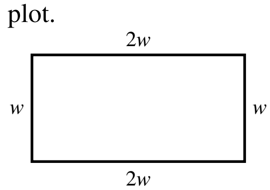
Chapter 1

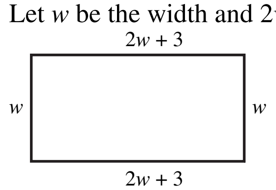
Problems 1.1

1. Let w be the width and $2w$ be the length of the plot.



Then area = 800.
$(2w)w = 800$
$2w^2 = 800$
$w^2 = 400$
$w = 20$ ft
Thus the length is 40 ft, so the amount of fencing needed is 2(40) + 2(20) = 120 ft.

2. Let w be the width and $2w + 3$ be the length.



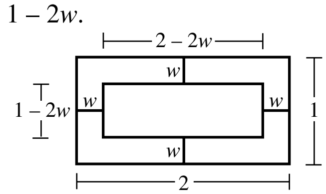
Then perimeter = 300.
$2w + 2(2w + 3) = 300$
$6w + 6 = 300$
$6w = 294$
$w = 49$ ft
Thus the length is 2(49) + 3 = 101 ft.
The dimensions are 49 ft by 101 ft.

3. Let n = number of ounces in each part. Then we have
$4n + 5n = 145$
$9n = 145$
$n = 16\frac{1}{9}$
Thus there should be $4\left(16\frac{1}{9}\right) = 64\frac{4}{9}$ ounces of A and $5\left(16\frac{1}{9}\right) = 80\frac{5}{9}$ ounces of B.

4. Let n = number of cubic feet in each part. Then we have
$1n + 3n + 5n = 765$
$9n = 765$
$n = 85$
Thus he needs $1n = 1(85) = 85$ ft^3 of portland cement, $3n = 3(85) = 255$ ft^3 of sand, and $5n = 5(85) = 425$ ft^3 of crushed stone.

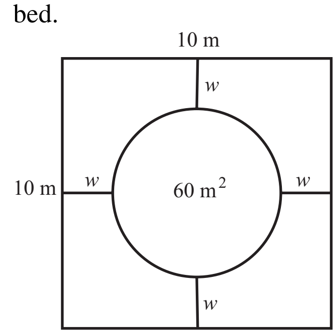
5. Let n = number of ounces in each part. Then we have
$2n + 1n = 16$
$3n = 16$
$n = \frac{16}{3}$
Thus the turpentine needed is
$(1)n = \frac{16}{3} = 5\frac{1}{3}$ ounces.

6. Let w = width (in miles) of strip to be cut. Then the remaining forest has dimensions $2 - 2w$ by $1 - 2w$.



Considering the area of the remaining forest, we have
$(2 - 2w)(1 - 2w) = \frac{3}{4}$
$2 - 6w + 4w^2 = \frac{3}{4}$
$8 - 24w + 16w^2 = 3$
$16w^2 - 24w + 5 = 0$
$(4w - 1)(4w - 5) = 0$
Hence $w = \frac{1}{4}, \frac{5}{4}$. But $w = \frac{5}{4}$ is impossible since one dimension of original forest is 1 mi. Thus the width of the strip should be $\frac{1}{4}$ mi.

7. Let w = "width" (in meters) of the pavement. Then $5 - w$ is the radius of the circular flower bed.



Thus

$$\pi r^2 = A$$
$$\pi(5-w)^2 = 60$$
$$w^2 - 10w + 25 = \frac{60}{\pi}$$
$$w^2 - 10w + \left(25 - \frac{60}{\pi}\right) = 0$$

$a = 1, b = -10, \ c = 25 - \dfrac{60}{\pi}$

$$w = \frac{-b \pm \sqrt{100 - 4(1)\left(25 - \frac{60}{\pi}\right)}}{2} \approx 9.37, \ 0.63$$

Since $0 < w < 5$, $w \approx 0.63$ m.

8. Since diameter of circular end is 140 mm, the radius is 70 mm. Area of circular end is $\pi(\text{radius})^2 = \pi(70)^2$. Area of square end is x^2. Equating areas, we have $x^2 = \pi(70)^2$. Thus $x = \pm\sqrt{\pi(70)^2} = \pm 70\sqrt{\pi}$. Since x must be positive, $x = 70\sqrt{\pi} \approx 124$ mm.

9. Let q = number of tons for \$560,000 profit.
Profit = Total Revenue – Total Cost

$$560{,}000 = 134q - (82q + 120{,}000)$$
$$560{,}000 = 52q - 120{,}000$$
$$680{,}000 = 52q$$
$$\frac{680{,}000}{52} = q$$
$$q \approx 13{,}076.9 \approx 13{,}077 \text{ tons.}$$

10. Let q = required number of units.
Profit = Total Revenue – Total Cost

$$150{,}000 = 50q - (25q + 500{,}000)$$
$$150{,}000 = 25q - 500{,}000$$

$650{,}000 = 25q$, from which

$$q = 26{,}000$$

11. Let x = amount at 6% and
$20{,}000 - x$ = amount at $7\frac{1}{2}\%$.

$x(0.06) + (20{,}000 - x)(0.075) = 1440$

$-0.015x + 1500 = 1440$

$-0.015x = -60$

$x = 4000$, so $20{,}000 - x = 16{,}000$. Thus the investment should be \$4000 at 6% and \$16,000 at $7\frac{1}{2}\%$.

12. Let x = amount at 4% and
$120{,}000 - x$ = amount at 5%.

$$0.04x + 0.05(120{,}000 - x) = 0.045(120{,}000)$$
$$-0.01x + 6000 = 5400$$
$$-0.01x = -600$$
$$x = 60{,}000$$

The investment consisted of \$60,000 at 5% and \$60,000 at 4%.

13. Let p = selling price. Then profit = $0.2p$.
selling price = cost + profit

$p = 3.40 + 0.2p$

$0.8p = 3.40$

$$p = \frac{3.40}{0.8} = \$4.25$$

14. Following the procedure in Example 6 we obtain the total value at the end of the second year to be $1{,}000{,}000(1+r)^2$.
So at the end of the third year, the accumulated amount will be $1{,}000{,}000(1+r)^2$ plus the interest on this, which is $1{,}000{,}000(1+r)^2 r$.
Thus the total value at the end of the third year will be $1{,}000{,}000(1+r)^2 + 1{,}000{,}000(1+r)^2 r$
$= 1{,}000{,}000(1+r)^3$.
This must equal \$1,125,800.

$$1{,}000{,}000(1+r)^3 = 1{,}125{,}800$$
$$(1+r)^3 = \frac{1{,}125{,}800}{1{,}000{,}000} = 1.1258$$
$$1 + r \approx 1.04029$$
$$r \approx 0.04029$$

Thus $r \approx 0.04029 \approx 4\%$.

15. Following the procedure in Example 6 we obtain

$$3{,}000{,}000(1+r)^2 = 3{,}245{,}000$$
$$(1+r)^2 = \frac{649}{600}$$
$$1 + r = \pm\sqrt{\frac{649}{600}}$$
$$r = -1 \pm \sqrt{\frac{649}{600}}$$
$$r \approx -2.04 \text{ or } 0.04$$

We choose $r \approx 0.04 = 4\%$.

16. Total revenue = variable cost + fixed cost

$100\sqrt{q} = 2q + 1200$

$50\sqrt{q} = q + 600$

$2500q = q^2 + 1200q + 360,000$

$0 = q^2 - 1300q + 360,000$

$0 = (q - 400)(q - 900)$

$q = 400$ or $q = 900$

17. Let n = number of bookings.

$0.90n = 81$

$n = 90$ seats booked

18. Let n = number of people polled.

$0.20p = 700$

$p = \frac{700}{0.20} = 3500$

19. Let s = monthly salary of deputy sheriff.

$0.30s = 200$

$s = \frac{200}{0.30}$

$\text{Yearly salary} = 12s = 12\left(\frac{200}{0.30}\right) = \$8000$

20. Yearly salary before strike = (7.50)(8)(260) = \$15,600

Lost wages = (7.50)(8)(46) = \$2760

Let P be the required percentage increase (as a decimal).

$P(15,600) = 2760$

$P = \frac{2760}{15,600} \approx 0.177 = 17.7\%$

21. Let q = number of cartridges sold to break even.

total revenue = total cost

$21.95q = 14.92q + 8500$

$7.03q = 8500$

$q \approx 1209.10$

1209 cartridges must be sold to approximately break even.

22. Let n = number of shares.

total investment = 5000 + 20n

$0.04(5000) + 0.50n = 0.03(5000 + 20n)$

$200 + 0.50n = 150 + 0.60n$

$-0.10n = -50$

$n = 500$

500 shares should be bought.

23. Let v = total annual vision-care expenses (in dollars) covered by program. Then

$35 + 0.80(v - 35) = 100$

$0.80v + 7 = 100$

$0.80v = 93$

$v = \$116.25$

24. **a.** $0.031c$

b. $c - 0.031c = 600,000,000$

$0.969c = 600,000,000$

$c \approx 619,195,046$

Approximately 619,195,046 bars will have to be made.

25. Revenue = (number of units sold)(price per unit)

Thus

$400 = q\left[\frac{80 - q}{4}\right]$

$1600 = 80q - q^2$

$q^2 - 80q + 1600 = 0$

$(q - 40)^2 = 0$

$q = 40$ units

26. If I = interest, P = principal, r = rate, and t = time, then $I = Prt$. To triple an investment of P at the end of t years, the interest earned during that time must equal $2P$. Thus

$2P = P(0.045)t$

$2 = 0.045t$

$t = \frac{2}{0.045} \approx 44.4$ years

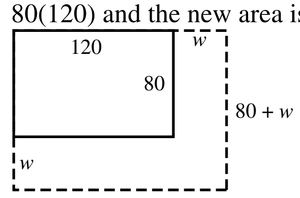
27. Let q = required number of units. Equate the incomes of each proposal.

$5000 + 0.50q = 50,000$

$0.50q = 45,000$

$q = 90,000$ units

28. Let w = width of strip. The original area is 80(120) and the new area is $(120 + w)(80 + w)$.



Thus
(120 + w)(80 + w) = 2(80)(120)
$9600 + 200w + w^2 = 19,200$
$w^2 + 200w - 9600 = 0$
$(w + 240)(w - 40) = 0$
$w = -240$ or $w = 40$
We choose $w = 40$ ft.

29. Let n = number of \$20 increases. Then at the rental charge of 400 + 20n dollars per unit, the number of units that can be rented is 50 – 2n. The total of all monthly rents is (400 + 20n)(50 – 2n), which must equal 20,240.
$20,240 = (400 + 20n)(50 - 2n)$
$20,240 = 20,000 + 200n - 40n^2$
$40n^2 - 200n + 240 = 0$
$n^2 - 5n + 6 = 0$
$(n - 2)(n - 3) = 0$
$n = 2, 3$
Thus the rent should be either
\$400 + 2(\$20) = \$440 or \$400 + 3(\$20) = \$460.

30. Let x = original value of the blue-chip investment, then 3,100,000 – x is the original value of the glamour stocks. Then the current value of the blue-chip stock is $x + \frac{1}{10}x$, or $\frac{11}{10}x$.
For the glamour stocks the current value is $(3,100,000 - x) - \frac{1}{10}(3,100,000 - x)$, which simplifies to $\frac{9}{10}(3,100,000 - x)$.
Thus for the current value of the portfolio,
$\frac{11}{10}x + \frac{9}{10}(3,100,000 - x) = 3,240,000$
$11x + 27,900,000 - 9x = 32,400,000$
$2x = 4,500,000$
$x = 2,250,000$
Thus the current value of the blue chip investment is $\frac{11}{10}(2,250,000)$ or \$2,475,000.

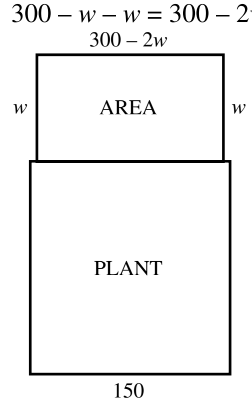
31. $10,000 = 800p - 7p^2$
$7p^2 - 800p + 10,000 = 0$
$$p = \frac{800 \pm \sqrt{640,000 - 280,000}}{14}$$
$$= \frac{800 \pm \sqrt{360,000}}{14} = \frac{800 \pm 600}{14}$$
For $p > 50$ we choose $p = \frac{800 + 600}{14} = \$100$.

32. Let p be the percentage change in market value.
$$(1 + 0.15)\left(\frac{P}{E}\right) = \frac{(1+p)P}{(1-0.10)E}$$
$$1.15 = \frac{1+p}{0.90}$$
$1.035 = 1 + p$
$p = 0.035 = 3.5\%$
The market value increased by 3.5%.

33. To have supply = demand,
$2p - 10 = 200 - 3p$
$5p = 210$
$p = 42$

34. $2p^2 - 3p = 20 - p^2$
$3p^2 - 3p - 20 = 0$
$a = 3$, $b = -3$, $c = -20$
$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$
$$= \frac{3 \pm \sqrt{249}}{6}$$
$p \approx 3.130$ or $p \approx -2.130$
The equilibrium price is $p \approx 3.13$.

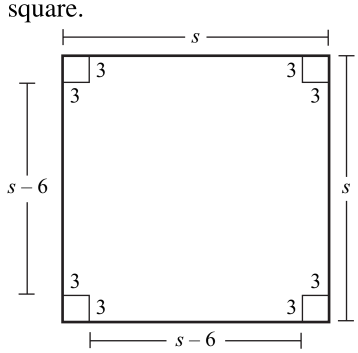
35. Let w = width (in ft) of enclosed area. Then length of enclosed area is
300 – w – w = 300 – 2w.



Thus
$w(300 - 2w) = 11,200$
$2w(150 - w) = 11,200$
$w(150 - w) = 5600$
$0 = w^2 - 150w + 5600$
$0 = (w - 80)(w - 70)$
Hence w = 80, 70. If w = 70, then length is 300 – 2w = 300 – 2(70) = 160. Since the building has length of only 150 ft, we reject

$w = 70$. If $w = 80$, then length is $300 - 2w = 300 - 2(80) = 140$. Thus the dimensions are 80 ft by 140 ft.

36. Let s = length in inches of side of original square.



Considering the volume of the box, we have (length)(width)(height) = volume

$(s - 4)(s - 4)(2) = 50$

$(s-4)^2 = 25$

$s-4 = \pm\sqrt{25} = \pm 5$

$s = 4 \pm 5$

Hence $s = -1, 9$. We reject $s = -1$ and choose $s = 9$. The dimensions are 9 in. by 9 in.

37. Original volume = (10)(5)(2) = 100 cm^3

Volume increase = 0.50(100) = 50 cm^3

Volume of new bar = 100 + 50 = 150 cm^3

Let x = number of centimeters that the length and width are each increased. Then

$$2(x+10)(x+5) = 150$$
$$x^2 + 15x + 50 = 75$$
$$x^2 + 15x - 25 = 0$$

$a = 1$, $b = 15$, $c = -25$

$$x = \frac{-15 \pm \sqrt{15^2 - 4(1)(-25)}}{2} \approx 1.51, -16.51$$

We reject −16.51 as impossible. The new length is approximately 11.51 cm, and the new width is approximately 6.51 cm.

38. Volume of old style candy

$$= \pi(7.1)^2(2.1) - \pi(2)^2(2.1)$$
$$= 97.461\pi \text{ mm}^3$$

Let r = inner radius (in millimeters) of new style candy. Considering the volume of the new style candy, we have

$$\pi(7.1)^2(2.1) - \pi r^2(2.1) = 0.78(97.461\pi)$$
$$29.84142\pi = 2.1\pi r^2$$
$$14.2102 = r^2$$
$$r \approx \pm 3.7696$$

Since r is a radius, we choose $r = 3.77$ mm.

39. Let x = amount of loan. Then the amount actually received is $x - 0.16x$. Hence,

$$x - 0.16x = 195{,}000$$
$$0.84x = 195{,}000$$
$$x \approx 232{,}142.86$$

To the nearest thousand, the loan amount is \$232,000. In the general case, the amount received from a loan of L with a compensating balance of $p\%$ is $L - \frac{p}{100}L.$

$$L - \frac{p}{100}L = E$$
$$\frac{100-p}{100}L = E$$
$$L = \frac{100E}{100-p}$$

40. Let n = number of machines sold over 600. Then the commission on each of $600 + n$ machines is $40 + 0.04n$. Equating total commissions to 30,800 we obtain

$(600 + n)(40 + 0.04n) = 30{,}800$

$$24{,}000 + 24n + 40n + 0.04n^2 = 30{,}800$$
$$0.02n^2 + 32n - 3400 = 0$$
$$n = \frac{-32 \pm \sqrt{1024+272}}{0.04} = \frac{-32 \pm 36}{0.04}$$

We choose $n = \frac{-32+36}{0.04} = 100.$ Thus the number of machines that must be sold is $600 + 100 = 700$.

41. Let n = number of acres sold. Then $n + 20$ acres were originally purchased at a cost of $\frac{7200}{n+20}$ each. The price of each acre sold was $30 + \left[\frac{7200}{n+20}\right]$. Since the revenue from selling n acres is \$7200 (the original cost of the parcel), we have

$$n\left[30+\frac{7200}{n+20}\right]=7200$$

$$n\left[\frac{30n+600+7200}{n+20}\right]=7200$$

$n(30n + 600 + 7200) = 7200(n + 20)$

$30n^2+7800n=7200n+144,000$

$30n^2+600n-144,000=0$

$n^2+20n-4800=0$

$(n + 80)(n – 60) = 0$

$n = 60$ acres (since $n > 0$), so 60 acres were sold.

42. Let q = number of units of product sold last year and $q + 2000$ = the number sold this year. Then the revenue last year was $3q$ and this year it is $3.5(q + 2000)$. By the definition of margin of profit, it follows that

$$\frac{7140}{3.5(q+2000)}=\frac{4500}{3q}+0.02$$

$$\frac{2040}{q+2000}=\frac{1500}{q}+0.02$$

$2040q = 1500(q + 2000) + 0.02q(q + 2000)$

$2040q=1500q+3,000,000+0.02q^2+40q$

$0=0.02q^2-500q+3,000,000$

$$q=\frac{500\pm\sqrt{250,000-240,000}}{0.04}$$

$$=\frac{500\pm\sqrt{10,000}}{0.04}$$

$$=\frac{500\pm100}{0.04}$$

= 10,000 or 15,000

So that the margin of profit this year is not greater than 0.15, we choose q = 15,000. Thus 15,000 units were sold last year and 17,000 this year.

43. Let q = number of units of B and $q + 25$ = number of units of A produced.

Each unit of B costs $\frac{1000}{q}$, and each unit of A costs $\frac{1500}{q+25}$. Therefore,

$$\frac{1500}{q+25}=\frac{1000}{q}+2$$

$1500q = 1000(q + 25) + 2(q)(q + 25)$

$0=2q^2-450q+25,000$

$0=q^2-225q+12,500$

$0 = (q – 100)(q – 125)$

$q = 100$ or $q = 125$

If $q = 100$, then $q + 25 = 125$; if $q = 125$, $q + 25 = 150$. Thus the company produces either 125 units of A and 100 units of B, or 150 units of A and 125 units of B.

Apply It 1.2

1. $200 + 0.8S \geq 4500$

$0.8S \geq 4300$

$S \geq 5375$

He must sell at least 5375 products per month.

2. Since $x_1\geq0,\ x_2\geq0,\ x_3\geq0,$ and $x_4\geq0,$ we have the inequalities

$150-x_4\geq0$

$3x_4-210\geq0$

$x_4+60\geq0$

$x_4\geq0$

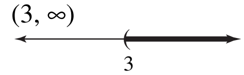
Problems 1.2

1. $5x>15$

$x>\frac{15}{5}$

$x>3$

$(3, \infty)$

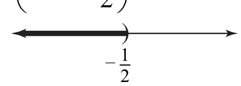


2. $4x<-2$

$x<\frac{-2}{4}$

$x<-\frac{1}{2}$

$\left(-\infty,-\frac{1}{2}\right)$

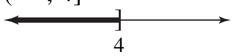


3. $5x - 11 \le 9$

$5x \le 20$

$x \le 4$

$(-\infty, 4]$

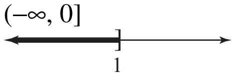


4. $5x \le 0$

$x \le \frac{0}{5}$

$x \le 0$

$(-\infty, 0]$

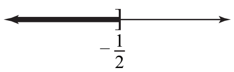


5. $-4x \ge 2$

$x \le \frac{2}{-4}$

$x \le -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right]$

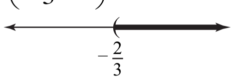


6. $3z + 2 > 0$

$3z > -2$

$z > -\frac{2}{3}$

$\left(-\frac{2}{3}, \infty\right)$

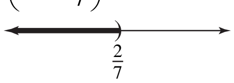


7. $5 - 7s > 3$

$-7s > -2$

$s < \frac{2}{7}$

$\left(-\infty, \frac{2}{7}\right)$

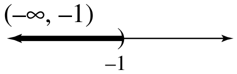


8. $4s - 1 < -5$

$4s < -4$

$s < -1$

$(-\infty, -1)$



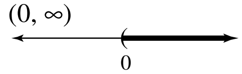
9. $3 < 2y + 3$

$0 < 2y$

$0 < y$

$y > 0$

$(0, \infty)$



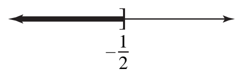
10. $4 \le 3 - 2y$

$1 \le -2y$

$-\frac{1}{2} \ge y$

$y \le -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right]$



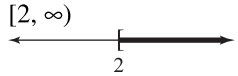
11. $t + 6 \le 2 + 3t$

$4 \le 2t$

$2 \le t$

$t \ge 2$

$[2, \infty)$



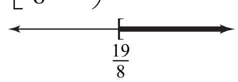
12. $-3 \ge 8(2 - x)$

$-3 \ge 16 - 8x$

$8x \ge 19$

$x \ge \frac{19}{8}$

$\left[\frac{19}{8}, \infty\right)$



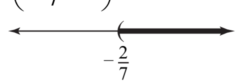
13. $3(2 - 3x) > 4(1 - 4x)$

$6 - 9x > 4 - 16x$

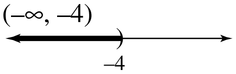
$7x > -2$

$x > -\frac{2}{7}$

$\left(-\frac{2}{7}, \infty\right)$



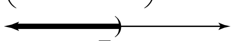
14. $8(x + 1) + 1 < 3(2x) + 1$
$8x + 9 < 6x + 1$
$2x < -8$
$x < -4$
$(-\infty, -4)$



15. $2(4x-2) > 4(2x+1)$
$8x-4 > 8x+4$
$-4 > 4$, which is false for all x.
Thus the solution set is $\varnothing$.

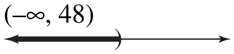
16. $5-(x+2) \le 2(2-x)$
$5-x-2 \le 4-2x$
$x \le 1$
$(-\infty, 1]$

17. $x+2 < \sqrt{3}-x$
$2x < \sqrt{3}-2$
$x < \dfrac{\sqrt{3}-2}{2}$
$\left(-\infty, \dfrac{\sqrt{3}-2}{2}\right)$

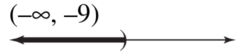


18. $\sqrt{2}(x+2) > \sqrt{8}(3-x)$
$\sqrt{2}(x+2) > 2\sqrt{2}(3-x)$
$x + 2 > 2(3 - x)$
$x + 2 > 6 - 2x$
$3x > 4$
$x > \dfrac{4}{3}$
$\left(\dfrac{4}{3}, \infty\right)$

19. $\dfrac{5}{6}x < 40$
$5x < 240$
$x < 48$
$(-\infty, 48)$



20. $-\dfrac{2}{3}x > 6$
$-x > 9$
$x < -9$
$(-\infty, -9)$



21. $\dfrac{5y+2}{4} \le 2y-1$
$5y+2 \le 8y-4$
$-3y \le -6$
$y \ge 2$
$[2, \infty)$

22. $\dfrac{3y-2}{3} \ge \dfrac{1}{4}$
$12y-8 \ge 3$
$12y \ge 11$
$y \ge \dfrac{11}{12}$
$\left[\dfrac{11}{12}, \infty\right)$

23. $-3x+1 \le -3(x-2)+1$
$-3x+1 \le -3x+7$
$1 \le 7$, which is true for all x. The solution is $-\infty < x < \infty$.
$(-\infty, \infty)$

24. $0x \le 0$
$0 \le 0$, which is true for all x. The solution is $-\infty < x < \infty$.
$(-\infty, \infty)$

25. $\dfrac{1-t}{2} < \dfrac{3t-7}{3}$
$3(1-t) < 2(3t-7)$
$3-3t < 6t-14$
$-9t < -17$
$t > \dfrac{17}{9}$
$\left(\dfrac{17}{9}, \infty\right)$