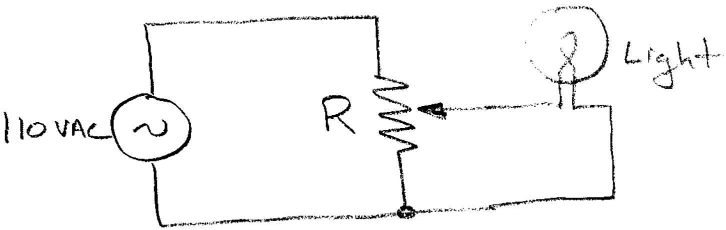
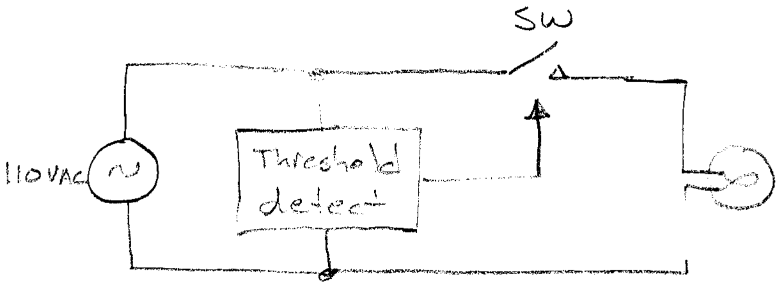
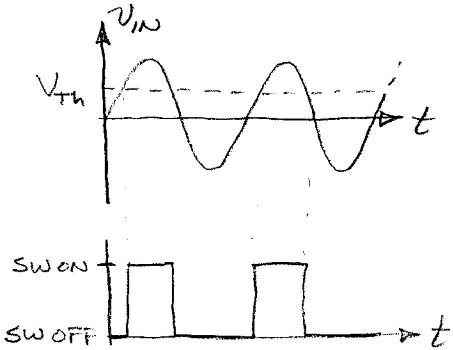
P1.1

One method would be to use a variable resistor as shown below. As the wiper on the resistor moves from the top to the bottom, the light varies from full brightness to completely off. This circuit is just a variable impedance divider.



A second method would be to use a variable threshold and only allow current to flow in the light when the input voltage is above the threshold. For this method to work we need a circuit to generate the threshold, which must depend on the setting of some control, and we need an electronically controllable switch. A simplified diagram is shown below along with the input waveform and an example of when the switch would be on for a given threshold setting.





The first method is simpler, but a significant amount of power is wasted in the variable resistor. The second method is close to what is used in solid-state dimmer switches; they typically use solid-state switches known as Triacs (see Chapter Two) which are turned on for some portion of each cycle of the sine wave.

P1.2

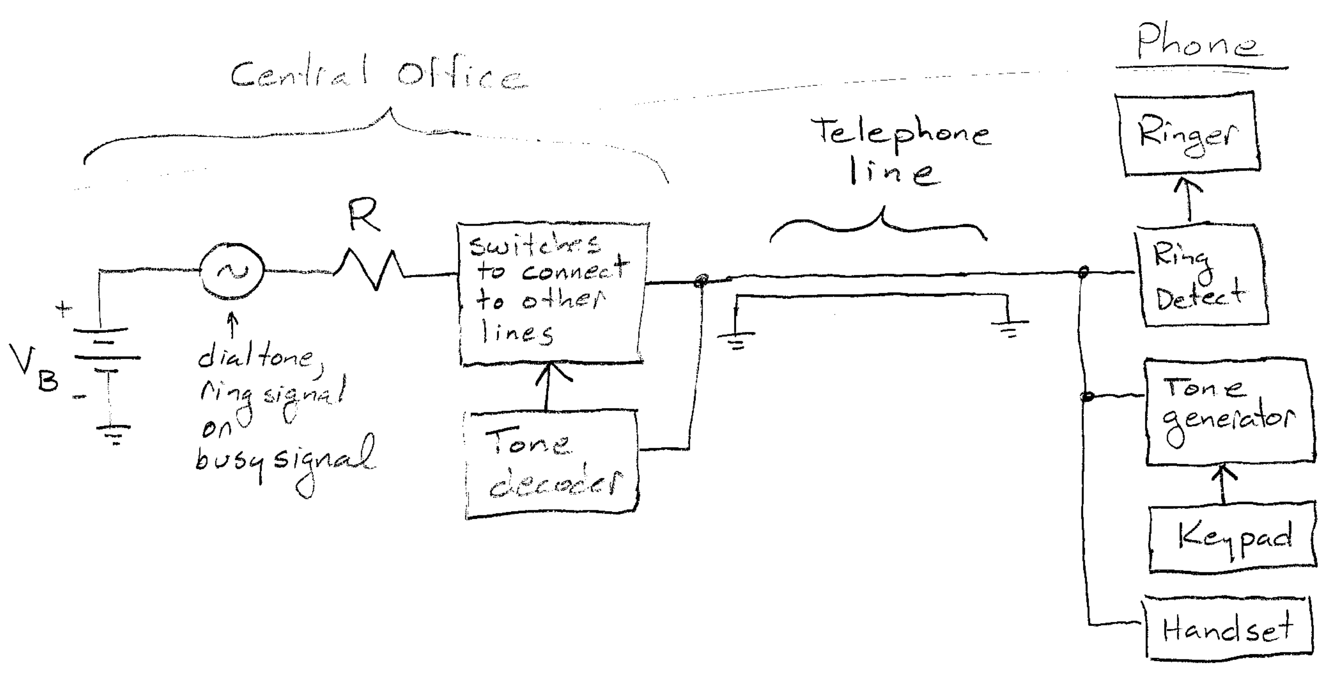
The switch is only capable of turning the power to the light on or off. Therefore, the light must detect some pattern of on-off-on switching in order to know what state to be in. For example, you could set it up so that if the light has been off for at least a few seconds and is then turned on once, it comes on steadily. If, on the other hand, it has been turned off for some time and is then turned on, off, and immediately back on again, it will come on in the motion detector mode. Alternatively, you could require that the light be turned on for some period of time, say 3 seconds, then off and back on again in order to trigger the motion detector mode. The first scheme is easier to use since it does not require the user to count for three seconds.

P1.3

One possible block diagram is shown below. The DC voltage, V_B, is used to power the phone. The central office (CO) can detect whether the phone is "on hook" or "off hook" by measuring the current flow in the line (one way to do this would be to monitor the voltage on the right-hand end of the series resistor). If the phone in on hook and the CO wants to ring the phone, an AC signal is added to the DC supply. This AC signal must be detected at the phone and used to trigger the ringer. When the phone is picked up to answer the call (i.e., it goes "off hook") the current consumed increases, the CO detects this increase, stops the ring signal and establishes the connection between this phone and the caller's phone. If no incoming call is present and the phone is taken off hook, the CO generates a dial tone and adds it to the DC supply. The phone then sends a sequence of tones to the CO to indicate the number it wants to be connected with. The CO must then decode these tones, ring the other line (or supply a busy signal if the phone is in use) and establish the connection if the other phone is picked up.

In order to implement the protocol described above, the CO must be able to generate ring signals, dial tones, and busy signals. In addition, the CO must be able to detect the current drawn from the line, decode the tones for dialing, and connect different phones together. The telephone must be able to sense the ring signal, send tones for dialing and must guarantee that the off-hook and on-hook DC current consumption is within specified limits.

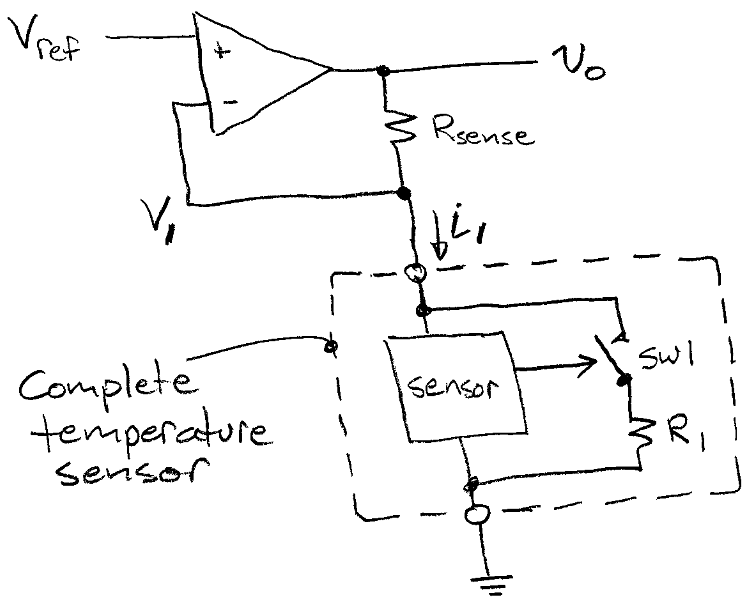
In the United states, the DC supply voltage is 48 ± 6 V when the phone is on hook and between 43 and 79 V when the phone is off hook. The ring signal is a 75 Vrms 20 Hz sine wave that is repeatedly pulsed on for 2 seconds and off for 4 seconds. The dial tone is actually two tones, one at 350 Hz and one at 440 Hz and the tones used for dialing are a mixture of one low frequency (697, 770, 852, or 941 Hz) and one high frequency (1,209, 1,336, or 1,477 Hz). Each low frequency corresponds to a given row on the keypad and each high frequency corresponds to a given column. Using two frequencies minimizes the chances of a misdial due to background noise. The maximum current a single phone is supposed to draw when on hook is 1 μA. The current can vary considerably when off hook, but is typically 10-30 mA.



P1.4

One method would be to use the pulse output to modulate the current used by the sensor. The external circuit would then supply the sensor with a constant voltage and would simultaneously monitor the current.

One way of accomplishing this is shown below.



The op amp keeps the the voltage supplied to the sensor constant:

$$V_1 = V_{ref}$$

The sensor output pulse controls SW1. When SW1 is closed, i_1 increases by V_1/R_1. When SW1 is open, i_1 is less.

Since i_1 flows through R_{sense} and V_1 is constant, v_o has steps with magnitude:

$$\Delta v_o = \Delta i_1 \cdot R_{sense} = \frac{V_1}{R_1} R_{sense}$$

P1.5

We can accomplish the desired function using electronically controllable switches and capacitors. In the first state, the switches connect the capacitors in parallel and charge them up to the available supply voltage as illustrated for two capacitors in Figure 1 below (all switches in position 1). After the capacitors are charged up, the switches are changed to stack the capacitors in series and generate a larger voltage as shown in Figure 2 (all switches in position 2). They dump their charge on an external holding cap and then repeat this process over and over to charge that capacitor up to the desired output voltage and to replenish charge removed by some external load. By changing which end of the capacitor string is connected to ground we can generate output voltages of the opposite polarity. One commercial product that works in just this way is the LM7660.

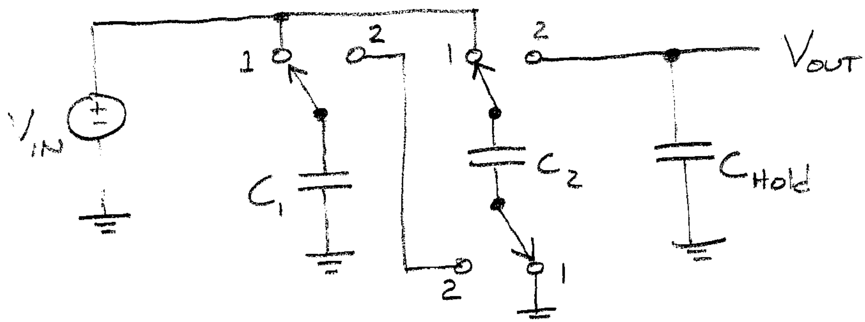


Figure 1

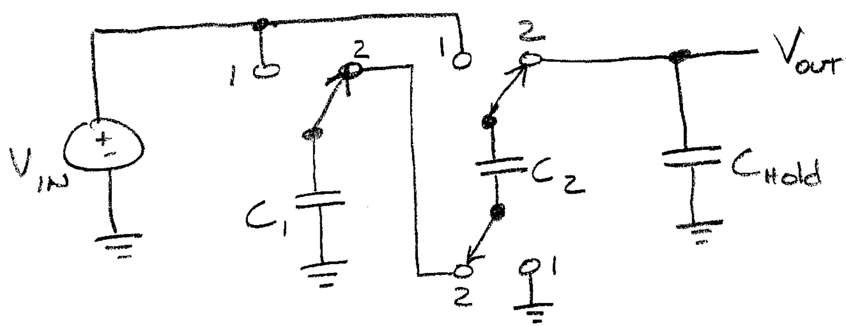


Figure 2

P1.6

(a)

$$V_{Th} = \frac{xR_P}{xR_P + (1-x)R_P} = 10x \qquad R_{Th} = xR_P \| (1-x)R_P = \frac{xR_P(1-x)R_P}{xR_P + (1-x)R_P} = x(1-x)R_P$$

(b) V_{Th} is a linear function of x (see part (a) result). If R_L is infinite, then $V_{out} = V_{Th}$. Hence, $V_{out} = 10x$ Volts, a linear function of x.

(c)

$V_{out} = 10x\dfrac{R_L}{R_{Th} + R_L}$, so maximum attenuation occurs when R_{Th} is maximum.

$\dfrac{dR_{Th}}{dx} = \dfrac{d}{dx}x(1-x)R_P = (1-2x)R_P$ and $\dfrac{dR_{Th}}{dx} = 0$ when $x = \dfrac{1}{2}$. At this value of x, $R_{Th} = \dfrac{R_P}{4}$.

(d)

Let $V_{out} = kV_{Th}$ where $k = \dfrac{R_L}{R_{Th} + R_L}$. Relative error $= \dfrac{kV_{Th} - V_{Th}}{V_{Th}} = k - 1$.

When error is maximum, $x = \dfrac{1}{2}$, $R_{Th} = \dfrac{R_P}{4}$, and $k = \dfrac{R_L}{(R_P/4) + R_L}$.

Therefore, relative error $= k - 1 = \dfrac{-R_P}{R_P + 4R_L}$

(e)

Let $Z = \dfrac{R_L}{R_P}$ (we want to solve for Z). Then relative

$$\text{error} = \frac{-R_P}{R_P + 4R_L} = \frac{-R_P}{R_P + 4ZR_P} = \frac{-1}{1+4Z}.$$

Set $-0.05 = \dfrac{-1}{1+4Z}$, hence $1 + 4Z = 20$ and $Z = \dfrac{19}{4} = 4.75$. So $\dfrac{R_L}{R_P} > 4.75$ to meet conditions.

(f)

Repeat steps of part (e) for 10% maximum error:

Set $-0.1 = \dfrac{-1}{1+4Z}$, hence $1 + 4Z = 10$ and $Z = \dfrac{9}{4} = 2.25$. So $\dfrac{R_L}{R_P} > 2.25$ to meet conditions.

The minimum power dissipation in the pot will occur when the current through it is lowest, hence the resistance of the pot should be the maximum value that does not exceed the error specification. Given that $R_L = 1000\Omega$, R_P must be less than 444Ω. The largest standard value not over 444Ω is 250Ω.

P1.7)

$$\frac{V_o}{V_s} = \frac{V_2}{V_s}\,\frac{V_o}{V_2}$$

$$\frac{V_2}{V_s} = \frac{R_2}{R_1 + R_2} \qquad \frac{V_o}{V_2} = -G_M(R_3 \| R_4)$$

$$\therefore \quad \frac{V_o}{V_s} = -G_M(R_3 \| R_4)\left(\frac{R_2}{R_1 + R_2}\right)$$

P1.8)

$$\frac{i_o}{V_s} = \frac{V_2}{V_s}\,\frac{i_o}{V_2}$$

$$\frac{V_2}{V_s} = \frac{R_2}{R_1 + R_2} \qquad \frac{i_o}{V_2} = \frac{-G_M R_3}{R_3 + R_4}$$

$$\frac{i_o}{V_s} = \frac{-G_M R_2 R_3}{(R_1+R_2)(R_3+R_4)}$$

P1.9)

$$\frac{v_o}{i_s} = \frac{v_o}{v_2}\frac{v_2}{i_s}$$

$$\frac{v_o}{v_2} = \frac{R_4 \| R_L}{R_4 \| R_L + R_3} A \qquad \frac{v_2}{i_s} = R_1 \| R_2$$

$$\therefore \frac{v_o}{i_s} = (R_1 \| R_2)\left(\frac{R_4 \| R_L}{R_4 \| R_L + R_3}\right) A$$

P1.10)

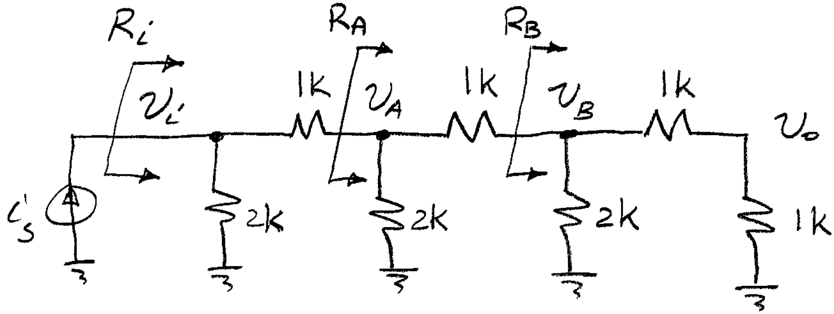
$$\frac{i_o}{i_s} = \frac{i_o}{v_o}\,\frac{v_o}{v_2}\,\frac{v_2}{i_s}$$

$$\frac{i_o}{v_o} = \frac{1}{R_L} \qquad \frac{v_o}{v_2} = \frac{R_4 \| R_L}{R_4 \| R_L + R_3} A \qquad \frac{v_2}{i_s} = R_1 \| R_2$$

so

$$\frac{i_o}{i_s} = (R_1 \| R_2)\left(\frac{R_4 \| R_L}{R_4 \| R_L + R_3}\right)\frac{A}{R_L}$$

P1.11)



$$\frac{v_o}{i_s} = \frac{v_o}{v_B}\,\frac{v_B}{v_A}\,\frac{v_A}{v_i}\,\frac{v_i}{i_s}$$

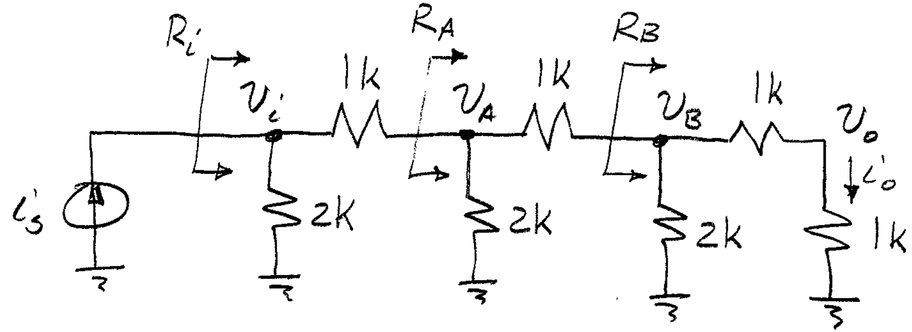
$R_B = 2k \| (1k + 1k) = 1k$ $\qquad$ $R_A = 2k \| (1k + R_B) = 1k$

$R_i = 2k \| (1k + R_A) = 1k$

$$\frac{v_o}{v_B} = \frac{1}{2} \qquad \frac{v_B}{v_A} = \frac{R_B}{R_B + 1k} = \frac{1}{2} \qquad \frac{v_A}{v_i} = \frac{R_A}{R_A + 1k} = \frac{1}{2}$$

$$\frac{v_i}{i_s} = R_i = 1k \qquad \therefore \quad \frac{v_o}{i_s} = \left(\frac{1}{2}\right)^3 \cdot 1k = 125\,\Omega$$

P1.12)



$$\frac{i_o}{i_s} = \frac{i_o}{v_o}\,\frac{v_o}{v_B}\,\frac{v_B}{v_A}\,\frac{v_A}{v_i}\,\frac{v_i}{i_s}$$

$R_B = (1k + 1k) \| 2k = 1k$ $\qquad$ $R_A = 2k \| (1k + R_B) = 1k$

$R_i = 2k(1K + R_A) = 1K$ $\qquad$ $\dfrac{i_o}{v_o} = \dfrac{1}{1K}$

$\dfrac{v_o}{v_B} = \dfrac{1}{2}$ $\qquad$ $\dfrac{v_B}{v_A} = \dfrac{R_B}{R_B + 1K} = \dfrac{1}{2}$ $\qquad$ $\dfrac{v_A}{v_i} = \dfrac{R_A}{R_A + 1K} = \dfrac{1}{2}$ $\qquad$ $\dfrac{v_i}{i_s} = R_i$

$$\therefore \quad \frac{i_o}{i_s} = \left(\frac{1}{2}\right)^3 \frac{1K}{1K} = \frac{1}{8}$$

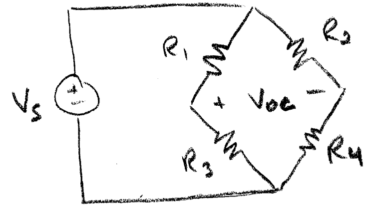
P1.13)

$$R_{th} = R_2 \parallel (R_1 + R_3)$$

$$V_{th} = V_{OC} = \frac{R_2 V_{IN}}{R_1 + R_2 + R_3}$$

$$i_N = \frac{V_{th}}{R_{th}} = \frac{V_{IN}}{R_1 + R_3}$$

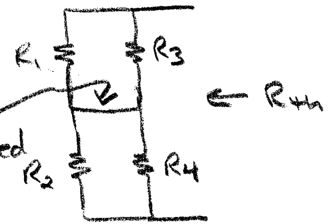
P1.14)



$$V_{th} = V_{oc}$$

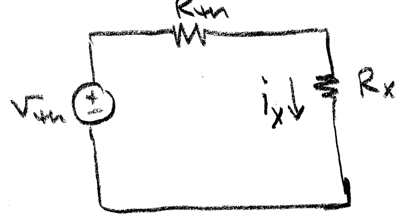
$$= \frac{R_3}{R_1 + R_3} V_s - \frac{R_4}{R_2 + R_4} V_s$$

$$\therefore \; V_{th} = \left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right) V_s$$



V_s is shorted

So,

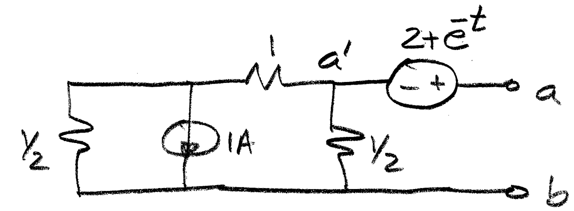


$$R_{th} = R_1 \| R_3 + R_2 \| R_4$$

(a) $$i_x = \frac{V_{th}}{R_x + R_{th}}$$

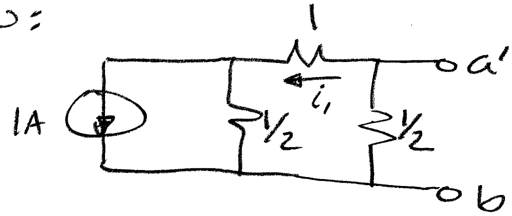
(b) $$R_x = \frac{V_{th} - i_x R_{th}}{i_x}$$

P1.15



1^{st} find open-ckt output voltage: $v_{ab} = v_{a'b} + 2 + e^{-t}$

Redraw:

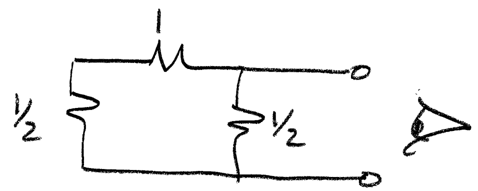


$$i_1 = \frac{1/2}{2} \cdot 1A = \frac{1}{4} A$$

$$v_{a'b} = -\tfrac{1}{2} i_1 = -\frac{1}{8} V$$

$$\therefore \quad \underline{v_{Th} = v_{ab} = 15/8 + e^{-t}}$$

Now find R_{Th}. Set all independent sources to zero and



$$R_{Th} = \frac{.5(1.5)}{2} = \underline{\underline{3/8\ \Omega}}$$

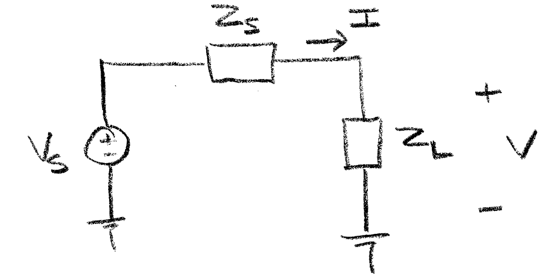
Now find the Norton current (i.e., the short-circuit output current):

$$i_N = \frac{v_{Th}}{R_{Th}} = \frac{15}{3} + \frac{8}{3} e^{-t}$$

$$= \underline{5 + \frac{8}{3} e^{-t}}$$

You can also find this directly from the circuit.

P1.20)



$$Z_S = R_S + jX_S$$
$$Z_L = R_L + jX_L$$

$$P = \frac{1}{2}\,\mathrm{Re}\{V \cdot I^*\} = \frac{1}{2}\,\mathrm{Re}\{I Z_L \cdot I^*\}$$

Remember that $I \cdot I^* = |I|^2$

$$P = \frac{1}{2}|I|^2\,\mathrm{Re}\{Z_L\}$$
$$= \frac{1}{2}\left|\frac{V}{Z_S + Z_L}\right|^2 \mathrm{Re}\{Z_L\}$$

Also, $|Z| = \sqrt{R^2 + I^2}$ ← Magnitude of a complex number is the square root of the sum of the squares of its real and imaginary parts.

$$\therefore\; P = \frac{|V|^2}{2}\,\frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

Looking at the formula for P, it can be seen that by having $R_S = R_L$ and $X_S = -X_L$ P_{MAX} is obtained

$$P_{MAX} = \frac{|V|^2}{4R_S}$$

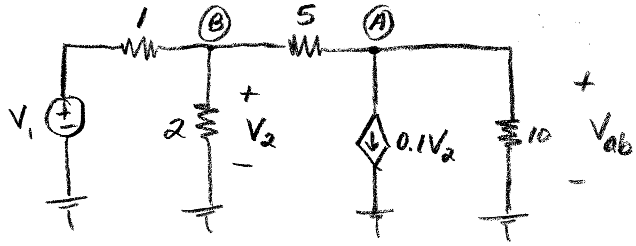
If $Z_L = R_S - jX_S$, then $Z_L = Z_S^*$

$R_S = R_L$ and $X_S = -X_L$ can also be shown by setting

$$\frac{\partial P}{\partial R_L} = 0 \quad \text{and} \quad \frac{\partial P}{\partial X_L} = 0$$

and solving for R_L and X_L, respectively.

P1.16)



Find V_{oc}:

$V_{ab} = V_{oc} = V_{TH}$

KCL at A: $\dfrac{V_2 - V_{ab}}{5} - 0.1V_2 - \dfrac{V_{ab}}{10} = 0$ ← multiply by 10

$2(V_2 - V_{ab}) - V_2 - V_{ab} = 0$

① $V_2 = 3V_{ab}$

KCL at B: $V_1 - V_2 - \dfrac{V_2}{2} + \dfrac{V_{ab} - V_2}{5} = 0$ ← multiply by 10

$10V_1 - 10V_2 - 5V_2 + 2(V_{ab} - V_2) = 0$

② $10V_1 - 17V_2 + 2V_{ab} = 0$

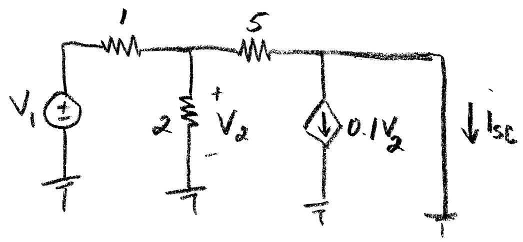
Substitute ① into ②:

$10V_1 - 17(3V_{ab}) + 2V_{ab} = 0$

$10V_1 - 49V_{ab} = 0$

$V_{ab} = \dfrac{10}{49}V_1 \Rightarrow \underline{\underline{V_{TH} = \dfrac{10}{49}V_1 \simeq 0.2V_1}}$

Find i_{sc}: $i_N = i_{sc}$



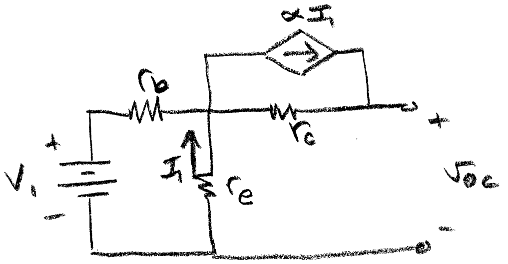
$i_{sc} = \dfrac{V_2}{5} - 0.1V_2 = 0.1V_2 = \dfrac{V_2}{10}$

$V_2 = \left(\dfrac{2\parallel 5}{2\parallel 5 + 1}\right)V_1 = \dfrac{10}{17}V_1$

$i_{sc} = \dfrac{1}{10}\left(\dfrac{10}{17}V_1\right) = \dfrac{V_1}{17} \Rightarrow \underline{\underline{i_N = \dfrac{V_1}{17} \simeq 0.06V_1}}$

$R_{TH} = \dfrac{V_{TH}}{i_N} = \dfrac{\frac{10}{49}V_1}{\frac{1}{17}V_1} = \dfrac{170}{49}\,\Omega \simeq \underline{\underline{3.47\,\Omega}}$

P1.17) Find v_{oc}: $\quad v_{oc} = V_{TH}$



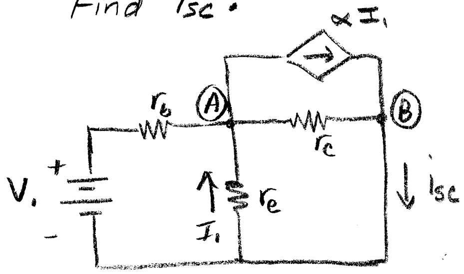
$$v_{oc} = \alpha I_1 r_c + \left(\frac{r_e}{r_e + r_b}\right) V_1$$

$$I_1 = \frac{-V_1}{r_e + r_b}$$

So, $$v_{oc} = \left(\frac{-\alpha r_c}{r_e + r_b}\right) V_1 + \left(\frac{r_e}{r_e + r_b}\right) V_1$$

$$\therefore \; V_{TH} = v_{oc} = \underline{\underline{\left(\frac{r_e - \alpha r_c}{r_e + r_b}\right) V_1}}$$

Find i_{sc}: $\quad i_{sc} = i_N$



KCL at Ⓐ:

① $$\frac{V_1 - V_A}{r_b} - \frac{V_A}{r_e} + \frac{0 - V_A}{r_c} - \alpha I_1 = 0$$

② $$I_1 = \frac{-V_A}{r_e}$$

Substituting ① into ②:

$$\frac{V_1 - V_A}{r_b} - \frac{V_A}{r_e} - \frac{V_A}{r_c} + \frac{\alpha V_A}{r_e} = 0$$

$$\frac{V_1}{r_b} = \left(\frac{1}{r_b} + \frac{1}{r_c} + \frac{(1-\alpha)}{r_e}\right) V_A = \left(\frac{r_c r_e + r_b r_e + (1-\alpha) r_b r_c}{r_b r_c r_e}\right) V_A$$

$$V_A = \left(\frac{r_c r_e}{r_c r_e + r_b r_e + (1-\alpha) r_b r_c}\right) V_1$$

$$i_{sc} = \alpha I_1 + \frac{V_A}{r_c} \quad \longleftarrow \text{substitute in ②}$$

$$i_{sc} = -\frac{\alpha V_A}{r_e} + \frac{V_A}{r_c} = \left(\frac{r_e - \alpha r_c}{r_e r_c}\right) V_A \quad \longleftarrow \text{substitute in ①}$$

$$\therefore \; i_N = \underline{\underline{\left(\frac{r_e - \alpha r_c}{r_c r_e + r_b r_e + (1-\alpha) r_b r_c}\right) V_1}}$$

$$R_{TH} = \frac{V_{TH}}{i_N} = \underline{\underline{\frac{r_c r_e + r_b r_e + (1-\alpha) r_b r_c}{r_e + r_b}}}$$

P1.21) $$\frac{V_o(j\omega)}{V_s(j\omega)} = \frac{V_{in}(j\omega)}{V_s(j\omega)}\,\frac{V_o(j\omega)}{V_{in}(j\omega)} = H_1(j\omega)\,H_2(j\omega)$$

$H_1(j\omega)$:

$$\left.\begin{aligned} H(j0) &= \frac{R_{in}}{R_{in}+R_s} \\ H(j\infty) &= 1 \end{aligned}\right\} \text{high-pass filter}$$

$$H_1(j\omega) = \frac{R_{in}}{R_{in}+Z} \quad \text{where } Z = R_s \,||\, Z_{C_s}$$

$$Z = \frac{R_s\left(\frac{1}{j\omega C_s}\right)}{R_s + \frac{1}{j\omega C_s}} = \frac{R_s}{1+j\omega R_s C_s}$$

$$H_1(j\omega) = \frac{R_{in}}{R_{in} + \frac{R_s}{1+j\omega R_s C_s}} = \frac{j\omega + \frac{1}{R_s C_s}}{j\omega + \frac{(R_s+R_{in})}{R_s R_{in} C_s}}$$

where $\omega_{z_1} = \dfrac{1}{R_s C_s}$ and $\omega_{p_1} = \dfrac{1}{(R_s || R_{in})\,C_s}$

$H_2(j\omega)$:

$$H_2(j\omega) = H_2(j0)\,\frac{1}{1+j\frac{\omega}{\omega_{p2}}} \quad \leftarrow \text{low-pass filter}$$

$$H_2(j0) = \frac{A R_L}{R_L + R_o}$$

$$\omega_{p2} = \frac{1}{(R_o || R_L)\,C_L}$$

$$\therefore\ \frac{V_o(j\omega)}{V_s(j\omega)} = \left(\frac{j\omega + \omega_{z_1}}{j\omega + \omega_{p_1}}\right)\left(\frac{1}{1+j\frac{\omega}{\omega_{p2}}}\right)\left(\frac{A R_L}{R_L + R_o}\right)$$

P1.18)

$$\frac{V_o(j\omega)}{V_s(j\omega)} = \frac{V_{in}(j\omega)}{V_s(j\omega)} \frac{V_o(j\omega)}{V_{in}(j\omega)} = H_1(j\omega) H_2(j\omega)$$

$H_1(j\omega)$:

$$H(j0) = \frac{R_{in}}{R_{in}+R_s} \quad \text{since } Z_{C_{in}} \to \infty$$

$$H(j\infty) = 0 \quad \text{since } Z_{C_{in}} \to 0$$

Since there is only one reactive element and the circuit acts as a low-pass filter => single-pole low-pass transfer characteristics.

Using equation A1.9 :

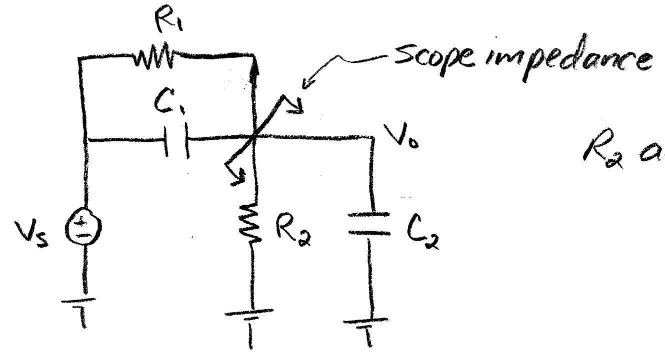
$$H_1(j0) = \frac{R_{in}}{R_{in}+R_s} \qquad \omega_{z_1} = \infty \qquad \omega_{p_1} = \frac{1}{C_{in}(R_s \parallel R_{in})}$$

$H_2(j\omega)$: Again, single-pole low-pass transfer characteristics

$$H_2(j0) = G_m(R_o \parallel R_L) \qquad \omega_{z_2} = \infty \qquad \omega_{p_2} = \frac{1}{C_L(R_L \parallel R_o)}$$

$$\therefore \frac{V_o(j\omega)}{V_s(j\omega)} = \frac{\frac{R_{in}}{R_{in}+R_s}}{1 + j\omega\frac{R_s R_{in} C_{in}}{R_{in}+R_s}} \left(\frac{G_m(R_o \parallel R_L)}{1 + j\omega\frac{R_L R_o C_L}{R_L + R_o}} \right)$$

P1.19) (a)



R_2 and C_2 are given as:

$R_2 = 1\,M\Omega$

$C_2 = 10\,pF$

$$\frac{V_o}{V_s} = \frac{R_2}{R_1 + R_2} = 0.1 \longleftarrow \text{attenuates signal by factor of 10.}$$

$$\therefore \; R_1 = 9\,M\Omega$$

For flat frequency response

$$R_1 C_1 = R_2 C_2$$

$$C_1 = \left(\frac{R_2}{R_1}\right) C_2 = \left(\frac{1}{9}\right)(10pF)$$

$$\therefore \; C_1 = \frac{10}{9}\,pF \triangleq 1.1\,pF$$

(b) Scope resistance and capacitance vary $\pm 10\%$.
For flat frequency, $R_1C_1 = R_2C_2$ must still hold.
Probe resistance R_1 is constant.

$$\therefore \; R_1 C_1 = R_2' C_2' \longrightarrow \begin{array}{l} R_2' = R_2 \pm 0.1 R_2 \\ C_2' = C_2 \pm 0.1 C_2 \end{array}$$

$$R_1 C_1 = (R_2 \pm 0.1 R_2)(C_2 \pm 0.1 C_2)$$

So,

Min. C_1:

$$R_1 C_1 = (0.9)^2 R_2 C_2$$

$$C_1 = (0.81) \frac{R_2 C_2}{R_1}$$

$\Rightarrow -19\%$
(19% decrease)

or

Max. C_1:

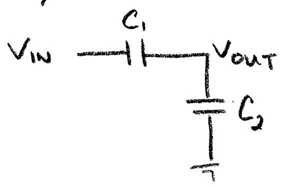
$$R_1 C_1 = (1.1)^2 R_2 C_2$$

$$C_1 = (1.21) \frac{R_2 C_2}{R_1}$$

$\Rightarrow +21\%$
(21% increase)

$\therefore$ C_1 must be adjustable between $\pm 21\%$ to ensure a flat frequency response.

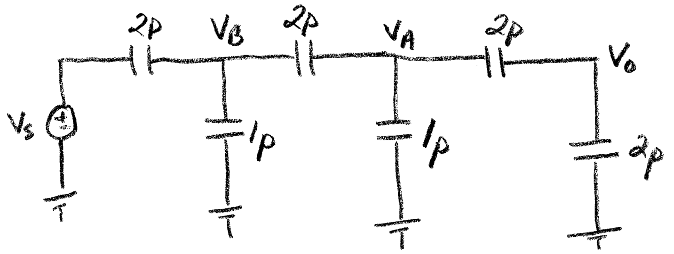
P1.22) A capacitive voltage divider can be described as



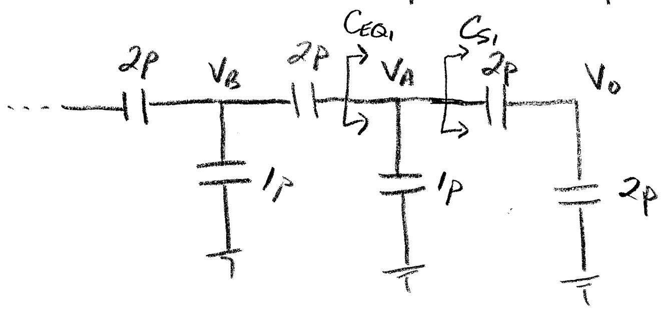
$$\frac{V_{OUT}}{V_{IN}} = \frac{Z_{C_2}}{Z_{C_2} + Z_{C_1}} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{C_1}{C_1 + C_2}$$

This relation can be used to find the transfer function of:



$$\frac{V_o}{V_s} = \frac{V_o}{V_A}\frac{V_A}{V_B}\frac{V_B}{V_s}$$



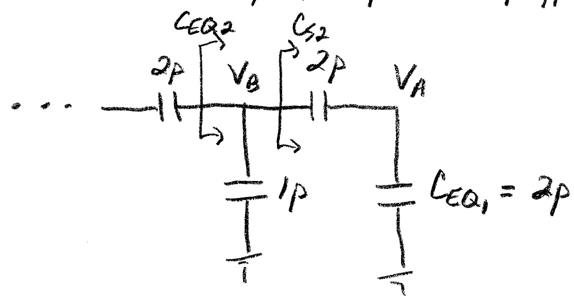
Remember: C_1 in series with C_2 is $C_{EQ} = \frac{C_1 C_2}{C_1 + C_2}$

If $C_1 = C_2$ then $C_S = \frac{1}{2} C_1$

$$\frac{V_o}{V_A} = \frac{2p}{2p + 2p} = \frac{1}{2}$$

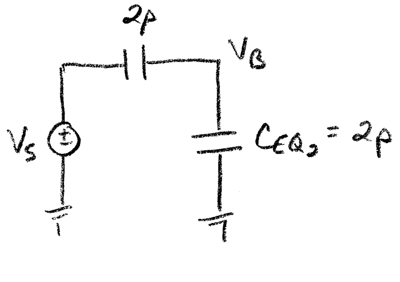
$C_{S1} = 1p$

$C_{EQ_1} = 1p \,||\, C_{S_1} = 1p \,||\, 1p = 2p \longrightarrow$ Capacitors in parallel add



$$\frac{V_A}{V_B} = \frac{2p}{2p + C_{EQ_1}} = \frac{1}{2}$$

$C_{S2} = 1p \qquad C_{EQ_2} = 2p$



$$\frac{V_B}{V_s} = \frac{2p}{2p + C_{EQ_2}} = \frac{1}{2}$$

$$\therefore \frac{V_o}{V_s} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$$

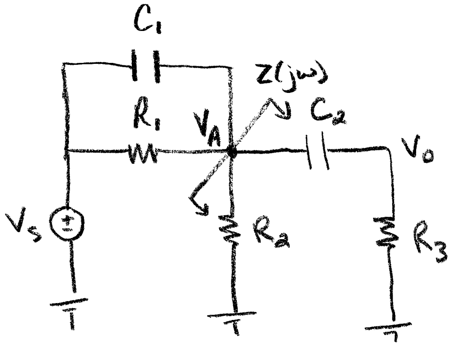
P1.23)

$$V_o(j\omega) = I_s(j\omega)\,(R_2 \parallel C)$$

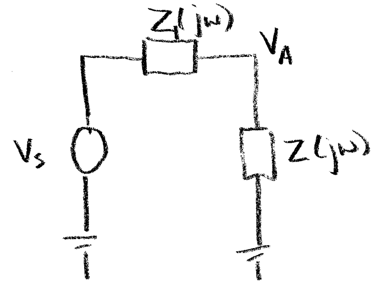
$$\frac{V_o(j\omega)}{I_s(j\omega)} = R_2 \parallel C$$

$$\therefore \quad \frac{V_o(j\omega)}{I_s(j\omega)} = \frac{R_2}{1 + j\omega R_2 C}$$

P1.24)



$$\frac{V_o(j\omega)}{V_s(j\omega)} = \frac{V_o(j\omega)}{V_A(j\omega)}\,\frac{V_A(j\omega)}{V_s(j\omega)}$$



$$Z_1(j\omega) = R_1 \,\|\, Z_{C_1} = \frac{R_1}{1+j\omega R_1}$$

$$Z(j\omega) = R_2 \,\|\, (R_3 + Z_{C_2})$$

$$= \frac{R_2\left(R_3 + \frac{1}{j\omega C_2}\right)}{R_2 + R_3 + \frac{1}{j\omega C_2}}$$

$$Z(j\omega) = \frac{R_2(1 + j\omega R_3 C_2)}{1 + j\omega(R_2+R_3)C_2}$$

$$\frac{V_A(j\omega)}{V_s(j\omega)} = \frac{Z(j\omega)}{Z(j\omega) + Z_1(j\omega)}$$

$$\frac{V_o(j\omega)}{V_A(j\omega)} = \frac{R_3}{R_3 + \frac{1}{j\omega C_2}} = \frac{j\omega R_3 C_2}{1 + j\omega R_3 C_2}$$

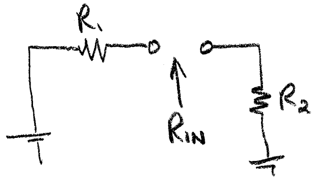
$$\therefore \quad \frac{V_o(j\omega)}{V_s(j\omega)} = \left(\frac{j\omega R_3 C_2}{1 + j\omega R_3 C_2}\right)\left(\frac{Z(j\omega)}{Z(j\omega) + Z_1(j\omega)}\right)$$

P1.25) (a) a) At $\omega = 0$, $Z_C = \infty \Rightarrow \dfrac{V_o(j0)}{V_s(j0)} = 0$

At $\omega = \infty$, $Z_C = 0 \Rightarrow \dfrac{V_o(j\infty)}{V_s(j\infty)} = \dfrac{R_2}{R_1 + R_2}$

$\therefore$ High-pass filter

b)



$\omega_p = \dfrac{1}{R_{IN} C}$ $\quad R_{IN} = R_1 + R_2$

$\therefore \omega_p = \dfrac{1}{(R_1 + R_2) C}$

c) Since $\dfrac{V_o(j\omega)}{V_s(j\omega)} = 0$ when $\omega = 0$, $\omega_z = 0$.

(b) a) At $\omega = 0$, $Z_C = \infty \Rightarrow \dfrac{V_o(j0)}{I_s(j0)} = R_1$

At $\omega = \infty$, $Z_C = 0 \Rightarrow \dfrac{V_o(j\infty)}{I_s(j\infty)} = 0$

$\therefore$ Low-pass filter

b) $\omega_p = \dfrac{1}{R_1 C}$

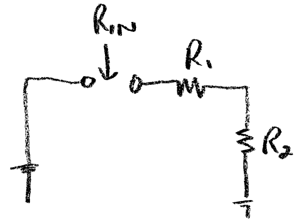
c) Since $\dfrac{V_o(j\omega)}{I_s(j\omega)} \rightarrow 0$ as $\omega \rightarrow \infty$, $\omega_z = \infty$.

(c) a) At $\omega = 0$, $Z_L = 0 \Rightarrow \dfrac{V_o(j0)}{V_s(j0)} = \dfrac{R_2}{R_1 + R_2}$

At $\omega = \infty$, $Z_L = \infty \Rightarrow \dfrac{V_o(j\infty)}{V_s(j\infty)} = 0$

$\therefore$ Low-pass filter

b)



$\omega_p = \dfrac{R_{IN}}{L}$ $\quad R_{IN} = R_1 + R_2$

$\therefore \omega_p = \dfrac{R_1 + R_2}{L}$

c) Since $\dfrac{V_o(j\omega)}{V_s(j\omega)} \rightarrow 0$ as $\omega \rightarrow \infty$, $\omega_z = \infty$.

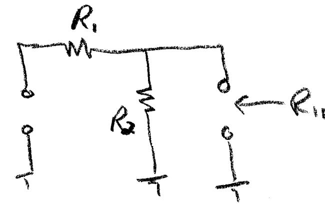
P1.25 Continued...

(d) a) At $\omega = 0$, $Z_L = 0 \Rightarrow \dfrac{V_o(j0)}{I_s(j0)} = 0$

At $\omega = \infty$, $Z_L = \infty \Rightarrow \dfrac{V_o(j\infty)}{I_s(j\infty)} = R_2$

$\therefore$ High-pass filter

b)



$\omega_p = \dfrac{R_{IN}}{L}$ $\quad R_{IN} = R_2$

$\therefore \omega_p = \dfrac{R_2}{L}$

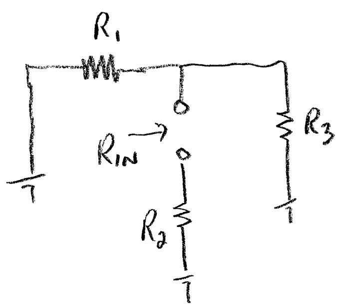
c) Since $\dfrac{V_o(j\omega)}{I_s(j\omega)} = 0$ when $\omega = 0$, $\omega_z = 0$.

(e) a) At $\omega = 0$, $Z_c = \infty \Rightarrow \dfrac{V_o(j0)}{V_s(j0)} = \dfrac{R_3}{R_1 + R_3}$

At $\omega = \infty$, $Z_c = 0 \Rightarrow \dfrac{V_o(j\infty)}{V_s(j\infty)} = \dfrac{R_2 \| R_3}{R_2 \| R_3 + R_1}$

$\therefore$ Low-pass filter

b)



$\omega_p = \dfrac{1}{R_{IN} C}$ $\quad R_{IN} = R_2 + R_1 \| R_3$

$\therefore \omega_p = \dfrac{1}{(R_2 + R_1 \| R_3) C}$

c) Since $\dfrac{V_o(j\omega)}{V_s(j\omega)}$ is non-zero at both $\omega = 0$ and $\omega = \infty$,

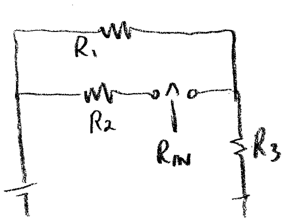
ω_z is zon-zero finite

(f) a) At $\omega = 0$, $Z_c = \infty \Rightarrow \dfrac{V_o(j0)}{V_s(j0)} = \dfrac{R_3}{R_1 + R_3}$

At $\omega = \infty$, $Z_c = 0 \Rightarrow \dfrac{V_o(j\infty)}{V_s(j\infty)} = \dfrac{R_3}{R_3 + R_1 \| R_2}$

$\therefore$ High-pass filter

b)



$\omega_p = \dfrac{1}{R_{IN} C}$ $\quad R_{IN} = R_2 + R_1 \| R_3$ $\quad \therefore \omega_p = \dfrac{1}{(R_2 + R_1 \| R_3) C}$

c) Since $\dfrac{V_o(j\omega)}{V_s(j\omega)}$ is non-zero at both $\omega = 0$ and $\omega = \infty$,

ω_z is non-zero finite.

P1.26

a) $$\frac{V_o}{V_i} = \frac{Z_L}{R + Z_c + Z_L}$$

$$= \frac{j\omega L}{R + \frac{1}{j\omega C} + j\omega L} = \frac{j\omega L}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\left|\frac{V_o}{V_i}\right| = \max \;@\; \left(\omega L - \frac{1}{\omega C}\right)^2 = 0$$

Solve : $$\boxed{\omega = \frac{1}{\sqrt{LC}} \quad \text{when} \quad \left|\frac{V_o}{V_i}\right| = \max}$$

b) $$\left|\frac{V_o}{V_i}\left(\omega = \frac{1}{\sqrt{LC}}\right)\right| = \left.\frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right|_{\omega = \frac{1}{\sqrt{LC}}} = \underline{\underline{\frac{1}{R}\sqrt{\frac{L}{C}}}}$$

← Note that this voltage gain can be much greater than one.

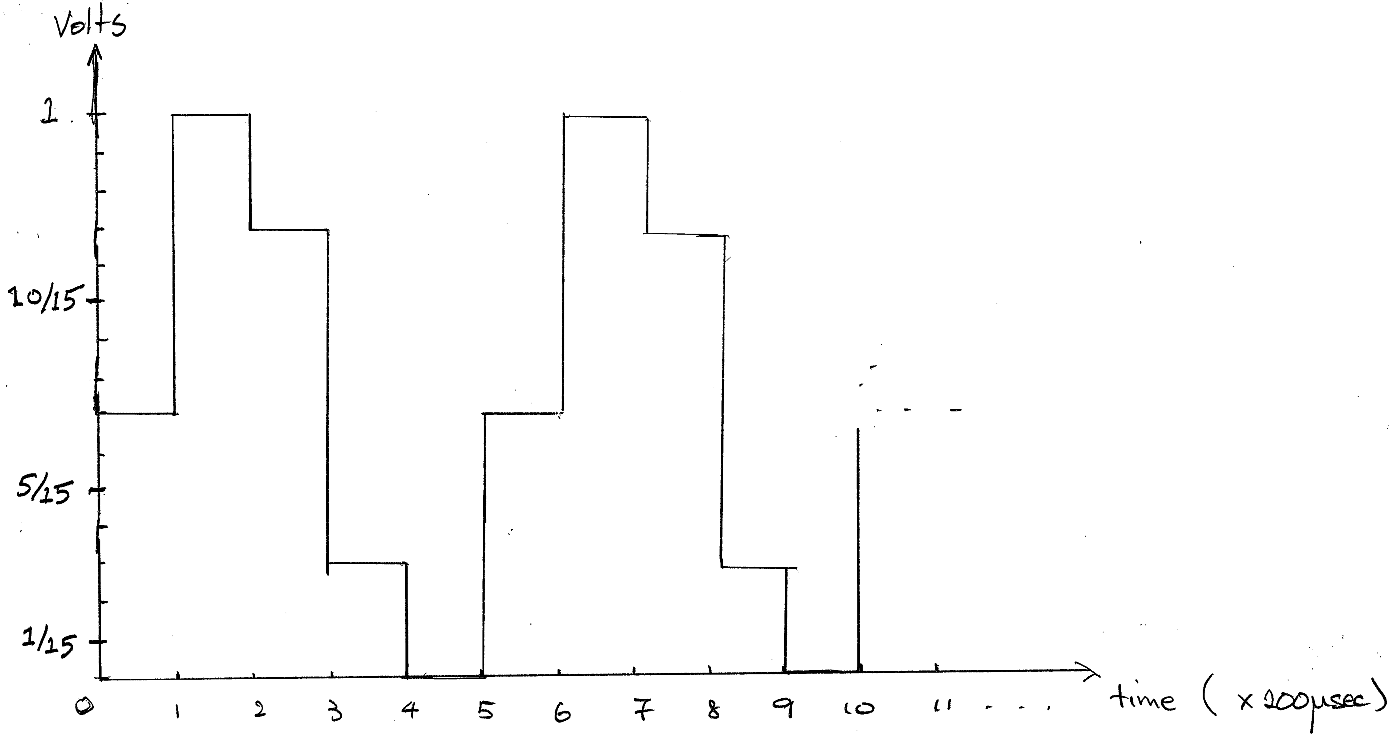
c) If more power was supplied to the load than was provided by the source, it would be possible to supply the "source" from the load (i.e., feedback some output) and still get power out of the modified circuit with no source. We could then extract power indefinitely from a closed system, which is the equivalent of perpetual motion and is a violation of the second law of thermodynamics.

P1.27

a) 0111, 1111, 1100, 0011, 0000 — binary word

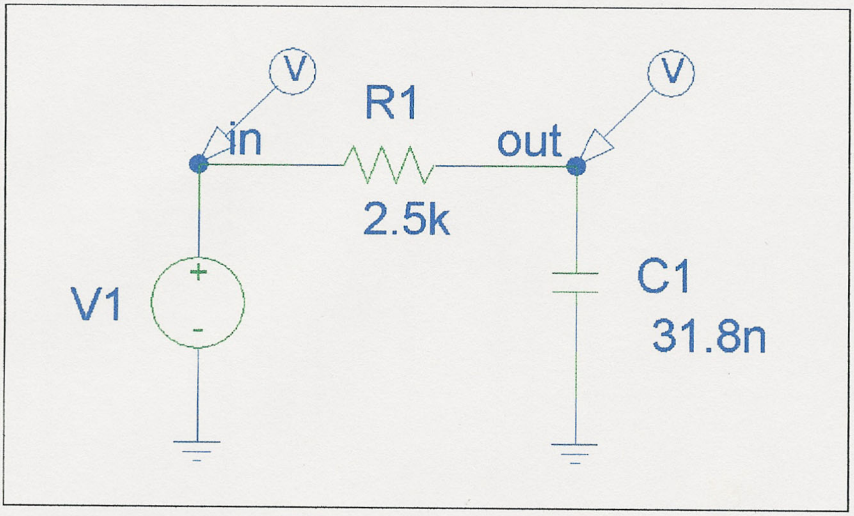
$\frac{7}{15}$, 1, $\frac{12}{15}$, $\frac{3}{15}$, 0 — voltage (V)

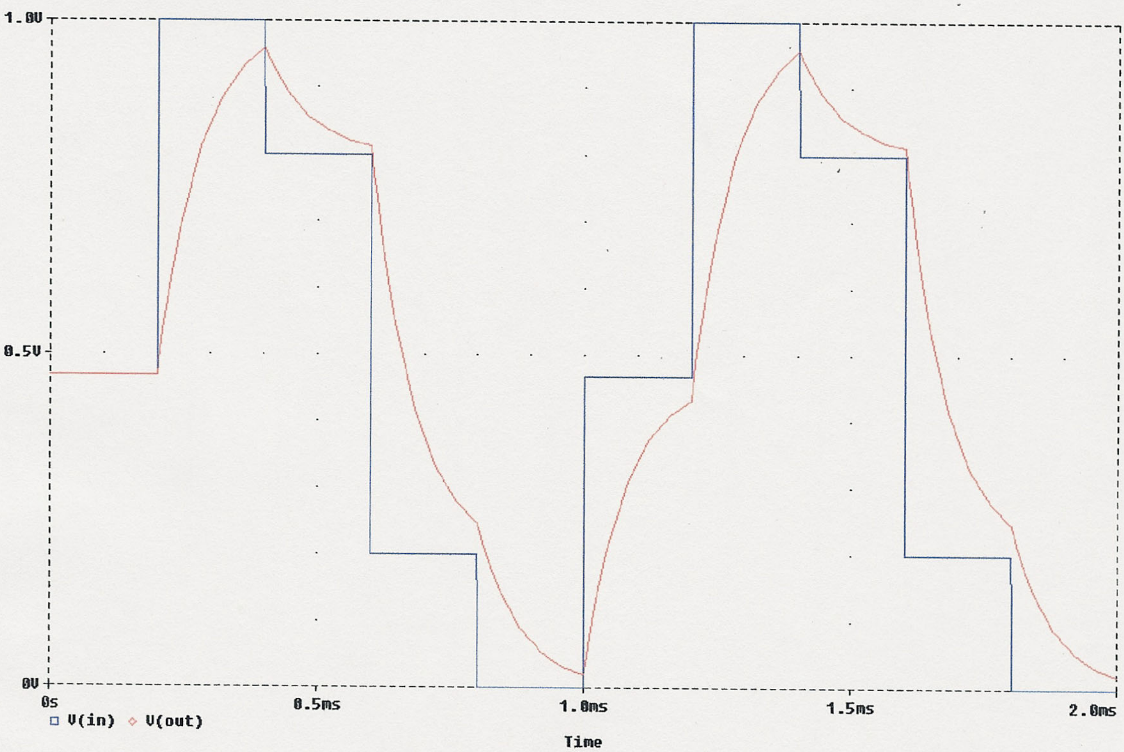
b)

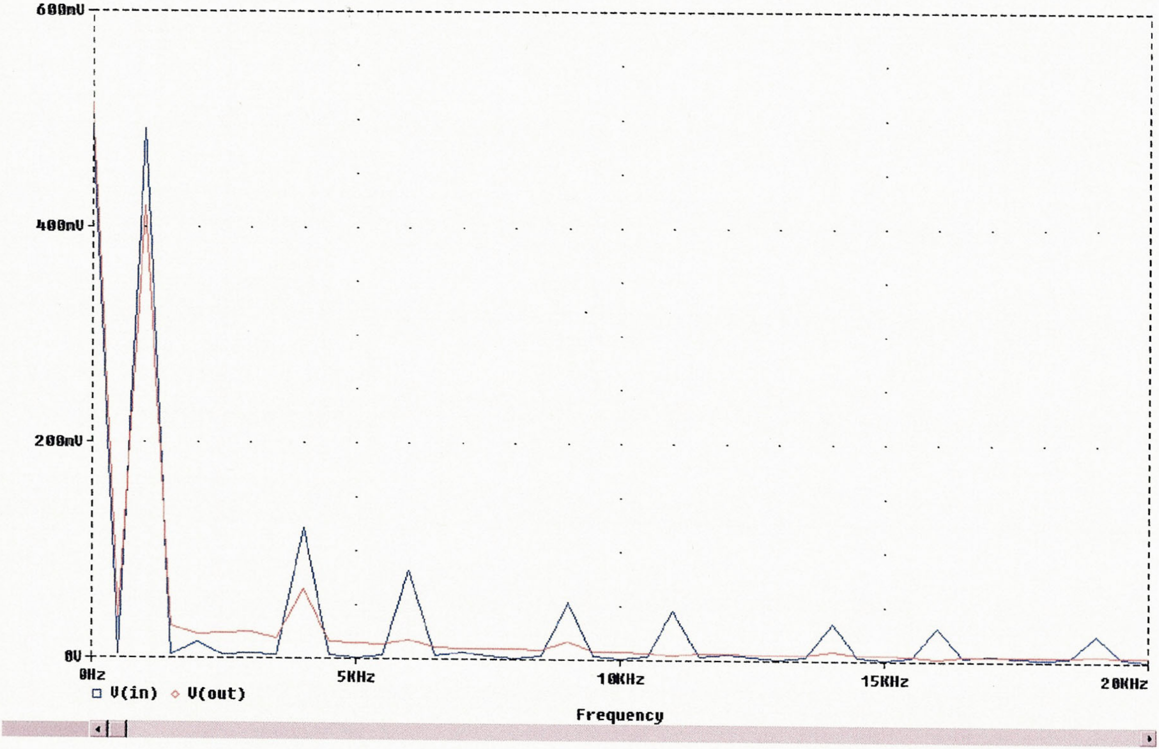


P1.28

The PSPICE schematic is shown below. The source was specified with a transient value of: PWL(200u .467 201u 1 400u 1 401u .8 600u .8 601u .2 800u .2 801u 0 1m 0 1001u .467 1.2m .467 1201u 1 1.4m 1 1401u .8 1.6m .8 1601u .2 1.8m .2 1801u 0 2m 0). The filter has a cutoff frequency of $1/(2\pi RC) = 2$ kHz as requested. The plots below show the input and output as a function of time and frequency (using the FFT of the time waveforms). The filter cuts off the high-frequency components in the input as seen in the frequency domain and this leads to a rounding off of the fast edges in the time domain.






600mV
400mV
200mV
0V
0Hz
5KHz
10KHz
15KHz
20KHz
V(in)
V(out)
Frequency

P1.29

$P = 500$ Torr $\quad V_{OUT} = 2V \quad \Delta V_{OUT} / \Delta P = 1\,mV/Torr$

Operating range $\quad 500$ Torr $\longleftrightarrow 3000$ Torr

V_{OUT} range $\quad 2V \longleftrightarrow 2V + 1\,mV/Torr \times (3000 - 500)\,Torr$

$= 4.5$

with 0.5 Torr resolution

we have $\dfrac{4.5 - 2.0}{0.5\,m} = 5000$ possible values

we need min. of <u>13 bits</u> because $2^{12} - 1 < 5000 < 2^{13} - 1$

P1.30

@ $t<0$ switch open
$t=0$ switch close
examine at $t=\infty$

a) Assume energy conservation

$t=0$ $\quad E_1 = \frac{1}{2}C_1V_1^2 = \frac{25}{2}C_1$ & $E_2 = \frac{1}{2}C_2V_2^2 = 0$

$t=\infty$ $\quad V_1 = V_2 = V_f$

$E_1 + E_2 = \frac{25}{2}C_1$ $\quad\because$ energy conserves

$$\frac{1}{2}C_1V_f^2 + \frac{1}{2}C_2V_f^2 = \frac{25}{2}C_1$$

$$V_f^2 = \frac{25\,C_1}{C_1+C_2} \Rightarrow \underline{\underline{V_f = 5\sqrt{\frac{C_1}{C_1+C_2}}}}$$

b) Assume charge conservation

$t=0$ $\quad Q_1 = C_1V_1 = 5C_1$ & $Q_2 = 0$

$t=\infty$ $\quad V_f = V_1 = V_2$ $\quad\because$ C_1 is in parallel C_2

$Q_f = Q_1 + Q_2 = 5C_1$ $\quad\because$ charge conserves

Total Capacitance $C_T = C_1 + C_2$

$$V_f = \frac{Q_f}{C_T}$$

$$\underline{\underline{V_f = 5\cdot\frac{C_1}{C_1+C_2}}}$$

c) V_1 in a) is larger than V_1 in b) because the model did not include resistance in the circuit. Energy will be lost during the redistiibution of charge through the resistance in the circuit.

V_1 in b) is the correct result since charge <u>must</u> be conserved (there is nowhere for it to go and it can't be destroged). Energy, on the other hand, can be lost due to radiation ($\Rightarrow$ radiation resistance) or heat (electrical resistance).

P1.31

a) No. Any two elements in parallel should have the same voltage across them. V_1 cannot be 5V and 3V at the same time. The series resistances of the batteries are not modeled.

b) No. Any two elements in series should have the same current through them. Current through 3Ω resistor cannot be 1A and 2A at the same time. The shunt resistances of the current sources are not modeled.