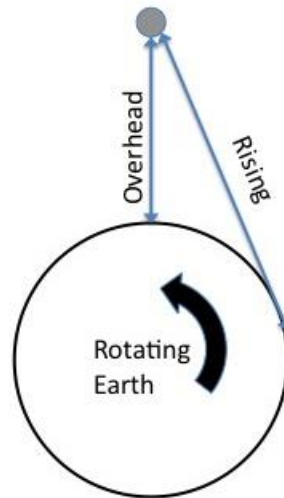


## CHAPTER 2 THE RISE OF ASTRONOMY

### Answers to Thought Questions

1. Despite the Moon illusion, you are actually closest to the Moon when it is at its highest point in the sky. As seen from over the North Pole,

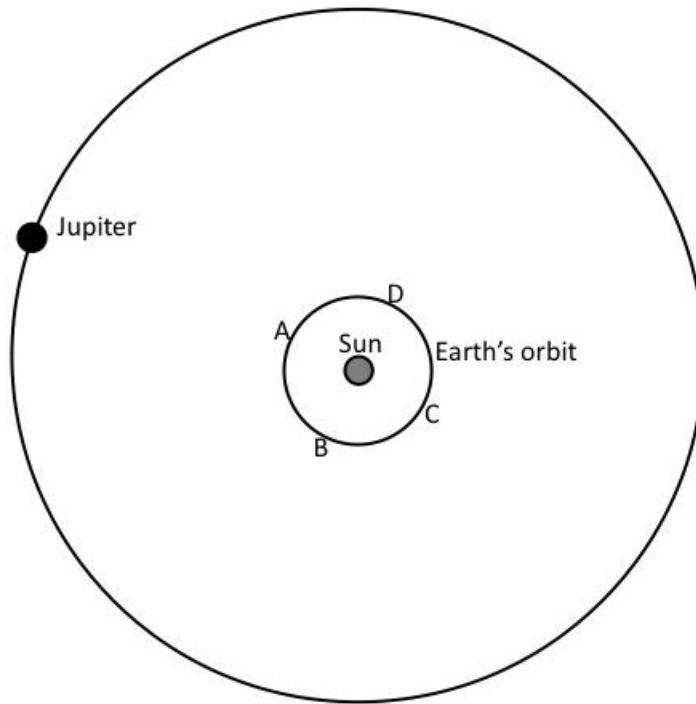


2. If the stars were much closer than they really are, Aristarchus would have been able to demonstrate the stellar parallax caused by the Earth's orbital motion around the Sun.

3. The phases of Venus are caused by Venus orbiting the Sun, so keeping all distances and periods constant, it wouldn't matter whether Earth and Venus were orbiting the Sun or Venus was orbiting the Sun and the Sun (and Venus) were orbiting the Earth.

4. The Sun has a slightly larger angular diameter in January than in July because the Earth's orbit is an ellipse. The idea that orbits are ellipses is Kepler's First Law.

5. The apparent motion of Jupiter is primarily a result of the Earth's motion around the Sun, not Jupiter's motion. A sketch like the one below can help show this—consider the view from Earth over the year, with Jupiter moving only a little along its orbit. At position A, Jupiter is at opposition, and rises when the Sun sets. By the time the Earth is at B, Jupiter has not moved very far (so it's motion is neglected in this sketch), but now it is only 90 degrees away from the Sun—already high in the sky at Sunset. By position C, Jupiter would be “up” in the daytime. Moving to position D, Jupiter is moving past the Sun on the opposite side as before.



6. If the same comet is only visible every 50 years or more (or much, much more!) then the comet must have a much longer  $p$  than the Earth's orbit. Consequently by Kepler's third law it must have a corresponding larger  $a$ . If the comet needs to be close to the Earth and Sun to be seen, then at least some of the time it must be at only 1 or 2 AU from the Sun; for this to fit with a large  $a$  and  $p$ , it must have an elliptical orbit with high eccentricity.

7. This should be less about technology and more about philosophy; and about getting students to look up contemporary work. One key difference is less personal, government and patron-sponsored *religious/mystic* motivation—astronomy is no longer in the game of predicting planetary positions as portents of war and peace. Also, astronomy has become increasingly based on fact and less influenced by philosophy (Ptolemy's and Kepler's motivations looking for perfection in the heavens; Kepler was conflicted but notably broke with his philosophical position (circular orbits) when confronted with scientific evidence). Increased technology has revealed entire branches of astronomy previously invisible to scientists (radio, x-ray, infrared, gravitational waves, etc.).

8. (Students must research the astronomers in question).

Answers to Problems

1. This problem is a modern version of the method Eratosthenes used to measure the size of the Earth. Given that the shadow length is 15 degrees, the distance in latitude between the two points on the asteroid must be 15 degrees, or  $15^\circ/360^\circ = 1/24$ th the circumference of the asteroid. If the 15 degrees corresponds to 10 km, then the total distance around the asteroid must be  $10 \text{ km} \times 24 = 240 \text{ km}$ . The radius,  $R$ , of the asteroid is related to its circumference,  $C$ , by  $C = 2\pi R$ . Thus  $R = C/2\pi = 240/2\pi = 38 \text{ km}$ .

2. This can be directly calculated, but it is easier to just look at the proportionalities involved. Since  $C = 360^\circ/\text{angle} \times \text{distance}$ , the circumference is directly proportional to the distance between Alexandria and Syene. If the distance were three times as much, and the angle the same, Eratosthenes would have calculated that the circumference of the Earth was three times larger:  $250,000 \text{ stadia} \times 3 = 750,000 \text{ stadia}$  or  $25,000 \text{ miles} \times 3 = 75,000 \text{ miles}$ . ("Three times larger" should be an acceptable answer.)

3. From Earth, the Sun has an angular diameter of  $0.5^\circ$ . Angular size varies as the inverse of the distance, so if Mercury is 0.387 times as far, the Sun is  $0.5^\circ/0.387 = 1.29^\circ$ . Pluto is 39.53 times the Earth's distance, so the Sun is  $0.5^\circ/39.53 = 0.0126^\circ$ .

4. The Andromeda galaxy has an angular diameter of 5 degrees at a distance of  $2.2 \times 10^6 \text{ ly}$ . We can find its true size by using the angular diameter formula  $L = 2\pi dA/360^\circ$ , where  $d$  = distance,  $A$  = angle subtended, and  $L$  = linear diameter. Thus,  $L = 2\pi \times 2.2 \times 10^6 \text{ ly} \times 5^\circ/360^\circ = 1.92 \times 10^5 \text{ ly}$ .

5. If  $P = 64$  years, you can use Kepler's 3rd Law to estimate its distance from the Sun.  $P^2 = a^3$ , where  $P$  = period in years and  $a$  = average orbital radius in AU. We can find  $a$  by taking the cube root of both sides to get  $P^{2/3} = a$ , or  $a = P^{2/3} = (64)^{2/3} = (64 \times 64)^{1/3} = (8 \times 8 \times 8 \times 8)^{1/3} = 2 \times 2 \times 2 \times 2 = 16 \text{ AU}$ . This is the radius of the orbit if the orbit is circular.

6.  $a^3 = P^2$ .  $(526)^3 = 1.4553 \times 10^8 = P^2$ , so  $P = 12,063 \text{ years}$ .

7. This is an application of Kepler's third law,  $P^2 = a^3$ , where  $a$  is in AU and  $P$  is in years. If  $P = 125 \text{ yrs}$ , then  $a^3 = 125^2$ . Solving for  $a$ , we take the cube root of both sides to get  $a = (125^2)^{1/3}$ , where we have used the fact that the cube root of a number is the number to the  $1/3$  power. This can be solved with a calculator or by noticing that

$(125 \times 125)^{1/3} = (25 \times 5 \times 5 \times 25)^{1/3} = (25 \times 25 \times 25)^{1/3} = 25$ , so  $a = 25$  AU. If the planet's orbit is circular, then that is also the planet's orbital radius.

8. This problem is another application of Kepler's third law,  $P^2 = a^3$ , where  $a$  is in AU and  $P$  is in years. In this case, we are given  $a$ , and are asked to find  $P$ . Thus  $P^2 = a^3 = 16^3$ . Solving for  $P$  by taking the square root and recalling that the square root is the number to the  $1/2$  power, we find that  $P = (16^3)^{1/2} = 64$  yrs. (Note: in solving this problem, you can simplify the math by reversing the order of the power and the square root. That is,  $(16^3)^{1/2} = (16^{1/2})^3 = 4^3 = 64$ .)

### Answers to Test Yourself

1. (d) Angular size is inversely proportional to distance so  $L_M/L_S = 1/400$ .
2. (b) Retrograde motion causes planets to stop their regular eastward motion with respect to the stars and move westward for a time.
3. (b) simplicity of models
4. (d)  $P^2 = a^3$ .  $4^3 = 4 \times 4 \times 4 = 4 \times 2 \times 2 \times 4 = 8^2$
5. (a) Kepler's 3rd Law relates a planet's orbital period to the size of its orbit.
6. (e) Venus orbits the Sun.
7. (c) Parallax was conjectured but impossible to measure without high quality telescopes.