ ly}^2 - 36 \text{ ly}^2 = -36 \text{ ly}^2$ . The results are the same, as they should be, because the spacetime interval is the same for all inertial frames.

91. (a) From Table 2.1  $\text{number} = \frac{fL}{v}(1-\beta) = \frac{(52\text{y}^{-1})(4.3 \text{ ly})}{0.8c}(1-0.8) = 55.9 \approx 56$

(b)  $t_1 = \frac{L}{v} + \frac{L}{c} = \frac{4.3 \text{ ly}}{0.8c} + \frac{4.3 \text{ ly}}{c} = 9.68 \text{ y}$ ;  $\text{number} = f' t_1 = \frac{fL}{v} \sqrt{1-\beta^2} = 167.7 \approx 168$

(c) Frank:  $f'' t'_2 = \frac{fL}{v} \sqrt{1-\beta^2} \approx 168$  so the total is  $168 + 168 = 336$ .

Mary:  $\text{number} = 2fL/v = 559$

(d) Frank:  $T = 2L/v = 10.75 \text{ y}$       Mary:  $T' = 2L/\gamma v = 6.45 \text{ y}$

(e) From part (c), 559 weeks = 10.75 years and 336 weeks = 6.46 years, which agrees with part (d).

92. Notice that the radar is shifted twice, once upon receipt by the speeding car and again upon reemission (reflection) of the beam. For this double shift, the received frequency  $f$  is

$$\frac{f}{f_0} = \sqrt{\frac{1-\beta}{1+\beta}} \sqrt{\frac{1-\beta}{1+\beta}} = \frac{1-\beta}{1+\beta}$$

For speeds much less than  $c$ , a Taylor series approximation gives an excellent result:

$$\frac{f}{f_0} = \frac{1-\beta}{1+\beta} = (1-\beta)(1+\beta)^{-1} \approx 1-2\beta.$$

Converting 80 mph to 35.8 m/s gives  $\beta = 1.19 \times 10^{-7}$ , so the received frequency is approximately

$$f = f_0(1-2\beta) = 10.4999975 \text{ GHz}, \text{ which is } 2.5 \text{ kHz less than the original frequency.}$$

93. For this transformation  $v = 0.8c$  (so  $\gamma = 5/3$ ),  $u'_x = 0$  and  $u'_y = 0.8c$ . Applying the transformations,

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}} = \frac{0 + 0.8c}{1 + 0} = 0.8c ; \quad u_y = \frac{u'_y}{\gamma \left(1 + \frac{vu'_x}{c^2}\right)} = \frac{0.8c}{\frac{5}{3}(1 + 0)} = 0.48c$$

The speed is  $u = \sqrt{u_x^2 + u_y^2} = 0.93c$ , safely under the speed of light.

94. (a)  $K = E + E_0 = 250E_0$ . Thus  $K = 249E_0 = 249(511 \text{ keV}) = 127.2 \text{ MeV}$

(b)  $\gamma = 250$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.999992$$

$$v = 0.999992c$$

(c)  $p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(250 \times 511 \text{ keV})^2 - (511 \text{ keV})^2}}{c} = 128 \text{ MeV}/c$

95. (a) For the proton:  $p = \gamma mu = \frac{1}{\sqrt{1 - 0.9^2}} (938 \text{ MeV}/c^2)(0.9c) = 1940 \text{ MeV}/c$ .

For the electron:  $E = \sqrt{p^2 c^2 + E_0^2} = \sqrt{(1940 \text{ MeV})^2 + (0.511 \text{ MeV})^2} = 1940 \text{ MeV}$ .

$$\gamma = \frac{E}{E_0} = \frac{1940 \text{ MeV}}{0.511 \text{ MeV}} = 3797$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{3797^2}} = \sqrt{1 - 6.94 \times 10^{-8}} \approx 1 - 3.97 \times 10^{-8}$$

$$v = (1 - 3.97 \times 10^{-8})c$$

(b) For the proton:  $K = (\gamma - 1)E_0 = \left(\frac{1}{\sqrt{1 - 0.9^2}} - 1\right)(938 \text{ MeV}) = 1214 \text{ MeV}$ .

For the electron:  $\gamma = \frac{K + E_0}{E_0} = \frac{1214 \text{ MeV} + 0.511 \text{ MeV}}{0.511 \text{ MeV}} = 2377$ .

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{2377^2}} = \sqrt{1 - 1.77 \times 10^{-7}} \approx 1 - 8.85 \times 10^{-8}$$

$$v = (1 - 8.85 \times 10^{-8})c$$

96. In the frame of the decaying  $K^0$  meson, the pi mesons must recoil with equal speeds in opposite directions in order to conserve momentum. In that reference frame the available kinetic energy is  $498 \text{ MeV} - 2(140 \text{ MeV}) = 218 \text{ MeV}$ . The pi mesons share this equally,

so each one has a kinetic energy of 109 MeV in that frame. The speed of each pi meson can be found:

$$\gamma = \frac{K + E_0}{E_0} = \frac{109 \text{ MeV} + 140 \text{ MeV}}{140 \text{ MeV}} = 1.779 \text{ so } u = \sqrt{1 - \frac{1}{\gamma^2}} c = 0.827c.$$

The greatest and least speeds in the lab frame are obtained when the pi mesons are released in the forward and backward directions. Then by the velocity addition laws:

$$v_{\max} = \frac{0.9c + 0.827c}{1 + (0.9)(0.827)} = 0.990c$$

$$v_{\min} = \frac{0.9c - 0.827c}{1 - (0.9)(0.827)} = 0.285c$$

97. (a) The round-trip distance is  $L_0 = 8.60 \text{ ly}$ . Assume the same constant speed  $v = \beta c$  for the entire trip. In the rocket's frame, the distance is only  $L = L_0 \gamma^{-1} = L_0 \sqrt{1 - \beta^2}$ .

Mary will age in the rocket's reference frame a total of 22 y, and in that frame

$$v = \frac{\text{distance}}{\text{time}} = \frac{L}{22\text{y}} = \frac{8.60 \text{ ly} \sqrt{1 - \beta^2}}{22\text{y}} = 0.39c \sqrt{1 - \beta^2}. \text{ Therefore, } \beta = \frac{v}{c} = 0.39 \sqrt{1 - \beta^2}.$$

Solving for  $\beta$  we find  $\beta = \sqrt{\frac{0.39^2}{1.00 + 0.39^2}}$ , or  $\beta = 0.36$  so Mary's speed is  $v = 0.36c$ .

- (b) To find the elapsed time on Earth, we know that  $T_0 = 22\text{y}$ , so

$$T = \gamma T_0 = \frac{1}{\sqrt{1 - \beta^2}} 22\text{y} = 23.6 \text{ y}. \text{ Frank will be 53.6 years old when Mary returns at the age of 52 y.}$$

98. With  $v = 0.995c$  then  $\beta = 0.995$  and  $\gamma = 10.01$ . The distance out to the star is

$L_0 = 5.98 \text{ ly}$ . In the rocket's reference frame, the distance is only

$$L = L_0 \gamma^{-1} = \frac{5.98 \text{ ly}}{10.01} = 0.597 \text{ ly}.$$

- (a) The time out to the star in Mary's frame is  $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{0.597 \text{ ly}}{0.995c} = 0.6 \text{ y}$ . The

time for the return journey will be the same. When you include the 3 years she spends at the star, her total journey will take 4.2 y.

- (b) To find the elapsed time on Earth for the outbound journey, we know that  $T_0 = 0.6\text{y}$  so  $T = \gamma T_0 = (10.01)0.6 \text{ y} = 6.006 \text{ y}$ . The return journey will take an equal time. The 3 years the spaceship orbits the star will be equivalent for both observers. Therefore Frank will measure a total elapsed time of 15.012 y, which makes him 10.8 years older than her.



99. (a) The Earth to moon distance is  $3.84 \times 10^8$  m. The rotation rate is

$\omega = 2\pi \text{ rad} \times 0.030 \text{ s}^{-1} = 1.885 \times 10^{-1} \text{ rad/s}$ . Then the speed across the moon's surface is

$$v = \omega R = (1.885 \times 10^{-1} \text{ rad/s})(3.82 \times 10^8 \text{ m}) = 7.24 \times 10^7 \text{ m/s}.$$

- (b) The required speed can be found using  $c = \omega R = 2\pi f R$  which requires the frequency

$$\text{to be } f = \frac{c}{2\pi R} = \frac{3.00 \times 10^8 \text{ m/s}}{2\pi(3.84 \times 10^8 \text{ m/s})} = 0.124 \text{ Hz}.$$

100. With the data given,  $\beta = 3.28 \times 10^{-6}$ , which is very small. We will use the binomial approximation theorem. From Equation (2.21), we know that

$$L = L_0 \gamma^{-1} \cdot \gamma^{-1} = \sqrt{1 - \beta^2} \approx 1 - \beta^2 / 2.$$

- (a) The percentage of length contraction would be:

$$\begin{aligned} \% \text{ change} &= \frac{[L_0 - L]}{L_0} \times 100\% = \frac{[L_0 - L_0 \gamma^{-1}]}{L_0} \times 100\% \\ &= [1 - \gamma^{-1}] \times 100\% = [1 - (1 - \beta^2 / 2)] \times 100\% \\ &= \frac{\beta^2}{2} \times 100\% = 5.37 \times 10^{-10} \end{aligned}$$

- (b) The clocks' rates differ by a factor  $\gamma = 1 / \sqrt{1 - \beta^2}$ . The clock on the SR-71 measures the proper time and Equation 2.19 tells us that  $T' = \gamma T_0$  so the time difference is  $\Delta t = T' - T_0 = \gamma T_0 - T_0 = T_0(\gamma - 1)$ . Using  $\gamma - 1 \approx \beta^2 / 2$  and the fact that the time for the trip in the SR-71 equals the distance divided by the speed

$$\begin{aligned} \Delta t &= t(\beta^2 / 2) = \frac{3.2 \times 10^6 \text{ m}}{983 \text{ m/s}} \frac{\left( \frac{983 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2}{2} \\ &= 1.75 \times 10^{-8} \text{ s} = 17.5 \text{ ns} \end{aligned}$$

101. As the spaceship is approaching the observer, we will make use of Equation (2.32),

$$f' = \frac{\sqrt{1 + \beta}}{\sqrt{1 - \beta}} f_0 \text{ with the prime indicating the Doppler shifted frequency (or wavelength).}$$

This equation indicates that the Doppler shifted frequency will be larger than the frequency measured in a frame where the observer is at rest with respect to the source. Since  $c = \lambda f$ , this means the Doppler shifted wavelengths will be lower. As given in the problem, we see that the difference in wavelengths for an observer at rest with respect to the source is  $\Delta \lambda_0 = 0.5974 \text{ nm}$ . We want to find a speed so that the Doppler shifted difference is reduced to  $\Delta \lambda' = 0.55 \text{ nm}$ . We have

$$\begin{aligned}
\Delta\lambda' &= \lambda'_2 - \lambda'_1 = \frac{c}{f'_2} - \frac{c}{f'_1} = c \left( \frac{f'_1 - f'_2}{f'_1 f'_2} \right) \\
&= c \left[ \frac{\left( \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \right) f_{0,1} - \left( \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \right) f_{0,2}}{\frac{1+\beta}{1-\beta} f_{0,1} f_{0,2}} \right] = c \left[ \left( \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \right) \left( \frac{f_{0,1} - f_{0,2}}{f_{0,1} f_{0,2}} \right) \right] \\
&= \left[ \left( \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \right) \left( \frac{c}{f_{0,2}} - \frac{c}{f_{0,1}} \right) \right] = \left[ \left( \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \right) (\lambda_{0,2} - \lambda_{0,1}) \right] \\
&= \left[ \left( \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \right) (\Delta\lambda_0) \right]
\end{aligned}$$

We can complete the algebra to show that  $\beta = \frac{\Delta\lambda_0^2 - (\Delta\lambda')^2}{\Delta\lambda_0^2 + (\Delta\lambda')^2} = 0.0825$  so that

$$v = 2.47 \times 10^7 \text{ m/s}.$$

102. As we know that the quasars are moving away at high speeds, we make use of Equation (2.33) and the equation  $c = \lambda f$ . Using a prime to indicate the Doppler shifted frequency

(or wavelength), Equation (2.33) indicates that frequency is given by  $f' = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$ , or

$$\begin{aligned}
\frac{f_0}{f'} &= \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \text{ so} \\
z &= \frac{(\lambda' - \lambda_0)}{\lambda_0} = \left( \frac{\lambda'}{\lambda_0} - 1 \right) = \left( \frac{c/f'}{c/f_0} - 1 \right) \\
&= \left( \frac{f_0}{f'} - 1 \right) = \left( \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} - 1 \right)
\end{aligned}$$

Therefore  $(z+1)^2 = \frac{1+\beta}{1-\beta}$ . We can complete the algebra to show that

$$\beta = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \text{ and thus } v = \left[ \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \right] c. \text{ For the values of } z \text{ given, } v = 0.787c \text{ for}$$

$$z = 1.9 \text{ and } v = 0.944c \text{ for } z = 4.9.$$

103. (a) In the frame of the decaying  $K^0$  meson, the pi mesons must recoil with equal momenta in opposite directions in order to conserve momentum. In that reference frame the available kinetic energy is  $498 \text{ MeV} - 2(135 \text{ MeV}) = 228 \text{ MeV}$ .

(b) The pi mesons share this equally, so each one has a kinetic energy of 114 MeV in that frame. The energy of each pi meson is  $E = K + E_0 = 114 \text{ MeV} + 135 \text{ MeV} = 249 \text{ MeV}$ .

The momentum of each pi meson can be found:

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(249 \text{ MeV})^2 - (135 \text{ MeV})^2}}{c} = 209.2 \text{ MeV}/c$$

104. (a) For each second, the energy used is  $3.9 \times 10^{26} \text{ J}$ . The mass used in each second is

$$m = \frac{E}{c^2} = \frac{3.9 \times 10^{26} \text{ J}}{(3.0 \times 10^8 \text{ m/s})^2} = 4.3 \times 10^9 \text{ kg}$$

(b) The time to use this mass is

$$\Delta t = \frac{m_{\text{sun}}}{\Delta m / \Delta t} \times \text{efficiency} = \frac{2.0 \times 10^{30} \text{ kg}}{4.3 \times 10^9 \text{ kg/s}} \times 0.007 = 3.3 \times 10^{18} \text{ s} \approx 1.0 \times 10^{11} \text{ y.}$$

That is 20 times longer than the expected lifetime of the sun.

105. a) The planet's orbital speed is  $v_p = \frac{2\pi r}{T} = 1.30 \times 10^4 \text{ m/s}$ . If the system's center of mass is at rest, conservation of momentum gives the star's speed:

$$m_p v_p = m_s v_s, \text{ so } v_s = \frac{m_p v_p}{m_s} = \frac{1.90 \times 10^{27}}{1.99 \times 10^{30}} 1.30 \times 10^4 \text{ m/s} = 12.4 \text{ m/s}$$

(b) The redshifted wavelength is  $\lambda = \lambda_0 \sqrt{\frac{1+\beta}{1-\beta}} \approx 550 \text{ nm} + 2.3 \times 10^{-5} \text{ nm}$

Similarly, the blueshifted wavelength is  $\lambda = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}} \approx 550 \text{ nm} - 2.3 \times 10^{-5} \text{ nm}$

These small differences can be detected using modern spectroscopic tools.

106. To a good approximation, the shift is the same in each direction, so the redshift for the approaching light is half the difference, or 0.0045 nm. Using this in the equation

$\lambda = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}}$  leads to a speed parameter  $\beta = 6.9 \times 10^{-6}$ , or  $v = \beta c = 2070 \text{ m/s}$ . The speed

is equal to the circumference  $2\pi R$  divided by the period, so the period is

$$T = \frac{2\pi R}{v} = \frac{2\pi (6.96 \times 10^8 \text{ m})}{2070 \text{ m/s}} = 2.11 \times 10^6 \text{ s} = 24.5 \text{ days.}$$