CHAPTER 3

3.1 Output = 5 Volts = V_o

Input = 5 μV = 5×10^{-6} volts = V_i

$$Gain = G = \frac{V_o}{V_i} = \frac{5}{5\times10^{-6}} = 10^6$$

$$G_{dB} = 20\log_{10} G = 20\log_{10} 10^6 = 120dB$$

3.2 G_{dB}=60dB

V_i=3mV=3×10^{-3} volts

G_{dB}=60dB=20$\log_{10}$G

3dB= $\log_{10}$G

$\Rightarrow$ G = 10^3

$\Rightarrow$ G = V_o/V_i

$V_o = G\times V_i = 10^3(3\times10^{-3})$

= 3 volts

3.3 Eq. 3.2 applies. For G =10, GdB = log10(10) = 20. Similarly, for G = 100 and 500, the decibel gains are 40 and 54.

3.4 The circuit resembles Fig. 3.9 (a). For this problem we want the voltage drop across the resistor Rs to be 0.01xV_s. The current in the loop is $I = V_s/(R_s + R_i)$ and the voltage drop across the resistor is $V_{drop} = I_s x R_s$.
Combining these:
$0.01\times V_s = V_s/(R_s+R_i)\times R_s = V_s/(120+R_i)120$. Solving for R_i, we get
R_i = 11,880 Ω.

3.5 The circuit resembles Fig. 3.9(a). The input voltage, V_i, is I$\times R_i$. The current is $V_s/(R_s+R_i)$. Combining, $V_i = R_i x V_s/(R_s+R_i)$. In the first case:

$0.005=5\text{x}10^6\text{x}V_s/(R_s+5\text{x}10^6)$ For the second case:

$0.0048=10{,}000\text{x}V_s/(R_s+10{,}000)$ These can be solved simultaneously to give
R_s = 416 Ω.

3.6 a) From Eq. 3.14,

$$G = 1 + \frac{R_2}{R_1}$$

$$100 = 1 + \frac{R_2}{R_1}$$

$$99 = \frac{R_2}{R_1}$$

Since R_1 and R_2 typically range from 1kΩ to 1MΩ, we arbitrarily choose:

R_2=99kΩ

$\Rightarrow$ R_1 = 1kΩ

b) f = 10 kHz = 10^4 Hz

GPB = 10^6 Hz   for 741

G = 100

From Eq. 3.15,

$$f_c = \frac{GPB}{G} = \frac{10^6\, Hz}{100} = 10^4\, Hz$$

This is the corner frequency so signal is -3dB from dc gain.

dc gain = 100 = 40dB. Gain at 10^4 Hz is then 37 dB.

From Eq. 3.16,

$$\phi = -\tan^{-1}\left(\frac{f}{f_c}\right) = -\tan^{-1}\left(\frac{10^4}{10^4}\right) = -\frac{\pi}{4} = -45^\circ$$

3.7

$$G = 1 + \frac{R_2}{R_1}$$

$$100 = 1 + \frac{R_2}{R_1}$$

$$99 = \frac{R_2}{R_1}$$

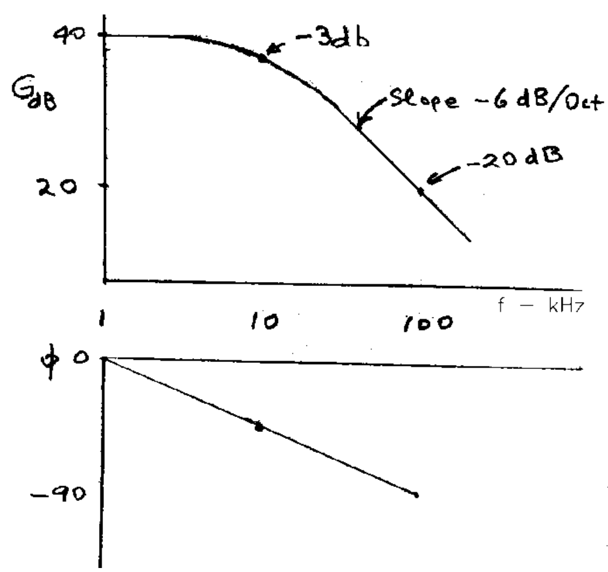
Selecting R_1 = 1kΩ, R_2 can be evaluated as 99kΩ..

Since GBP = 1 MHz = 100(Bandwidth)
⇒ bandwidth = 10 kHz = f_c

Gain will decrease 6dB from DC value for each octave above 10 kHz.
The phase angle can be determined from Eq. 3.16,

$$\phi = -\tan^{-1}\left(\frac{f}{f_c}\right)$$

f(Hz)	0	5k	10k	100k
ϕ	0	-26.6	-45	-84.3



3.8

$$G = 1000 = 1 + \frac{R_2}{R_1}$$

$$999 = \frac{R_2}{R_1}$$

Selecting R_2 = 999 kΩ, R_1 can be evaluated as 1 kΩ.
Since GBP = 1MHz for the μA741C op-amp and G = 1000 at low frequencies,

GBP = 1MHz = 1000(Bandwidth)

⇒ Bandwidth = 1 kHz = f_c

If f = 10 kHz and f_c = 1 kHz, we must calculate the number of times f_c doubles before reaching f.

$$f_c \times 2^x = f$$

$$1000 \times 2^x = 10000$$

$$\therefore x = 3.32$$

Now the gain can be calculated knowing that for each doubling the gain decreases by 6dB (i.e. per octave)

$$\text{Gain(dB)} = 20\log_{10} 1000 - 3.32(6\text{dB})$$

$$= 40\text{dB}$$

From Eq. 3.16,

$$\phi = -\tan^{-1}\left(\frac{f}{f_c}\right)$$

$$= -\tan^{-1}\left(\frac{10000}{1000}\right)$$

$$= -84.3^\circ$$

3.9 G = 100 (Actually -100 since signal inverted)

Input impedance = 1000Ω ≈ R_1
From Eq. 3.17,

$$G = -\frac{R_2}{R_1}$$

$$-100 = \frac{R_2}{1000}$$

$$\Rightarrow R_2 = 100k\Omega$$

Since $GPB_{noninv} = 10^6$ Hz, from Eq. 3.18,

$$GPB_{inv} = \frac{R_2}{R_1 + R_2} GPB_{noninv}$$

$$= \frac{100000}{1000 + 100000}\left(10^6\right)$$

$$= 9.9 \times 10^5 Hz$$

From Eq. 3.15,

$$f_c = \frac{GPB}{G} = \frac{9.9 \times 10^5}{100} = 9.9kHz$$

3.10 G = 10 (Actually -10 since output inverted)
Input impedance = 10 kΩ = 10000 Ω ≈ R_1
From Eq. 3.17,

$$G = -\frac{R_2}{R_1}$$

$$-10 = \frac{R_2}{10000}$$

$$\Rightarrow R_2 = 100k\Omega$$

Since $GPB_{noninv} = 10^6$Hz, from Eq. 3.18,

$$GPB_{inv} = \frac{R_2}{R_1 + R_2} GPB_{noninv}$$

$$= \frac{100000}{10000 + 100000}\left(10^6\right)$$

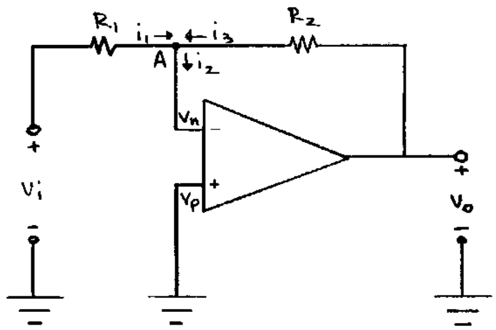
$$= 909kHz$$

From Eq. 3.15,

$$f_c = \frac{GPB}{G} = \frac{0.909 \times 10^3}{10} = 90.9kHz$$

3.11 **(a)** $10 = 2^N$. N = ln10/ln2 = 3.33
(b) dB/decade = NxdB/octave = 3.33x6 = 20 dB/decade

3.12



The gain of the op-amp itself is

$V_o = g(V_p - V_n)$ [A]

V_p is grounded so $V_p = 0$ [B]

The current through the loop including V_i, R_1, R_2, and V_o is

$$I_L = \frac{\sum V}{\sum R} = \frac{V_i - V_o}{R_1 + R_2}$$

V_n can then be evaluated as

$$V_n = V_i - I_L R_1 = V_i - \frac{R_1(V_i - V_o)}{R_1 + R_2} \quad [C]$$

Substituting [C] and [B] into [A]

$$V_o = g\left[-V_i + \frac{R_1(V_i - V_o)}{R_1 + R_2}\right]$$

Rearranging:

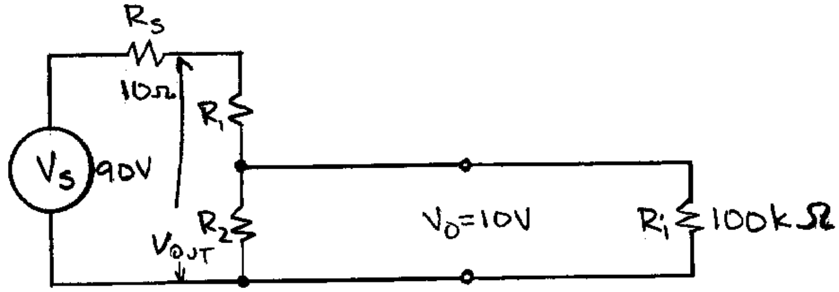
$$V_o(R_1 + R_2 + gR_1) = V_i(-R_1 - R_2 + R_1)g$$

$$\frac{V_o}{V_i} = G = \frac{-R_2 g}{R_1 + R_2 + gR_1}$$

Noting the g is very large

$$G = -\frac{R_2}{R_1}$$

3.13 The complete circuit is as follows,



For a loading error of 0.1%, the voltage drop across R_s should be $90 \times 0.001 = 0.09$ V. The current through R_s is then:

$$I_{R_S} = \frac{V_{R_s}}{R_s} = \frac{0.09}{10} = 0.009A$$

I_{Rs} also flows through R_1 and the combination of R_2 and R_i. For R_2 and R_i, we have:

$$V_o = 10 = IR = 0.009\left(\frac{1}{\frac{1}{R_2} + \frac{1}{R_i}}\right) = 0.009\left(\frac{1}{\frac{1}{R_2} + \frac{1}{100k}}\right)$$

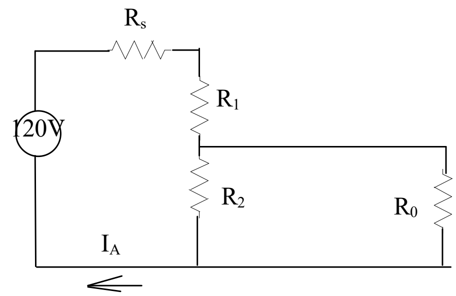
$$R_2 = 1124\Omega$$

The voltage drop across R_1=90–0.09–10= 79.91V

$$R_1 = \frac{V}{I} = \frac{79.91}{0.009} = 8879.0\Omega$$

3.14

a) If we ignore the effects of R_s and R_0, we can use Eq. 3.19:



$$\frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

$$\frac{8}{120} = \frac{R_2}{100000 + R_2}$$

$$\Rightarrow R_2 = 7142.9\Omega$$

(If we include R_s and R_0 ,the value of R_2 is 7193 Ω, less than 1% different.)

b) $I = \frac{V}{\sum R} = \frac{V_s}{R_1 + R_2} = \frac{120}{100000 + 7142.9} = 0.00112A$ (neglecting load effects)

$\Rightarrow$ P = I^2R = $(0.0012)^2(100000 + 7142.9)$ = 0.13 W

c)

$$I_A = \frac{V}{\sum R} = \frac{120}{0.5 + 100000 + \left(\frac{1}{\frac{1}{7142.9} + \frac{1}{10^6}}\right)}$$

$$= \frac{120}{0.5 + 100000 + 7092.2}$$

$$= 0.00112A$$

Voltage drop across line load resistor

$$V = I_A R = 0.00112 \times 0.5 = 0.00056V\ (small)$$

3.15 If f_1 = 7600 Hz and f_2 = 2100 Hz then the following equation may be used,

$f_2 \times 2^x = f_1$ where x = # octaves

Substituting,

$$2100 \times 2^x = 7600$$

$$2^x = 3.619$$

$$x \log 2 = \log 3.619$$

$$\Rightarrow x = 1.856\ octaves$$

3.16 f_c = 1kHz = 1000Hz , Butterworth
Rolloff = 24 dB/octave
$A_{1out} = 0.10V$
$f_1 = 3kHz = 3000Hz$
$f_2 = 20kHz = 20000Hz$
Since Rolloff = 24 dB/octave = 6n dB/octave,
$\Rightarrow$ n = 4
From Eq. 3.20,

$$G_1 = \frac{1}{\sqrt{1+\left(f_1/f_c\right)^{2n}}}$$

$$= \frac{1}{\sqrt{1+\left(3000/1000\right)^{2\times 4}}}$$

$$= 0.01234$$

$$\Rightarrow \mathrm{A}_{1in} = \frac{\mathrm{A}_{1out}}{\mathrm{G}_1} = \frac{0.10}{0.01234} = 8.1\mathrm{V} = \mathrm{A}_{2in}$$

From Eq. 3.20,

$$G_2 = \frac{1}{\sqrt{1+\left(20000/1000\right)^{2\cdot 4}}}$$

$$= 0.00000625$$

$$\Rightarrow A_{2out} = G_2 A_{2in} = 0.00000625(8.1) = 0.051mV$$

3.17 Using Eq. 3.2, $-2 = 20\log_{10}(V_o/5.6)$. Solving, V_o = 4.45

3.18 We want a low-pass filter with a constant gain up to 10 Hz but a gain of 0.1 at 60 Hz. Using Eq. 3.20:

$$G = \frac{1}{\sqrt{1+\left(f_1/f_c\right)^{2n}}}$$

$$0.1 = \frac{1}{\sqrt{1+\left(60/10\right)^{2n}}}$$

Solving for n, we get 1.28. Since this is not an integer, we select n = 2. With this filter, the 10 Hz signal will be attenuated 3 dB. If this is a problem, then a higher corner frequency and possibly a higher filter order might be selected.

3.19 We want a low-pass filter with a constant gain up to 100 Hz but an attenuation at 1000 Hz of $20\log_{10} 0.01 = -40$ dB (G = 0.01). Using Eq. 3.20:

$$G = \frac{1}{\sqrt{1+\left(f_1/f_c\right)^{2n}}}$$

$$0.01 = \frac{1}{\sqrt{1+\left(1000/100\right)^{2n}}}$$

Solving for n, we get n = 2

With the selected corner frequency, the 100 Hz signal will be attenuated 3dB. If this were to be a problem, a higher corner frequency would be required and also a higher order filter.

3.20 f_c = 1500 Hz
f = 3000 Hz

a) For a fourth-order Butterworth filter
n = 4
From Eq. 3.20,

$$G = \frac{1}{\sqrt{1+\left(f/f_c\right)^{2n}}}$$

$$= \frac{1}{\sqrt{1+\left(3000/1500\right)^{2\times 4}}}$$

$$= 0.0624 = 6.24\%$$

$$= -24dB$$

b) For a fourth-order Chebeshev filter with 2 dB ripple width

n = 4

Frequency Ratio $\frac{f}{f_c} = \frac{3000}{1500} = 2$

From Fig. 3.18 we see that for n = 4 and f/f_c = 2,
G(dB) = –34dB

c) For a fourth-order Bessel filter

n = 4

Frequency Ratio $\frac{f}{f_c} = \frac{3000}{1500} = 2$

From Fig. 3.20 we see that for n = 4 and f/f_c = 2,
G(dB) = –14 dB

3.21

$n = 1$

$G = 1$

$f_c = 12kHz$

$R_1 = 1000\Omega$

At dc, Eqs. 3.21 and 3.17 are equivalent. Since we require no gain, set $R_1 = R_2$.
Thus, $R_1 = R_2 = 1000\Omega$
From Eq. 3.26, we can calculate C,

$$f_c = \frac{1}{2\pi R_2 C}$$

$$12kHz = \frac{1}{2\pi(1000)C}$$

$$\Rightarrow C = 0.013\mu F$$

3.22 It would not be possible to solve problem 3.16 using a simple Butterworth filter based on the inverting amplifier. This is because R_1 would have to be on the order 10 MΩ. Such a resistance is higher than resistances normally used for such circuits because it is on the order of various capacitive impedances associated with the circuit. The signal should first be input to an amplifier with a very high input impedance such as a non-inverting amplifier and the signal then passed through a filter.

3.23

$n = 4$

$G = 1$

$f_c = 1500Hz$

$f = 25kHz$

From Eq. 3.20,

$$G = \frac{1}{\sqrt{1 + \left(f/f_c\right)^{2n}}}$$

$$= \frac{1}{\sqrt{1 + \left(25000/1500\right)^{2\times 4}}}$$

$$= 1.30 \times 10^{-5}$$

$G_{dB} = 20\log_{10}(1.3\times10^{-5})$

3.24 $V_{in} = Deflection \times V/div = 4.3 \times 2 = 8.6V$

3.25 $Range = Maximum\,Deflection \times V\,/\,div = 8 \times 100mV = 800mV$

3.26 The visual resolution is on the order of the beam thickness (for thick beams is may be on the order of ½ the beam thickness since one can interpolate within the beam. Taking the resolution as the beam thickness, the fractional error in reading is 0.05/1 = 0.05 (5%). In volts the resolution is 0.05x5 mV = 0.25 mV.

3.21

$n = 1$

$G = 1$

$f_c = 12kHz$

$R_1 = 1000\Omega$

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3.23

$n = 4$

$G = 1$

$f_c = 1500 Hz$

$f = 25 kHz$

From Eq. 3.20,

$$G = \frac{1}{\sqrt{1+\left(f/f_c\right)^{2n}}}$$

$$= \frac{1}{\sqrt{1+\left(25000/1500\right)^{2\times 4}}}$$

$$= 1.30 \times 10^{-5}$$

$G_{dB} = 20\log_{10}(1.3\times10^{-5})$
$= -97.7$ dB