Solutions Manual for

Fluid Mechanics for Chemical Engineers

Second Edition

with Microfluidics and CFD

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1.1

Units Conversion

(a) $\dfrac{1}{640} \times 5280^2 = \underline{\underline{4.36 \times 10^4 \text{ ft}^3}}$

acre ft $\quad \dfrac{\text{mile}^2}{\text{acre}} \quad \dfrac{\text{ft}^2}{\text{miles}^2}$

(b) $4.36 \times 10^4 \times 7.48 = \underline{\underline{3.26 \times 10^5 \text{ gal}}}$

$\text{ft}^3 \quad \dfrac{\text{gal}}{\text{ft}^3}$

(c) $4.36 \times 10^4 \times \dfrac{1}{3.281^3} = \underline{\underline{1{,}233 \text{ m}^3}}$

$\text{ft}^3 \quad \dfrac{\text{m}^3}{\text{ft}^3}$

(d) $M = \rho V$

$$\begin{array}{c} V \\ = 1{,}233 \\ \text{m}^3 \end{array} \times \begin{array}{c} \rho \\ 1{,}000 \\ \frac{\text{kg}}{\text{m}^3} \end{array} = 1.233 \times 10^6 \text{ kg}$$

$= \underline{\underline{1{,}233 \text{ tonnes}}}$ (t)

Units Conversion

Viscosity

$$\mu = 10 \text{ centipoise} = \underbrace{10 \times 0.01 \times 10^{-1} \frac{kg}{m\,s}}_{1 \text{ poise}} = \underline{\underline{0.01 \frac{kg}{m\,s}}}$$

$$= \underset{\frac{kg}{m\,s}}{0.01} \times \frac{\overset{}{0.3048}}{\underset{}{0.4536}} = \underline{\underline{6.72 \times 10^{-3} \frac{lbm}{ft\,s}}}$$

$$\frac{kg}{m\,s} \quad \frac{lbm}{kg} \quad \frac{m}{ft}$$

(Useful conversion factors: $1 \text{ cp} = 0.000672 \text{ lbm/ft s}$
$= 2.42 \text{ lbm/(ft hr)}$)

Density

$$\rho = 0.8 \times 1 \times \frac{(100)^3}{1000} = \underline{\underline{800 \frac{kg}{m^3}}}$$

$$\frac{g}{cm^3} \quad \frac{kg}{g} \quad \left(\frac{cm}{m}\right)^3$$

$$= 0.8 \times 62.4 = \underline{\underline{49.9 \frac{lbm}{ft^3}}}$$

1.3 Units Conversion

Gravitational Acceleration

$$g = \frac{981}{100} \frac{cm}{s^2} \frac{m}{cm} = 9.81 \frac{m}{s^2}$$

Pressure

$$p = \frac{14.7 \times 32.2 \times 144 \times 3.281}{2.205}$$

$$\frac{lbf}{in^2} \frac{lbm\ ft}{lbf\ s^2} \frac{in^2}{ft^2} \frac{kg}{lbm} \frac{ft}{m}$$

$$= 1.01 \times 10^5 \frac{kg}{m\,s^2} = \frac{N}{m^2} = Pa$$

Since $1\ bar = 10^5\ Pa$

$$p = 1.01\ bar$$

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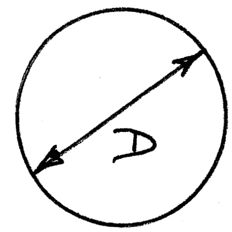
Meteorite Density



Mass $M = \dfrac{\pi D^3}{6}\rho$

$$\rho = \frac{6M}{\pi D^3} = \frac{6 \times 10^6 \times 1000}{\pi \times 60^3}$$

$$\rho = 8,842 \ \frac{kg}{m^3}$$

$$s = \frac{8,842}{1,000} = 8.84$$

Specific gravities of some elements are

Fe	7.86	Co	8.9	Ag	10.5
Ni	8.9	Pb	11.3	Au	19.3
				U	18.5

Most likely candidate is iron, the deviation being due to the "ball park" figures in the article.

Kinetic Energy

$$\frac{1}{2}Mu^2 = \frac{1}{2}10^9 \times (15000)^2 = 1.125 \times 10^{17} \text{ J}$$

$$\text{TNT equivalent} = \frac{1.125 \times 10^{17}}{5 \times 10^9} = 2.25 \times 10^7 \text{ tonnes}$$

Reynolds Number

Cross-sectional area

$$A = \frac{\pi D^2}{4} = \frac{\pi \left(\frac{1.05}{12}\right)^2}{4} = 0.00601 \ ft^2$$

Volumetric flow rate

$$Q = \frac{35}{7.48 \times 60} = 0.0780 \ \frac{ft^3}{s}$$

Mean velocity

$$u_m = \frac{Q}{A} = \frac{0.0780}{0.00601} = 12.98 \ \frac{ft}{s}$$

Reynolds number

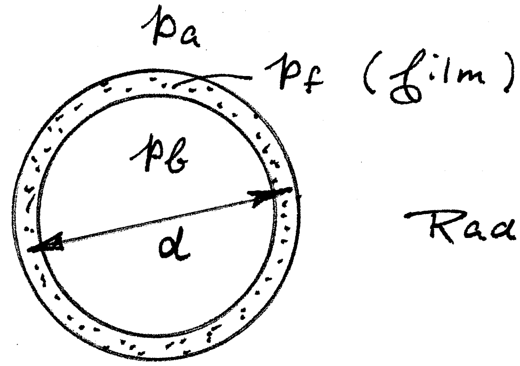
$$Re = \frac{\rho u_m D}{\mu} = \frac{62.3 \times 12.98 \times 1.05/12}{1.2 \times 0.000672}$$

$$\frac{lbm}{ft^3} \quad \frac{ft}{s} \quad ft \qquad \frac{cP}{cP \ \ lbm/ft\,s} \quad \Bigg\} \text{ All units cancel}$$

$$\doteq \underline{\underline{87,740}} \quad (\text{dimensionless})$$

Pressure in Bubble

Atmosphere



Radius $a = \frac{d}{2}$

From notes, the increase in pressure as we go inwards across a convex surface is

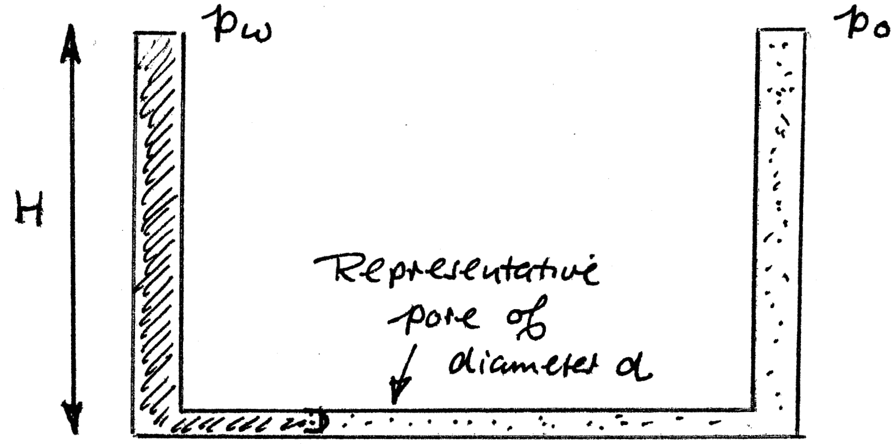
$$\left.\begin{aligned} p_f - p_a &= \frac{2\sigma}{a} \\ p_b - p_f &= \frac{2\sigma}{a} \end{aligned}\right\} \text{Two surfaces are involved}$$

Hence, by addition,

$$p_b - p_a = \frac{4\sigma}{a} = \frac{8\sigma}{d}$$

$$p_b = p_a + \frac{8\sigma}{d}$$

Reservoir Water-Flooding



$$p_o + \rho_o g H + \underbrace{\frac{4\sigma}{d}}_{\substack{\text{Increase in pressure}\\ \text{from oil into water}}} = p_w + \rho_w g H$$

Hence required water inlet pressure is

$$p_w = p_o - \underbrace{(\rho_w - \rho_o)}_{\text{Positive}} g H + \frac{4\sigma}{d}$$

Barometer Reading

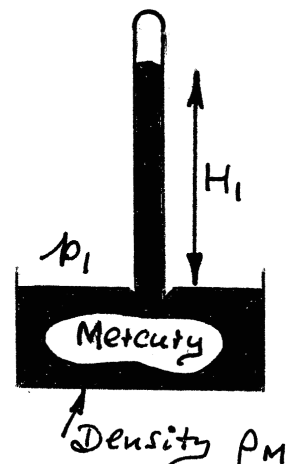
② • House $\quad z_2 = 950$ ft $\quad p_2 = ?$

$H_2 = ?$

① • Weather Station $\quad z_1 = 700$ ft $\quad p_1 = 0.966$ bar

$H_1 = ?$

At the weather station



The atmospheric pressure p_1 is balanced by a column of mercury of height H_1:

$$p_1 = \rho_M g H_1$$

Hence

$$H_1 = \frac{p_1}{\rho_M g} = \frac{0.966 \times 10^5 \times 3.281 \times 12}{13.57 \times 1000 \times 9.81}$$

$$\frac{kg}{m^2}\frac{m}{s^2} \quad \frac{m^3}{kg} \quad \frac{s^2}{m} \quad \frac{in}{m} = in$$

$$= \underline{\underline{28.57 \text{ in mercury}}}$$

(Note: $g = 9.81\ m/s^2$, $3.281\ ft = 1\ m$,

$\rho_W = 1000\ \frac{kg}{m^3}$, $1\ bar = 10^5$ pascal $= 10^5\ \frac{kg\frac{m}{s^2}}{m^2}$)

<u>Correction for elevation increase</u>. Since $z_2 - z_1$ is "small" for air, we can take ρ_A as essentially constant between ① and ②. Now pressure at weather station is

$$0.966 \text{ bar} \times \frac{14.7 \text{ psia}}{1.01 \text{ bar}} \left(\begin{matrix}\text{see}\\ \text{Problem}\\ 3\end{matrix}\right) = 14.06 \text{ psia}$$

Hence the appropriate mean pressure between ① and ② for purposes of estimating the density can be taken as 14.06 or (as done here, with a trifling change in the answer) slightly less — say 14.0 psia.

$$\rho_A = \frac{M_A p}{RT} = \frac{28.8 \times 14.0}{10.73 \times (460 + \underset{\uparrow \text{ Average for January}}{25})} = 0.0775 \ \frac{\text{lbm}}{\text{ft}^3}$$

<u>Change in Pressure</u> $\quad p_2 - p_1 = -\rho_A g (z_2 - z_1)$

<u>Change in Barometer Reading</u> $\quad H_2 - H_1 = \dfrac{p_2 - p_1}{\rho_M g} = -\dfrac{\rho_A (z_2 - z_1)}{\rho_M}$

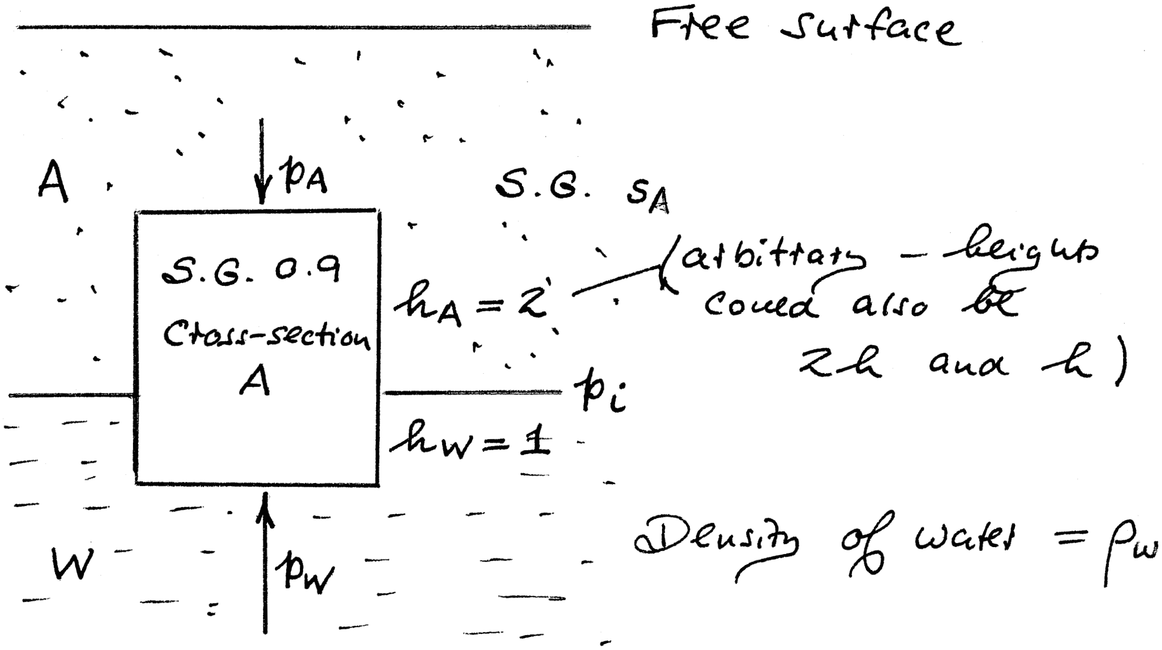
$$H_2 - H_1 = -\frac{0.0775\,(950 - 700) \times 12}{13.57 \times 62.4} = -0.27 \text{ in}$$

Hence $H_2 = H_1 - 0.27 = 28.57 - 0.27 = \underline{\underline{28.3 \text{ in Hg}}}$

<u>Pressure</u> $\quad p_2 = \rho_M g H_2 = \dfrac{13.57 \times 62.4 \times 32.2 \times 28.3}{32.2 \times 144 \times 12}$

$$\underline{\underline{p_2 = 13.87 \text{ psia}}} \qquad \frac{\text{lbm}}{\text{ft}^3}\ \frac{\text{ft}}{\text{sec}^2}\ \text{in}\ \frac{\text{lbf sec}^2}{\text{lbm ft}}\ \frac{\text{ft}^2}{\text{in}^2}\ \frac{\text{ft}}{\text{in}}$$

Two-Layer Buoyancy



Method 1

Weight displaced (upwards buoyant force) = weight of cylinder downwards

$$\rho_w A g \,(1 + 2 s_A) = 0.9\, \rho_w A\, 3 g$$

$$s_A = 0.85$$

Method 2

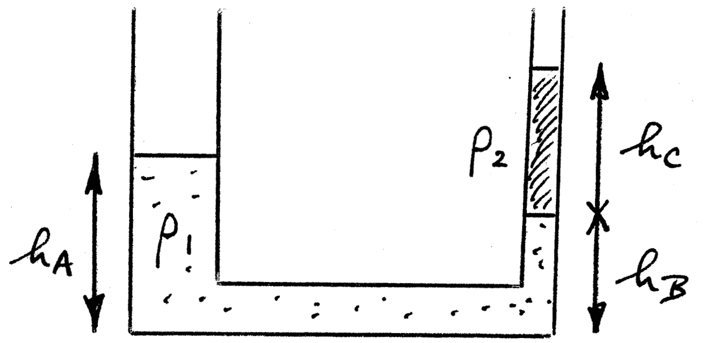
Force balance on cylinder ↓

$$p_A A + \underbrace{0.9 \rho_w A\, 3 g}_{\text{weight}} - p_W A = 0$$

$$p_i - 2 s_A \rho_w g + 2.7 \rho_w g - (p_i + \rho_w g) = 0$$

$$s_A = 0.85$$

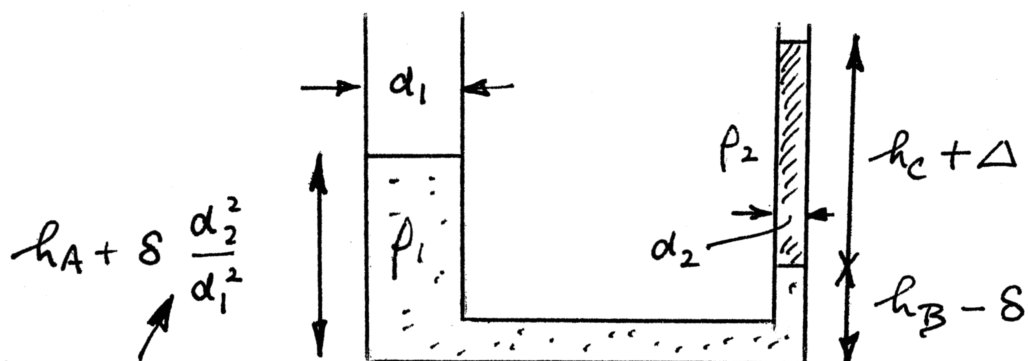
Differential Manometer



Before

$$\rho_1 (h_A - h_B) = \rho_2 h_C \quad (1)$$

Now add Δ to h_C and let h_B decline by δ



From volume conservation

After

$$\rho_1 \left(h_A + \delta \frac{d_2^2}{d_1^2} - (h_B - \delta)\right) = \rho_2 (h_C + \Delta) \quad (2)$$

Subtract (1) from (2)

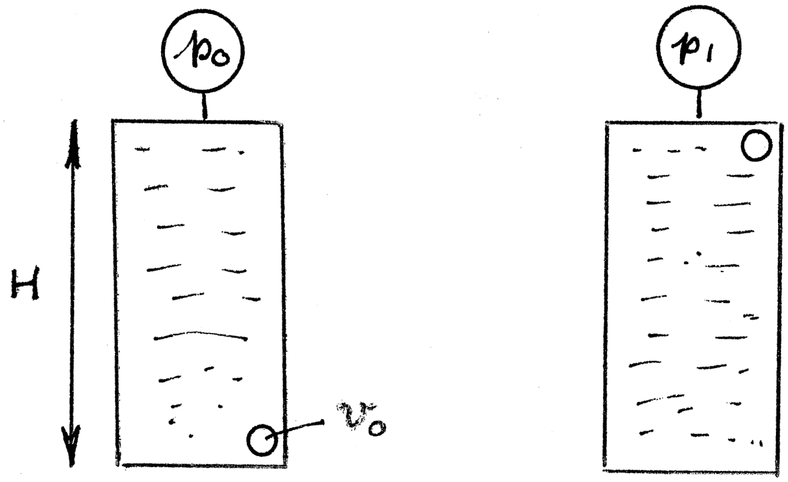
$$\rho_1 \delta \left(\frac{d_2^2}{d_1^2} + 1 \right) = \rho_2 \Delta = \rho_2 \frac{v_2}{\frac{\pi}{4} d_2^2} \quad (3)$$

Equation (3) gives δ as a function of v_2, and also enables ρ_2 to be found by observing the value of δ.

Solution for δ gives:

$$\delta = \frac{\rho_2}{\rho_1} \frac{4 v_2}{\pi d_2^2} \left(\frac{d_1^2}{d_1^2 + d_2^2} \right)$$

Ascending Bubble



Since the cylinder and oil volumes don't change, the bubble volume must remain constant at v_0.

But

$$p V_0 = nRT$$

Therefore, since T is constant, p within the bubble does not change. Hence

$$p = \underbrace{p_0 + \rho g H}_{\text{Before}} = \underset{\text{After}}{p_1}$$

Thus $p_1 = p_0 + \rho g H$

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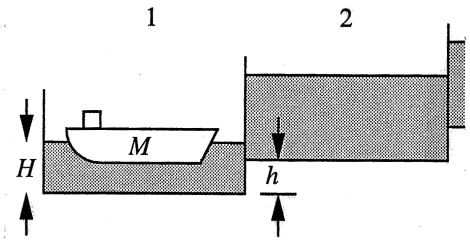
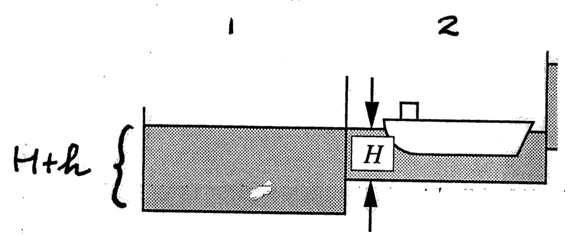
Ship Passing Through Locks

Uphill The ship must increase its elevation by an amount h as it passes from lock 1 to lock 2. Consider the water in lock 1 before and after;

Before



After



Mass of water in lock $= \rho AH - M$

Mass of water in lock $= \rho A(H+h)$

From Archimedes law, the ship displaces a mass M of water

Hence mass of water to be supplied to lock 1 is

$$\rho A(H+h) - (\rho AH - M) = \rho Ah + M$$

Downhill A similar analysis gives the water loss from a lock as

$$\underbrace{\rho AH - M}_{\text{mass at start}} - \underbrace{\rho A(H-h)}_{\text{mass at end}} = \rho Ah - M$$

(note that final depth of water in lock is $H-h$)

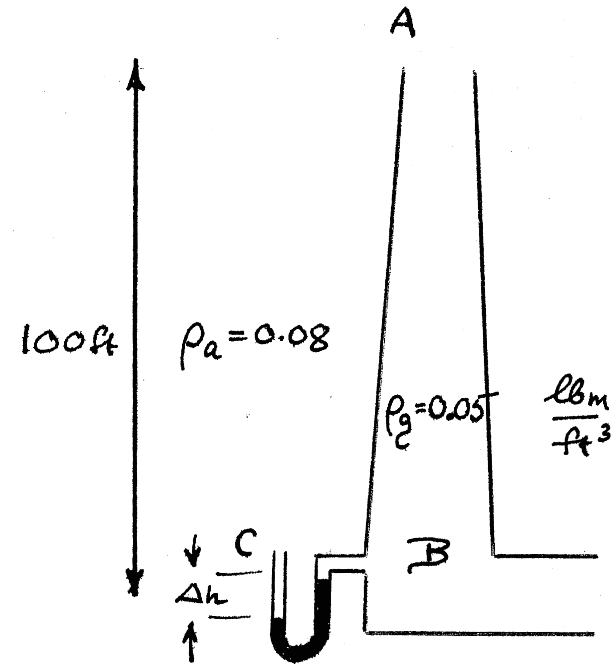
Total water supply

(i) Uphill only: $\rho Ah + M$ (depends on M)

(ii) Up and down: $\rho Ah + M + \rho Ah - M = 2\rho Ah$ (independent of M)

Furnace Stack

Start from point A and consider hydro-static increase of pressure in both cases:



$$p_B = p_A + \rho_g g H$$

$$p_C = p_A + \rho_a g H$$

Hence $p_C = p_B + \underbrace{(\rho_a - \rho_g)}_{\text{positive}} g H$

Hence the water moves <u>up</u> in the right-hand leg by Δh given by

$$\rho_w g \Delta h = (\rho_a - \rho_g) g H$$

$$\Delta h = \frac{\rho_a - \rho_g}{\rho_w} H = \frac{(0.08 - 0.05) \times 100 \times 12}{62.4} = \underline{\underline{0.58 \text{ in.}}}$$

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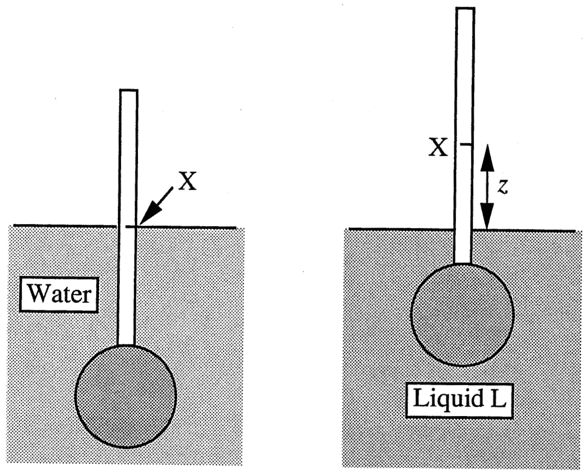
Hydrometer



Since the same weight (that of the hydrometer) is supported by the displaced liquid in both cases:

$$Mg = V \rho_w g = (V - Az)\rho_w s g$$

mass of hydrometer   Density of water

Cancellation of $\rho_w g$ and solution for s gives

$$s = \frac{1}{1 - \frac{Az}{V}}$$