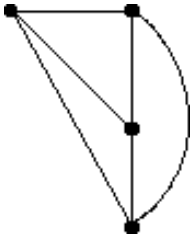
Free-Response

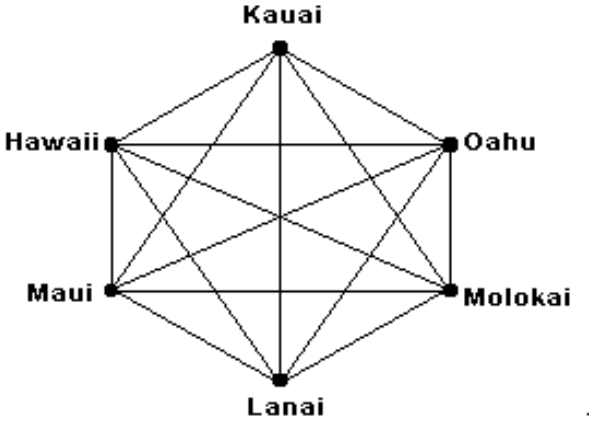
Format: Short Answer

1. Construct a complete graph on four vertices.
   Ans: Answers will vary. One solution is:



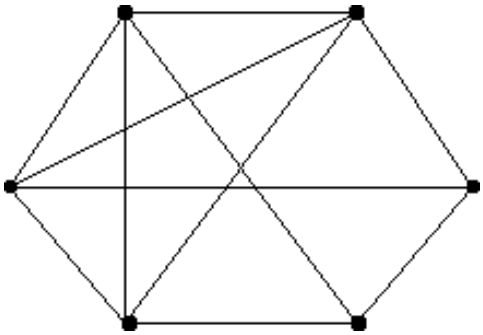
Format: Short Answer

2. Construct a complete graph whose vertices represent the six largest islands of Hawaii: Kauai, Oahu, Molokai, Lanai, Maui, and Hawaii.
   Ans: Answers will vary. One solution is:

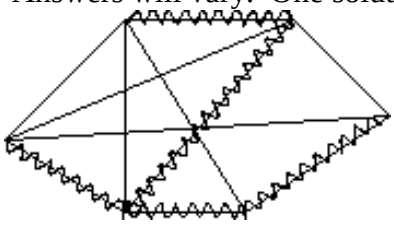


Format: Short Answer

3. Construct an example of a spanning tree on the graph given below.



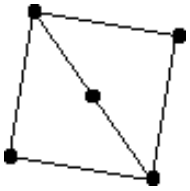
Ans: Answers will vary. One solution is:



Format: Short Answer

4. Construct an example of a graph with no Hamiltonian circuit.

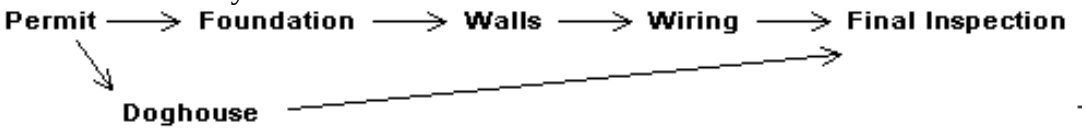
Ans: Answers will vary. One solution is:



Format: Short Answer

5. Construct a digraph for the following tasks necessary when building a house: get a building permit, install wiring, pour foundation, build walls, build doghouse, pass final inspection.

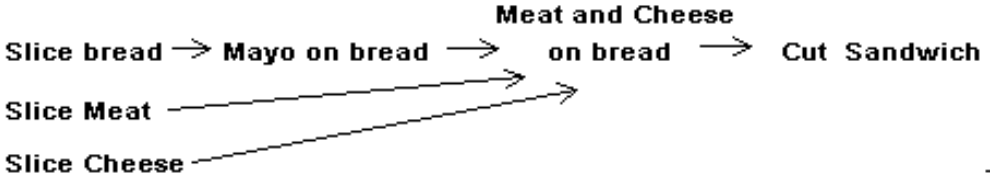
Ans: Answers will vary. One solution is:



Format: Short Answer

6. Identify six tasks necessary when building a sandwich, and construct a digraph for these tasks.

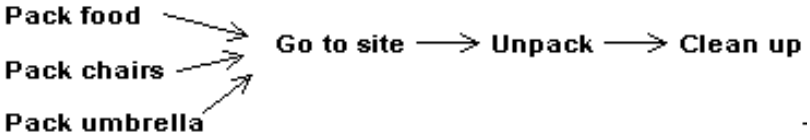
   Ans: Answers will vary. One solution is:



Format: Short Answer

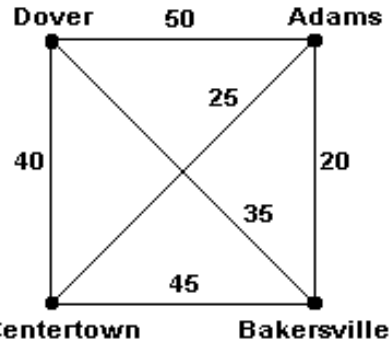
7. Identify six tasks necessary when preparing for a picnic, and construct a digraph for these tasks.

   Ans: Answers will vary. One solution is:



Format: Short Answer

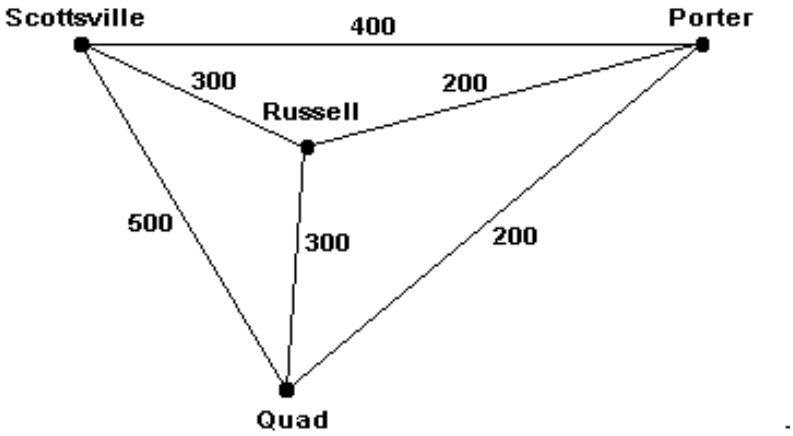
8. Use the brute force algorithm to solve the traveling salesman problem for the graph of the four cities shown below.



   Ans: Route ABCDA and ACBDA have cost 155. Route ABDCA has (minimum) cost 120.

Format: Short Answer

9. Use the brute force algorithm to solve the traveling salesman problem for the graph of the four cities shown below.



Ans: Route PQRSP and PQSRP have (minimum) cost 1200. Route PRQSP has cost 1400.

Format: Short Answer

10. If a graph of nine vertices is complete, how many edges are there?
Ans: (9)(8)/2=36 edges

Format: Short Answer

11. You own a chain of 12 apartment complexes (including your residence) and you want to plan a trip to visit each of your properties. If it takes 1/2 minute to compute the total length of a tour, how long will it take to apply the brute force algorithm to find the optimal tour?
Ans: (11!/2)(1/2) = 9,979,200 minutes, or approximately 19 years

Format: Short Answer

12. You own a chain of 10 one-day photo development kiosks and a lab where the photos are developed. Each morning and evening a delivery truck leaves the lab, visits each kiosk, and returns to the lab. If it takes 1/3 minute to compute the total length of a tour, how long will it take to apply the brute force algorithm to find the optimal tour for the delivery truck?
Ans: (9!/2)(1/3) = 60,480 minutes, or 42 days

Format: Short Answer

13. You want to create a mileage grid showing the distance between every pair of the 50 U.S. state capitals. How many numbers will you have to compute?
Ans: (50)(49)/2 = 1225

Format: Short Answer

14. You want to create a mileage grid showing the distance between every pair of the 10 Canadian provincial and territorial capitals. How many numbers will you have to compute?
Ans: (10)(9)/2 = 45

Format: Short Answer

15. The local cafe offers three different entrees, 10 different vegetables, and four different salads. A "blue plate special" includes an entree, a vegetable, and a salad. How many different ways can a special be constructed?
Ans: 120

Format: Short Answer

16. A nearby ice cream shop offers 31 different flavors and three different types of cones. How many different single scoop cones can be ordered?
Ans: 93

Format: Short Answer

17. In some states, license plates use a mixture of letters and numerals. How many possible plates could be constructed using three letters followed by three numerals?
Ans: $26^3 \times 10^3 = 17{,}576{,}000$

Format: Short Answer

18. In some states, license plates use a mixture of letters and numerals. How many possible plates could be constructed using three letters followed by four numerals?
Ans: $26^3 \times 10^4 = 175{,}760{,}000$

Format: Short Answer

19. What is an advantage of a *heuristic* algorithm?
Ans: Fast

Format: Short Answer

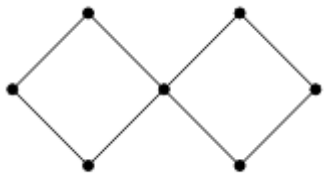
20. What is a disadvantage of a *heuristic* algorithm?
Ans: Not always optimal

Format: Short Answer

21. What is *critical* about the *critical path* of an order-requirement digraph?
    Ans: It requires the critical or essential amount of time required to complete the project.

Format: Short Answer

22. Construct a graph which has an Euler circuit, but not a Hamiltonian circuit.
    Ans: Answers may vary. One solution is:



Format: Short Answer

23. Can a graph have a Hamiltonian circuit, but not an Euler circuit?
    Ans: Yes

Format: Short Answer

24. Will the nearest-neighbor algorithm ever use the most expensive edge of a graph?
    Ans: Yes

Format: Short Answer

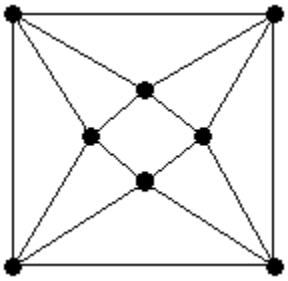
25. The route of a neighborhood garbage truck generally follows an Euler circuit. Under what circumstances should it instead follow a Hamiltonian circuit?
    Ans: If it only picks up at the intersection of streets

Format: Short Answer

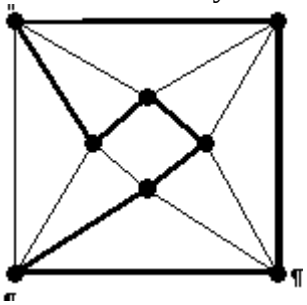
26. The route of a delivery truck generally follows a Hamiltonian circuit. Under what circumstances should it instead follow an Euler circuit?
    Ans: If it delivers to houses on the sides of streets

Format: Short Answer

27. In the graph below, construct a Hamiltonian circuit.

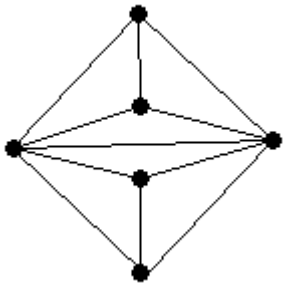


Ans: Answers will vary. One solution is:

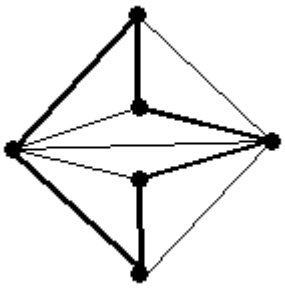


Format: Short Answer

28. In the graph below, construct a Hamiltonian circuit.

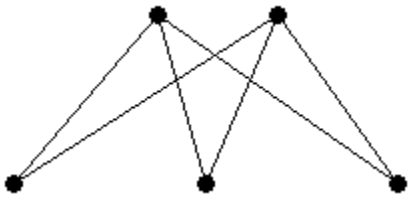


Ans: Answers will vary. One solution is:



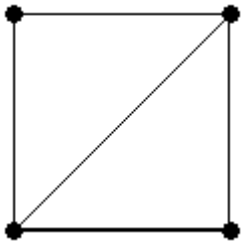
Format: Short Answer

29. Construct an example of a connected graph that does not have a Hamiltonian circuit.
Ans: Answers will vary. One solution is:



Format: Short Answer

30. Construct an example of a connected graph that has a Hamiltonian circuit but does not have an Euler circuit.
Ans: Answers will vary. One solution is:

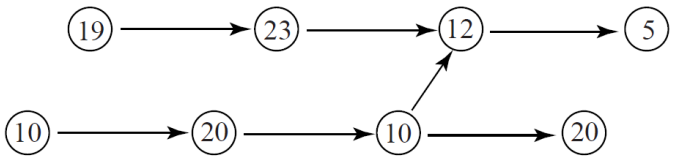


Format: Short Answer

31. A connected graph H has a spanning tree with 50 edges. How many vertices does the spanning tree have? How many vertices does H have? What can one say about the number of edges H has?
Ans: The spanning tree has 51 vertices. H also has 51 vertices. H must have at least 50 edges.

Format: Short Answer

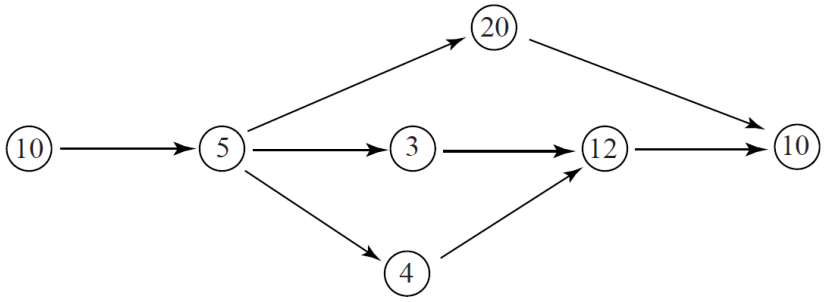
32. Find the earliest completion time for the following order-requirement digraph.



Ans: 60

Format: Short Answer

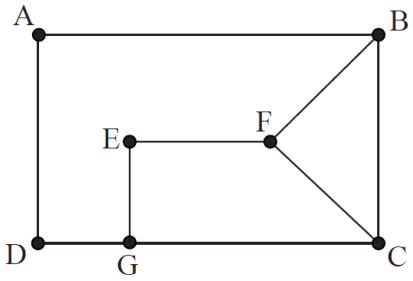
33. Find the earliest completion time for the following order-requirement digraph.



Ans: 45

Format: Short Answer

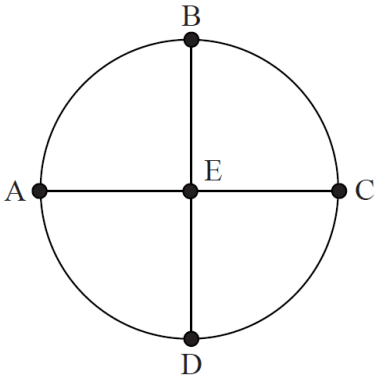
34. How many distinct Hamiltonian circuits can you find on the following graph?



Ans: One: ADGEFCBA

Format: Short Answer

35. How many distinct Hamiltonian circuits can you find on the following graph? (Do not count a circuit and the reverse of the same circuit as distinct.)



Ans: Four: ABECDA, ABCEDA, AEDCBA, AEBCDA

Format: Short Answer

36. If you add a new vertex to a complete graph of 10 vertices, how many new edges are needed to make the new graph complete?
Ans: 10