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Solutions Manual to accompany AN INTRODUCTION TO MECHANICS 2nd edition

February 2014 corrected November 2014

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ACKNOWLEDGEMENT

We thank Prof. Tom Donnelly and his physics students at Harvey Mudd College for pointing out a number of errors and omissions in the Solutions Manual.

NOTE

Corrected solutions through November 2014 are collected in the Appendix.

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1.1 Vector algebra 1

 $\mathbf{A} = (2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}) \quad \mathbf{B} = (5\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\,\hat{\mathbf{k}})$ (a) $\mathbf{A} + \mathbf{B} = (2 + 5)\,\hat{\mathbf{i}} + (-3 + 1)\,\hat{\mathbf{j}} + (7 + 2)\,\hat{\mathbf{k}} = 7\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} + 9\,\hat{\mathbf{k}}$ (b) $\mathbf{A} - \mathbf{B} = (2 - 5)\,\hat{\mathbf{i}} + (-3 - 1)\,\hat{\mathbf{j}}(7 - 2)\,\hat{\mathbf{k}} = -3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}$ (c) $\mathbf{A} \cdot \mathbf{B} = (2)(5) + (-3)(1) + (7)(2) = 21$ (d) $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -3 & 7 \\ 5 & 1 & 2 \end{vmatrix}$ $= -13\,\hat{\mathbf{i}} + 31\,\hat{\mathbf{j}} + 17\,\hat{\mathbf{k}}$

1.2 Vector algebra 2

$$\mathbf{A} = (3\,\mathbf{\hat{i}} - 2\,\mathbf{\hat{j}} + 5\,\mathbf{\hat{k}}) \quad \mathbf{B} = (6\,\mathbf{\hat{i}} - 7\,\mathbf{\hat{j}} + 4\,\mathbf{\hat{k}})$$
(a) $A^2 = \mathbf{A} \cdot \mathbf{A} = 3^2 + (-2)^2 + 5^2 = 38$
(b) $B^2 = \mathbf{B} \cdot \mathbf{B} = 6^2 + (-7)^2 + 4^2 = 101$
(c) $(\mathbf{A} \cdot \mathbf{B})^2 = [(3)(6) + (-2)(-7) + (5)(4)]^2 = [18 + 14 + 20]^2 = 52^2 = 2704$

1.3 Cosine and sine by vector algebra

 $\mathbf{A} = (3\,\mathbf{\hat{i}} + \,\mathbf{\hat{j}} + \,\mathbf{\hat{k}}) \quad \mathbf{B} = (-2\,\mathbf{\hat{i}} + \,\mathbf{\hat{j}} + \,\mathbf{\hat{k}})$ (a)

$$\mathbf{A} \cdot \mathbf{B} = A B \cos(\mathbf{A}, \mathbf{B})$$

$$\cos(\mathbf{A}, \mathbf{B}) = \frac{\mathbf{A} \cdot \mathbf{B}}{A B}$$

$$= \frac{(-6+1+1)}{\sqrt{(9+1+1)}\sqrt{4+1+1)}} = \frac{-4}{\sqrt{11}\sqrt{6}} \approx 0.492$$

(b) *method 1*:

$$|\mathbf{A} \times \mathbf{B}| = A B \sin(\mathbf{A}, \mathbf{B})$$

 $\sin(\mathbf{A}, \mathbf{B}) = \frac{|\mathbf{A} \times \mathbf{B}|}{A B}$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & 1 \\ -2 & 1 & 1 \end{vmatrix}$$

= $(1 - 1)\hat{\mathbf{i}} - (3 + 2)\hat{\mathbf{j}} + (3 + 2)\hat{\mathbf{k}} = -5\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
 $|\mathbf{A} \times \mathbf{B}| = \sqrt{5^2 + 5^2} = 5\sqrt{2}$
 $\sin(\mathbf{A}, \mathbf{B}) = \frac{|\mathbf{A} \times \mathbf{B}|}{AB} = \frac{5\sqrt{2}}{\sqrt{11}\sqrt{6}} \approx 0.870$

(c) *method 2 (simpler) – use:*

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin (\mathbf{A}, \mathbf{B}) = \sqrt{1 - \cos^2 (\mathbf{A}, \mathbf{B})}$$

$$= \sqrt{1 - (0.492)^2} \quad \text{from (a)} \approx 0.871$$

1.4 Direction cosines

Note that here α , β , γ stand for direction cosines, not for the angles shown in the figure: $\theta_x = \cos^{-1} \alpha$, $\theta_y = \cos^{-1} \beta$, $\theta_z = \cos^{-1} \gamma$.

continued next page \implies



$$\mathbf{A} = A_x \, \mathbf{\hat{i}} + A_y \, \mathbf{\hat{j}} + A_z \, \mathbf{\hat{k}}$$
$$A_x = \mathbf{A} \cdot \mathbf{\hat{i}} = A \, \cos\left(\mathbf{A}, \mathbf{\hat{i}}\right) \equiv A \, \alpha$$
$$\alpha = \cos\left(\mathbf{A}, \mathbf{\hat{i}}\right) = \cos \theta_x.$$

Similarly,

$$A_{y} = A \cos(\mathbf{A}, \mathbf{\hat{j}}) \equiv A\beta$$
$$\beta = \cos(\mathbf{A}, \mathbf{\hat{j}}) = \cos\theta_{y}$$
$$A_{z} = A \cos(\mathbf{A}, \mathbf{\hat{k}}) \equiv A\gamma$$
$$\gamma = \cos(\mathbf{A}, \mathbf{\hat{k}}) = \cos\theta_{z}$$

Using these results,

$$A^{2} = A_{x}^{2} + A_{y}^{2} + A_{z}^{2}$$
$$= A^{2} \left(\alpha^{2} + \beta^{2} + \gamma^{2}\right)$$

from which it follows that

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Another way to see this is

$$A^{2} = \rho^{2} + A_{z}^{2} = A_{x}^{2} + A_{y}^{2} + A_{z}^{2} = A^{2} (\alpha^{2} + \beta^{2} + \gamma^{2})$$

and it follows as before that

$$\alpha^2 + \beta^2 + \gamma^2 = 1.$$

1.5 Perpendicular vectors

Given $|\mathbf{A} - \mathbf{B}| = |\mathbf{A} + \mathbf{B}|$ with **A** and **B** nonzero. Evaluate the magnitudes by squaring.

$$A^{2} - 2\mathbf{A} \cdot \mathbf{B} + B^{2} = A^{2} + 2\mathbf{A} \cdot \mathbf{B} + B^{2}$$
$$-2\mathbf{A} \cdot \mathbf{B} = +2\mathbf{A} \cdot \mathbf{B}.$$
$$\mathbf{A} \cdot \mathbf{B} = 0$$

and it follows that $\mathbf{A} \perp \mathbf{B}$.

1.6 Diagonals of a parallelogram



1.7 Law of sines

The area \mathcal{A} of the triangle is

$$\mathcal{A} = \frac{1}{2}Ah = \frac{1}{2}AB\sin\gamma = \frac{1}{2}|\mathbf{A} \times \mathbf{B}|$$

Similarly,

$$\mathcal{A} = \frac{1}{2} |\mathbf{B} \times \mathbf{C}| = \frac{1}{2} BC \sin \alpha$$
$$\mathcal{A} = \frac{1}{2} |\mathbf{C} \times \mathbf{A}| = \frac{1}{2} AC \sin \beta.$$



Hence $AB \sin \gamma = BC \sin \alpha = AC \sin \beta$, from which it follows

$$\frac{\sin\gamma}{C} = \frac{\sin\alpha}{A} = \frac{\sin\beta}{B}$$

Introducing the cross product makes the notation convenient, and emphasizes the relation between the cross product and the area of the triangle, but it is not essential for the proof.

1.8 Vector proof of a trigonometric identity

Given two unit vectors $\hat{\mathbf{a}} = \cos \theta \, \hat{\mathbf{i}} + \sin \theta \, \hat{\mathbf{j}}$ and $\hat{\mathbf{b}} = \cos \phi \, \hat{\mathbf{i}} + \sin \phi \, \hat{\mathbf{j}}$, with a = 1, b = 1. First evaluate their scalar product using components:

 $\mathbf{a} \cdot \mathbf{b} = ab \, \cos\theta \cos\phi + ab \, \sin\theta \sin\phi$

 $= \cos\theta\cos\phi + \sin\theta\sin\phi$

then evaluate their scalar product geometrically.

$$\mathbf{a} \cdot \mathbf{b} = ab \cos(\mathbf{a}, \mathbf{b}) = ab \cos(\phi - \theta) = \cos(\phi - \theta)$$

Equating the two results,

 $\cos\left(\phi - \theta\right) = \cos\phi\cos\theta + \sin\phi\sin\theta$

1.9 Perpendicular unit vector

Given $\mathbf{A} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ and $\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$, find C such that $\mathbf{A} \cdot \mathbf{C} = \mathbf{0}$ and $\mathbf{B} \cdot \mathbf{C} = \mathbf{0}$.

$$\mathbf{C} = C_x \, \mathbf{\hat{i}} + C_y \, \mathbf{\hat{j}} + C_z \, \mathbf{\hat{k}}$$

= $C_x (\mathbf{\hat{i}} + (C_y/C_x) \, \mathbf{\hat{j}} + (C_z/C_x) \, \mathbf{\hat{k}})$
$$\mathbf{A} \cdot \mathbf{C} = C_x (1 + (C_y/C_x) - (C_z/C_x)) = 0$$

$$\mathbf{B} \cdot \mathbf{C} = C_x (2 + (C_y/C_x) - 3(C_z/C_x)) = 0$$

We have two equations for the two unknowns (C_y/C_x) and (C_z/C_x) .

$$1 + (C_y/C_x) - (C_z/C_x) = 0$$

2 + (C_y/C_x) - 3(C_z/C_x) = 0.

The solutions are $(C_y/C_x) = -\frac{1}{2}$ and $(C_z/C_x) = \frac{1}{2}$, so that $\mathbf{C} = \mathbf{C}_x(\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}} + \frac{1}{2}\hat{\mathbf{k}})$. To evaluate C_x , apply the condition that \mathbf{C} is a unit vector.

$$C^{2} = \frac{3}{2} C_{x}^{2} = 1$$

$$C_{x} = \pm \sqrt{(2/3)}$$

$$\hat{\mathbf{C}} = \pm \sqrt{(2/3)} (\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}} + \frac{1}{2}\hat{\mathbf{k}})$$

continued next page \implies



which can be written

$$\hat{\mathbf{C}} = \pm \frac{1}{\sqrt{6}} \left(2\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} \right)$$

Geometrically, **C** can be perpendicular to both **A** and **B** only if **C** is perpendicular to the plane determined by **A** and **B**. From the standpoint of vector algebra, this implies that $\mathbf{C} \propto \mathbf{A} \times \mathbf{B}$. To prove this, evaluate $\mathbf{A} \times \mathbf{B}$.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{\hat{i}} & \mathbf{\hat{j}} & \mathbf{\hat{k}} \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{vmatrix}$$
$$= -2 \,\mathbf{\hat{i}} + \mathbf{\hat{j}} - \mathbf{\hat{k}}$$
$$\propto \mathbf{C}.$$

1.10 Perpendicular unit vectors

Given $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, find a unit vector $\hat{\mathbf{B}}$ perpendicular to \mathbf{A} .

(a)

$$\mathbf{B} = B_x \mathbf{\hat{i}} + B_y \mathbf{\hat{j}} = B_x [\mathbf{\hat{i}} + (B_y/B_x)\mathbf{\hat{j}}]$$
$$\mathbf{A} \cdot \mathbf{B} = B_x [3 + 4(B_y/B_x)] = 0$$
$$B_y/B_x = -3/4$$
$$\mathbf{B} = B_x [\mathbf{\hat{i}} - \frac{3}{4}\mathbf{\hat{j}}]$$

To evaluate B_x , note that **B** is a unit vector, $B^2 = 1$.

$$1 = B_x^2 \left[(1)^2 + \left(\frac{3}{4}\right)^2 \right] = \left(\frac{25}{16}\right) B_x^2$$

which gives

$$B_x = \pm (4/5)$$

$$\hat{\mathbf{B}} = \pm (4/5)(\hat{\mathbf{i}} - (3/4)\hat{\mathbf{j}}) = \pm \frac{1}{5}(4\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$$

continued next page \Longrightarrow

(b)

$$\mathbf{C} = C_x \, \mathbf{\hat{i}} + C_y \, \mathbf{\hat{j}} + C_z \, \mathbf{\hat{k}}$$

= $C_x [\mathbf{\hat{i}} + (C_y/C_x) \, \mathbf{\hat{j}} + (C_z/C_x) \, \mathbf{\hat{k}}]$
$$\mathbf{A} \cdot \mathbf{C} = 0 \implies C_x [3 + 4(C_y/C_x) - 4(C_z/C_x)] = 0$$

$$\mathbf{B} \cdot \mathbf{C} = 0 \implies \frac{1}{5} C_x [4 - 3(C_y/C_x)] = 0$$

 $C_y/C_x = 4/3 \quad C_z/C_x = 25/12$

To make C a unit vector,

$$C^{2} = C_{x}^{2} \left[(1)^{2} + \left(\frac{4}{3}\right)^{2} + \left(\frac{25}{12}\right)^{2} \right] = 1$$

$$C_{x} \approx \pm 0.348$$

(c) The vector **B** × **C** is perpendicular (normal) to the plane defined by **B** and **C**, so we want to prove

$$\mathbf{A} \propto \mathbf{B} \times \mathbf{C}$$
$$\mathbf{B} \times \mathbf{C} = C_x \begin{vmatrix} \mathbf{\hat{i}} & \mathbf{\hat{j}} & \mathbf{\hat{k}} \\ \frac{4}{5} & -\frac{3}{5} & 0 \\ 1 & \frac{4}{3} & \frac{25}{12} \end{vmatrix}$$
$$= C_x \left[-\left(\frac{75}{60}\right) \mathbf{\hat{i}} - \left(\frac{100}{60}\right) \mathbf{\hat{j}} + \left(\frac{25}{15}\right) \mathbf{\hat{k}} \right]$$
$$= \left(\frac{5}{12}\right) C_x (-3 \mathbf{\hat{i}} - 4 \mathbf{\hat{j}} + 4 \mathbf{\hat{k}}) \propto \mathbf{A}.$$

1.11 Volume of a parallelepiped

With reference to the sketch, the height is $A \cos \alpha$, so the frontal area is $AB \cos \alpha$. The depth is $C \sin \beta$, so the volume V is

$$V = (AB\cos\alpha)(C\sin\beta) = (A\cos\alpha)(BC\sin\beta) = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$$

The same approach can be used starting with a different face.

$$V = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \quad V = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$

Note that A, B, C are arbitrary vectors. This proves the vector identity

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$



1.12 Constructing a vector to a point

Applying vector addition to the lower triangle in the sketch,

 $\mathbf{A} = \mathbf{r}_1 + x(\mathbf{r}_2 - \mathbf{r}_1)$ $= (1 - x)\mathbf{r}_1 + x\mathbf{r}_2$



1.13 Expressing one vector in terms of another

We will express vector **A** in terms of a unit vector $\hat{\mathbf{n}}$. As shown in the sketch, we can write **A** as the vector sum of a vector \mathbf{A}_{\parallel} parallel to $\hat{\mathbf{n}}$ and a vector \mathbf{A}_{\perp} perpendicular to $\hat{\mathbf{n}}$, so that $\mathbf{A} = \mathbf{A}_{\parallel} + \mathbf{A}_{\perp}$.



 $|\mathbf{A}_{\parallel}| = A \cos \alpha$

The direction of A_{\parallel} is along $\hat{\mathbf{n}}$, so it follows that

 $\mathbf{A}_{\parallel} = (\mathbf{A} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.$ $|\mathbf{A}_{\perp}| = A \sin \alpha = |\hat{\mathbf{n}} \times \mathbf{A}|$

The direction of $(\hat{\mathbf{n}} \times \mathbf{A})$ is into the paper, so taking its cross product with $\hat{\mathbf{n}}$ gives a vector $(\hat{\mathbf{n}} \times \mathbf{A}) \times \hat{\mathbf{n}}$ along \mathbf{A}_{\perp} and with the correct magnitude. Hence

 $\mathbf{A} = (\mathbf{A} \cdot \mathbf{\hat{n}})\mathbf{\hat{n}} + (\mathbf{\hat{n}} \times \mathbf{A}) \times \mathbf{\hat{n}}$

1.14 Two points

 $\mathbf{S} = \mathbf{r}_2 - \mathbf{r}_1 \qquad \mathbf{B} = x\mathbf{S} \qquad \mathbf{A} = \mathbf{r}_1 + \mathbf{B}$ $x = 0 \text{ at } t = 0; \ x = 1 \text{ at } t = T$ so that x = t/T, linear in t $\mathbf{A} = \mathbf{r}_1 + x\mathbf{S} = \mathbf{r}_1 + \frac{t}{T}(\mathbf{r}_2 - \mathbf{r}_1) = \left(1 - \frac{t}{T}\right)\mathbf{r}_1 + \frac{t}{T}\mathbf{r}_2$



1.15 Great circle

Consider vectors \mathbf{R}_1 and \mathbf{R}_2 from the center of a sphere of radius *R* to points on the surface. To avoid complications, the sketch shows the geometry of a generic vector \mathbf{R}_i (*i* = 1 or 2) making angles λ_i and ϕ_i . The magnitude of \mathbf{R}_i is *R*, so $R_1 = R_2 = R$. The coordinates of a point on the surface are

 $\mathbf{R}_{i} = R \cos \lambda_{i} \cos \phi_{i} \, \mathbf{\hat{i}} + R \cos \lambda_{i} \sin \phi_{i} \, \mathbf{\hat{j}} + R \sin \lambda_{i} \, \mathbf{\hat{k}}$

The angle between two points can be found using the dot product.

$$\theta(1,2) = \arccos\left(\frac{\mathbf{R}_1 \cdot \mathbf{R}_2}{R_1 R_2}\right) = \arccos\left(\frac{\mathbf{R}_1 \cdot \mathbf{R}_2}{R^2}\right)$$

Note that $\theta(1, 2)$ is in radians.

The great circle distance between \mathbf{R}_1 and \mathbf{R}_2 is $S = R\theta(1, 2)$.

 $\mathbf{R_1} \cdot \mathbf{R_2} = R^2(\cos \lambda_1 \cos \phi_1 \cos \lambda_2 \cos \phi_2 + \cos \lambda_1 \sin \phi_1 \cos \lambda_2 \sin \phi_2 + \sin \lambda_1 \sin \lambda_2)$

Hence

$$S = R \theta(1, 2)$$

= R arccos [cos λ_1 cos λ_2 (cos ϕ_1 cos ϕ_2 + sin ϕ_1 sin ϕ_2) + sin λ_1 sin λ_2]
= R arccos $\left\{ \frac{1}{2} \cos(\lambda_1 + \lambda_2) \left[\cos(\phi_1 - \phi_2) - 1 \right] + \frac{1}{2} \cos(\lambda_1 - \lambda_2) \left[\cos(\phi_1 - \phi_2) + 1 \right] \right\}$



1.16 Measuring g

The motion is free fall with uniform acceleration, so the trajectory is a parabola, as shown in the sketch. Take the initial conditions at T=0 to be $z = z_A$ and $v = v_A$. The height z is then

$$z = z_A + v_A T - \frac{1}{2}gT^2$$

The height is again z_A when $T = T_A$.

$$z_A = z_A + v_A T_A - \frac{1}{2}gT_A^2$$

so that

$$0 = v_A T_A - \frac{1}{2}gT_A^2 \implies v_A = \frac{1}{2}gT_A$$

By the symmetry of the trajectory, the body reaches height z_B for the second time at $T = \frac{1}{2}(T_A + T_B)$.

$$h = z_B - z_A$$

$$= \left[z_A + \frac{1}{2} v_A (T_A + T_B) - \frac{1}{2} g [\frac{1}{2} (T_A + T_B)]^2 \right] - \left[z_A + v_A T_A - \frac{1}{2} g T_A^2 \right]$$

$$= \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) g T_A (T_A + T_B) - \frac{1}{8} g (T_A + T_B)^2$$

$$= \frac{1}{8} g (T_A^2 - T_B^2)$$

$$g = \frac{8h}{T_A^2 - T_B^2}$$



1.17 Rolling drum

The drum rolls without slipping, so that when it has rotated through an angle θ , it advances down the plane by a distance *x* equal to the arc length $s = R\theta$ laid down.

$$\begin{aligned} x &= R\theta \\ a &= \ddot{x} = R\ddot{\theta} = R\alpha \end{aligned}$$

so that

$$\alpha = \frac{a}{R}$$

1.18 Elevator and falling marble

Starting at t = 0, the elevator moves upward with uniform speed v_0 , so its height above the ground at time t is $z = v_0 t$.

At time T_1 , $h = v_0T_1$, so that $T_1 = h/v_0$. At the instant T_1 when the marble is released, the marble is at height hand has an instantaneous speed v_0 . Its height z at a later time t is then

$$z = h + v_0(t - T_1) - \frac{1}{2}g(t - T_1)^2$$

The marble hits the ground h = 0 at time $t = T_2$.

$$0 = h + v_0(T_2 - T_1) - \frac{1}{2}g(T_2 - T_1)^2$$

= $h + \frac{h}{T_1}(T_2 - T_1) - \frac{1}{2}g(T_2 - T_1)^2$
= $h\frac{T_2}{T_1} - \frac{1}{2}g(T_2 - T_1)^2$
 $h = \frac{1}{2}\frac{T_1}{T_2}g(T_2 - T_1)^2$



1.19 Relative velocity



1.20 Sportscar

With reference to the sketch, the distance D traveled is the area under the plot of speed vs. time. The goal is to minimize the time while keeping D constant. This involves accelerating with maximum acceleration a_a for time t_0 and then braking with maximum (negative) acceleration a_b to bring the car to rest.

$$v_{max} = a_{a}t_{0} = a_{b}(T - t_{0})$$

$$t_{0} = \frac{a_{b}T}{a_{a} + a_{b}}$$

$$D = \frac{1}{2}v_{max}T = \frac{1}{2}a_{a}t_{0}T = \frac{1}{2}\left(\frac{a_{a}a_{b}}{a_{a} + a_{b}}\right)T^{2}$$

$$T = \sqrt{\frac{2D(a_{a} + a_{b})}{a_{a}a_{b}}}$$

$$a_{a} = \frac{100 \text{ km/hr}}{3.5 \text{ s}} = \left(\frac{100 \text{ km}}{\text{hr}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)\left(\frac{1}{3.5 \text{ s}}\right) \approx 7.94 \text{ m/s}^{2}$$

$$a_{b} = 0.7g = 0.7(9.80 \text{ m/s}^{2}) \approx 6.86 \text{ m/s}^{2}$$

$$T = \sqrt{\frac{(2000 \text{ m})(6.86 + 7.94) \text{ m/s}^{2}}{(6.86 \text{ m/s}^{2})(7.94 \text{ m/s}^{2})}} \approx 23.5 \text{ s}$$

1.21 Particle with constant radial velocity

(a)

$$\mathbf{v} = \dot{r} \, \hat{\mathbf{r}} + r \dot{\theta} \, \hat{\theta} = (4.0 \,\mathrm{m/s}) \, \hat{\mathbf{r}} + (3.0 \,\mathrm{m})(2.0 \,\mathrm{rad/s}) \, \hat{\theta}$$

(Note that radians are dimensionless.)
 $\mathbf{v} = (4.0 \, \hat{\mathbf{r}} + 6.0 \, \hat{\theta}) \,\mathrm{m/s}$ $\mathbf{v} = \sqrt{v_r^2 + v_{\theta}^2} = \sqrt{16.0 + 36.0} \approx 7.2 \,\mathrm{m/s}$
(b)
 $\mathbf{a} = (\ddot{r} - r \dot{\theta}^2) \, \hat{\mathbf{r}} + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \, \hat{\theta}$
 $\ddot{r} = 0 \,\mathrm{and} \, \ddot{\theta} = 0$
 $a_r = -r \dot{\theta}^2 = -(3.0 \,\mathrm{m})(2.0 \,\mathrm{rad/s})^2 = -12.0 \,\mathrm{m/s}^2$
 $a_{\theta} = 2\dot{r} \dot{\theta} = 2(4.0 \,\mathrm{m/s})(2.0 \,\mathrm{rad/s}) = 16.0 \,\mathrm{m/s}^2$
 $a = \sqrt{a_r^2 + a_{\theta}^2} = \sqrt{144.0 + 256.0} = 20.0 \,\mathrm{m/s}^2$

1.22 Jerk

Refer to the Appendix for a corrected solution.

1.23 Smooth elevator ride

(a) Let
$$a(t) \equiv acceleration$$

 $a(t) = \frac{1}{2}a_m[1 - \cos(2\pi t/T)] \quad 0 \le t \le T$
 $a(t) = -\frac{1}{2}a_m[1 - \cos(2\pi t/T)] \quad T \le t \le 2T$
Let $j(t) \equiv jerk$
 $j(t) = \frac{da}{dt}$
 $j(t) = a_m(\pi/T)\sin(2\pi t/T) \quad 0 \le t \le T$
 $j(t) = -a_m(\pi/T)\sin(2\pi t/T) \quad T \le t \le 2T$

Let $v(t) \equiv speed$

$$v(t) = v(0) + \int_0^t a(t')dt' \quad 0 \le t \le T$$

= $\frac{1}{2}a_m[t - (T/2\pi)\sin(2\pi t/T)]$
$$v(t) = v(T) + \int_T^t a(t')dt' \quad T \le t \le 2T$$

= $\frac{1}{2}a_mT - \frac{1}{2}a_m[(t - T) - (T/2\pi)\sin(2\pi t/T)]$
= $\frac{1}{2}a_m[(2T - t) + (T/2\pi)\sin(2\pi t/T)]$

The sketch (in color) shows the jerk j(t) (red), the acceleration a(t) (green), and the speed v(t) (black) versus time t.



(b) The speed v(t) is the area under the curve of a(t). As the sketch indicates, v(t) increases with time up to t = T, and then decreases. The maximum speed v_{max} therefore occurs at t = T, so that $v_{max} = v(T)$.

$$v_{max} = v(0) + \int_0^T a(t')dt' = \frac{1}{2}a_m \int_0^T [1 - \cos(2\pi t'/T)]dt'$$
$$= \frac{1}{2}a_m \left[t' - (T/2\pi)\sin(2\pi t'/T)\right]|_0^T = \frac{1}{2}a_m T$$

(c) For $t \ll T$, we can use the small angle approximation:

$$\sin \theta = \left[\theta - \frac{1}{3!}\theta^3 + \ldots\right]$$

$$v(t) = \int_0^t a(t')dt' = \frac{1}{2}a_m[t - (T/2\pi)\sin(2\pi t/T)]$$

$$= \frac{a_m}{2}\{t - (T/2\pi)[(2\pi t/T) - \frac{1}{3!}(2\pi t/T)^3 + \ldots\}$$

$$\approx \frac{a_m}{2}\{\frac{1}{3!}(2\pi/T)^2t^3\} \approx a_m\left(\frac{\pi^2}{3}\right)\left(\frac{t^3}{T^2}\right)$$

(d) *direct method:*

Let the distance at time t be x(t).

$$x(t) = \int v(t')dt'$$

where

$$\begin{aligned} v(t) &= \frac{1}{2} \int_0^t a(t')dt' \quad 0 \le t \le T \\ &= \frac{a_m}{2} [t - (T/2\pi)\sin(2\pi t/T)] \quad 0 \le t \le T \\ v(t) &= \int_0^T a(t')dt' + \int_T^t a(t')dt' \quad T \le t \le 2T \\ &= \frac{a_m}{2} [T - t + T + (T/2\pi)\sin(2\pi t/T)] \quad T \le t \le 2T \end{aligned}$$

(Note that v(2T) = 0.) Then

$$D = x(2T)$$

= $\frac{a_m}{2} \int_0^T [t' - (T/2\pi)\sin(2\pi t'/T)]dt' + \frac{a_m}{2} \int_T^{2T} [2T - t' + (T/2\pi)\sin(2\pi t'/T)]dt'$
= $\frac{a_m}{2}T^2$

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(e) symmetry method:

By symmetry, the distance from x(0) to x(T) and the distance from x(T) to x(2T) are equal. The distance from x(0) to x(T) is

$$\begin{aligned} x(T) &= \int_0^T v(t')dt' \\ &= \frac{a_m}{2} \int_0^T [t - (T/2\pi)\sin(2\pi t'/T)]dt' \\ &= \frac{a_m}{2} \left[t'^2/2 + (T/2\pi)^2\cos(2\pi t'/T) \right]_0^T = \frac{a_m}{4}T^2 \end{aligned}$$

By symmetry

$$D = 2x(T) = \frac{1}{2}a_m T^2$$

as before.

1.24 Rolling tire

Let *x*, *y* be the coordinates of the pebble measured from the stationary origin. Let ρ be the vector from the stationary origin to the center of the rolling tire, and let **R**' be the vector from the center of the tire to the pebble.

$$\rho = R\theta \,\hat{\mathbf{i}} + R \,\hat{\mathbf{j}}$$
$$\mathbf{R}' = -R \sin\theta \,\hat{\mathbf{i}} - R \cos\theta \,\hat{\mathbf{j}}$$



From the diagram, the vector from the origin to the pebble is

$$x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boldsymbol{\rho} + \mathbf{R}' = R\theta\hat{\mathbf{i}} + R\hat{\mathbf{j}} - R\sin\theta\hat{\mathbf{i}} - R\cos\theta\hat{\mathbf{j}}$$
$$x = R\theta - R\sin\theta \quad \dot{x} = R\dot{\theta} - R\cos\theta\dot{\theta}$$
$$y = R - R\cos\theta \quad \dot{y} = R\sin\theta\dot{\theta}$$

The tire is rolling at constant speed without slipping: $\theta = \omega t = (V/R)t$.

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 $\dot{x} = R\omega - R\omega\cos\theta \quad \ddot{x} = R\omega^2\sin\theta$ $\dot{y} = R\omega\sin\theta \quad \ddot{y} = R\omega^2\cos\theta$

Note that

$$\ddot{x}\hat{i} + \ddot{y}\hat{j} = \ddot{\rho} + \ddot{R'} = \ddot{R'}$$

The pebble on the tire experiences an inward radial acceleration V^2/R , and from the results for \ddot{x} and \ddot{y}

$$\sqrt{\ddot{x}^2 + \ddot{y}^2} = R\omega^2$$
$$= \frac{V^2}{R}$$

as expected.

This result shows that the acceleration measured in the stationary system is the same as measured in the system moving uniformly along with the tire.

1.25 Spiraling particle

$$r = \frac{\theta}{\pi} \qquad \theta = \frac{\alpha t^2}{2}$$

$$r = \frac{\alpha t^2}{2\pi}$$

$$\dot{r} = \frac{\alpha t}{\pi} \qquad \dot{\theta} = \alpha t$$

$$\ddot{r} = \frac{\alpha}{\pi} \qquad \ddot{\theta} = \alpha$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{\hat{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{\hat{\theta}} = \left(\frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi}\right)\,\mathbf{\hat{r}} + \left(\frac{5\alpha^2 t^2}{2\pi}\right)\,\mathbf{\hat{\theta}}$$

(b)

$$a_r = \frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi} = 0 \text{ at time t'}$$
$$\frac{\alpha}{\pi} = \frac{\alpha^3 t'^4}{2\pi} \implies t'^2 = \frac{\sqrt{2}}{\alpha}$$
$$\theta(t') = \frac{\alpha t'^2}{2} = \frac{1}{\sqrt{2}} \text{ rad}$$

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(c)

 $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{\hat{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{\hat{\theta}}$

Using the expression for θ from part (a),

$$\mathbf{a} = \left(\frac{\alpha}{\pi}\right) \left[(1 - 2\theta^2) \,\hat{\mathbf{r}} + 5\theta \,\hat{\boldsymbol{\theta}} \right]$$

Setting $|a_r| = |a_{\theta}|$, then $|1 - 2\theta^2| = |5\theta|$

If
$$\theta < \frac{1}{\sqrt{2}}$$
, then $1 - 2\theta^2 = 5\theta$

Because $\theta \ge 0$, the only allowable root is

$$\theta = \frac{-5 + \sqrt{33}}{4} \approx 0.186 \text{ rad} \approx 10^{\circ}$$

If $\theta > \frac{1}{\sqrt{2}}$, then $2\theta^2 - 1 = 5\theta$
 $\theta = \frac{5 + \sqrt{33}}{4} \approx 2.69 \text{ rad} \approx 154^{\circ}$



In the sketch, the velocity vectors are in scale to one another, as are the acceleration vectors.

NOTE: The figure is an example of an *Archimedean spiral*. In polar coordinates, the equation of an Archimedean spiral is $r = A \theta$, where A is a constant. A fundamental property of an Archimedean spiral is that the radial spacing between adjacent turns is the same everywhere on the spiral. Consider a point (r, θ) on the spiral. The point on the adjacent turn along the same radial line thus has coordinates $(r', \theta + 2\pi)$. Then

$$\Delta r = r' - r = A \left(\theta + 2\pi\right) - A \theta$$
$$= 2\pi A$$

a constant, the same at any point of the spiral.

1.26 Range on a hill

The trajectory of the rock is described by coordinates x and y, as shown in the sketch. Let the initial velocity of the rock be v_0 at angle θ .

$$x = (v_0 \cos \theta) t$$
 $y = (v_0 \sin \theta) t - \frac{1}{2}g t^2$

The locus of the hill is $y = -x \tan \phi$

Let the rock land on the hill at time t'.

$$t' = \frac{x}{v_0 \cos \theta}$$

The locus of the hill and the trajectory of the rock intersect at t'.

$$-x \tan \phi = x \tan \theta - \frac{1}{2} \left(\frac{g}{v_0^2} \right) \left(\frac{x^2}{\cos^2 \theta} \right)$$

Solving for *x*,

$$x = \left(\frac{2v_0^2}{g}\right) \left[\cos\theta\sin\theta + (\cos^2\theta)\tan\phi\right] = \left(\frac{2v_0^2}{g}\right) \left[\frac{1}{2}\sin 2\theta + (\cos^2\theta)\tan\phi\right]$$

The condition for maximum range is $dx/d\theta = 0$. Note that ϕ is a constant.

$$\frac{dx}{d\theta} = 0 = \cos 2\theta - 2\sin \theta \cos \theta \tan \phi = \cos 2\theta - (\sin 2\theta) \tan \phi$$
$$t 2\theta = \tan \phi$$

 $\cot 2\theta = \tan \phi$

$$\tan 2\theta = \tan\left(\frac{\pi}{2} - \phi\right)$$

 $\theta = \frac{\pi}{4} - \frac{\phi}{2}$ for maximum range

The sketch is drawn for the case $\phi = 20^{\circ}$ and $v_0 = 5.0$ m/s.



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VECTORS AND KINEMATICS

1.27 Peaked roof

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Let the initial speed at t = 0 be v_0 . A straightforward way to solve this problem is to write the equations of motion in a uniform gravitational field, as follows:

$$x = -h + v_{0x}t \qquad y = v_{0y}t - \frac{1}{2}gt^{2}$$
$$v_{x} = v_{0x} \qquad v_{y} = v_{0y} - gt$$

At time T, the ball is at the peak, where y = h and $v_y = 0$.

$$0 = v_{0y} - gT \implies T = \frac{v_{0y}}{g}$$
$$h = v_0 yT - \frac{1}{2}gT^2 = \frac{v_{0y}^2}{g} - \frac{1}{2}\frac{v_{0y}^2}{g}$$
$$v_{0y}^2 = 2gh$$

At time T, x = 0.

$$0 = -h + v_{0x}T \implies v_{0x} = \frac{h}{T} = \frac{\sqrt{gh}}{2}$$

We then have

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{2 + \frac{1}{2}}\sqrt{gh} = \sqrt{\frac{5}{2}}\sqrt{gh}$$

A more physical approach is to note that the vertical speed needed to reach the peak is the same as the speed v_{0y} a mass acquires falling a distance h: $v_{0y} = \sqrt{2gh}$. The time T to fall that distance is $T = v_{0y}/g$. The horizontal distance traveled in the time T is

$$h = v_{0x}T = v_{0x}\left(\frac{v_{0y}}{g}\right) = v_{0x}\sqrt{\frac{2h}{g}}$$
$$v_{0x} = \sqrt{\frac{gh}{2}}$$

The initial speed v_0 is therefore

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{2 + \frac{1}{2}}\sqrt{gh} = \sqrt{\frac{5}{2}}\sqrt{gh}$$