INSTRUCTOR'S SOLUTIONS MANUAL

AN INTRODUCTION TO MATHEMATICAL STATISTICS AND ITS APPLICATIONS

SIXTH EDITION

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Pearson

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Publishing as Pearson, 330 Hudson Street, NY NY 10013

Pearson

ISBN-13: 978-0-13-411427-9
ISBN-10: 0-13-411427-2

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Chapter 2: Probability

Section 2.2: Sample Spaces and the Algebra of Sets

2.2.1 $S = \{(s,s,s), (s,s,f), (s,f,s), (f,s,s), (s,f,f), (f,s,f), (f,f,s), (f,f,f)\}$
$A = \{\ (s,f,s), (f,s,s)\}$; $B = \{(f,f,f)\}$

2.2.2 Let (x, y, z) denote a red x, a blue y, and a green z.
Then $A = \{(2,2,1), (2,1,2), (1,2,2), (1,1,3), (1,3,1), (3,1,1)\}$

2.2.3 (1,3,4), (1,3,5), (1,3,6), (2,3,4), (2,3,5), (2,3,6)

2.2.4 There are 16 ways to get an ace and a 7, 16 ways to get a 2 and a 6, 16 ways to get a 3 and a 5, and 6 ways to get two 4's, giving 54 total.

2.2.5 The outcome sought is (4, 4). It is "harder" to obtain than the set {(5, 3), (3, 5), (6, 2), (2, 6)} of other outcomes making a total of 8.

2.2.6 The set N of five card hands in hearts that are not flushes are called *straight flushes*. These are five cards whose denominations are consecutive. Each one is characterized by the lowest value in the hand. The choices for the lowest value are A, 2, 3, …, 10. (Notice that an ace can be high or low). Thus, N has 10 elements.

2.2.7 P = {right triangles with sides $(5, a, b)$: $a^2 + b^2 = 25$}

2.2.8 A = {*SSBBBB*, *SBSBBB*, *SBBSBB*, *SBBBSB*, *BSSBBB*, *BSBSBB*, *BSBBSB*, *BBSSBB*, *BBSBSB*, *BBBSSB*}

2.2.9 (a) S = {(0, 0, 0, 0) (0, 0, 0, 1), (0, 0, 1, 0), (0, 0, 1, 1), (0, 1, 0, 0), (0, 1, 0, 1), (0, 1, 1, 0), (0, 1, 1, 1), (1, 0, 0, 0), (1, 0, 0, 1), (1, 0, 1, 0), (1, 0, 1, 1,), (1, 1, 0, 0), (1, 1, 0, 1), (1, 1, 1, 0), (1, 1, 1, 1,)}
(b) A = {(0, 0, 1, 1), (0, 1, 0, 1), (0, 1, 1, 0), (1, 0, 0, 1), (1, 0, 1, 0), (1, 1, 0, 0,)}
(c) $1 + k$

2.2.10 (a) S = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (4, 1), (4, 2), (4, 4)}
(b) {2, 3, 4, 5, 6, 8}

2.2.11 Let p_1 and p_2 denote the two perpetrators and i_1, i_2, and i_3, the three in the lineup who are innocent.
Then $S = \{(p_1,i_1), (p_1,i_2), (p_1,i_3), (p_2,i_1), (p_2,i_2), (p_2,i_3), (p_1,p_2), (i_1,i_2), (i_1,i_3), (i_2,i_3)\}$.
The event A contains every outcome in S except (p_1, p_2).

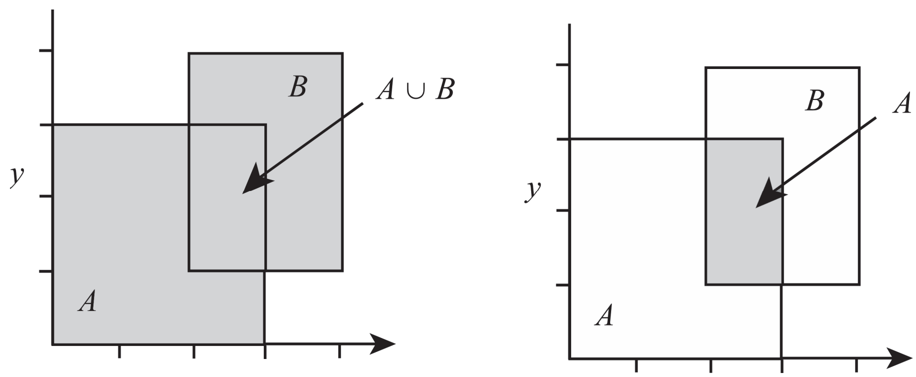
2.2.12 The quadratic equation will have complex roots—that is, the event A will occur—if $b^2 - 4ac < 0$.

2.2.13 In order for the shooter to win with a point of 9, one of the following (countably infinite) sequences of sums must be rolled: (9,9), (9, no 7 or no 9,9), (9, no 7 or no 9, no 7 or no 9,9), …

2.2.14 Let (x, y) denote the strategy of putting x white chips and y red chips in the first urn (which results in $10 - x$ white chips and $10 - y$ red chips being in the second urn). Then $S = \{(x, y): x = 0,1,...,10,\ \ y = 0,1,...,10,\ \text{and } 1 \le x + y \le 19\}$. Intuitively, the optimal strategies are (1, 0) and (9, 10).

2.2.15 Let A_k be the set of chips put in the urn at $1/2^k$ minute until midnight. For example, $A_1 = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$. Then the set of chips in the urn at midnight is $\bigcup_{k=1}^{\infty}(A_k - \{k+1\}) = \varnothing$.

2.2.16 move arrow on first figure raise B by 1

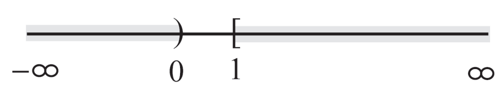


2.2.17 If $x^2 + 2x \le 8$, then $(x + 4)(x - 2) \le 0$ and $A = \{x: -4 \le x \le 2\}$. Similarly, if $x^2 + x \le 6$, then $(x + 3)(x - 2) \le 0$ and $B = \{x: -3 \le x \le 2)$. Therefore, $A \cap B = \{x: -3 \le x \le 2\}$ and $A \cup B = \{x: -4 \le x \le 2\}$.

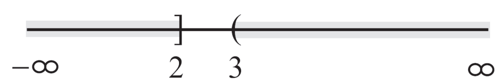
2.2.18 $A \cap B \cap C = \{x: x = 2, 3, 4\}$

2.2.19 The system fails if either the first pair fails or the second pair fails (or both pairs fail). For either pair to fail, though, both of its components must fail. Therefore, $A = (A_{11} \cap A_{21}) \cup (A_{12} \cap A_{22})$.

2.2.20 (a) 

(b) 

(c) empty set

(d) 

2.2.21 40

2.2.22 (a) $\{E1, E2\}$ (b) $\{S1, S2, T1, T2\}$ (c) $\{A, I\}$

2.2.23 (a) If s is a member of $A \cup (B \cap C)$ then s belongs to A or to $B \cap C$. If it is a member of A or of $B \cap C$, then it belongs to $A \cup B$ and to $A \cup C$.
Thus, it is a member of $(A \cup B) \cap (A \cup C)$.
Conversely, choose s in $(A \cup B) \cap (A \cup C)$. If it belongs to A, then it belongs to $A \cup (B \cap C)$. If it does not belong to A, then it must be a member of $B \cap C$.
In that case it also is a member of $A \cup (B \cap C)$.

(b) If s is a member of $A \cap (B \cup C)$ then s belongs to A and to $B \cup C$. If it is a member of B, then it belongs to $A \cap B$ and, hence, $(A \cap B) \cup (A \cap C)$. Similarly, if it belongs to C, it is a member of $(A \cap B) \cup (A \cap C)$. Conversely, choose s in $(A \cap B) \cup (A \cap C)$. Then it belongs to A. If it is a member of $A \cap B$ then it belongs to $A \cap (B \cup C)$. Similarly, if it belongs to $A \cap C$, then it must be a member of $A \cap (B \cup C)$.

2.2.24 Let $B = A_1 \cup A_2 \cup \ldots \cup A_k$. Then $A_1^C \cap A_2^C \cap \ldots \cap A_k^C = (A_1 \cup A_2 \cup \ldots \cup A_k)^C = B^C$. Then the expression is simply $B \cup B^C = S$.

2.2.25 (a) Let s be a member of $A \cup (B \cup C)$. Then s belongs to either A or $B \cup C$ (or both). If s belongs to A, it necessarily belongs to $(A \cup B) \cup C$. If s belongs to $B \cup C$, it belongs to B or C or both, so it must belong to $(A \cup B) \cup C$. Now, suppose s belongs to $(A \cup B) \cup C$. Then it belongs to either $A \cup B$ or C or both. If it belongs to C, it must belong to $A \cup (B \cup C)$. If it belongs to $A \cup B$, it must belong to either A or B or both, so it must belong to $A \cup (B \cup C)$.

(b) Suppose s belongs to $A \cap (B \cap C)$, so it is a member of A and also $B \cap C$. Then it is amember of A and of B and C. That makes it a member of $(A \cap B) \cap C$. Conversely, if s is a member of $(A \cap B) \cap C$, a similar argument shows it belongs to $A \cap (B \cap C)$.

2.2.26 (a) $A^C \cap B^C \cap C^C$
(b) $A \cap B \cap C$
(c) $A \cap B^C \cap C^C$
(d) $(A \cap B^C \cap C^C) \cup (A^C \cap B \cap C^C) \cup (A^C \cap B^C \cap C)$
(e) $(A \cap B \cap C^C) \cup (A \cap B^C \cap C) \cup (A^C \cap B \cap C)$

2.2.27 A is a subset of B.

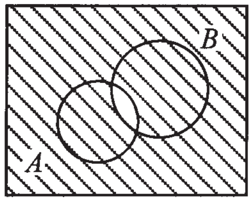
2.2.28 (a) $\{0\} \cup \{x: 5 \le x \le 10\}$
(b) $\{x: 3 \le x < 5\}$
(c) $\{x: 0 < x \le 7\}$
(d) $\{x: 0 < x < 3\}$
(e) $\{x: 3 \le x \le 10\}$
(f) $\{x: 7 < x \le 10\}$

2.2.29 (a) B and C
(b) B is a subset of A.

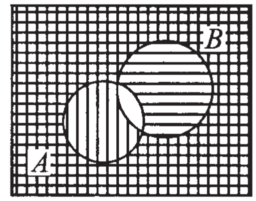
2.2.30 (a) $A_1 \cap A_2 \cap A_3$
(b) $A_1 \cup A_2 \cup A_3$
The second protocol would be better if speed of approval matters. For very important issues, the first protocol is superior.

2.2.31 Let A and B denote the students who saw the movie the first time and the second time, respectively. Then $N(A) = 850$, $N(B) = 690$, and $N[(A \cup B)^C] = 4700$ (implying that $N(A \cup B) = 1300$). Therefore, $N(A \cap B)$ = number who saw movie twice $= 850 + 690 - 1300 = 240$.

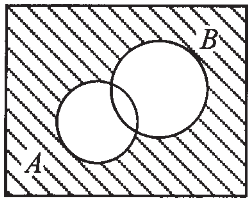
2.2.32 (a)



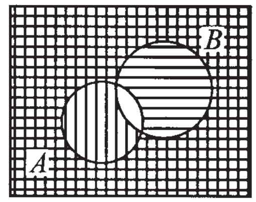
$(A \cap B)^C = A^C \cup B^C$



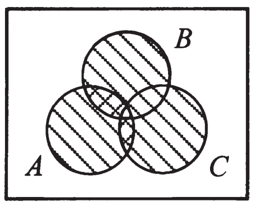
(b)



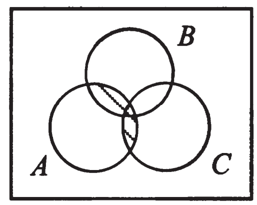
$(A \cup B)^C = A^C \cap B^C$



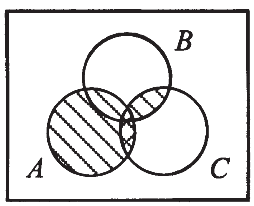
2.2.33 (a)



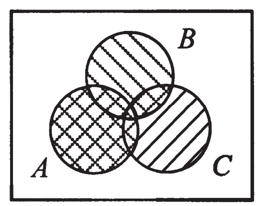
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



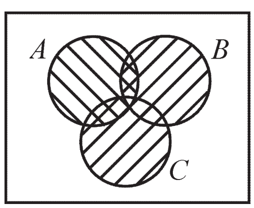
(b)



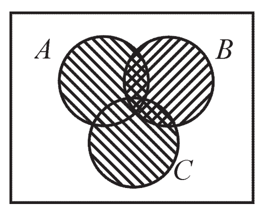
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



2.2.34 (a)

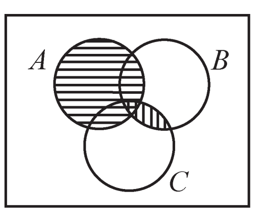


$A \cup (B \cup C)$

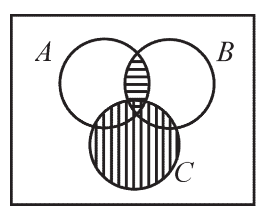


$(A \cup B) \cup C$

(b)



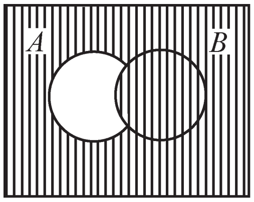
$A \cap (B \cap C)$



$(A \cap B) \cap C$

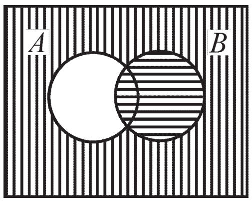
2.2.35 A and B are subsets of $A \cup B$.

2.2.36 (a)



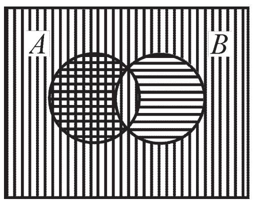
$(A \cap B^C)^C = A^C \cup B$

(b)



$B \cup (A \cup B)^C = A^C \cup B$

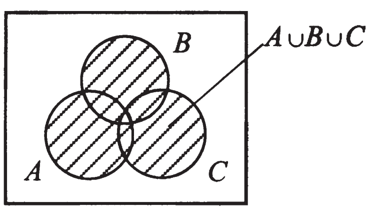
(c)



$A \cap (A \cap B)^C = A \cap B^C$

2.2.37 Let A be the set of those with MCAT scores ≥ 27 and B be the set of those with GPAs ≥ 3.5. We are given that $N(A) = 1000$, $N(B) = 400$, and $N(A \cap B) = 300$.
Then $N(A^C \cap B^C) = N[(A \cup B)^C] = 1200 - N(A \cup B) = 1200 - [(N(A) + N(B) - N(A \cap B)]$
$= 1200 - [(1000 + 400 - 300] = 100$. The requested proportion is 100/1200.

2.2.38



$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)$

2.2.39 Let A be the set of those saying "yes" to the first question and B be the set of those saying "yes" to the second question. We are given that $N(A) = 600$, $N(B) = 400$, and
$N(A^C \cap B) = 300$. Then $N(A \cap B) = N(B) - N(A^C \cap B) = 400 - 300 = 100$. $N(A \cap B^C)$
$= N(A) - N(A \cap B) = 600 - 100 = 500$.

2.2.40 $N[(A \cap B)^C] = 120 - N(A \cup B) = 120 - [N(A^C \cap B) + N(A \cap B^C) + N(A \cap B)]$
$= 120 - [50 + 15 + 2] = 53$

Section 2.3: The Probability Function

2.3.1 Let L and V denote the sets of programs with offensive language and too much violence, respectively. Then $P(L) = 0.42$, $P(V) = 0.27$, and $P(L \cap V) = 0.10$.
Therefore, $P(\text{program complies}) = P((L \cup V)^C) = 1 - [P(L) + P(V) - P(L \cap V)] = 0.41$.

2.3.2 $P(A \text{ or } B \text{ but not both}) = P(A \cup B) - P(A \cap B) = P(A) + P(B) - P(A \cap B) - P(A \cap B)$
$= 0.4 + 0.5 - 0.1 - 0.1 = 0.7$

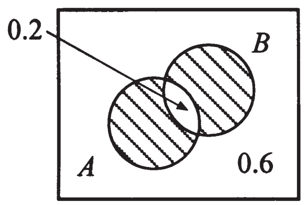
2.3.3 (a) $1 - P(A \cap B)$
(b) $P(B) - P(A \cap B)$

2.3.4 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3$; $P(A) - P(A \cap B) = 0.1$. Therefore, $P(B) = 0.2$.

2.3.5 No. $P(A1 \cup A2 \cup A3) = P(\text{at least one "6" appears}) = 1 - P(\text{no 6's appear}) = 1 - \left(\frac{5}{6}\right)^3 \neq \frac{1}{2}$.

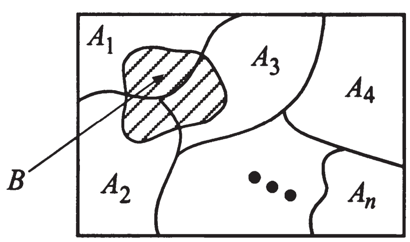
The Ai's are not mutually exclusive, so $P(A_1 \cup A_2 \cup A_3) \neq P(A_1) + P(A_2) + P(A_3)$.

2.3.6



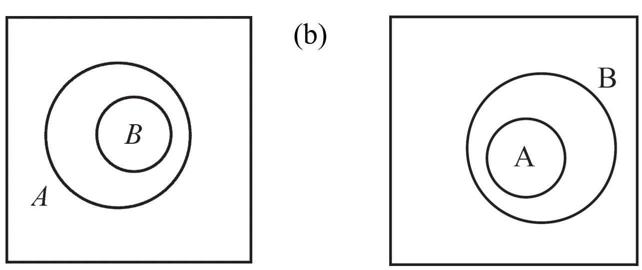
$P(A \text{ or } B \text{ but not both}) = 0.5 - 0.2 = 0.3$

2.3.7



By inspection, $B = (B \cap A_1) \cup (B \cap A_2) \cup \ldots \cup (B \cap A_n)$.

2.3.8 (a) (b)



2.3.9 $P(\text{odd man out}) = 1 - P(\text{no odd man out}) = 1 - P(HHH \text{ or } TTT) = 1 - \frac{2}{8} = \frac{3}{4}$

2.3.10 $A = \{2, 4, 6, \ldots, 24\}$; $B = \{3, 6, 9, \ldots, 24)$; $A \cap B = \{6, 12, 18, 24\}$.
Therefore, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{12}{24} + \frac{8}{24} - \frac{4}{24} = \frac{16}{24}$.

2.3.11 Let A: State wins Saturday and B: State wins next Saturday. Then $P(A) = 0.10$, $P(B) = 0.30$, and $P(\text{lose both}) = 0.65 = 1 - P(A \cup B)$, which implies that $P(A \cup B) = 0.35$. Therefore, $P(A \cap B) = 0.10 + 0.30 - 0.35 = 0.05$, so $P(\text{State wins exactly once}) = P(A \cup B) - P(A \cap B)$ $= 0.35 - 0.05 = 0.30$.

2.3.12 Since A_1 and A_2 are mutually exclusive and cover the entire sample space, $p_1 + p_2 = 1$.
But $3p_1 - p_2 = \frac{1}{2}$, so $p_2 = \frac{5}{8}$.

2.3.13 Let F: female is hired and T: minority is hired. Then $P(F) = 0.60$, $P(T) = 0.30$, and $P(F^C \cap T^C) = 0.25 = 1 - P(F \cup T)$. Since $P(F \cup T) = 0.75$, $P(F \cap T)$ $= 0.60 + 0.30 - 0.75 = 0.15$.

2.3.14 The smallest value of $P[(A \cup B \cup C)^C]$ occurs when $P(A \cup B \cup C)$ is as large as possible. This, in turn, occurs when A, B, and C are mutually disjoint. The largest value for $P(A \cup B \cup C)$ is $P(A) + P(B) + P(C) = 0.2 + 0.1 + 0.3 = 0.6$. Thus, the smallest value for $P[(A \cup B \cup C)^C]$ is 0.4.

2.3.15 (a) $X^C \cap Y = \{(H, T, T, H), (T, H, H, T)\}$, so $P(X^C \cap Y) = 2/16$
(b) $X \cap Y^C = \{(H, T, T, T), (T, T, T, H), (T, H, H, H), (H, H, H, T)\}$ so $P(X \cap Y^C) = 4/16$

2.3.16 $A = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$
$A \cap B^C = \{(1, 5), (3, 3), (5, 1)\}$, so $P(A \cap B^C) = 3/36 = 1/12$.

2.3.17 $A \cap B$, $(A \cap B) \cup (A \cap C)$, A, $A \cup B$, S

2.3.18 Let A be the event of getting arrested for the first scam; B, for the second. We are given $P(A) = 1/10$, $P(\mathrm{B}) = 1/30$, and $P(A \cap B) = 0.0025$. Her chances of not getting arrested are $P[(A \cup B)^C] = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] = 1 - [1/10 + 1/30 - 0.0025]$ $= 0.869$

Section 2.4: Conditional Probability

2.4.1 $P(\text{sum} = 10|\text{sum exceeds } 8) = \dfrac{P(\text{sum} = 10 \text{ and sum exceeds } 8)}{P(\text{sum exceeds } 8)}$

$$= \frac{P(\text{sum} = 10)}{P(\text{sum } = 9, 10, 11, \text{ or } 12)} = \frac{3/36}{4/36 + 3/36 + 2/36 + 1/36} = \frac{3}{10}.$$

2.4.2 $P(A|B) + P(B|A) = 0.75 = \frac{P(A \cap B)}{P(B)} + \frac{P(A \cap B)}{P(A)} = \frac{10P(A \cap B)}{4} + 5P(A \cap B)$, which implies that $P(A \cap B) = 0.1$.

2.4.3 If $P(A|B) = \frac{P(A \cap B)}{P(B)} < P(A)$, then $P(A \cap B) < P(A) \cdot P(B)$.

It follows that $P(B|A) = \frac{P(A \cap B)}{P(A)} < \frac{P(A) \cdot P(B)}{P(A)} = P(B)$.

2.4.4 $P(E|A \cup B) = \frac{P(E \cap (A \cup B))}{P(A \cup B)} = \frac{P(E)}{P(A \cup B)} = \frac{P(A \cup B) - P(A \cap B)}{P(A \cup B)} = \frac{0.4 - 0.1}{0.4} = \frac{3}{4}$.

2.4.5 The answer would remain the same. Distinguishing only three family types does not make them equally likely; (girl, boy) families will occur twice as often as either (boy, boy) or (girl, girl) families.

2.4.6 $P(A \cup B) = 0.8$ and $P(A \cup B) - P(A \cap B) = 0.6$, so $P(A \cap B) = 0.2$.
Also, $P(A|B) = 0.6 = \frac{P(A \cap B)}{P(B)}$, so $P(B) = \frac{0.2}{0.6} = \frac{1}{3}$ and $P(A) = 0.8 + 0.2 - \frac{1}{3} = \frac{2}{3}$.

2.4.7 Let R_i be the event that a red chip is selected on the ith draw, $i = 1, 2$.
Then $P(\text{both are red}) = P(R_1 \cap R_2) = P(R_2 \mid R_1)P(R_1) = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$.

2.4.8 $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) + P(B) - P(A \cup B)}{P(B)} = \frac{a + b - P(A \cup B)}{b}$.

But $P(A \cup B) \le 1$, so $P(A|B) \ge \frac{a + b - 1}{b}$.

2.4.9 Let W_i be the event that a white chip is selected on the ith draw, $i = 1,2$.
Then $P(W_2|W_1) = \frac{P(W_1 \cap W_2)}{P(W_1)}$. If both chips in the urn are white, $P(W_1) = 1$;

if one is white and one is black, $P(W_1) = \frac{1}{2}$.

Since each chip distribution is equally likely, $P(W_1) = 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$.

Similarly, $P(W_1 \cap W_2) = 1 \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{8}$, so $P(W_2|W_1) = \frac{5/8}{3/4} = \frac{5}{6}$.

2.4.10 $P[(A \cap B) \mid (A \cup B)^C] = \frac{P[(A \cap B) \cap (A \cup B)^C]}{P[(A \cup B)^C]} = \frac{P(\varnothing)}{P[(A \cup B)^C]} = 0$

2.4.11 (a) $P(A^C \cap B^C) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] =$
$1 - [0.65 + 0.55 - 0.25] = 0.05$

(b) $P[(A^C \cap B) \cup (A \cap B^C)] = P(A^C \cap B) + P(A \cap B^C) =$
$[P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = [0.65 - 0.25] + [0.55 - 0.25] = 0.70$
(c) $P(A \cup B) = 0.95$
(d) $P[(A \cap B)^C] = 1 - P(A \cap B) = 1 - 0.25 = 0.75$
(e) $P\{[(A^C \cap B) \cup (A \cap B^C)] | A \cup B\} = \dfrac{P[(A^C \cap B) \cup (A \cap B^C)]}{P(A \cup B)} = 0.70/0.95 = 70/95$
(f) $P(A \cap B) | A \cup B) = P(A \cap B)/P(A \cup B) = 0.25/0.95 = 25/95$
(g) $P(B|A^C) = P(A^C \cap B)/P(A^C)\ ] = [P(B) - P(A \cap B)]/[1 - P(A)] = [0.55 - 0.25]/[1 - 0.65]$
$= 30/35$

2.4.12 P(No. of heads ≥ 2| No. of heads ≤ 2)
$= P$(No. of heads ≥ 2 and No. of heads ≤ 2)/P(No. of heads ≤ 2)
$= P$(No. of heads $= 2$)/P(No. of heads ≤ 2) $= (3/8)/(7/8) = 3/7$

2.4.13 P(first die ≥ 4|sum $= 8$) $= P$(first die ≥ 4 and sum $= 8$)/P(sum $= 8$)
$= P(\{(4, 4), (5, 3), (6, 2)\}/P(\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}) = 3/5$

2.4.14 There are 4 ways to choose three aces (count which one is left out). There are 48 ways to choose the card that is not an ace, so there are $4 \times 48 = 192$ sets of cards where exactly three are aces. That gives 193 sets where there are at least three aces. The conditional probability is (1/270,725)/(193/270,725) = 1/193.

2.4.15 First note that $P(A \cup B) = 1 - P[(A \cup B)^C] = 1 - 0.2 = 0.8$.
Then $P(B) = P(A \cup B) - P(A \cap B^C) - P(A \cap B) = 0.8 - 0.3 - 0.1 = 0.5$.
Finally $P(A|B) = P(A\cap B)/P(B) = 0.1/0.5 = 1/5$

2.4.16 $P(A|B) = 0.5$ implies $P(A \cap B) = 0.5P(B)$. $P(B|A) = 0.4$ implies $P(A \cap B) = (0.4)P(A)$.
Thus, $0.5P(B) = 0.4P(A)$ or $P(B) = 0.8P(A)$.
Then, $0.9 = P(A) + P(B) = P(A) + 0.8P(A)$ or $P(A) = 0.9/1.8 = 0.5$.

2.4.17 $P[(A \cap B)^C] = P[(A \cup B)^C] + P(A \cap B^C) + P(A^C \cap B) = 0.2 + 0.1 + 0.3 = 0.6$
$P(A \cup B|(A \cap B)^C) = P[(A \cap B^C) \cup (A^C \cap B)]/P((A \cap B)^C) = [0.1 + 0.3]/0.6 = 2/3$

2.4.18 P(sum ≥ 8|at least one die shows 5)
$= P$(sum ≥ 8 and at least one die shows 5)/P(at least one die shows 5)
$= P(\{(5, 3), (5, 4), (5, 6), (3, 5), (4, 5), (6, 5), (5, 5)\})/(11/36) = 7/11$

2.4.19 P(Outandout wins|Australian Doll and Dusty Stake don't win)
$= P$(Outandout wins and Australian Doll and Dusty Stake don't win)/P(Australian Doll and Dusty Stake don't win) = 0.20/0.55 = 20/55

2.4.20 Suppose the guard will randomly choose to name Bob or Charley if they are the two to go free. Then the probability the guard will name Bob, for example, is
P(Andy, Bob) + (1/2)P(Bob, Charley) = 1/3 + (1/2)(1/3) = 1/2.
The probability Andy will go free given the guard names Bob is P(Andy, Bob)/P(Guard names Bob) = (1/3)/(1/2) = 2/3. A similar argument holds for the guard naming Charley. Andy's concern is not justified.