SOLUTIONS MANUAL

INTRODUCTION TO DIGITAL COMMUNICATION

SECOND EDITION

RODGER E. ZIEMER ▪ ROGER L. PETERSON

Prentice Hall
Upper Saddle River, NJ 07458

Associate Editor: Alice Dworkin
Special Projects Manager: Barbara A. Murray
Production Editor: Blake Cooper
Supplement Cover Manager: Paul Gourhan
Supplement Cover Designer: PM Workshop Inc.
Manufacturing Buyer: Lisa McDowell

Prentice Hall

Upper Saddle River, NJ 07458

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-015924-7

Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada, Inc., Toronto
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Pearson Education Asia Pte. Ltd., Singapore
Prentice-Hall of Japan, Inc., Tokyo
Editora Prentice-Hall do Brazil, Ltda., Rio de Janeiro

Table of Contents

Chapter 1

Problem 1-1

With $\log_2(q)$ plotted on the abscissa and R_m = source rate plotted on the ordinate in kbps, the plot is a straight line starting at ($\log_2 q = 1$, R_m = 8 kbps) ending at ($\log_2 q = 8$, R_m = 64 kbps).

Problem 1-2

Using (1-2),

$$R_m = 2W\log_2 q \text{ or } 18{,}000 = 2(3{,}000)\log_2 q \text{ or } \log_2 q = \frac{18000}{6000} = 3$$

Thus, $q = 8$ levels.

Problem 1-3

(a) Raise both sides of (1-4) to the power of 2:

$$2^{C/W} = 1 + \frac{E_b}{N_0}\left(\frac{C}{W}\right)$$

Solve for E_b/N_0 to get (1-5).
(c) The required transmission bandwidth is $B_T = 2W = 2/T_b = 2R$ Hz (R in bps). Therefore, the point to be plotted is $R/B_T = ½$ and $E_b/N_0 = 6$ dB.

Problem 1-4

Use Shannon's capacity formula, (1-4) rewritten as

$$C = W\log_2\left[1 + \frac{E_b/T_b}{N_0 W}\right] \text{ bits per second where } \frac{E_b}{T_b} = \text{carrier power, } R = \frac{1}{T_b}$$

Thus,

$$C = 10^4 \log_2\left[1 + \frac{10^{-12}}{(10^{-19})(10^4)}\right] = 10^4 \log_2[1 + 10^3] = 99.67 \text{ kbps}$$

Hence, C = 99.67 kbps is geather than the required R = 60 kbps and it is theoretically possible to achieve the desired performance. Therefore, your company is safe in submitting a bid in terms of the project being theoretically possible.

Problem 1-5

Using the rules in the problem statement written for $n = 3$, we have $g_1 = b_1$, $g_2 = b_2 \oplus b_1$, and $g_3 = b_3 \oplus b_2$. Application of these rules give the results in the table below:

Decimal number	Binary representation	Gray code, $g_1g_2g_3$
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

Problem 1-6

(a) Equation (1-6) gives

$$\begin{bmatrix} P(Y=0) \\ P(Y=1) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.34 \\ 0.66 \end{bmatrix}$$

(b) By Bayes' rule

$$P(X=1 \mid Y=1) = \frac{P(Y=1 \mid X=1)P(X=1)}{P(Y=1)} = \frac{(0.9)(0.6)}{0.66} = 0.818$$

where $P(Y=1) = P(Y=1 \mid X=0)P(X=0) + P(Y=1 \mid X=1)P(X=1)$

(c) Since $P(X=0 \mid Y=1) + P(X=1 \mid Y=1) = 1$, $P(X=0 \mid Y=1) = 1 - 0.818 = 0.182$. As a check, apply Bayes' rule to find the same answer.

Problem 1-7

(a) Expand the two expressions for $y(t)$ using trigonometric identities:

$$
\begin{aligned}
y(t) &= A\left[a_1 \cos\omega_0 t + a_2 \cos\omega_0(t - \tau)\right] \\
&= A\left[\left(a_1 + a_2 \cos\omega_0\tau\right)\cos\omega_0 t + a_2 \sin\omega_0\tau \sin\omega_0 t\right] \\
&\equiv AB\cos(\omega_0 t + \theta) \\
&\equiv A\left[B\cos\theta \cos\omega_0 t - B\sin\theta \sin\omega_0 t\right]
\end{aligned}
$$

Equate coefficients of $\cos \omega_0 t$ and $\sin \omega_0 t$ on the second and fourth lines to get

$$B\cos\theta = a_1 + a_2 \cos\omega_0\tau \text{ and } -B\sin\theta = a_2 \sin\omega_0\tau$$

Solve for B and θ to get

$$B = \sqrt{a_1^2 + 2a_1 a_2 \cos\omega_0\tau + a_2^2} \text{ and } \tan\theta = -\frac{a_2 \sin\omega_0\tau}{a_1 + a_2\cos\omega_0\tau}$$

(b) Substitute values in the equations immediately above to get $\max(B) = |a_1 + a_2| = 1.2$ for $\cos(\omega_0\tau) = 1$, and $\min(B) = |a_1 - a_2| = 0.8$ for $\cos(\omega_0\tau) = -1$. Maxima and minima are spaced by 1 MHz.

(c) For 10 kHz, negligible unless signal spectrum is located at a notch of B; for 100 kHz, negligible to moderate; for 1 MHz, severe; for 10 MHz, very severe to unusable unless steps are taken to undo the effects of the channel.

Problem 1-8

(a) Given that $a_2 = 0.5a_1$. Thus,

$$E_b = (a_1 + a_2)^2 T_b = 1.5^2 a_1^2 T_b$$

Therefore

$$P_b = \frac{1}{2}\exp\left(-2.25 a_1^2 T_b/N_0\right) = \frac{1}{2}\exp(-2.25k) = 10^{-5}$$

Solve to find that $k = -\ln(2\times10^{-5})/2.25 = 4.81$.

(b) For a reinforcing case (i.e., 1 followed by a 1 or a -1 followed by a -1), we have an ampltude of $1.5a_1$ or $-1.5a_1$ for an energy of $E_{br} = 2.25a_1^2T_b$. For a partial cancellation case, half the pulse has amplitude $0.5a_1$ and half of it has amplitude $1.5a_1$ for an energy of $E_{bc} = 0.5[0.25a_1^2 + 2.25a_1^2] = 1.25a_1^2T_b$. We leave out the first bit from the average because it is a transient situation. From Figure 1-8, we have six cancellation cases and three reinforcement cases. From our previous normalization procedure, we have an error probability of 10^{-5} for each of the reinforcement cases. For the cancellation cases, we have an error probability of

$$P_{bc} = \frac{1}{2}\exp(-1.25\times 4.81) = 0.0012$$

The average bit error probability averaged over bits 2 through 10 (bit 1 was left out because it is a transient situation) is

$$\overline{P_b} = \frac{3\times 10^{-5} + 6\times 0.0012}{9} = 8.033\times 10^{-4}$$

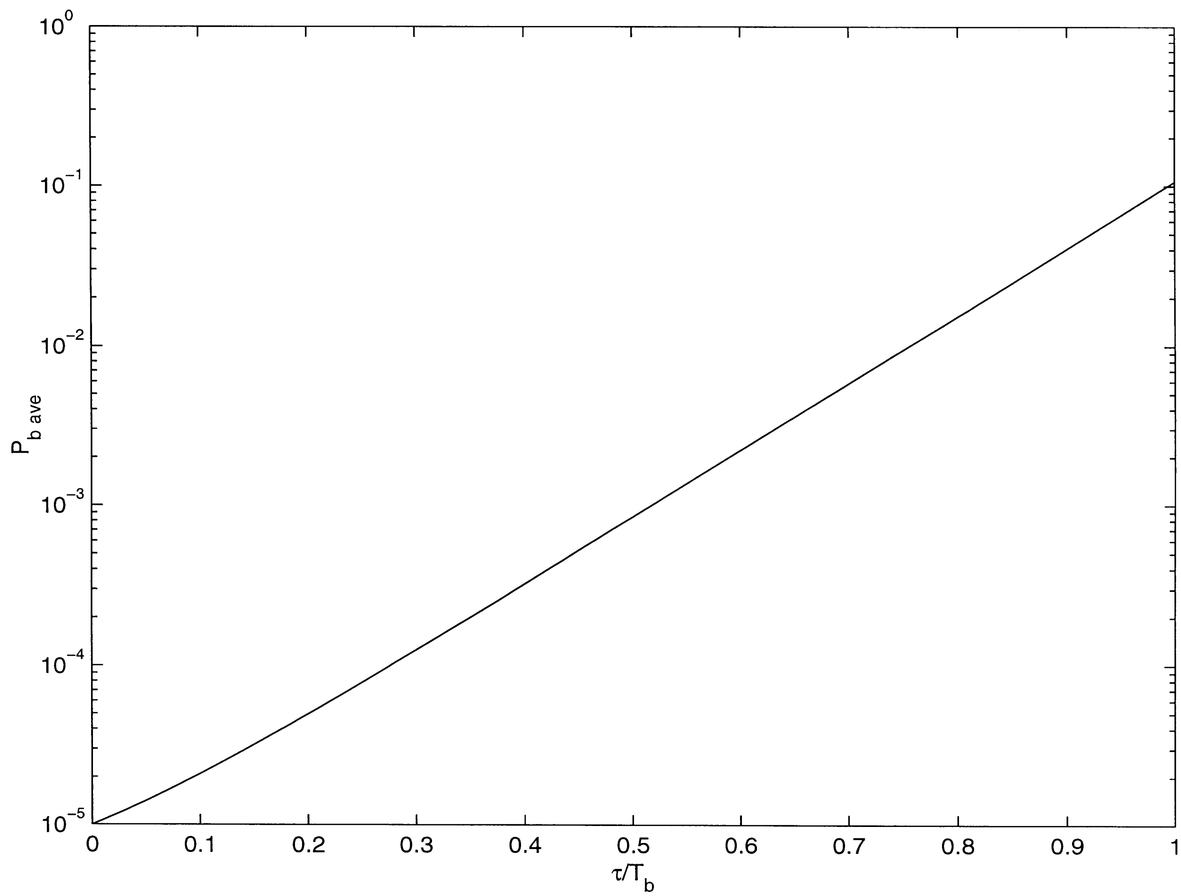
Note that as more bits are included in the average, it will approach a limit which is the average bit error probability for the intersymbol interference being considered.
(c) For arbitrary $\tau \leq T$, we can show with the aid of a sketch like the one in part (b) that

$$\begin{aligned} E_{br} &= 2.25a_1^2T_b \\ \text{and } E_{bc} &= (0.5a_1)^2\tau + (1.5a_1)^2(T_b - \tau) \\ &= 2.25a_1^2T_b(1 - 0.89\tau/T_b) \end{aligned}$$

Therefore,

$$\begin{aligned} P_{br} &= 10^{-5} \text{ (due to calibration)} \\ P_{bc} &= \exp[-2.25(a_1^2T_b)(1 - 0.89\tau/T_b) \\ &= \exp[-2.25(4.81)(1 - 0.89\tau/T_b) = 10^{-5}\times\exp(9.63\tau/T_b), \ \tau \leq T_b \end{aligned}$$

A plot is shown below:



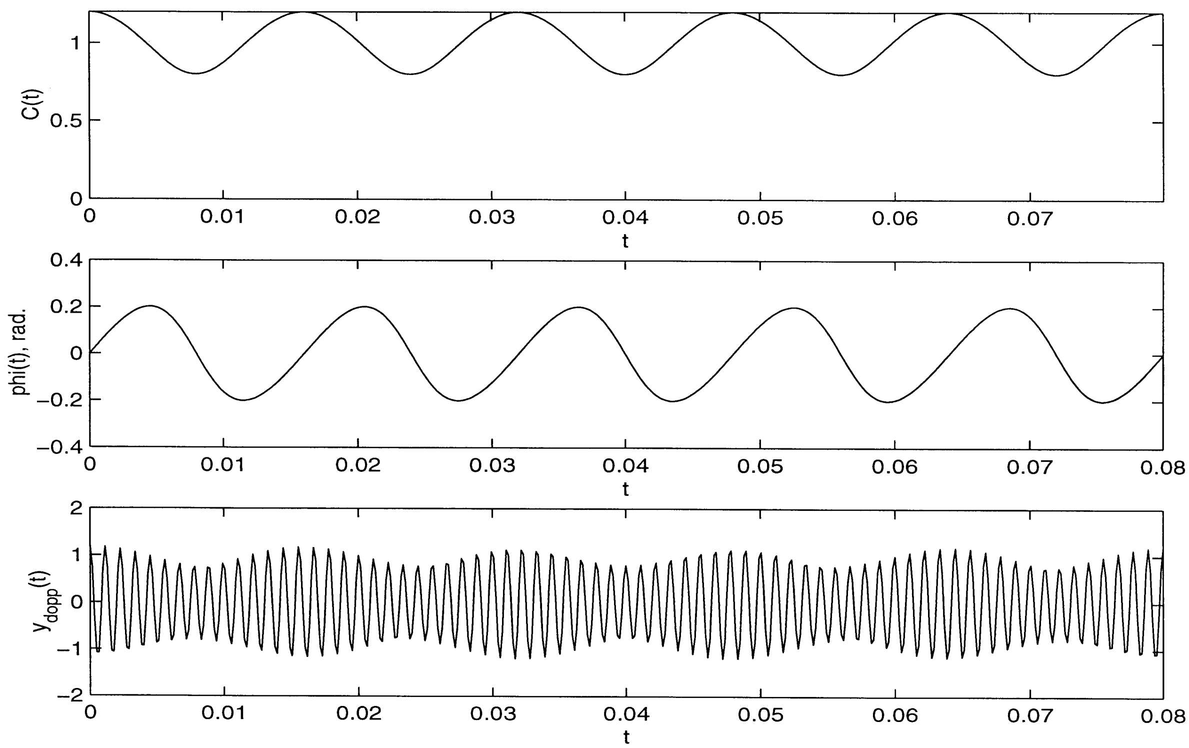
Problem 1-9

(a) From the information given in the problem, take $\alpha_1 = 1$ and $\alpha_2 = 0.2$. The Doppler shift is

$$f_d = \frac{(75\times10^3/3600\ \text{km/s})(900\times10^6\ \text{Hz})}{3\times10^8\ \text{m/s}} = 62.5\ \text{Hz or } \omega_d = 392.7\ \text{rad/s}$$

Since the automobile is traveling away from the base station the reflected component is at 900 MHz minus 62.5 Hz. This only makes a difference on the phase. Plots will be furnished after the answer to part (b).

(b) The period of the fading envelope is $f_d^{-1} = 1/62.5 = 16$ ms. The bit periods corresponding to the various data rates given are 1 ms, 0.1 ms, and 20 μs, respectively. Thus, the degradations would be moderate, negligible, and negligible, respectively.



Problem 1-10

(a) $\alpha = 0.0215(1)^{1.136} = 0.0215$ dB/km; (b) $\alpha = 0.362(1)^{0.972} = 0.362$ dB/km; (c) $\alpha = 0.0215(25)^{1.136} = 0.833$ dB/km; (d) $\alpha = 0.362(25)^{0.972} = 8.27$ dB/km; (e) $\alpha = 0.0719(10)^{1.097} = 0.8989$ dB/km.

Problem 1-11

The available power is

$$P_{\text{avail}} = kT = (1.38\times10^{-23})(290) = 4.002\times10^{-21} = -204 \text{ dBw} = -174 \text{ dBm}$$

Problem 1-12

The received power in dBW is

$$P_{R,\,\mathrm{dBW}} = 20\log_{10}\left(\frac{\lambda}{4\pi d}\right) + P_{T,\,\mathrm{dBW}} + G_{T,\,\mathrm{dB}} + G_{R,\,\mathrm{dB}} - L_{o,\,\mathrm{dB}}$$

The wavelength is $\lambda = c/f = 3\times10^8/12\times10^9 = 0.025$ m. The antenna gains are

$$G_R = G_T = \frac{4\pi(1)(0.7)}{(0.025)^2} = 14{,}074.3 = 41.48 \text{ dB}$$

The free-space loss is

$$L_{o,\,\mathrm{dB}} = -20\log_{10}\left[\frac{0.025}{4\pi\times35{,}784{,}000}\right] = 205.1 \text{ dB}$$

Therefore, the received power is

$$\begin{aligned} P_{R,\,\mathrm{dBW}} &= -205.1 + 10\log_{10}(100) + 2(41.48) - 6 \\ &= -108.1 \text{ dBW} = -78.1 \text{ dBm} \\ &= 1.549\times10^{-8} \text{ mW} = 1.549\times10^{-5}\ \mu\text{W} \end{aligned}$$

Problem 1-13

(a) The transmit and receive antenna gains in this case are

$$G_R = G_T = \frac{4\pi(2)(0.75)}{0.015^2} = 49.23 \text{ dB}; \ \lambda = \frac{3\times10^8}{20\times10^9} = 0.015 \text{ m}$$

The attenuation due to rain is

$$L_{\mathrm{rain}} = 2(0.8989) = 1.7978 \text{ dB} - \text{from Problem 1-10e}$$

The total attenuation (other than free space) is

$$L_{o,\,\mathrm{dB}} = L_{\mathrm{rain}} + L_{\mathrm{hardware}} + L_{\mathrm{atmos}} = 1.7978 + 2 + 0.4 = 4.1978 \text{ dB}$$

Solve (1-25) for $P_{T,\,\mathrm{dBw}}$:

$$P_{T,\,\mathrm{dBW}} = -20\log_{10}\frac{\lambda}{4\pi d} - G_{T,\,\mathrm{dB}} - G_{R,\,\mathrm{dB}} + L_{o,\,\mathrm{dBW}} + P_{R,\,\mathrm{dBW}}$$

where

$$P_{R,\,\mathrm{dBw}} = 10\log_{10}10^{-12} = -120 \text{ dBW}$$

and

$$\text{Free space loss} = -20\log_{10}\left(\frac{0.015}{4\pi\times 35{,}784{,}000}\right) = 209.54 \text{ dB}$$

Therefore,

$$P_{T,\,\mathrm{dBW}} = 209.54 - 2(49.23) + 4.1978 - 120 = -4.7222 \text{ dBW} = 0.3371 \text{ watts}$$

(b) The rain attenuation is

$$\alpha_{\mathrm{rain}} = 0.0368(15)^{1.118} = 0.7598 \text{ dB/km and } L_{\mathrm{rain}} = 2(0.7598) = 1.5196 \text{ dB}$$

The atmospheric loss is

$$L_{\mathrm{atmos}} = \left(\frac{15}{20}\right)^2(0.4) = 0.255 \text{ dB}$$

and the total loss is

$$L_{o,\,\mathrm{dB}} = 2 + 0.966 + 0.225 = 3.7446 \text{ dB}$$

The antenna gains are

$$G_R = G_T = 10\log_{10}\left(\frac{4\pi(2)(0.75)}{0.02^2}\right) = 46.73 \text{ dB}; \; \lambda = \frac{3\times 10^8}{15\times 10^9} = 0.02 \text{ m}$$

The free-space loss is

$$\text{Free space loss} = -20\log_{10}\left(\frac{0.02}{4\pi\times 35{,}784{,}000}\right) = 207.04 \text{ dB}$$

Thus, the transmit power is

$$P_{T,\,\text{dBW}} = 207.04 - 2(46.73) + 3.7446 - 120 = -2.6754 \text{ dBW} = 0.5401 \text{ watts}$$

Problem 1-14

For the uplink, $\lambda = 3\times10^8/12\times10^9 = 0.025$ m. The transmit and receive antenna gains are

$$G_{R,\,\text{up}} = G_{T,\,\text{up}} = 10\log_{10}\left[\frac{4\pi(1.5)(0.7)}{(0.025)^2}\right] = 43.25 \text{ dB}$$

The free-space loss is

$$\text{Free space loss}_{\text{up}} = -20\log_{10}\left(\frac{0.025}{4\pi\times 40{,}000{,}000}\right) = 206.07 \text{ dB}$$

Overall losses are $L_{o,\,\text{up}} = 3 + 0.2 = 3.2$ dB. The effective noise temperature is $T_e = T_o(F - 1) = 290(10^{0.45} - 1) = 527.33$ K. The overall equivalent noise temperature for the uplink is $T_{\text{up}} = T_e + T_{\text{ant}} = 527.33 + 300 = 827.33$ K. The noise power for the uplink is

$$P_{n,\,\text{up}} = 10\log_{10}(kT_{\text{up}}B) = 10\log_{10}[(1.38\times10^{-23})(827.33)(10^5)] = -149.42 \text{ dBW}$$

The uplink signal-to-noise power ratio is to be 20 dB, so

$$P_{R,\,\text{dBW, sat}} - P_{n,\,\text{dBW}} = 20 \text{ dB} \text{ or } P_{R,\,\text{dBW, sat}} = 20 - 149.42 = -129.42 \text{ dBW}$$

From (1-27),

$$\begin{aligned} P_{T,\,\text{dBW, up}} &= P_{R,\,\text{dBW, sat}} + \text{FSL}_{\text{up}} - G_{T,\,\text{dB}} - G_{R,\,\text{dB}} + L_{o,\,\text{up, dB}} \\ &= -129.42 + 206.07 - 2(43.25) + 3.2 \\ &= -6.65 \text{ dBW} = 0.216 \text{ W} \end{aligned}$$

On the downlink, $\lambda = 3\times10^8/10\times10^9 = 0.03$ m. The transmit and receive antenna gains are

$$G_{R,\text{ down}} = G_{T,\text{ down}} = 10\log_{10}\left[\frac{4\pi(1.5)(0.7)}{(0.03)^2}\right] = 41.66 \text{ dB}$$

The free-space loss is

$$\text{Free space loss}_{\text{down}} = -20\log_{10}\left(\frac{0.03}{4\pi\times 40{,}000{,}000}\right) = 204.48 \text{ dB}$$

Overall losses are the same as on the uplink, and the effective noise temperature is the same. The overall equivalent noise temperature for the downlink is $T_{\text{down}} = T_e + T_{\text{ant}} = 527.33 + 50 = 577.33$ K. The noise power for the downlink is

$$P_{n,\text{ down}} = 10\log_{10}(kT_{\text{down}}B) = 10\log_{10}[(1.38\times 10^{-23})(577.33)(10^5)] = -151 \text{ dBW}$$

The received signal power is

$$P_{R,\text{ dBW, ground}} = 20 - 151 \text{ dB} = -131 \text{ dBW}$$

From (1-27), the transmit power on the downlink may be calculated as

$$P_{T,\text{ dBW, grnd, dB}} = P_{R,\text{ dBW}} - 20\log_{10}\left(\frac{\lambda}{4\pi d}\right) - G_{T,\text{ dB}} - G_{R,\text{ dB}} + L_{o,\text{ dB}}$$
$$= -131 + 204.48 - 2(41.66) + 3.2 = -6.64 \text{ dBW} = 0.217 \text{ W}$$

Problem 1-15

(a) The effective aperture for a circular antenna of diameter d is

$$A_{\text{eff}} = \rho\left(\frac{\pi d^2}{4}\right)$$

Therefore, (1-22 becomes

$$G = 4\pi\rho\left(\frac{\pi d^2}{4}\right)\frac{1}{\lambda^2} = \rho\left(\frac{\pi d}{\lambda}\right)^2 = \rho\left(\frac{\pi f d}{c}\right)^2$$

(b) Putting the numbers into the above equation and expressing in dB gives $G_{\text{dB}} = 19.15 + 20\log_{10}(f \text{ in GHz})$, and $\phi_{3\text{ dB}} = 0.346/(f \text{ in Ghz})$ radians. The following table results:

f, GHz	G_{dB}	$\phi_{3\,dB}$, radians (degrees)
10	39.5	0.035 (2)
15	42.7	0.023 (1.32)
20	45.2	0.017 (0.97)
30	48.7	0.012 (0.69)

Problem 1-16

The gain in terms of frequency is

$$G_{dB} = 10\log_{10}\left[\rho\left(\frac{\pi f d}{c}\right)^2\right]$$

so

$$30 = 10\log_{10}\left[0.75(5\pi d)^2\right] \text{ or } d = \frac{1}{5\pi}\left(\frac{10^3}{0.75}\right)^{1/2} = 2.325 \text{ m}$$

Also,

$$\phi_{3\text{ dB}} = \frac{0.2}{2.325\sqrt{0.75}} = 0.1 \text{ rad} = 5.69 \text{ degrees}$$

Problem 1-17

(a) For 20 GHz, the wavelength is $\lambda = 0.015$ m. We want $\phi_{3\text{ dB}} = 2$ degrees = 0.035 radians. Solve the 3 dB beamwidth equation given in Problem 1-15 for d to get

$$d = \frac{\lambda}{\phi_{3\text{ dB}}\sqrt{\rho}} = \frac{0.015}{0.035\sqrt{0.65}} = 0.533 \text{ m}$$

The gain is

$$G_{dB} = 10\log_{10}\left[\rho\left(\frac{\pi d}{\lambda}\right)^2\right] = 10\log_{10}\left[0.65\left(\frac{0.533\pi}{0.015}\right)^2\right] = 39.1 \text{ dB} = 8.1\times10^3 \text{ ratio}$$

(b) A 150 mile spot at d = 35,784 km is desired. Note that 1 mi = 1.6093 km, so 150 mi = 241.4 km. Therefore,

$$\phi_{3\ \text{dB}} = \frac{241.4}{35{,}784} = 0.00675 \ \text{rad} = \frac{\lambda}{d\sqrt{\rho}}$$

Solving for *d* gives

$$d = \frac{\lambda}{\phi_{3\ \text{dB}}\sqrt{\rho}} = \frac{0.015}{0.00675\sqrt{0.65}} = 2.76 \ \text{m}$$

The required gain is

$$G_{\text{dB}} = 10\log_{10}\left[\rho\left(\frac{\pi d}{\lambda}\right)^2\right] = 10\log_{10}\left[0.65\left(\frac{2.76\pi}{0.015}\right)^2\right] = 53.36 \ \text{dB} = 2.167\times10^5 \ \text{ratio}$$

Chapter 2

Problem 2-1

Part	Energy or Power	Periodic?	Power, W	Energy, J
a	Power	Yes; period = 1 s	1	∞
b	Energy	No	0	1/10
c	Energy	No	0	2
d	Energy	No	0	π
e	Power	No	1	∞

Problem 2-2

(a) Linear, fixed, noncausal; (b) Linear, fixed, causal; (c) Linear, time varying, causal; (d) Linear, fixed, noncausal; (e) Nonlinear, fixed, causal.

Problem 2-3

(a) The response is

$$y(t) = \int_{-\infty}^{\infty} 10 e^{-10(t-\tau)} u(\tau) u(t-\tau)\, d\tau = \int_{0}^{t} 10 e^{-10(t-\tau)} d\tau, \; t \ge 0$$

$$= \left(1 - e^{-10t}\right) u(t)$$

(b) Using the superposition integral, the response is

$$y(t) = \int_{-1}^{t} 10 e^{-10(t-\tau)} d\tau, \; |t| \le 1$$

$$= \begin{cases} 0, & t < -1 \\ 1 - e^{-10} e^{-10t}, & |t| \le 1 \\ (e^{10} - e^{-10}) e^{-10t}, & t > 1 \end{cases}$$

Another way is to write the input as $x(t) = u(t+1) - u(t-1)$ and use superposition with the result of (a).
(c) Consider the input $x(t) = \exp(-\alpha t)\, u(t)$ so that it can be used in both parts (c) and (d). For $t > 0$,

$$y(t) = \int_0^t e^{-\alpha\tau} 10 e^{-10(t-\tau)} d\tau = \frac{10}{10-\alpha}\left(e^{-\alpha t} - e^{-10t}\right), \ t \geq 0$$

Now let $\alpha = 3$ to obtain the answer for part (c):

$$y(t) = \frac{10}{7}\left(e^{-3t} - e^{-10t}\right)u(t)$$

(d) Use the general result for part (c) with $\alpha = -j6\pi$ to get the result

$$y(t) = \frac{10}{10 + j6\pi}\left(e^{j6\pi t} - e^{-10t}\right)u(t)$$

(e) Use superposition with $x(t) = 0.5\exp(j6\pi t) + 0.5\exp(-j6\pi t)$ and the result for part (d) to get

$$\begin{aligned} y(t) &= \frac{5}{10 + j6\pi}\left(e^{j6\pi t} - e^{-10t}\right)u(t) + \frac{5}{10 - j6\pi}\left(e^{-j6\pi t} - e^{-10t}\right)u(t) \\ &= \frac{10}{100 + 36\pi^2}\left[10\cos 6\pi t + 6\pi \sin 6\pi t - 10e^{-10t}\right]u(t) \end{aligned}$$

Problem 2-4

(a) Sketches show that $\phi_1(t)$ is 1 between 0 and 1 and 0 elsewhere, $\phi_2(t)$ is 1 between 1 and 2 and 0 elsewhere, $\phi_3(t)$ is 1 between 2 and 3 and 0 elsewhere, and $\phi_4(t)$ is 1 between 3 and 4 and 0 elsewhere. Clearly, the area under the square of each is 1. Since there is no overlap between differing functions, the set is clearly orthogonal.

(b) The expansion of (2-17) with four terms can be found by applying (2-22). The expansion coefficients are

$$d_1 = \int_0^1 e^{-t/2} dt = 2(1 - e^{-1/2}), \ d_2 = \int_1^2 e^{-t/2} dt = 2(1 - e^{-1/2})e^{-1/2},$$

$$d_3 = \int_2^3 e^{-t/2} dt = 2(1 - e^{-1/2})e^{-1}, \ d_4 = \int_3^4 e^{-t/2} dt = 2(1 - e^{-1/2})e^{-3/2}$$

Therefore,

$$\tilde{x}(t) = 2(1 - e^{-1/2})\left[\phi_1(t) + e^{-1/2}\phi_2(t) + e^{-1}\phi_3(t) + e^{-3/2}\phi_4(t)\right]$$

The minimum integral-squared error is

$$\epsilon_{4,\min} = \int_0^4 e^{-t}dt - \sum_{n=1}^{4} d_n^2 = 1 - e^{-4} - 4(1 - e^{-1/2})^2(1 + e^{-1} + e^{-2} + e^{-3}) = 0.02$$

Problem 2-5

Use Euler's theorem to expand the complex exponential Fourier series expression:

$$x(t) = \sum_{n=-\infty}^{\infty} X_n \exp(jn\omega_0 t) = \sum_{n=-\infty}^{\infty} X_n\left[\cos(n\omega_0 t) + j\sin(n\omega_0 t)\right]$$

$$= X_0 + \sum_{n=1}^{\infty} X_n \cos n\omega_0 t + \sum_{n=-\infty}^{-1} X_n \cos n\omega_0 t$$

$$+ \sum_{n=1}^{\infty} jX_n \sin n\omega_0 t + \sum_{n=-\infty}^{-1} jX_n \sin n\omega_0 t$$

Let $m = -n$ in the second and fourth sums:

$$x(t) = X_0 + \sum_{n=1}^{\infty} (X_n + X_{-n})\cos n\omega_0 t + \sum_{n=1}^{\infty} j(X_n - X_{-n})\sin n\omega_0 t$$

$$\equiv a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \quad \text{(Fourier sine-cosine series)}$$

Equate coefficients to get

$$a_0 = X_0;\ a_n = X_n + X_{-n};\ b_n = j(X_n - X_{-n})$$

Problem 2-6

Write the Fourier sum in terms of

$$|X_n|\exp(j\theta_n) = X_n = \frac{1}{T_0}\int_{T_0} x(t)\exp(-jn\omega_0 t)\,dt$$

to get

$$x(t) = \sum_{n=-\infty}^{\infty} |X_n| \exp[j(n\omega_0 t + \theta_n)]$$

If $x(t)$ is real, $X_n = X_{-n}^*$ as can be seen by conjugating the integral for X_n and replacing n by $-n$. This implies that $|X_n| = |X_{-n}|$ and $\theta_n = \theta_{-n}$. Now write the Fourier series sum as

$$x(t) = X_0 + \sum_{n=0}^{\infty} |X_n| \exp[j(n\omega_0 t + \theta_n)] + \sum_{n=-\infty}^{-1} |X_n| \exp[j(n\omega_0 t + \theta_n)]$$

In the last sum, replace n by $-m$ and then change back to n. Put both sums together and use Euler's theorem to get

$$x(t) = X_0 + \sum_{n=1}^{\infty} 2|X_n| \cos(n\omega_0 t + \theta_n) \equiv A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_n)$$

Problem 2-7

In $\sum_n |X_n|^2 = \sum_n X_n X_n^*$, replace X_n by the integral expression for it:

$$\sum_{n=-\infty}^{\infty} |X_n|^2 = \sum_{n=-\infty}^{\infty} X_n^* \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0}\int_{T_0} x(t) \sum_{n=-\infty}^{\infty} X_n^* e^{-jn\omega_0 t} dt$$

But

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} \text{ so } x^*(t) = \sum_{n=-\infty}^{\infty} X_n^* e^{-jn\omega_0 t}$$

Therefore,

$$\sum_{n=-\infty}^{\infty} |X_n|^2 = \frac{1}{T_0}\int_{T_0} x(t) x^*(t) dt = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$$

Problem 2-8

(a) For the half-rectified sine wave

$$X_n = \frac{1}{T_0}\int_0^{T_0/2} A\sin(\omega_0 t)e^{-jn\omega_0 t}\,dt + \frac{1}{T_0}\int_{T_0/2}^{T_0}(0)e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} A\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}e^{-jn\omega_0 t}\,dt$$

$$= \frac{A}{j2T_0}\left[\left.\frac{e^{j(1-n)\omega_0 t}}{j(1-n)\omega_0}\right|_0^{T_0/2} + \left.\frac{e^{-j(1+n)\omega_0 t}}{-j(1+n)\omega_0}\right|_0^{T_0/2}\right] = \begin{cases} 0, & n \text{ odd} \\ \dfrac{A}{\pi(1-n^2)}, & n \text{ even and } \neq \pm 1 \\ \dfrac{A}{4j}, & n = \pm 1 \end{cases}$$

(b) For a full-rectified sine wave, the period is really $T_0/2$. Furthermore, it is an even signal, so

$$X_n = \frac{2A}{T_0}\int_0^{T_0/2}\sin(\omega_0 t)\cos(n\omega_0' t)\,dt,\ \omega_0' = 2\pi/(T_0/2) = 2\omega_0$$

$$= \frac{A}{T_0}\int_0^{T_0/2}\{\sin[(1-2n)\omega_0 t] + \sin[(1+2n)\omega_0 t]\}\,dt$$

$$= \frac{A}{\omega_0 T_0}\left[-\frac{\cos[(1-2n)\pi] - 1}{1-2n} - \frac{\cos[(1+2n)\pi] - 1}{1+2n}\right]$$

$$= \frac{2A}{\pi(1-4n^2)}$$

(c) For the pulse train:

$$X_n = \frac{1}{T_0}\int_{t_0-\tau/2}^{t_0+\tau/2} A\exp(-jn\omega_0 t)\,dt = \frac{A}{T_0}\left[-\frac{\exp(-jn\omega_0 t)}{jn\omega_0}\right]_{t_0-\tau/2}^{t_0+\tau/2}$$

$$= \frac{A\tau}{T_0}\mathrm{sinc}(n\tau/T_0)\exp(-j2\pi n t_0/T_0)$$

(d) For an even square wave

$$X_n = \frac{2A}{T_0}\left[\int_0^{T_0/4} \cos(n\omega_0 t)\, dt - \int_{T_0/4}^{T_0/2} \cos(n\omega_0 t)\, dt\right]$$

$$= \frac{2A}{T_0}\left[\frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_0^{T_0/4} - \frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_{T_0/4}^{T_0/2}\right] = \frac{2A\sin(n\pi/2)}{n\pi}$$

$$= \begin{cases} \dfrac{2A}{\pi|n|}, & n = \pm 1, \pm 5, \cdots \\ -\dfrac{2A}{\pi|n|}, & n = \pm 3, \pm 7, \cdots \\ 0, & n \text{ even} \end{cases}$$

(e) For an even triangle signal,

$$X_n = \frac{2A}{T_0}\left[\int_0^{T_0/2}\left(1 - \frac{4t}{T_0}\right)\cos(n\omega_0 t)\, dt\right]$$

$$= \frac{2A}{T_0}\left[\frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_0^{T_0/2} - \frac{4}{T_0}\int_0^{T_0/2} t\cos(n\omega_0 t)\, dt\right]$$

The first term in the brackets is zero upon substitution of limits. The second term must be integrated by parts or looked up in a table. We get

$$X_n = \frac{2A}{T_0}\left(-\frac{4}{T_0}\right)\left[\frac{t\sin(n\omega_0 t)}{n\omega_0}\Bigg|_0^{T_0/2} - \int_0^{T_0/2}\frac{\sin(n\omega_0 t)}{n\omega_0}\, dt\right]$$

The first term in the brackets again evaluates to 0. The remaining integral evaluates to

$$X_n = -\frac{4A}{n\pi T_0}\frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_0^{T_0/2} = \begin{cases} \dfrac{4A}{(n\pi)^2}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

Problem 2-9

(a) Using the notation of (2-39), $p(t)$ is a triangle function centered at $t = 0$, of peak amplitude 1, and with half width 2. From pair 3, Table F-7,

$$\Lambda(t/2) \overset{\mathscr{F}}{\leftrightarrow} 2\,\mathrm{sinc}^2(2f)$$

Again, in the notation of (2-40), $f_0 = 1/T_0 = 1/4 = 0.25$, so (2-40) becomes

$$X(f) = 0.25 \sum_{n=-\infty}^{\infty} 2\,\mathrm{sinc}^2(0.5n)\,\delta(t - 0.25n)$$

(b) In this case, $p(t) = \Pi(t/4)$ and $T_0 = 8$, so (2-40) becomes

$$X(f) = 0.125 \sum_{n=-\infty}^{\infty} 4\,\mathrm{sinc}(0.5n)\,\delta(t - 0.125n)$$

where use has been made of the transform pair

$$\Pi(t/4) \leftrightarrow 4\,\mathrm{sinc}(4f)$$

Problem 2-10

Let

$$y(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$$

By the multiplication theorem of Fourier transforms and pair 17 of Table F-7,

$$Y(f) = X(f) * f_s \sum_{m=-\infty}^{\infty} \delta(f - mT_s) = f_s \sum_{m=-\infty}^{\infty} X(f) * \delta(f - mf_s) = f_s \sum_{m=-\infty}^{\infty} X(f - mf_s)$$

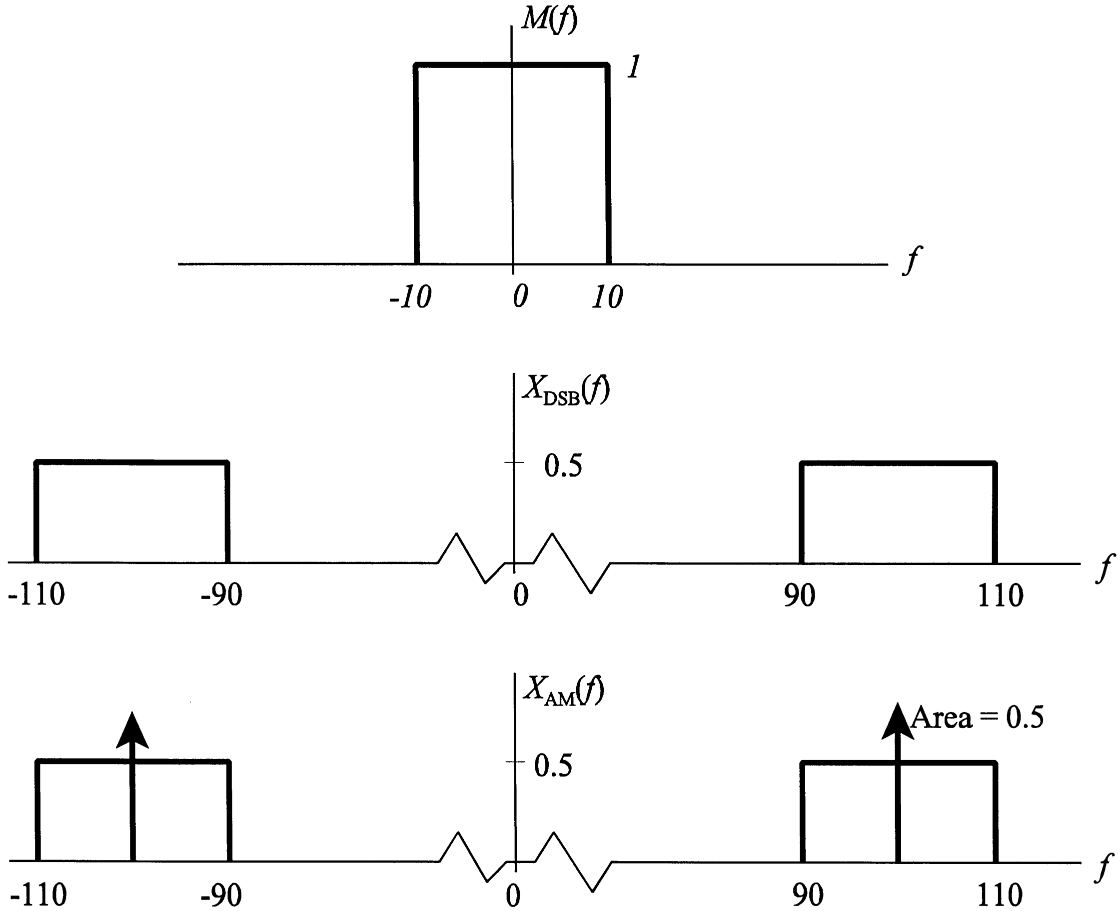
Problem 2-11

Use the delay theorem followed by the frequency translation theorem to get

$$\mathscr{F}\left[\Pi\left(\frac{t-2}{4}\right) e^{j2\pi(t-2)}\right] = 4\,\mathrm{sinc}[4(f-1)]\,e^{-j4\pi f}$$

Problem 2-12

The spectra are shown below:



The single sideband spectra consist of the portion of the DSB spectrum from -110 to -100 and 100 to 110 Hz for USB, or the portion from -100 to -90 and from 90 to 100 for LSB.

Problem 2-13

For DSB and AM, we need $m(t) = \mathscr{F}^{-1}[M(f)] = 20\,\mathrm{sinc}(20t)$. The modulated wave for for AM is

$$x_{\mathrm{AM}}(t) = A[1 + am_n(t)]\cos(200\pi t)$$

where, for convenience, we take $A = 1$. It was given that $a = 1/2$. From above, we have $m_n(t) = \mathrm{sinc}(20t)$. Thus,

$$x_{\mathrm{AM}}(t) = [1 + 0.5\,\mathrm{sinc}(20t)]\cos(200\pi t)$$

For DSB,

$$x_{\text{DSB}}(t) = m(t)\cos(2\pi f_c t) = 20\,\text{sinc}(20t)\cos(200\pi t)$$

where the carrier amplitude = 1 for convenience. For single sideband, we need the Hilbert transform of the modulating signal. Its Fourier transform is $-j\text{sgn}(f)M(f)$, so by inverse Fourier transformation

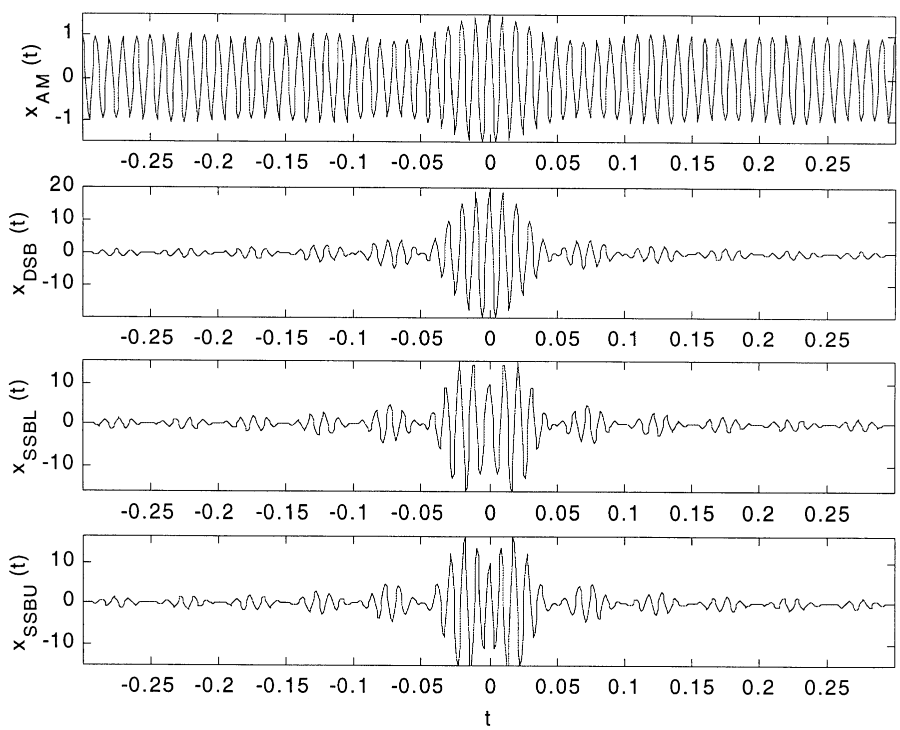
$$\hat{m}(t) = \mathscr{F}^{-1}[\hat{M}(f)] = \mathscr{F}^{-1}[-j\text{sgn}(f)M(f)] = \int_{-10}^{0} je^{j2\pi ft}\,df + \int_{0}^{10} -je^{j2\pi ft}\,df$$

$$= \frac{1}{\pi t}\left[1 - \cos(20\pi t)\right] = \frac{2}{\pi t}\sin^2(20\pi t) = (800\pi t)\,\text{sinc}^2(20t)$$

where $2\sin^2 u = 1 - \cos(2u)$. The single-sideband signals, from (2-65), are

$$x_{\text{SSBL, SSBU}}(t) = \frac{A}{2}[m(t)\cos(200\pi t) \pm \hat{m}(t)\sin(200\pi t)]$$

$$= \frac{1}{2}[20\,\text{sinc}(20t)\cos(200\pi t) \pm (800\pi t)\,\text{sinc}^2(20t)\sin(200\pi t)]$$



Problem 2-14

For AM:

$$x_{AM}(t)\cos(\omega_0 t) = A[1 + am_n(t)]\cos^2(\omega_0 t)$$

$$= \frac{A}{2}[1 + am_n(t)] + \frac{A}{2}[1 + am_n(t)]\cos(2\omega_0 t)$$

$$= \text{signal with spectrum at 0 freq.} + \text{signal with spectrum at } 2\times\text{carrier}$$

If $f_0 >> W$ (the modulating signal bandwidth), the lowpass filter's cutoff frequency can be chosen to reject the second term. A dc blocking capacitor rejects the $A/2$ term. The output of the lowpass filter is then proportional to $m(t)$. The proof for DSB is similar to this.

For SSB:

$$x_{AM}(t)\cos(\omega_0 t) = \frac{A}{2}[m(t)\cos(\omega_0 t) \pm \hat{m}(t)\sin(\omega_0 t)]\cos(\omega_0 t)$$

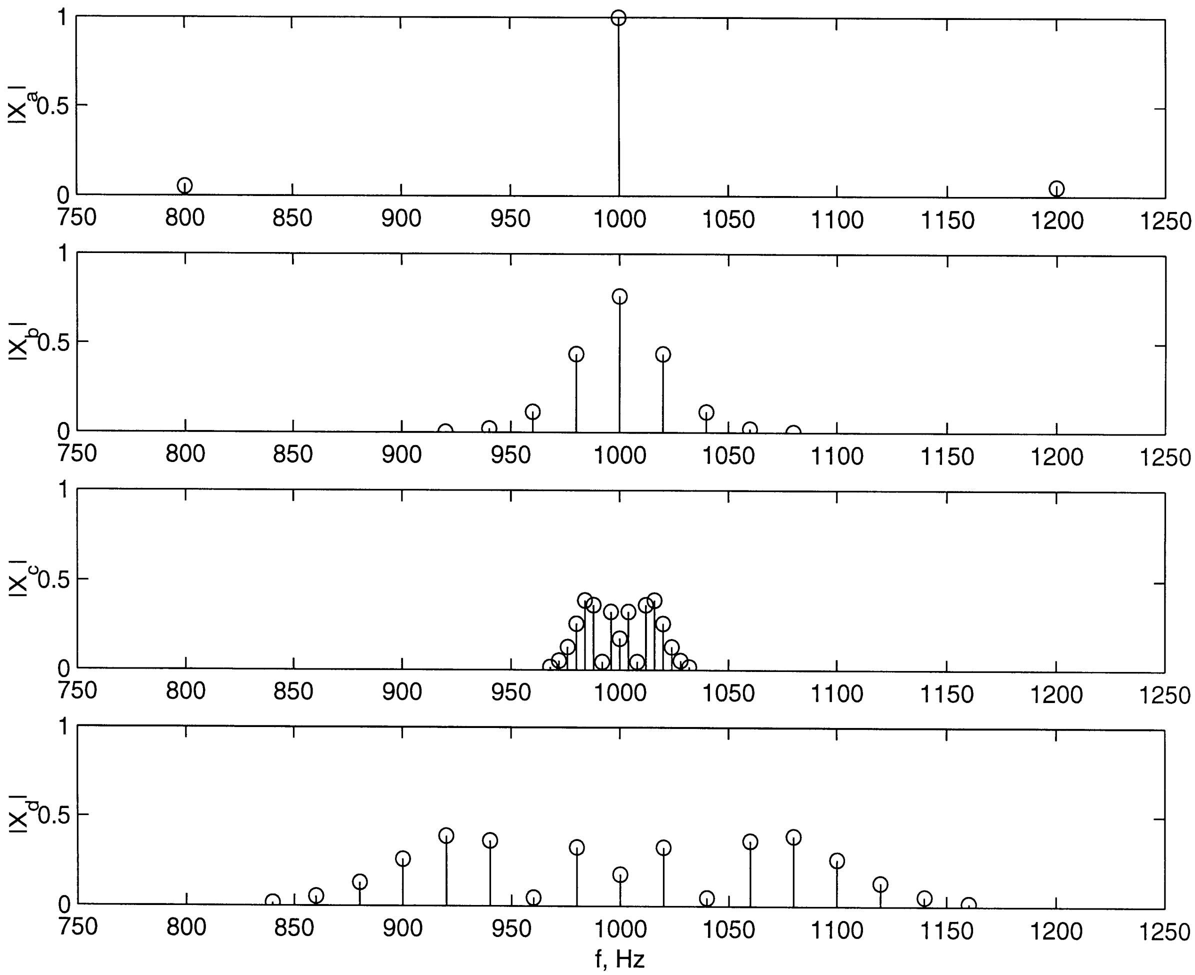
$$= \frac{A}{2}[m(t)\cos^2(\omega_0 t) \pm \hat{m}(t)\sin(\omega_0 t)\cos(\omega_0 t)$$

$$= \frac{A}{4}[m(t) + m(t)\cos(2\omega_0 t) \pm \hat{m}(t)\sin(2\omega_0 t)]]$$

The lowpass filter will reject the last two terms of the last equation and pass the first, which gives an output proportional to $m(t)$.

Problem 2-15

The spectrum may be found by applying (2-75) with $\beta = A_m f_d / f_m$ as given by (2-72).
(a) $\beta = (2)(10)/200 = 0.1$; hence, only the first lower and upper sideband lines and the carrier are significant. We find that $J_0(0.1) \approx 1$, $J_1(0.1) \approx 0.05 = -J_{-1}(0.1)$. The lower sideband line is at 800 Hz, the carrier line is at 100 Hz, and the upper sideband line is at 1200 Hz. The spectrum is shown at the end of the problem.
(b) $\beta = (2)(10)/20 = 1$; Use the recursion relationship (2-76) and Table 2-5 to compute $J_0(1) = 0.765$, $J_1(1) = 0.44$, $J_2(1) = 0.115$, $J_3(1) = 0.02$, and $J_4(1) = 0.002$; also, use (2-76) to get negative-indexed Bessel functions. Lines are spaced by 20 Hz. The spectrum is shown at the end of the problem.
(c) $\beta = (2)(10)/4 = 5$; Use the recursion relationship (2-76) and Table 2-5 to compute $J_0(5) = -0.178$, $J_1(5) = -0.328$, $J_2(5) = 0.047$, $J_3(5) = 0.365$, $J_4(5) = 0.391$, $J_5(5) = 0.261$, $J_6(5) = 0.131$, $J_7(5) = 0.053$, and $J_8(5) = 0.018$; also, use (2-76) to get negative-indexed Bessel functions. Lines are spaced by 5 Hz. The spectrum is shown at the end of the problem.
(d) $\beta = (10)(10)/20 = 5$; use the same J_n's as in part (c). Lines are now spaced by 20 Hz. See the next page for the amplitude spectrum.



Problem 2-16

(a) Using the overbar as average, we get for DSB:

$$P_{\mathrm{DSB}} = \overline{[A\,m(t)\cos\omega_0 t]^2} = A^2\overline{m^2(t)}\,\overline{\cos^2\omega_0 t}$$

$$= A^2\overline{m^2(t)}\left[\frac{1}{2} + \frac{1}{2}\cos 2\omega_0 t\right] = \frac{1}{2}A^2\overline{m^2(t)}$$

(b) For AM, we get

$$P_{\text{AM}} = \overline{\{A[1 + m(t)]\cos\omega_0 t\}^2} = A^2\overline{[1 + m^2(t)]^2}\,\overline{\cos^2\omega_0 t}$$

$$= A^2\left[1 + 2a\overline{m(t)} + \overline{m^2(t)}\right]\overline{\left[\frac{1}{2} + \frac{1}{2}\cos 2\omega_0 t\right]} = \frac{1}{2}A^2\left[1 + a^2\overline{m^2(t)}\right]$$

$$= \frac{1}{2}A^2[1 + a^2 P_m]$$

(c) For SSB, note that

$$\overline{\cos^2\omega_0 t} = \overline{\sin^2\omega_0 t} = \frac{1}{2} \text{ and } \overline{m^2(t)} = \overline{\hat{m}^2(t)} = P_m$$

We find that

$$P_{\text{SSB}} = A^2 P_m$$

(d) For PM and FM, the power may be calculated as

$$P_{\text{PM, FM}} = \overline{\{A\cos[\omega_0 t + \beta\phi(t)]\}^2} = \frac{1}{2}A^2 + \frac{1}{2}A^2\overline{\cos[2(\omega_0 t + \beta\phi(t)]} = \frac{1}{2}A^2$$

which follows as long as $\phi(t)$ is not such that a spectral line of the $\cos[2(\omega_0 t + \beta\phi(t)]$ terms ends up at zero frequency. This is possible, for example, if $\phi(t)$ is sinusoidal with frequency a subharmonic of ω_0.

<u>Problem 2-17</u>

(a) By applying (2-58), the Hilbert transform is

$$\hat{m}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(t)}{t - \lambda}d\lambda = \frac{1}{\pi}\int_{-1}^{1}\frac{d\lambda}{t - \lambda} = \frac{1}{\pi}\ln\left|\frac{t + 1}{t - 1}\right|$$

(b) For nonzero spectrum for $f > 0$,

$$z_p(t) = \Pi(t/2) + j\ln\left|\frac{t + 1}{t - 1}\right|$$

For nonzero spectrum for $f < 0$,

$$z_p(t) = \Pi(t/2) - j\ln\left|\frac{t+1}{t-1}\right|$$

(c) The Fourier transform of $m(t)$ is 2 sinc(2f). $Z_p(f)$ is the portion to the right of $f = 0$ (the left-hand part is zero), $Z_n(f)$ is the portion to the left of $f = 0$ (the right-hand part is zero).

Problem 2-18

(a) Use the transform pair

$$\exp(-\alpha|t|) \leftrightarrow \frac{2\alpha}{\alpha^2 + (2\pi f)^2}$$

and the modulation theorem to obtain

$$X(f) = \frac{1}{1 + [2\pi(f - 500)]^2} + \frac{1}{1 + [2\pi(f + 500)]^2}$$

Take the positive frequency part and shift left 500 Hz to get

$$\tilde{X}(f) = \frac{2}{1 + (2\pi f)^2}$$

Take the inverse Fourier transform to get (in this case the answer is obvious from the time domain waveform)

$$\tilde{x}(t) = \exp[-|t|]$$

(b) Using (2-104), the lowpass transfer function is

$$\tilde{H}(f) = \frac{2}{1 + j2(100)f/500} = \frac{2}{1 + j2\pi f/5\pi}$$

The envelope of the output waveform is

$$\tilde{y}(t) = \mathscr{F}^{-1}[\tilde{H}(f)\tilde{X}(f)] = \mathscr{F}^{-1}\left[\frac{1}{1 + j2\pi f/5\pi}\,\frac{1}{1 + (2\pi f)^2}\right]$$

Let $s = j2\pi f$ and $\alpha = 5\pi$. We expand

$$F(s) = \frac{2\alpha}{(1+s)(1-s)(\alpha+s)}$$

using partial fractions to

$$F(s) = -\frac{\alpha}{1-\alpha}\frac{1}{1+s} + \frac{\alpha}{1+\alpha}\frac{1}{1-s} + \frac{2\alpha}{1-\alpha^2}\frac{1}{\alpha+s}$$

Now substitute $s = j2\pi f$:

$$Y(f) = F(j2\pi f) = -\frac{\alpha}{1-\alpha}\frac{1}{1+j2\pi f} + \frac{\alpha}{1+\alpha}\frac{1}{1-j2\pi f} + \frac{2\alpha}{1-\alpha^2}\frac{1}{\alpha+j2\pi f}, \quad \alpha = 5\pi$$

Now use

$$\exp(-\alpha t)\,u(t) \leftrightarrow \frac{1}{\alpha + j2\pi f}$$

to get

$$\tilde{y}(t) = -\frac{\alpha}{1-\alpha}\exp(-t)\,u(t) + \frac{\alpha}{1+\alpha}\exp(t)\,u(-t) + \frac{2\alpha}{1-\alpha^2}\exp(-\alpha t)\,u(t)$$

Then find the real signal using

$$y(t) = \mathrm{Re}[\tilde{y}(t)\exp(j1000\pi t)], \quad \alpha = 5\pi$$

Problem 2-19

The phase response is linear for all frequency. Therefore, there will be no phase distortion.
(a) Cosinusoidal components at 2.5 Hz and 7.5 Hz get different gains ⇒ amplitude distortion.
(b) Both components get the same gain ⇒ no distortion.
(c) Cosinusoidal components at 9 Hz and 15 Hz get different gains ⇒ amplitude distortion.