Solution Manual

to accompany the textbook

Fixed Income Securities:
Valuation, Risk, and Risk Management

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Chapters 2 - 8

Version 1

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Solutions to Chapter 2

Exercise 1.

Compute the discount factors implied by the STRIPS:

$$Z(0,3) = \exp(-3 \times 0.1) = 0.741 \quad (1)$$

$$Z(0,5) = \exp(-5 \times 0.05) = 0.779. \quad (2)$$

Since $Z(0,3) < Z(0,5)$, there is an arbitrage opportunity due to the violation of the positive time discount rate. To exploit it: buy the 3-year bond and sell the 5-year bond.

Exercise 2.

Compute the quoted price P of the T-bill as:

$$P = 100 \times \left[1 - \frac{n}{360} \times d\right], \quad (3)$$

using the discount rate given, d. The simple (bond equivalent) yield measures your annualized return as:

$$BEY = \frac{100 - P}{P} \times \frac{365}{n}. \quad (4)$$

Let $\tau = \frac{n}{365}$ be the time to maturity expressed as fraction of a year, and let T denote the maturity date of a given T-bill. The continuously compounded yield follows as:

$$r(t,T) = -\frac{1}{\tau} \ln \frac{P}{100}. \quad (5)$$

Finally, to obtain the semi-annually compounded yield for the 1-year T-bill, use:

$$r_2(0,1) = 2 \times \left(\frac{1}{(P/100)^{1/2}} - 1 \right) \quad (6)$$

	Maturity	n	$T-t$	Discount, d	Price, P	BEY	Cont. comp. yield	Semi-annual comp.	Date
a.	4-week	28	0.083	3.48%	99.7293	3.5379%	3.53%	–	12/12/2005
b.	4-week	28	0.083	0.13%	99.9899	0.13%	0.13%	–	11/6/2008
c.	3-month	90	0.25	4.93%	98.7675	5.06%	5.03%	–	7/10/2006
d.	3-month	90	0.25	4.76%	98.8100	4.88%	4.86%	–	5/8/2007
e.	3-month	90	0.25	0.48%	99.8800	0.49%	0.49%	–	11/4/2008
f.	6-month	180	0.5	4.72%	97.6400	4.90%	4.84%	–	4/21/2006
g.	6-month	180	0.5	4.75%	97.6250	4.93%	4.87%	–	6/6/2007
h.	6-month	180	0.5	0.89%	99.5550	0.91%	0.90%	–	11/11/2008
i.	360-day	360	1	1.73%	98.2700	1.78%	1.77%	1.75%	9/30/2008
j.	360-day	360	1	1.19%	98.8100	1.22%	1.21%	1.20%	11/5/2008

Exercise 3.

Compute respective discount factors taking into account the convention on which the interest rate is given:

$$\begin{aligned}
1.\ \ Z(t,t+1.5) &= \exp(-0.02\times 1.5) = 0.97045 \\
2.\ \ Z(t,t+1.5) &= \exp(-0.03) = 0.97045 \\
3.\ \ Z(t,t+1.5) &= \frac{1}{(1+0.021)^{1.5}} = 0.96931 \\
4.\ \ Z(t,t+1.5) &= \frac{1}{(1+0.0201/2)^{2\times 1.5}} = 0.97045
\end{aligned}$$

Bond 3 is mispriced.

Exercise 4.

Using Table 2.4, obtain the discount factor $Z(t,T)$ for each maturity $T-t$ from 0.25 to 7.5 years:

$$Z(t,T) = \frac{1}{\left(1+\frac{r_2(t,T)}{2}\right)^{2(T-t)}}. \tag{7}$$

Use Z to price each bond:

a. $P_z(0,5) = 100\times Z(0,5) = 72.80$

b. $P_{c=15\%,n=2}(0,7) = \frac{15}{2}\times\sum_{i=1}^{14} Z(0,i/2) + 100\times Z(0,7) = 151.23$

c. $P_{c=7\%,n=4}(0,4) = \frac{7}{4}\times\sum_{i=1}^{16} Z(0,i/4) + 100\times Z(0,4) = 101.28$

d. $P_{c=9\%,n=2}(0,3.25) = \frac{9}{2}\times\sum_{i=1}^{7} Z(0,i/2-0.25) + 100\times Z(0,3.25) = 108.55$

e. 100 (see Fact 2.11)

f. $P_{FR,n=1,s=0} = Z(0,0.5)\times 100\times(1+\frac{6.8\%}{1}) = 103.44$, where we assume that $r_1(0) = 6.8\%$

g. $P_{FR,n=4,s=0.35\%}(0,5.5) = 100 + \frac{0.35}{4}\sum_{i=1}^{22} Z(0,i/4) = 101.6$

h. $P_{FR,n=2,s=0.40\%} = Z(0,0.25)\times 100\times(1+\frac{6.4\%}{2}) + \frac{0.40}{2}\sum_{i=1}^{15} Z(0,i/2-0.25) = 104$, where we assume that $r_2(0) = 6.4\%$

Exercise 5.

a. When coupon c is equal to the yield to maturity y the bond trades at par; when coupon is below (above) the yield to maturity the bond trades above (below) par. Obtain bond prices given yield and the coupon using:

$$P_c(0,T) = \sum_{i=1}^{20}\frac{c/2\times 100}{(1+y/2)^i} + \frac{100}{(1+y/2)^{20}} \tag{8}$$

It follows:

c	y	P
5%	6%	107.79
6%	6%	100
7%	6%	92.89

b. Figure 1 plots bond prices implied by different yields to maturity.

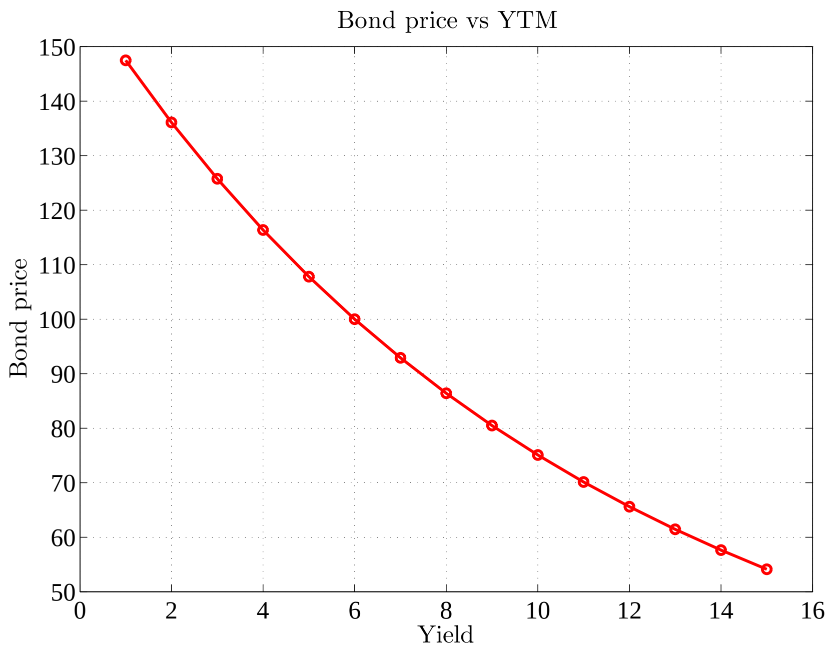

Fig. 1. Bond price as function of yield to maturity

Exercise 6.

a. To obtain bond prices use the expression:

$$P_c(0,T) = \frac{c}{2} \times \sum_{i=1}^{2T} Z(0, i/2) + 100 \times Z(0,T), \tag{9}$$

To compute the yield to maturity, solve equation (8) for y using a numerical solver.

c	$T-t$	P	y
15%	7	151.2306	5.9461%
3%	7	84.3482	5.7474%

b. The yields to maturity are different since bonds have different coupons, despite having the same time to maturity. Both bonds are priced using a no arbitrage discount curve. Therefore, their prices are fair.

Exercise 7.

a. Bootstrap the discount factors $Z(t,T)$ using the expression (9), and substituting recursively for the 6-month, 1-year, 1.5-year, and 2-year bonds. E.g., given $Z(0,0.5)$ and $Z(0,1)$, for the 1.5-year bond you have:

$$Z(0,1.5) = \frac{100.86 - \frac{7.5}{2}(Z(0,0.5)+Z(0,1))}{100+\frac{7.5}{2}}. \tag{10}$$

This yields:

$T-t$	Coupon, c	Price, P	Issued	$Z(t,T)$
0.5	0.00%	$96.80	5/15/2000	0.9680
1	5.75%	$99.56	5/15/1998	0.9407
1.5	7.50%	$100.86	11/15/1991	0.9032
2	7.50%	$101.22	5/15/1992	0.8740

b. Compute the no-arbitrage price of the two bonds given the discount function obtained above. The prices of the two bonds are:

$$P_{c=8\%} = \$101.71 \tag{11}$$

$$P_{c=13.13\%} = \$106.60, \tag{12}$$

i.e. both are higher than the market prices. There is an arbitrage opportunity. You could make riskless profit by buying the underpriced bond at the traded price and selling the corresponding portfolio of zeros that replicates the cash flows from the bond.

Exercise 8.

The quotes are obtained on May 15, 2000. Use the mid bid-ask quote to compute the price. You want to obtain the semi-annual curve. The provided maturities of bonds are spaced semi-annually. We can assume that the clean (quoted) price is equal to the dirty (invoice) price, i.e. the accrual is zero. For each maturity, bootstrap the discount factors $Z(t,T)$ as in Exercise 7.a. The continuously compounded zero coupon yield is given as:

$$r(t,T) = -\frac{1}{T-t}\ln Z(t,T). \tag{13}$$

Cusip	$T-t$	Mid bid-ask quote	Coupon, c	Accrual	$Z(t,T)$	Yield, $r(t,T)$
912827ZN	0.5	100.9063	8.500%	0	0.96793	6.520%
912810CU	1	105.9961	13.125%	0	0.93508	6.713%
912810CX	1.5	112.4102	15.750%	0	0.90312	6.793%
912827F4	2	101.2188	7.500%	0	0.87418	6.724%
912810DA	2.5	110.6836	11.625%	0	0.84387	6.790%
912810DD	3	110.3438	10.750%	0	0.81638	6.762%
912810DG	3.5	115.3242	11.875%	0	0.78928	6.761%
912810DH	4	118.9141	12.375%	0	0.76267	6.773%
912810DM	4.5	118.3125	11.625%	0	0.73951	6.706%
912810DQ	5	121.6289	12.000%	0	0.71544	6.697%
912827V8	5.5	96.0000	5.875%	0	0.69440	6.631%
912827X8	6	100.6211	6.875%	0	0.67229	6.618%
912827Z6	6.5	98.7656	6.500%	0	0.65080	6.609%
9128272U	7	99.5781	6.625%	0	0.63152	6.566%
9128273X	7.5	93.1484	5.500%	0	0.61225	6.542%
9128274F	8	93.8008	5.625%	0	0.59478	6.494%
9128274V	8.5	87.9922	4.750%	0	0.57640	6.482%
9128275G	9	92.8398	5.500%	0	0.56151	6.413%