CHAPTER 2

Exercises

E2.1 (a) R_2, R_3, and R_4 are in parallel. Furthermore R_1 is in series with the combination of the other resistors. Thus we have:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 3\,\Omega$$

(b) R_3 and R_4 are in parallel. Furthermore, R_2 is in series with the combination of R_3, and R_4. Finally R_1 is in parallel with the combination of the other resistors. Thus we have:

$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5\,\Omega$$
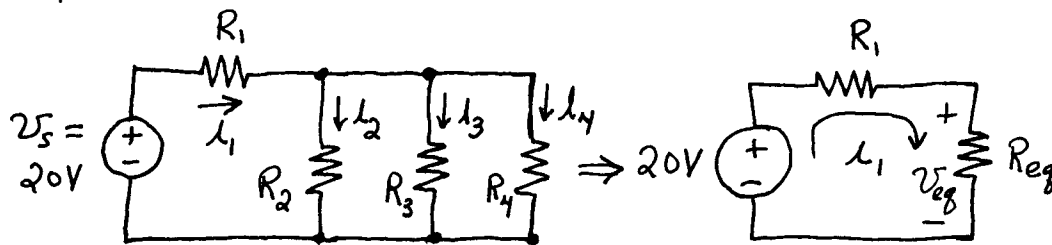
(c) R_1 and R_2 are in parallel. Furthermore, R_3, and R_4 are in parallel. Finally, the two parallel combinations are in series.

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} + \frac{1}{1/R_3 + 1/R_4} = 52.1\,\Omega$$

(d) R_1 and R_2 are in series. Furthermore, R_3 is in parallel with the series combination of R_1 and R_2.

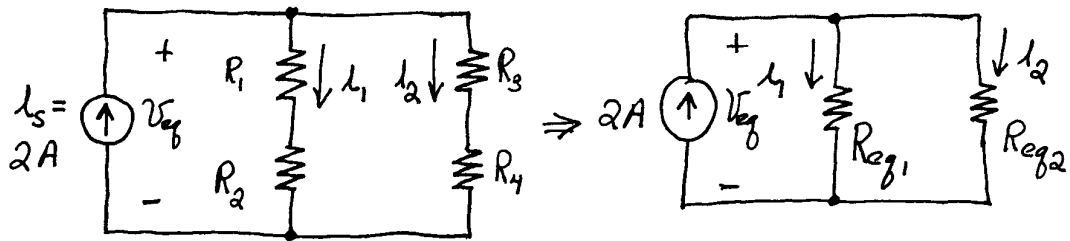$$R_{eq} = \frac{1}{1/R_3 + 1/(R_1 + R_2)} = 1.5\,k\Omega$$

E2.2 (a) First we combine R_2, R_3, and R_4 in parallel. Then R_1 is in series with the parallel combination.

$$R_{eq} = \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 9.231\,\Omega \qquad i_1 = \frac{20\text{ V}}{R_1 + R_{eq}} = \frac{20}{10 + 9.231} = 1.04\text{ A}$$

$$v_{eq} = R_{eq}i_1 = 9.600\text{ V} \qquad i_2 = v_{eq}/R_2 = 0.480\text{ A} \qquad i_3 = v_{eq}/R_3 = 0.320\text{ A}$$

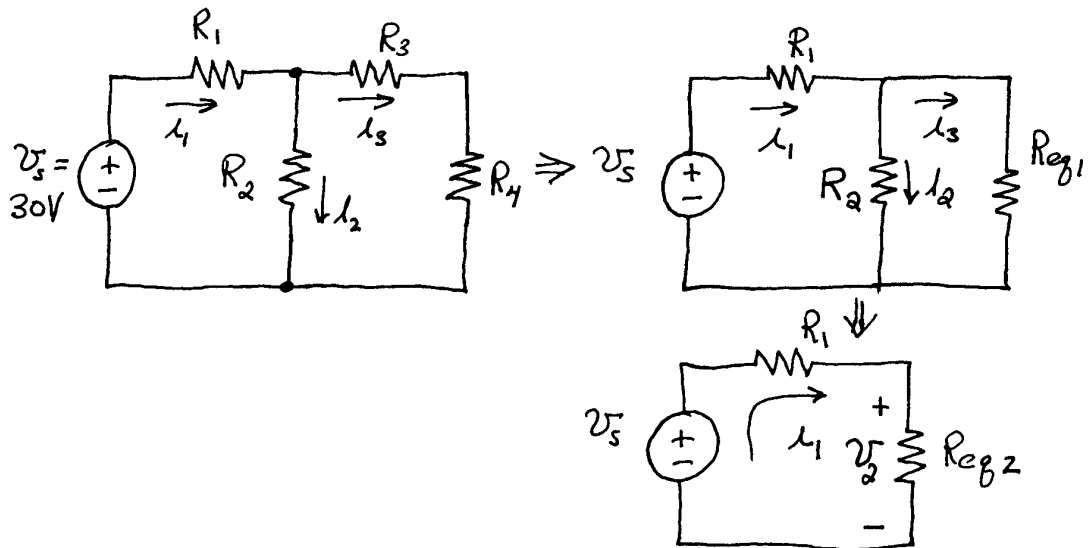$$i_4 = v_{eq}/R_4 = 0.240\text{ A}$$

(b) R_1 and R_2 are in series. Furthermore, R_3, and R_4 are in series.
Finally, the two series combinations are in parallel.

$$R_{eq1} = R_1 + R_2 = 20\ \Omega \quad R_{eq2} = R_3 + R_4 = 20\ \Omega \quad R_{eq} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 10\ \Omega$$

$$V_{eq} = 2 \times R_{eq} = 20\ V \quad i_1 = V_{eq}/R_{eq1} = 1\ A \quad i_2 = V_{eq}/R_{eq2} = 1\ A$$

(c) R_3, and R_4 are in series. The combination of R_3 and R_4 is in parallel
with R_2. Finally the combination of R_2, R_3, and R_4 is in series with R_1.

$$R_{eq1} = R_3 + R_4 = 40\ \Omega \quad R_{eq2} = \frac{1}{1/R_{eq1} + 1/R_2} = 20\ \Omega \quad i_1 = \frac{V_s}{R_1 + R_{eq2}} = 1\ A$$

$$v_2 = i_1 R_{eq2} = 20\ V \quad i_2 = v_2/R_2 = 0.5\ A \quad i_3 = v_2/R_{eq1} = 0.5\ A$$

E2.3   (a) $v_1 = V_s \dfrac{R_1}{R_1 + R_2 + R_3 + R_4} = 10\ V.$ $v_2 = V_s \dfrac{R_2}{R_1 + R_2 + R_3 + R_4} = 20\ V.$

Similarly, we find $v_3 = 30\ V$ and $v_4 = 60\ V$.

(b) First combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 2.917 \ \Omega$.

Then we have $v_1 = v_s \dfrac{R_1}{R_1 + R_{eq} + R_4} = 6.05 \text{ V}$. Similarly, we find

$v_2 = v_s \dfrac{R_{eq}}{R_1 + R_{eq} + R_4} = 5.88 \text{ V}$ and $v_4 = 8.07 \text{ V}$.

E2.4 (a) First combine R_1 and R_2 in series: $R_{eq} = R_1 + R_2 = 30 \ \Omega$. Then we have

$i_1 = i_s \dfrac{R_3}{R_3 + R_{eq}} = \dfrac{15}{15 + 30} = 1 \text{ A}$ and $i_3 = i_s \dfrac{R_{eq}}{R_3 + R_{eq}} = \dfrac{30}{15 + 30} = 2 \text{ A}$.

(b) The current division principle applies to two resistances in parallel. Therefore, to determine i_1, first combine R_2 and R_3 in parallel: $R_{eq} =$

$1/(1/R_2 + 1/R_3) = 5 \ \Omega$. Then we have $i_1 = i_s \dfrac{R_{eq}}{R_1 + R_{eq}} = \dfrac{5}{10 + 5} = 1 \text{ A}$.

Similarly, $i_2 = 1$ A and $i_3 = 1$ A.

E2.5 Write KVL for the loop consisting of v_1, v_y, and v_2. The result is $-v_1 - v_y + v_2 = 0$ from which we obtain $v_y = v_2 - v_1$. Similarly we obtain $v_z = v_3 - v_1$.

E2.6 Node 1: $\dfrac{v_1 - v_3}{R_1} + \dfrac{v_1 - v_2}{R_2} = i_a$ Node 2: $\dfrac{v_2 - v_1}{R_2} + \dfrac{v_2}{R_3} + \dfrac{v_2 - v_3}{R_4} = 0$

Node 3: $\dfrac{v_3}{R_5} + \dfrac{v_3 - v_2}{R_4} + \dfrac{v_3 - v_1}{R_1} + i_b = 0$

E2.7 Following the step-by-step method in the book, we obtain

$$
\begin{bmatrix}
\dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_2} & 0 \\[2ex]
-\dfrac{1}{R_2} & \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} & -\dfrac{1}{R_4} \\[2ex]
0 & -\dfrac{1}{R_4} & \dfrac{1}{R_4} + \dfrac{1}{R_5}
\end{bmatrix}
\begin{bmatrix}
v_1 \\[2ex] v_2 \\[2ex] v_3
\end{bmatrix}
=
\begin{bmatrix}
-i_s \\[2ex] 0 \\[2ex] i_s
\end{bmatrix}
$$

E2.8 Instructions for various calculators vary. The MATLAB solution is given in the book following this exercise.

E2.9   (a) Writing the node equations we obtain:

Node 1: $\dfrac{V_1 - V_3}{20} + \dfrac{V_1}{5} + \dfrac{V_1 - V_2}{10} = 0$

Node 2: $\dfrac{V_2 - V_1}{10} + 10 + \dfrac{V_2 - V_3}{5} = 0$

Node 3: $\dfrac{V_3 - V_1}{20} + \dfrac{V_3}{10} + \dfrac{V_3 - V_2}{5} = 0$

(b) Simplifying the equations we obtain:

$$0.35v_1 - 0.10v_2 - 0.05v_3 = 0$$
$$-0.10v_1 + 0.30v_2 - 0.20v_3 = -10$$
$$-0.05v_1 - 0.20v_2 + 0.35v_3 = 0$$

(c) and (d) Solving using Matlab:

```
>>clear
>>G = [0.35 -0.1 -0.05; -0.10 0.30 -0.20; -0.05 -0.20 0.35];
>>I = [0; -10; 0];
>>V = G\I
V =
  -27.2727
  -72.7273
  -45.4545
>>Ix = (V(1) - V(3))/20
Ix =
    0.9091
```

E2.10   Using determinants we can solve for the unknown voltages as follows:

$$V_1 = \dfrac{\begin{vmatrix} 6 & -0.2 \\ 1 & 0.5 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \dfrac{3 + 0.2}{0.35 - 0.04} = 10.32 \text{ V}$$

$$V_2 = \dfrac{\begin{vmatrix} 0.7 & 6 \\ -0.2 & 1 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \dfrac{0.7 + 1.2}{0.35 - 0.04} = 6.129 \text{ V}$$

Many other methods exist for solving linear equations.

E2.11   First write KCL equations at nodes 1 and 2:

$$\text{Node 1:} \quad \frac{v_1 - 10}{2} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$

$$\text{Node 2:} \quad \frac{v_2 - 10}{10} + \frac{v_2}{5} + \frac{v_2 - v_1}{10} = 0$$

Then, simplify the equations to obtain:
$$8v_1 - v_2 = 50 \quad \text{and} \quad -v_1 + 4v_2 = 10$$

Solving manually or with a calculator, we find v_1 = 6.77 V and v_2 = 4.19 V.
The MATLAB session using the symbolic approach is:

```
>> clear
[V1,V2] = solve('(V1-10)/2+(V1)/5 +(V1 - V2)/10 = 0' , ...
                '(V2-10)/10 +V2/5 +(V2-V1)/10 = 0')
V1 =
210/31
V2 =
130/31
```

Next, we solve using the numerical approach.

```
>> clear
G = [8 -1; -1 4];
I = [50; 10];
V = G\I
V =
   6.7742
   4.1935
```

E2.12   The equation for the supernode enclosing the 15-V source is:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = \frac{v_1}{R_2} + \frac{v_2}{R_4}$$

This equation can be readily shown to be equivalent to Equation 2.37 in the book. (Keep in mind that v_3 = -15 V.)

E2.13 Write KVL from the reference to node 1 then through the 10-V source to node 2 then back to the reference node:

$$-v_1 + 10 + v_2 = 0$$

Then write KCL equations. First for a supernode enclosing the 10-V source, we have:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

Node 3:

$$\frac{v_3}{R_4} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} = 0$$
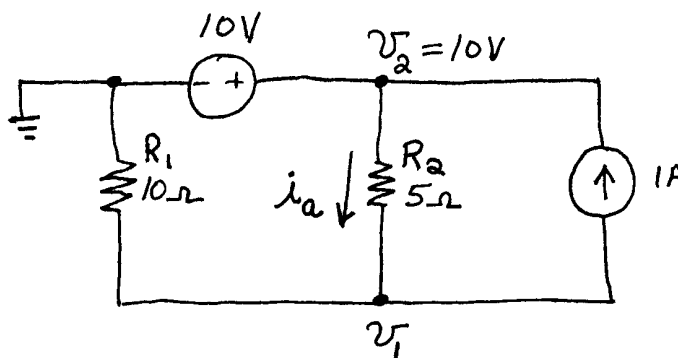
Reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

An independent set consists of the KVL equation and any two of the KCL equations.

E2.14 (a) Select the reference node at the left-hand end of the voltage source as shown at right.
Then write a KCL equation at node 1.

$$\frac{v_1}{R_1} + \frac{v_1 - 10}{R_2} + 1 = 0$$

Substituting values for the resistances and solving, we find $v_1 = 3.33$ V.

Then we have $i_a = \dfrac{10 - v_1}{R_2} = 1.333$ A.

(b) Select the reference node and assign node voltages as shown.
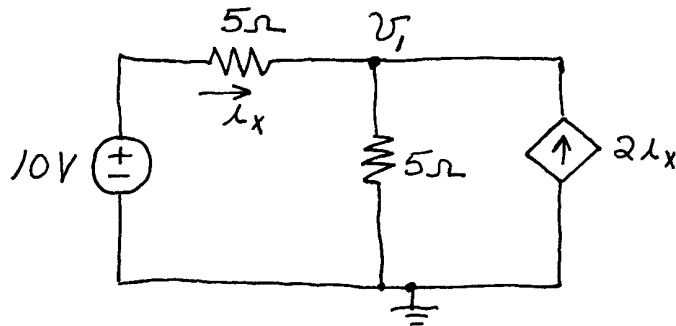

Then write KCL equations at nodes 1 and 2.
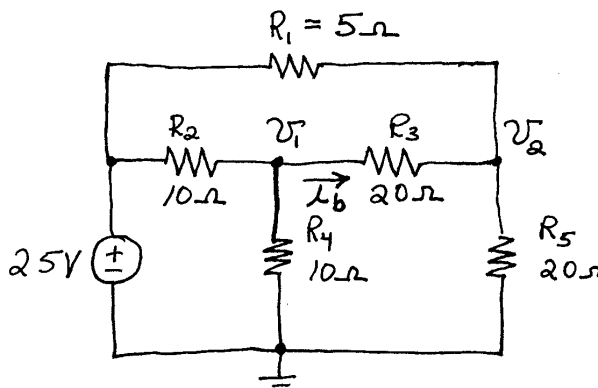
$$\frac{v_1 - 25}{R_2} + \frac{v_1}{R_4} + \frac{v_1 - v_2}{R_3} = 0$$

$$\frac{v_2 - 25}{R_1} + \frac{v_2 - v_1}{R_3} + \frac{v_2}{R_5} = 0$$

Substituting values for the resistances and solving, we find v_1 = 13.79 V
and v_2 = 18.97 V. Then we have $i_b = \frac{v_1 - v_2}{R_3} = $ -0.259 A.

E2.15 (a) Select the reference node and node voltage as shown. Then write a KCL equation at node 1, resulting in
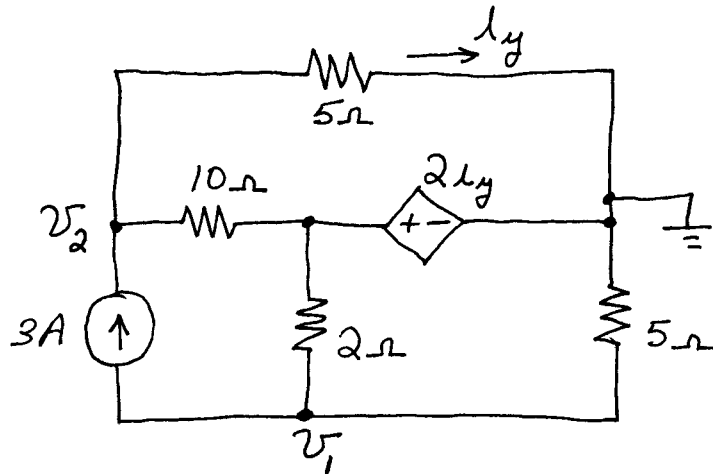
$$\frac{v_1}{5} + \frac{v_1 - 10}{5} - 2i_x = 0$$

Then use $i_x = (10 - v_1)/5$ to substitute and solve. We find v_1 = 7.5 V.
Then we have $i_x = \frac{10 - v_1}{5} = 0.5$ A.

(b) Choose the reference node and node voltages shown:

Then write KCL equations at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2i_y}{2} + 3 = 0 \qquad \frac{v_2}{5} + \frac{v_2 - 2i_y}{10} = 3$$

Finally use $i_y = v_2 / 5$ to substitute and solve. This yields $v_2 = 11.54$ V and $i_y = 2.31$ A.

E2.16 » clear
» [V1 V2 V3] = solve('V3/R4 + (V3 - V2)/R3 + (V3 - V1)/R1 = 0', ...
               'V1/R2 + V3/R4 = Is', ...
               'V1 = (1/2)*(V3 - V1) + V2' ,'V1','V2','V3');
» pretty(V1), pretty(V2), pretty(V3)

$$\frac{R2\ Is\ (2\ R3\ R1 + 3\ R4\ R1 + 2\ R4\ R3)}{2\ R3\ R1 + 3\ R4\ R1 + 3\ R1\ R2 + 2\ R4\ R3 + 2\ R3\ R2}$$

$$\frac{R2\ Is\ (3\ R3\ R1 + 3\ R4\ R1 + 2\ R4\ R3)}{2\ R3\ R1 + 3\ R4\ R1 + 3\ R1\ R2 + 2\ R4\ R3 + 2\ R3\ R2}$$

$$\frac{Is\ R2\ R4\ (3\ R1 + 2\ R3)}{2\ R3\ R1 + 3\ R4\ R1 + 3\ R1\ R2 + 2\ R4\ R3 + 2\ R3\ R2}$$

E2.17 Refer to Figure 2.33b in the book. (a) Two mesh currents flow through R_2: i_1 flows downward and i_4 flows upward. Thus the current flowing in R_2 referenced upward is $i_4 - i_1$. (b) Similarly, mesh current i_1 flows to the left through R_4 and mesh current i_2 flows to the right, so the total current referenced to the right is $i_2 - i_1$. (c) Mesh current i_3 flows downward through R_8 and mesh current i_4 flows upward, so the total current referenced downward is $i_3 - i_4$. (d) Finally, the total current referenced upward through R_8 is $i_4 - i_3$.

**E2.18** Refer to Figure 2.33b in the book. Following each mesh current in turn, we have

$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$
$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$
$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

In matrix form, these equations become

$$\begin{bmatrix} (R_1 + R_2 + R_4) & -R_4 & 0 & -R_2 \\ -R_4 & (R_4 + R_5 + R_6) & -R_6 & 0 \\ 0 & -R_6 & (R_6 + R_7 + R_8) & -R_8 \\ -R_2 & 0 & -R_8 & (R_2 + R_3 + R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} v_A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

E2.19   We choose the mesh currents as shown:
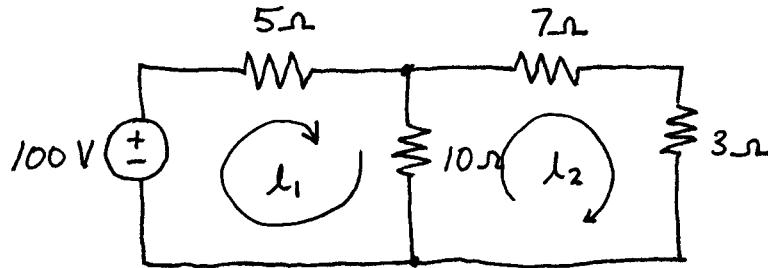
Then, the mesh equations are:

$$5i_1 + 10(i_1 - i_2) = 100 \quad \text{and} \quad 10(i_2 - i_1) + 7i_2 + 3i_2 = 0$$

Simplifying and solving these equations, we find that $i_1 = 10$ A and $i_2 = 5$ A. The net current flowing downward through the 10-Ω resistance is $i_1 - i_2 = 5$ A.

To solve by node voltages, we select the reference node and node voltage shown. (We do not need to assign a node voltage to the connection between the 7-Ω resistance and the 3-Ω resistance because we can treat the series combination as a single 10-Ω resistance.)
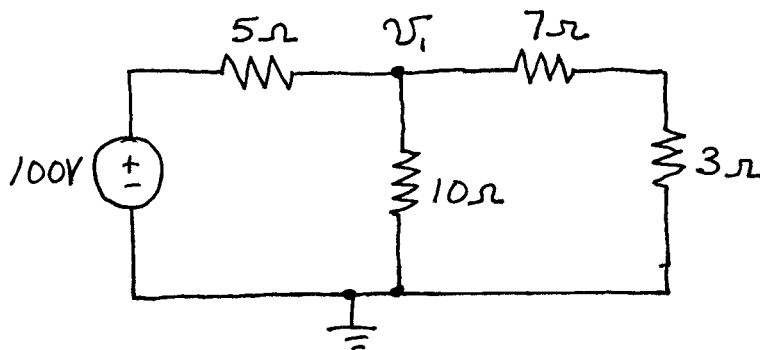
The node equation is $(v_1 - 10)/5 + v_1/10 + v_1/10 = 0$. Solving we find that v_1 = 50 V. Thus we again find that the current through the 10-Ω resistance is $i = v_1/10 = 5$ A.

Combining resistances in series and parallel, we find that the resistance "seen" by the voltage source is 10 Ω. Thus the current through the source and 5-Ω resistance is (100 V)/(10 Ω) = 10 A. This current splits equally between the 10-Ω resistance and the series combination of 7 Ω and 3 Ω.

E2.20 First, we assign the mesh currents as shown.
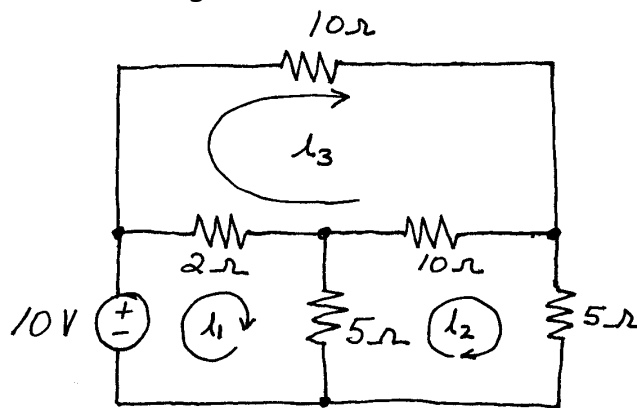
Then we write KVL equations following each mesh current:

$$2(i_1 - i_3) + 5(i_1 - i_2) = 10$$
$$5i_2 + 5(i_2 - i_1) + 10(i_2 - i_3) = 0$$
$$10i_3 + 10(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Simplifying and solving, we find that i_1 = 2.194 A, i_2 = 0.839 A, and i_3 = 0.581 A. Thus the current in the 2-Ω resistance referenced to the right is $i_1 - i_3$ = 2.194 - 0.581 = 1.613 A.

E2.21 Following the step-by-step process, we obtain

$$\begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

E2.22   Refer to Figure 2.39 in the book. In terms of the mesh currents, the current directed to the right in the 5-A current source is i_1, however by the definition of the current source, the current is 5 A directed to the left. Thus, we conclude that i_1 = -5 A. Then we write a KVL equation following i_2, which results in $10(i_2 - i_1) + 5i_2 = 100$.

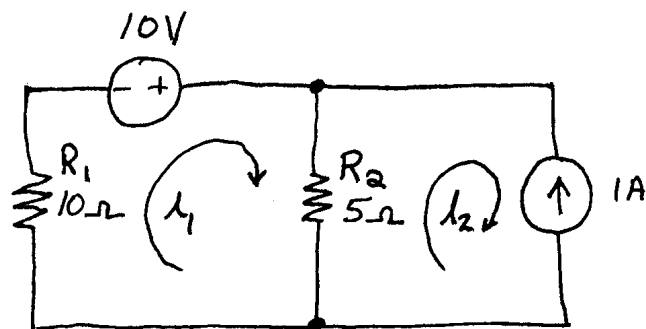**E2.23**   Refer to Figure 2.40 in the book. First, for the current source, we have

$$i_2 - i_1 = 1$$

Then, we write a KVL equation going around the perimeter of the entire circuit:

$$5i_1 + 10i_2 + 20 - 10 = 0$$

Simplifying and solving these equations we obtain i_1 = -4/3 A and i_2 = -1/3 A.

E2.24   (a) As usual, we select the mesh currents flowing clockwise around the meshes as shown. Then for the current source, we have i_2 = -1 A. This is because we defined the mesh current i_2 as the current referenced downward through the current source. However, we know that the current through this source is 1 A flowing upward. Next we write a KVL equation around mesh 1: $10i_1 - 10 + 5(i_1 - i_2) = 0$. Solving, we find that i_1 = 1/3 A. Referring to Figure 2.30a in the book we see that the value of the current i_a referenced downward through the 5 Ω resistance is to be found. In terms of the mesh currents, we have $i_a = i_1 - i_2 = 4/3\,A$.

(b) As usual, we select the mesh currents flowing clockwise around the meshes as shown.
Then we write a KVL equation for each mesh.

$R_1 = 5\,\Omega$
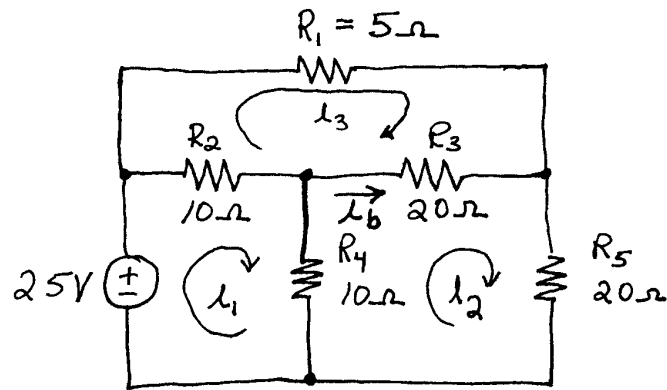
$$-25 + 10(i_1 - i_3) + 10(i_1 - i_2) = 0$$
$$10(i_2 - i_1) + 20(i_2 - i_3) + 20i_2 = 0$$
$$10(i_3 - i_1) + 5i_3 + 20(i_3 - i_2) = 0$$

Simplifying and solving, we find $i_1 = 2.3276$ A, $i_2 = 0.9483$ A, and $i_3 = 1.2069$ A. Finally, we have $i_b = i_2 - i_3 = -0.2586$ A.
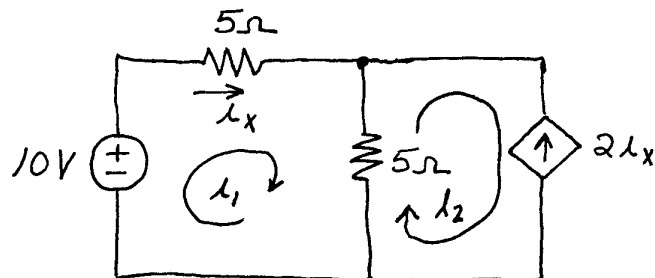
E2.25 (a) KVL mesh 1:
$$-10 + 5i_1 + 5(i_1 - i_2) = 0$$
For the current source:
$$i_2 = -2i_x$$
However, i_x and i_1 are the same current, so we also have $i_1 = i_x$.

Simplifying and solving, we find $i_x = i_1 = 0.5$ A.

(b) First for the current source, we have: $i_1 = 3$ A
Writing KVL around meshes 2 and 3, we have:

$$2(i_2 - i_1) + 2i_y + 5i_2 = 0$$
$$10(i_3 - i_1) + 5i_3 - 2i_y = 0$$
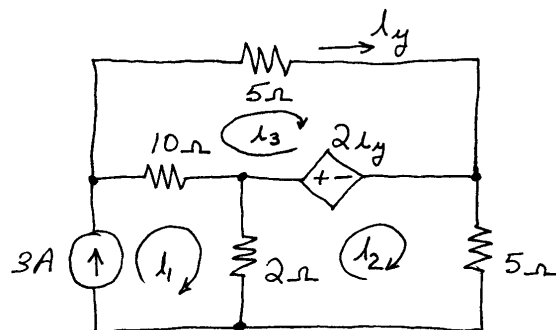
However i_3 and i_y are the same current: $i_y = i_3$. Simplifying and solving, we find that $i_3 = i_y = 2.31$ A.

E2.26 Under open-circuit conditions, 5 A circulates clockwise through the current source and the 10-Ω resistance. The voltage across the 10-Ω resistance is 50 V. No current flows through the 40-Ω resistance so the open circuit voltage is $V_t = 50$ V.

With the output shorted, the 5 A divides between the two resistances in parallel. The short-circuit current is the current through the 40-Ω resistance, which is $i_{sc} = 5\dfrac{10}{10+40} = 1$ A. Then, the Thévenin resistance is $R_t = v_{oc}/i_{sc} = 50$ Ω.

E2.27 Choose the reference node at the bottom of the circuit as shown:
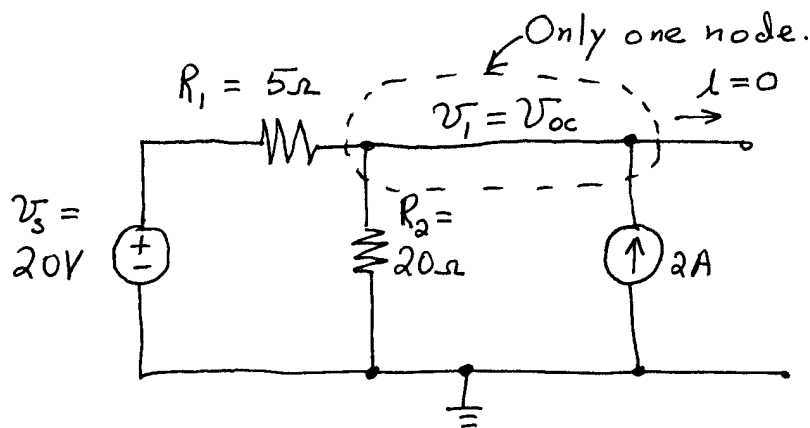
Notice that the node voltage is the open-circuit voltage. Then write a KCL equation:

$$\frac{V_{oc}-20}{5}+\frac{V_{oc}}{20}=2$$

Solving we find that v_{oc} = 24 V which agrees with the value found in Example 2.17.

E2.28 To zero the sources, the voltage sources become short circuits and the current sources become open circuits. The resulting circuits are :
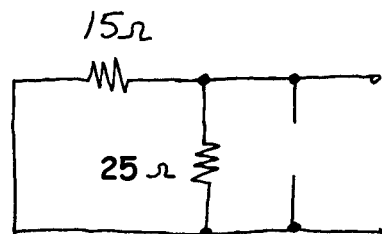
(a)

(b)

$R_{eq} = 4\Omega$

(c)
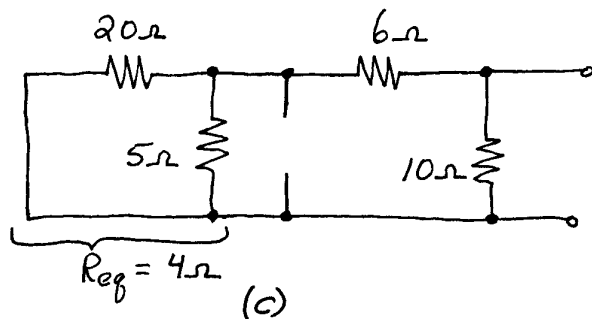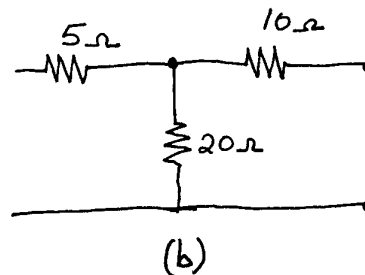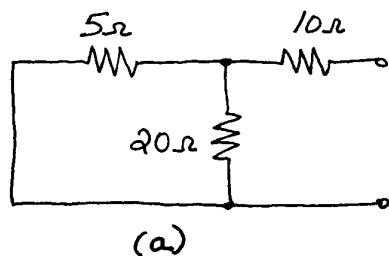
(a) $R_t = 10 + \dfrac{1}{1/5 + 1/20} = 14 \ \Omega$ 　　　(b) $R_t = 10 + 20 = 30 \ \Omega$
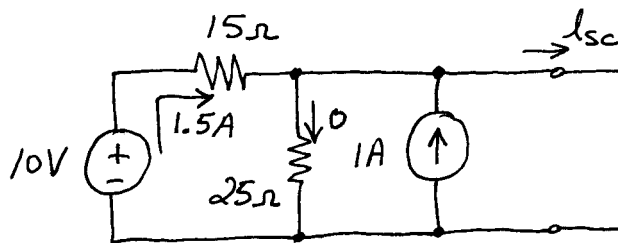
(c) $R_t = \dfrac{1}{\dfrac{1}{10} + \dfrac{1}{6 + \dfrac{1}{(1/5 + 1/20)}}} = 5 \ \Omega$

E2.29 (a) Zero sources to determine Thévenin resistance. Thus
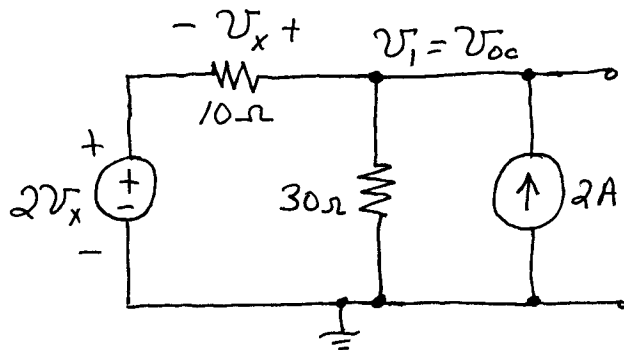
$R_t = \dfrac{1}{1/15 + 1/25} = 9.375 \ \Omega.$

Then find short-circuit current:

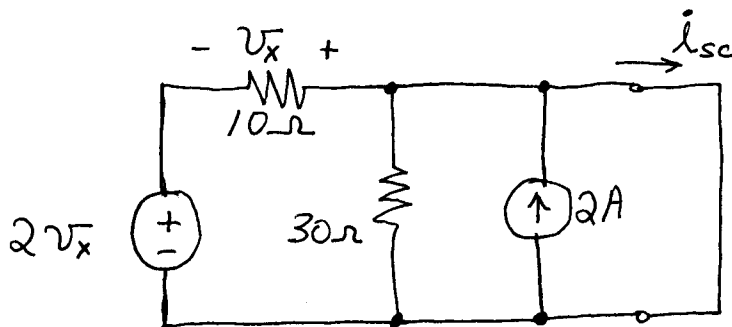$I_n = i_{sc} = 10/15 + 1 = 1.67 \ A$

(b) We cannot find the Thévenin resistance by zeroing the sources, because we have a controlled source. Thus, we find the open-circuit voltage and the short-circuit current.

$$\frac{V_{oc} - 2v_x}{10} + \frac{V_{oc}}{30} = 2 \qquad V_{oc} = 3v_x$$
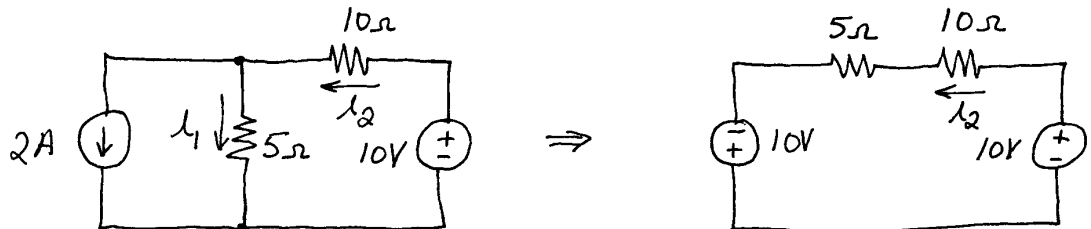
Solving, we find $V_t = V_{oc} = 30$ V.

Now, we find the short-circuit current:

$$2v_x + v_x = 0 \qquad \Rightarrow \qquad v_x = 0$$

Therefore $i_{sc} = 2$ A. Then we have $R_t = V_{oc} / i_{sc} = 15 \ \Omega$.

E2.30 First, we transform the 2-A source and the 5-Ω resistance into a voltage source and a series resistance:

Then we have $i_2 = \dfrac{10 + 10}{15} = 1.333$ A.

From the original circuit, we have $i_1 = i_2 - 2$, from which we find $i_1 = -0.667$ A.

The other approach is to start from the original circuit and transform the 10-Ω resistance and the 10-V voltage source into a current source and parallel resistance:
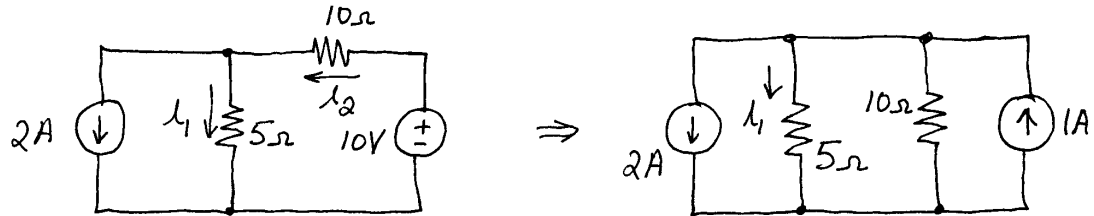
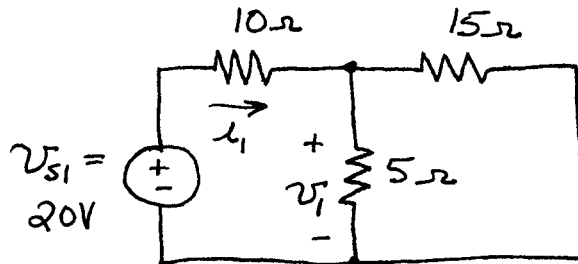Then we combine the resistances in parallel. $R_{eq} = \dfrac{1}{1/5 + 1/10} = 3.333\ \Omega$.

The current flowing upward through this resistance is 1 A. Thus the voltage across R_{eq} referenced positive at the bottom is 3.333 V and $i_1 = -3.333 / 5 = -0.667$ A. Then from the original circuit we have $i_2 = 2 + i_1 = 1.333$ A, as before.

E2.31 Refer to Figure 2.62b. We have $i_1 = 15 / 15 = 1$ A.

Refer to Figure 2.62c. Using the current division principle, we have

$i_2 = -2 \times \dfrac{5}{5 + 10} = -0.667$ A. (The minus sign is because of the reference direction of i_2.) Finally, by superposition we have $i_T = i_1 + i_2 = 0.333$ A.

E2.32 With only the first source active we have:

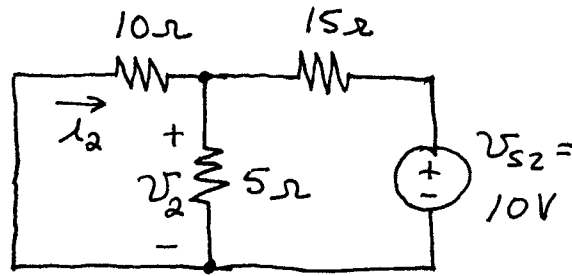Then we combine resistances in series and parallel:

$R_{eq} = 10 + \dfrac{1}{1/5 + 1/15} = 13.75\ \Omega$

Thus, $i_1 = 20 / 13.75 = 1.455$ A, and $v_1 = 3.75 i_1 = 5.45$ V.

With only the second source active, we have:

Then we combine resistances in series and parallel:

$$R_{eq2} = 15 + \frac{1}{1/5 + 1/10} = 18.33 \ \Omega$$

Thus, $i_s = 10/18.33 = 0.546$ A, and $v_2 = 3.33 i_s = 1.818$ V. Then, we have
$i_2 = (-v_2)/10 = -0.1818$ A

Finally we have $v_T = v_1 + v_2 = 5.45 + 1.818 = 7.27$ V and
$i_T = i_1 + i_2 = 1.455 - 0.1818 = 1.27$ A.

Problems

P2.1* (a) $R_{eq} = 20 \ \Omega$ (b) $R_{eq} = 23 \ \Omega$

**P2.2*** We have $4 + \dfrac{1}{1/20 + 1/R_x} = 8$ which yields $R_x = 5 \ \Omega$.

P2.3* The 20-Ω and 30-Ω resistances are in parallel and have an equivalent resistance of R_{eq1} = 12 Ω. Also the 40-Ω and 60-Ω resistances are in parallel with an equivalent resistance of R_{eq2} = 24 Ω. Next we see that R_{eq1} and the 4-Ω resistor are in series and have an equivalent resistance of R_{eq3} = 4 + R_{eq1} = 16 Ω. Finally R_{eq3} and R_{eq2} are in parallel and the overall equivalent resistance is

$$R_{ab} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 9.6 \ \Omega$$

P2.4* The 12-Ω and 6-Ω resistances are in parallel having an equivalent resistance of 4 Ω. Similarly, the 18-Ω and 9-Ω resistances are in parallel and have an equivalent resistance of 6 Ω. Finally, the two parallel combinations are in series, and we have

$$R_{ab} = 4 + 6 = 10 \ \Omega$$

P2.5*
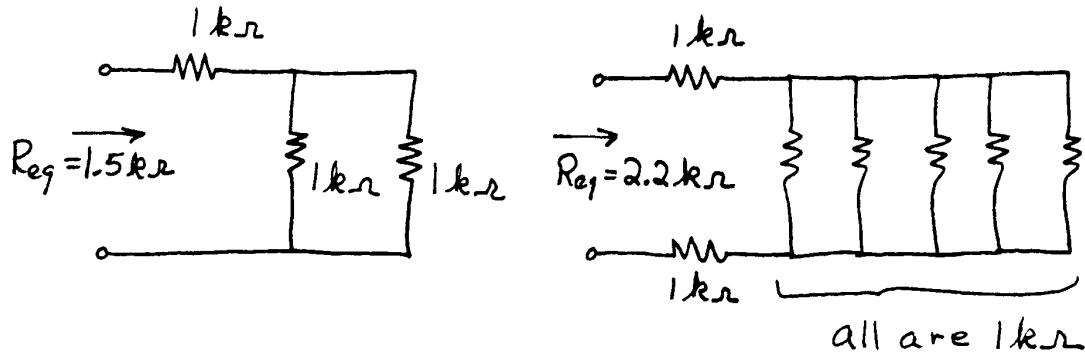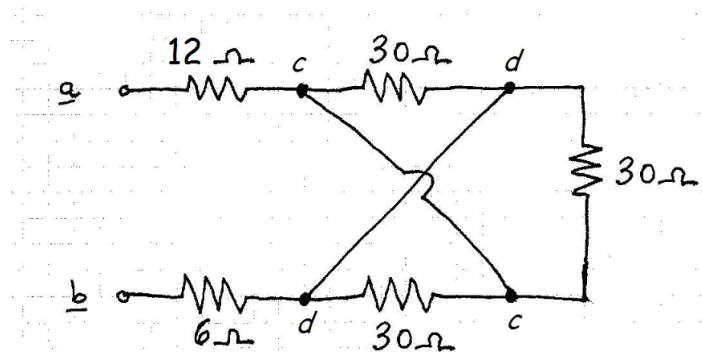

P2.6 (a) $R_{eq} = 8\,\Omega$ (b) $R_{eq} = 12.5\,\Omega$

P2.7 Because the resistances are in series, the same current i flows through both of them. The voltage across R_1 is $v_1 = 60i$. The voltage across R_2 is $v_2 = 3v_1 = 180i$. Thus, we have $R_2 = 180\,\Omega$.

P2.8 (a) $R_{eq} = 44\,\Omega$ (b) $R_{eq} = 32\,\Omega$
(c)


Notice that the points labeled c are the same node and that the points labeled d are another node. Thus, all three of the 30-Ω resistors are in parallel because they are each connected between nodes c and d. The equivalent resistance is 28 Ω.

P2.9 We have $\dfrac{1}{1/70 + 1/R_x} = 20$ which yields $R_x = 28\ \Omega.$

P2.10 We have $R_{eq} = \dfrac{2R(3R)}{2R+3R} = \dfrac{2R}{5}$. Clearly, for R_{eq} to be an integer, R must be an integer multiple of 5.

P2.11 $R_{ab} = 16\ \Omega$
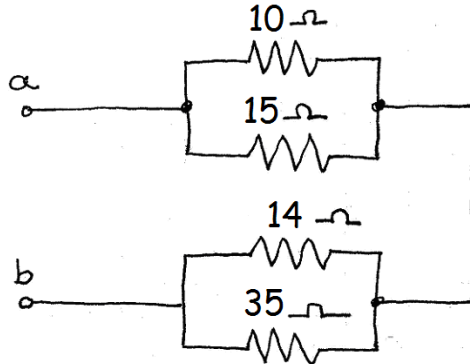
P2.12 In the lowest power mode, the power is $P_{lowest} = \dfrac{120^2}{R_1 + R_2} = 102.9\ \text{W}.$

For the highest power mode, the two elements should be in parallel with an applied voltage of 240 V. The resulting power is

$$P_{highest} = \dfrac{240^2}{R_1} + \dfrac{240^2}{R_2} = 1440 + 576 = 2016\ \text{W}.$$

Some other modes and resulting powers are:
- R_1 operated separately from 240 V yielding 1440 W
- R_2 operated separately from 240 V yielding 576 W
- R_1 in series with R_2 operated from 240 V yielding 411.4 W
- R_1 operated separately from 120 V yielding 360 W

P2.13 Combining the resistances shown in Figure P2.13b, we have

$$R_{eq} = 8 + \dfrac{1}{1/5 + 1/R_{eq}} + 8 = 16 + \dfrac{5R_{eq}}{5 + R_{eq}}$$

$$\left(R_{eq}\right)^2 - 16R_{eq} - 80 = 0$$

$$R_{eq} = 20\ \Omega$$

$(R_{eq} = -4\ \Omega$ is another root, but is not physically reasonable.)

P2.14 $R_{eq} = \dfrac{1}{\dfrac{1}{1000} + \dfrac{1}{1000} + \dfrac{1}{1000} + \dots} = \dfrac{1}{\dfrac{n}{1000}} = \dfrac{1000}{n}$

P2.15 For operation at the lowest power, we have

$$P = 180 = \frac{120^2}{R_1 + R_2}$$

At the high power setting, we have

$$P = 960 = \frac{120^2}{R_1} + \frac{120^2}{R_2}$$

These equations can be put in the form

$$R_1 + R_2 = 80$$

$$\frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2} = 15$$

Solving these equations, we find $R_1 = 20\,\Omega$ and $R_2 = 60\,\Omega$. (A second solution simply has the values of R_1 and R_2 interchanged.)

The intermediate power settings are obtained by operating one of the elements from 120 V resulting in powers of 240 W and 720 W.

P2.16 $R = 10\,\Omega.$

P2.17 By symmetry, we find the currents in the resistors as shown below:
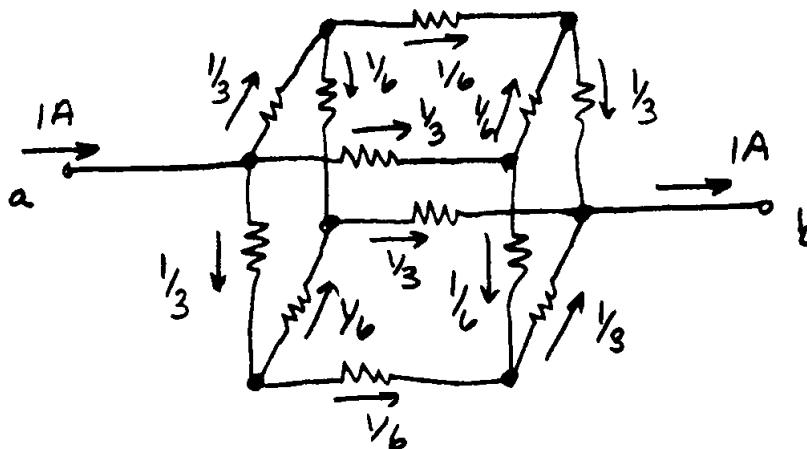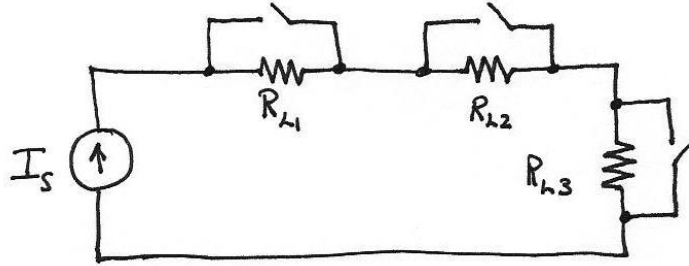

Then, the voltage between terminals a and b is

$$v_{ab} = R_{eq} = 1/3 + 1/6 + 1/3 = 5/6$$

P2.18   (a) For a series combination $G_{eq} = \dfrac{1}{1/G_1 + 1/G_2 + 1/G_3}$

           (b) For a parallel combination of conductances $G_{eq} = G_1 + G_2 + G_3$

P2.19   To supply the loads in such a way that turning one load on or off does not affect the other loads, we must connect the loads in series with a switch in parallel with each load:

           To turn a load on, we open the corresponding switch, and to turn a load off, we close the switch.

P2.20   The equations for the conductances are

$$G_b + G_c = \frac{1}{R_{as}} = \frac{1}{24} \qquad G_a + G_c = \frac{1}{R_{bs}} = \frac{1}{30} \qquad G_b + G_a = \frac{1}{R_{cs}} = \frac{1}{40}$$

           Adding respective sides of the first two equations and subtracting the respective sides of the third equation yields

$$2G_c = \frac{1}{24} + \frac{1}{30} - \frac{1}{40} = \frac{6}{120} \text{ from which we obtain } G_c = \frac{1}{40} \text{ S. Then we}$$

           have $R_c = 40\,\Omega$. Similarly, we find $R_a = 120\,\Omega$ and $R_b = 60\,\Omega$.

P2.21   We have $R_a + R_b = R_{ab} = 30$, $R_b + R_c = R_{bc} = 50$ and $R_a + R_c = R_{ca} = 40$. These equations can be solved to find that $R_a = 10\ \Omega$, $R_b = 20\ \Omega$, and $R_c = 30\ \Omega$. After shorting terminals b and c, the equivalent resistance between terminal a and the shorted terminals is

$$R_{eq} = R_a + \frac{1}{1/R_b + 1/R_c} = 22\ \Omega$$

P2.22   The steps in solving a circuit by network reduction are:
           1. Find a series or parallel combination of resistances.
           2. Combine them.

3. Repeat until the network is reduced to a single resistance and a single source (if possible).
4. Solve for the currents and voltages in the final circuit. Transfer results back along the chain of equivalent circuits, solving for more currents and voltages along the way.
5. Check to see that KVL and KCL are satisfied in the original network.

The method does not always work because some networks cannot be reduced sufficiently. Then, another method such as node voltages or mesh currents must be used.

P2.23* $\quad i_1 = \dfrac{10}{R_{eq}} = \dfrac{10}{10} = 1\ A$

$v_x = 4\ V$

$i_2 = \dfrac{v_x}{8} = 0.5\ A$

P2.24* $\quad R_{eq} = \dfrac{1}{1/5 + 1/15} = 3.75\ \Omega \qquad v_x = 2\ A \times R_{eq} = 7.5\ V$

$i_1 = v_x/5 = 1.5\ A \qquad i_2 = v_x/15 = 0.5\ A$

$P_{4A} = 4 \times 7.5 = 30\ W\ delivering$
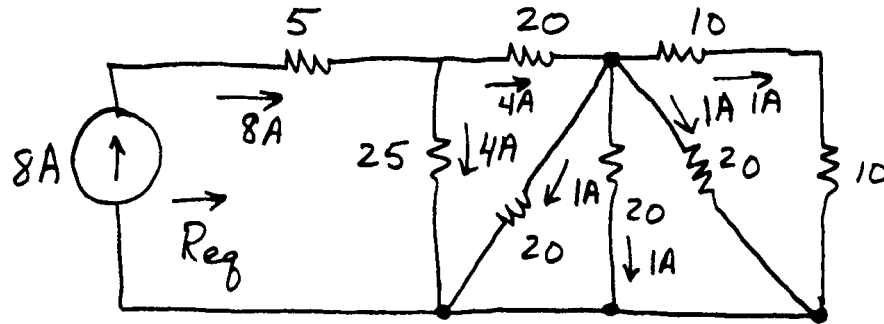
$P_{2A} = 2 \times 7.5 = 15\ W\ absorbing$

$P_{5\Omega} = 7.5^2/5 = 11.25\ W\ absorbing$

$P_{15\Omega} = (7.5)^2/15 = 3.75\ W\ absorbing$

P2.25* Combining resistors in series and parallel, we find that the equivalent resistance seen by the current source is $R_{eq} = 17.5\ \Omega$.

Thus, $v = 8 \times 17.5 = 140$ V. Also, $i = 1$ A.

P2.26* We combine resistances in series and parallel until the circuit becomes an equivalent resistance across the voltage source. Then, we solve the simplified circuit and transfer information back along the chain of equivalents until we have found the desired results.

$v_1 = 3V$

$v_2 = 0.5V$
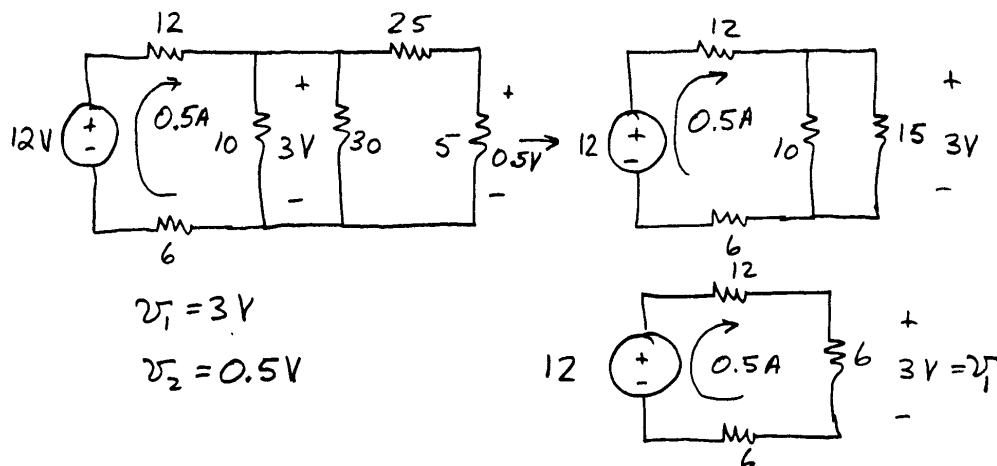
P2.27 Using Ohm's and Kirchhoff's laws, we work from right to left resulting in
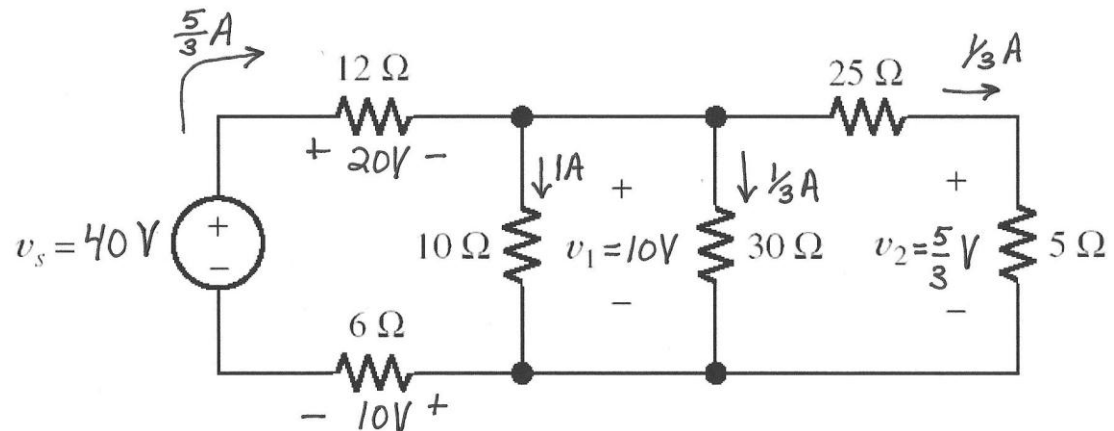
P2.28   The equivalent resistance seen by the current source is

$$R_{eq} = 6 + \frac{1}{1/20 + 1/(25 + 5)} = 18 \ \Omega$$

Then, we have $v_s = 3R_{eq} = 54$ V $\qquad v_1 = 3\frac{1}{1/20 + 1/(25 + 5)} = 36$ V

$$i_2 = \frac{v_1}{25 + 5} = 1.2 \ A$$

P2.29   The equivalent resistance seen by the voltage source is

$$R_{eq} = \frac{1}{1/18 + 1/(7 + 2)} + 4 = 10 \ \Omega$$

Then, we have

$$i_1 = \frac{30 \ V}{R_{eq}} = 3 \ A \qquad v_2 = i_1 \frac{1}{1/18 + 1/(7 + 2)} = 18 \ V$$

$$i_2 = \frac{v_2}{18} = 1 \ A \qquad\qquad i_3 = -\frac{v_2}{9} = -2 \ A$$

P2.30   The equivalent resistance seen by the current source is

$$R_{eq} = 6 + \frac{1}{1/9 + 1/18} + \frac{1}{1/10 + 1/40} = 20 \ \Omega. \text{ Then, we have } v = 5R_{eq} = 100 \ V,$$

$i_2 = 3.333$ A, and $i_1 = 4$ A.

P2.31 $i_2 = \frac{12 \ V}{4 \ \Omega} = 3 \ A \qquad i_1 = i_2 + 2 = 5 \ A$

Notice that i_1 is referenced into the negative reference for the voltage source, thus $P_{voltage-source} = -12i_1 = -60$ W. The 2-A of the current source flows from the positive reference for the voltage toward the negative reference, and we have
$P_{current-source} = 2 \ A \times 12 \ V = +24$ W.
Power is delivered by the voltage source and absorbed by the current source. The resistance absorbs 36 W so power is conserved in the circuit.

P2.32   With the switch open, the current flowing clockwise in the circuit is given by $i = \frac{16}{6 + R_2}$, and we have $v_2 = R_2 i = \frac{16R_2}{6 + R_2} = 10$. Solving, we find $R_2 = 10$ Ω.

With the switch closed, R_2 and R_L are in parallel with an equivalent

resistance given by $R_{eq} = \dfrac{1}{1/R_2 + 1/R_L} = \dfrac{1}{1/10 + 1/R_L}$. The current

through R_{eq} is given by $i = \dfrac{16}{6 + R_{eq}}$ and we have $v_2 = R_{eq}i = \dfrac{16R_{eq}}{6 + R_{eq}} = 8$.

Solving, we find $R_{eq} = 6\ \Omega$. Then, we can write $R_{eq} = \dfrac{1}{1/10 + 1/R_L} = 6$.

Solving, we find $R_L = 15\ \Omega$.

P2.33   The currents through the 3-Ω resistance and the 4-Ω resistance are zero, because they are series with an open circuit. Thus, we can consider the 8-Ω and the 7-Ω resistances to be in series. The current circulating clockwise in the left-hand loop is given by $i_1 = \dfrac{30}{7 + 8} = 2$ A, and we have v_1 $= 14$ V. The current circulating counterclockwise in the right-hand loop is 3 A. By Ohm's law, we have $v_2 = 6$ V. Then, using KVL, we have $v_{ab} = v_1 - v_2 = 3$ V, $v_{bc} = v_2 = 6\,V$, and $v_{ca} = -v_1 = -14$ V.

P2.34 $i = \dfrac{P}{v} = \dfrac{4\,W}{10\,V} = 0.4A$ $R_{eq} = R + \dfrac{1}{1/R + 1/R} + R = 2.5R$

$i = 0.4 = \dfrac{10}{R_{eq}} = \dfrac{10}{2.5R}$ $R = 10\ \Omega$

P2.35*
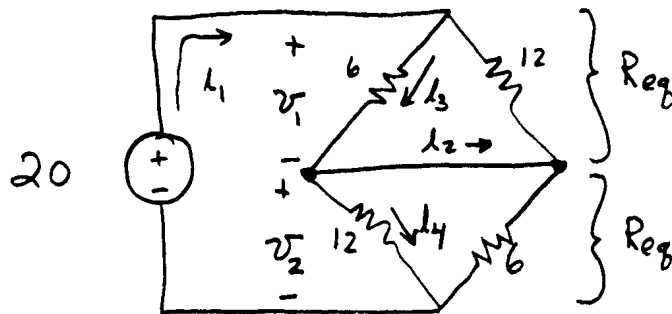
$R_{eq} = \dfrac{1}{1/6 + 1/12} = 4\,\Omega$ $i_1 = \dfrac{20\,V}{2R_{eq}} = 2.5$ A

$v_1 = v_2 = R_{eq}i_1 = 10$ V $i_3 = 10/6 = 1.667$ A

$i_4 = 10/12 = 0.8333$ A $i_2 = i_3 - i_4 = 0.8333$ A

P2.36* $V_1 = \dfrac{R_1}{R_1 + R_2 + R_3} \times V_s = 5\text{ V}$ $\qquad$ $V_2 = \dfrac{R_2}{R_1 + R_2 + R_3} \times V_s = 7\text{ V}$

$V_3 = \dfrac{R_3}{R_1 + R_2 + R_3} \times V_s = 13\text{ V}$

P2.37* $i_1 = \dfrac{R_2}{R_1 + R_2} i_s = 1\text{ A}$ $\qquad$ $i_2 = \dfrac{R_1}{R_1 + R_2} i_s = 2\text{ A}$

P2.38* Combining R_2 and R_3, we have an equivalent resistance

$R_{eq} = \dfrac{1}{1/R_2 + 1/R_3} = 10\ \Omega$. Then, using the voltage-division principle, we

have $v = \dfrac{R_{eq}}{R_1 + R_{eq}} \times V_s = \dfrac{10}{20 + 10} \times 10 = 3.333\text{ V}$.

P2.39 $i_2 = \dfrac{R_3}{R_2 + R_3} \times i_s = \dfrac{50}{25 + 50} \times 30 = 20\text{ mA}$

$i_3 = \dfrac{R_2}{R_2 + R_3} \times i_s = \dfrac{25}{25 + 50} \times 30 = 10\text{ mA}$

P2.40 (a) $R_1 + R_2 = \dfrac{9\text{ V}}{10\text{ mA}} = 900\ \Omega$ $\qquad\qquad$ $\dfrac{R_2}{R_1 + R_2} \times 9 = 5$

Solving, we find $R_2 = 500\ \Omega$ and $R_1 = 400\ \Omega$.

(b)
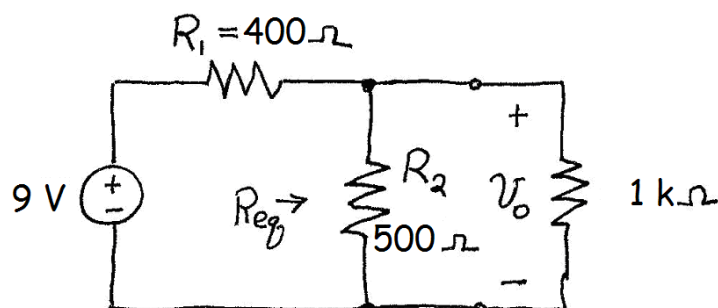
The equivalent resistance for the parallel combination of R_2 and the load is

$$R_{eq} = \frac{1}{1/500 + 1/1000} = 333.3\ \Omega$$

Then, using the voltage division principle, we have

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} \times 9\ \text{V} = 4.091\ \text{V}$$

(c) If we choose a larger current in part (a), resulting in smaller values for R_1 and R_2, the loaded voltage in part (b) would be closer to 5 V. However, this would result in shorter battery life.

P2.41 We have $240\dfrac{5}{10 + 5 + R_x} = 30$, which yields $R_x = 25\ \Omega$.

P2.42 First, we combine the 60 Ω and 20 Ω resistances in parallel yielding an equivalent resistance of 15 Ω, which is in parallel with R_x. Then, applying the current division principle, we have

$$30\dfrac{15}{15 + R_x} = 10$$

which yields $R_x = 30\ \Omega$.

P2.43 In a similar fashion to the solution for Problem P2.13, we can write the following expression for the resistance seen by the 16-V source.

$$R_{eq} = 2 + \frac{1}{1/R_{eq} + 1/4}\ \text{k}\Omega$$

The solutions to this equation are $R_{eq} = 4\ \text{k}\Omega$ and $R_{eq} = -2\ \text{k}\Omega$. However, we reason that the resistance must be positive and discard the negative root. Then, we have $i_1 = \dfrac{16\ \text{V}}{R_{eq}} = 4$ mA, $i_2 = i_1\dfrac{R_{eq}}{4 + R_{eq}} = \dfrac{i_1}{2} = 2$ mA, and
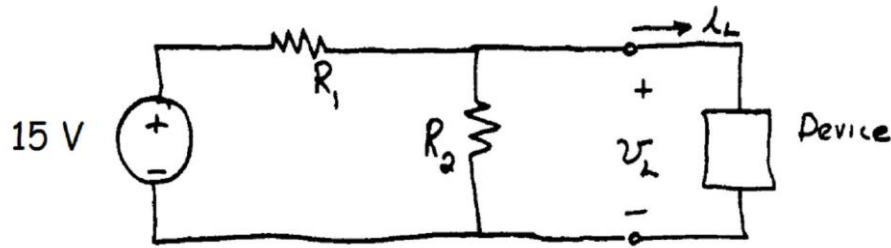
$i_3 = \dfrac{i_1}{2} = 2$ mA. Similarly, $i_4 = \dfrac{i_3}{2} = \dfrac{i_1}{2^2} = 1$ mA. Clearly, $i_{n+2} = i_n/2$. Thus,

$i_{18} = \dfrac{i_1}{2^9} = 7.8125\ \mu\text{A}$.

P2.44* $v = 0.1\,\text{mA} \times R_w = 50\,\text{mV}$

$$R_g = \frac{50\,\text{mV}}{2\,\text{A} - 0.1\,\text{mA}} = 25\,\text{m}\Omega$$

P2.45   The circuit diagram is:

With $i_L = 0$ and $v_L = 5\,\text{V}$, we must have $\dfrac{R_2}{R_1 + R_2} \times 15 = 5\,\text{V}$. Rearranging, this gives

$$\frac{R_1}{R_2} = 2 \qquad\qquad (1)$$

With $i_L = 100\,\text{mA}$ and $v_L = 4.7\,\text{V}$, we have $15 - R_1(4.7/R_2 + 100\,\text{mA}) = 4.7$. Rearranging, this gives

$$4.7\frac{R_1}{R_2} + R_1 \times 0.1 = 10.3. \qquad\qquad (2)$$

Using Equation (1) to substitute into Equation (2) and solving, we obtain $R_1 = 9\,\Omega$ and $R_2 = 4.5\,\Omega$.

Maximum power is dissipated in R_1 for $i_L = 100\,\text{mA}$, for which the voltage across R_1 is 10.3 V. Thus, $P_{\max R1} = \dfrac{10.3^2}{9} = 11.8\,\text{W}$. Thus, R_1 must be rated for at least 11.8 W of power dissipation.

Maximum power is dissipated in R_2 for $i_L = 0$, in which case the voltage across R_2 is 5 V. Thus, $P_{\max R2} = \dfrac{5^2}{4.5} = 5.56\,\text{W}$.

P2.46 We need to place a resistor in series with the load and the voltage source as shown:
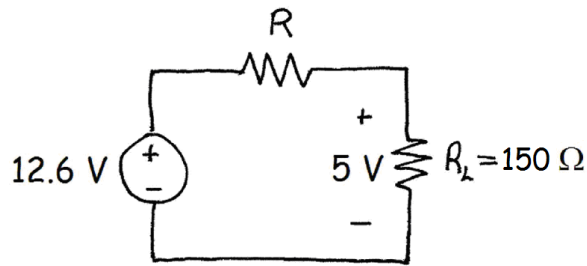
Applying the voltage-division principle, we have $12.6\dfrac{150}{150+R}=5$. Solving, we find $R=228\ \Omega$.

P2.47 We have $P=500\times10^{-3}=I_L^2 R_L=I_L^2\times200$. Solving, we find that the current through the load is $I_L=50$ mA. Thus, we must place a resistor in parallel with the current source and the load.
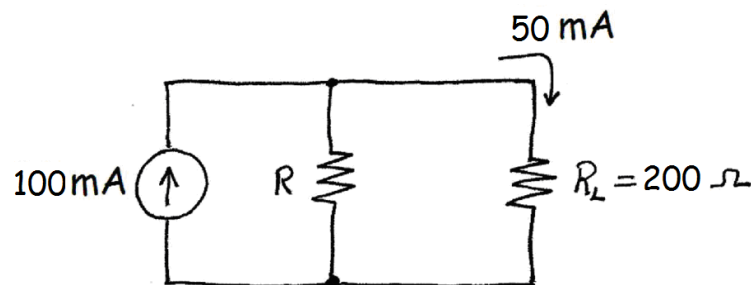
Then, we have $100\dfrac{R}{R+R_L}=50$ from which we find $R=200\ \Omega$.

P2.48. 1. Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.

2. Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.

3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the

network equations, and obtain equations having only the node voltages as unknowns.

4. Put the equations into standard form and solve for the node voltages.

5. Use the values found for the node voltages to calculate any other currents or voltages of interest.

P2.49*   At node 1 we have: $\dfrac{v_1}{20} + \dfrac{v_1 - v_2}{10} = 1$

At node 2 we have: $\dfrac{v_2}{5} + \dfrac{v_2 - v_1}{10} = 2$

In standard form, the equations become

$$0.15v_1 - 0.1v_2 = 1$$
$$-0.1v_1 + 0.3v_2 = 2$$

Solving, we find $v_1 = 14.29\,\text{V}$ and $v_2 = 11.43\,\text{V}$.

Then we have $i_1 = \dfrac{v_1 - v_2}{10} = 0.2857\ A$.

P2.50   Writing KCL equations, we have

$$\frac{v_1}{10} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{22} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} = 4$$

$$\frac{v_3}{4} + \frac{v_3 - v_1}{22} + = -4$$

In standard form, we have:

$$0.3455v_1 - 0.2v_2 - 0.04545v_3 = 0$$
$$-0.2v_1 + 0.3v_2 = 4$$
$$-0.04545v_1 + 0.2955v_3 = -4$$

The MATLAB commands needed to solve are:

G = [0.3455 -0.2 -0.04545; -0.2 0.3 0; -0.04545 0 0.2955]

I = [0; 4; -4]

V = G\I

From this, we find $v_1 = 10.0\,\text{V}$, $v_2 = 20.0\,\text{V}$, and $v_3 = -12.0\ \text{V}$.

If the source is reversed, the algebraic signs are reversed in the **I** matrix and consequently, the node voltages are reversed in sign.

P2.51 Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_4} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_1} = 0$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_3} = I_s$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

In standard form, we have:

$$0.6167v_1 - 0.20v_2 - 0.25v_3 = 0$$
$$-0.20v_1 + 0.325v_2 - 0.125v_3 = 4$$
$$-0.25v_1 - 0.125v_2 + 0.50v_3 = 0$$

Using Matlab, we have

G = [0.6167 -0.20 -0.25; -0.20 0.325 -0.125; -0.25 -0.125 0.500];
I = [0; 4; 0];
V = G\I
V =
   13.9016
   26.0398
   13.4608

P2.52 Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_3} + \frac{v_1 - v_2}{R_4} + I_s = 0$$

$$\frac{v_2 - v_1}{R_4} + \frac{v_2 - v_3}{R_6} + \frac{v_2}{R_5} = 0$$

$$\frac{v_3}{R_1 + R_2} + \frac{v_3 - v_2}{R_6} = I_s$$

In standard form, we have:

$$0.15v_1 - 0.10v_2 = -2$$
$$-0.10v_1 + 0.475v_2 - 0.25v_3 = 0$$
$$-0.25v_2 + 0.3v_3 = 2$$

Solving using Matlab, we have

G = [0.15 -0.10 0; -0.10 0.475 -0.25; 0 -0.25 0.3]
I = [-2; 0; 2]
V = G\I

$$v_1 = -12.22 \ \text{V} \qquad v_2 = 1.667 \ \text{V} \qquad v_3 = 8.055 \ \text{V}$$

P2.53* Writing a KVL equation, we have $v_1 - v_2 = 10$.

At the reference node, we write a KCL equation: $\dfrac{v_1}{5} + \dfrac{v_2}{10} = 1$.

Solving, we find $v_1 = 6.667$ and $v_2 = -3.333$.

Then, writing KCL at node 1, we have $i_s = \dfrac{v_2 - v_1}{5} - \dfrac{v_1}{5} = -3.333 \text{ A}$.

P2.54 To minimize the number of unknowns, we select the reference node at one end of the voltage source. Then, we define the node voltages and write a KCL equation at each node.
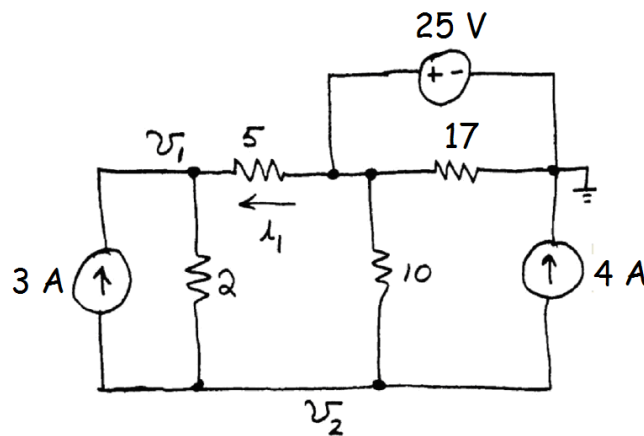
$$\frac{v_1 - 25}{5} + \frac{v_1 - v_2}{2} = 3 \qquad\qquad \frac{v_2 - v_1}{2} + \frac{v_2 - 25}{10} = -7$$

In Matlab, we have

```
G = [0.7 -0.5; -0.5 0.6]
I = [8; -4.5]
V = G\I
I1 = (25 - V(1))/5
```

Then, we have $i_1 = 2 \text{ A}$.

The 17-Ω resistance does not appear in the network equations and has no effect on the answer. The voltage at the top end of the 10-Ω resistance is 25 V regardless of the value of the 17-Ω resistance. Thus, any nonzero value could be substituted for the 17-Ω resistance without affecting the answer.

P2.55 We must not use all of the nodes (including those that are inside supernodes) in writing KCL equations. Otherwise, dependent equations result.

P2.56 The circuit with a 1-A source connected is:
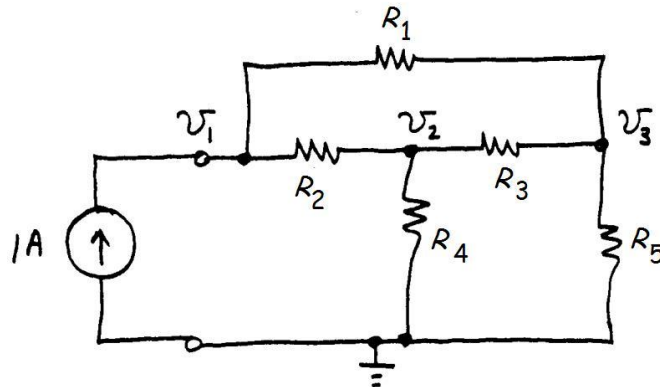
$$\frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_1} = 1 \qquad \frac{V_2}{R_4} + \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_3}{R_3} = 0$$

$$\frac{V_3}{R_5} + \frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_3} = 0$$

In Matlab, we use the commands

```
[V1,V2,V3] = solve('(V1 - V2)/R2 + (V1 - V3)/R1 = 1' , ...
          ' V2/R4 + (V2 - V1)/R2 + (V2 - V3)/R3 = 0' , ...
          ' V3/R5 + (V3 - V1)/R1 + (V3 - V2)/R3 = 0');
 pretty(V1)
```

After some clean up, this produces

$$R_{eq} = \frac{R_4 R_5 R_2 + R_2 R_3 R_5 + R_2 R_3 R_1 + R_5 R_2 R_1 + R_4 R_2 R_1 + R_4 R_3 R_1 + R_4 R_1 R_5 + R_4 R_3 R_5}{R_5 R_3 + R_5 R_1 + R_2 R_3 + R_4 R_3 + R_4 R_1 + R_1 R_3 + R_5 R_2 + R_4 R_2}$$

Then, the command
subs(V1, {'R1','R2','R3','R4','R5'}, {15,15,15,10,10})
yields $R_{eq} = 12.5 \ \Omega$.

P2.57* First, we can write: $i_x = \dfrac{v_1 - v_2}{5}$.

Then, writing KCL equations at nodes 1 and 2, we have:

$$\dfrac{v_1}{10} + i_x = 1 \quad \text{and} \quad \dfrac{v_2}{20} + 0.5i_x - i_x = 0$$

Substituting for i_x and simplifying, we have

$$0.3v_1 - 0.2v_2 = 1$$
$$-0.1v_1 + 0.15v_2 = 0$$

Solving, we have $v_1 = 6$ and $v_2 = 4$.

Then, we have $i_x = \dfrac{v_1 - v_2}{5} = 0.4 \text{ A}$.

P2.58* $$v_x = v_2 - v_1$$

Writing KCL at nodes 1 and 2:

$$\dfrac{v_1}{5} + \dfrac{v_1 - 2v_x}{15} + \dfrac{v_1 - v_2}{10} = 1$$

$$\dfrac{v_2}{5} + \dfrac{v_2 - 2v_x}{10} + \dfrac{v_2 - v_1}{10} = 2$$

Substituting and simplifying, we have

$$15v_1 - 7v_2 = 30 \quad \text{and} \quad v_1 + 2v_2 = 20 \, .$$

Solving, we find $v_1 = 5.405$ and $v_2 = 7.297$.

P2.59 First, we can write:

$$i_x = \dfrac{5i_x - v_2}{10}$$

Simplifying, we find $i_x = -0.2v_2$.

Then write KCL at nodes 1 and 2:

$$\dfrac{v_1 - 5i_x}{5} = 4 + 2 \qquad \dfrac{v_2}{10} - i_x = -6$$

Substituting for i_x and simplifying, we have

$$v_1 + v_2 = 30 \quad \text{and} \quad 0.3v_2 = -6$$

which yield $v_1 = 50 \text{ V}$ and $v_2 = -20 \text{ V}$.

P2.60   First, we can write $i_x = -\dfrac{v_1}{10}$. Then writing KVL, we have $v_1 - 5i_x - v_2 = 0$.

Writing KCL at the reference node, we have $\dfrac{v_2}{20} + \dfrac{v_2}{20} = i_x + 8$. Using the

first equation to substitute for i_x and simplifying, we have

$$1.5v_1 - v_2 = 0$$
$$v_1 + v_2 = 80$$

Solving, we find $v_1 = 32.0$ V, $v_2 = 48.0$ V, and $i_x = -\dfrac{v_1}{10} = -3.2$ A. Finally,

the power delivered to the 8-Ω resistance is $P = \dfrac{(v_1 - v_2)^2}{8} = 32.0$ W.

P2.61   The circuit with a 1-A current source connected is:
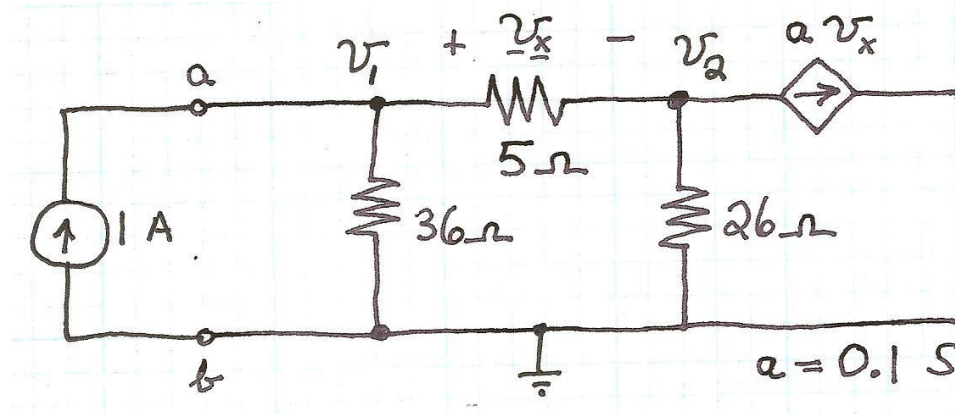
$$v_x = v_1 - v_2$$
$$\frac{v_1}{36} + \frac{v_1 - v_2}{5} = 1$$
$$\frac{v_2 - v_1}{5} + \frac{v_2}{26} + 0.1v_x = 0$$

Using the first equation to substitute for v_x and simplifying, we have

$$0.22778v_1 - 0.2v_2 = 1$$
$$-0.1v_1 + 0.13846v_2 = 0$$

Solving we find $v_1 = 12$. However, the equivalent resistance is equal in

value to v_1 so we have $R_{eq} = 12$ Ω.

P2.62   The circuit with a 1-A current source connected is
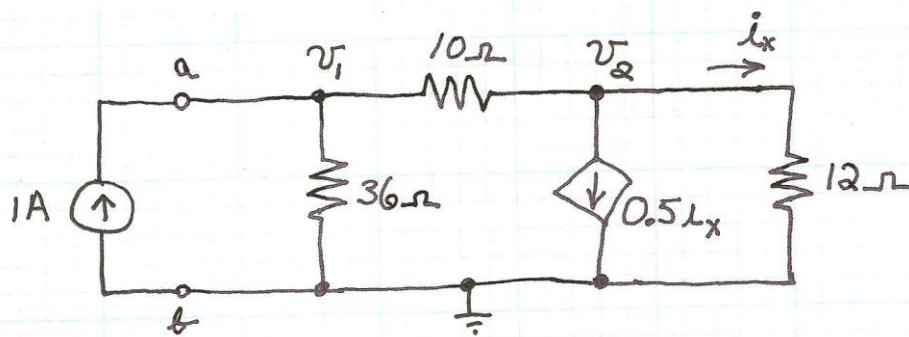
$$i_x = \frac{v_2}{12}$$

$$\frac{v_1}{36} + \frac{v_1 - v_2}{10} = 1$$

$$\frac{v_2 - v_1}{10} + \frac{v_2}{12} + 0.5i_x = 0$$

Using the first equation to substitute for i_x and simplifying, we have

$$0.12778v_1 - 0.1v_2 = 1$$
$$-0.1v_1 + 0.225v_2 = 0$$

Solving we find $v_1 = 12$ V. However, the equivalent resistance is equal in value to v_1, so we have R_{eq} = 12 Ω.

P2.63   Elements on the diagonal of **G** equal the sum of the conductances connected to any node, which is 3 S. Element g_{jk} off the diagonal is zero if no resistance is connected between nodes j and k and equal to –1 if there is a resistance connected between the nodes. **G** is the same for all three parts of the problem, only the node to which the current source is attached changes. (We used the MATLAB Array Editor to enter the elements of G.)

```
>> G
G =
   3   -1   -1    0    0    0    0
  -1    3    0   -1    0    0   -1
  -1    0    3   -1   -1    0    0
   0   -1   -1    3    0   -1    0
   0    0   -1    0    3   -1    0
   0    0    0   -1   -1    3   -1
   0   -1    0    0    0   -1    3
>> Ia = [1; 0; 0; 0; 0; 0; 0];
```

≫ Ib = [0; 1; 0; 0; 0; 0; 0];
≫ Ic = [0; 0; 0; 1; 0; 0; 0];
≫ Va = G\Ia;
≫ Vb = G\Ib;
≫ Vc = G\Ic;
≫ % Here are the answers:
≫ Ra = Va(1)
Ra =
    0.5833
≫ Rb = Vb(2)
Rb =
    0.7500
≫ Rc = Vc(4)
Rc =
    0.8333

By symmetry, shorting nodes with equal node voltages, and series parallel combination, we can obtain $R_a = 7/12$ Ω, $R_b = 3/4$ Ω, and $R_c = 5/6$ Ω.

P2.64   First, we enter the node voltage equations into the solve command and define the unknowns to be V1, V2 and Vout. Then, we use the pretty command to print the answer for Vout:

SV = solve('(V1 - Vin)/(2*R1) + (V1 - Vout)/R1 + (V1 - V2)/R1 = 0' , ...
       '(V2 - V1)/R1 + V2/R1 + (V2 - Vout)/R1 = 0', ...
       '(Vout - V1)/R1 + (Vout - V2)/R1 + Vout/R2 = 0' , 'V1','V2','Vout');
pretty(SV.Vout)

The result is

```
        4 R2 Vin
      -------------
       13 R1 + 11 R2
```
Thus, we have
$$\frac{V_{out}}{V_{in}} = \frac{4R_2}{13R_1 + 11R_2}$$

P2.65   We write equations in which voltages are in volts, resistances are in kΩ, and currents are in mA.

KCL node 2: $\dfrac{(v_2 - v_1)}{4} + \dfrac{(v_2 - v_3)}{3} + \dfrac{v_2}{2} = 0$

KCL node 3: $\dfrac{(v_3 - v_2)}{3} + \dfrac{(v_3 - v_1)}{1} + \dfrac{(v_3 - v_4)}{2} = 5$

KCL ref node: $\dfrac{v_2}{2} + \dfrac{v_4}{5} = 5$

KVL: $v_1 - v_4 = 20$

Then, using Matlab we have:
G = [-1/4 (1/2 + 1/3 + 1/4) -1/3 0; ...
-1 -1/3 (1 + 1/2 + 1/3) -1/2; 0 1/2 0 1/5; 1 0 0 -1];
I = [0; 5; 5; 20];
V = G\I

V =
          20.7317
          9.7073
          16.0000
          0.7317

P2.66.   1. If necessary, redraw the network without crossing conductors or
elements. Then, define the mesh currents flowing around each of the
open areas defined by the network. For consistency, we usually select a
clockwise direction for each of the mesh currents, but this is not a
requirement.
2. Write network equations, stopping after the number of equations is
equal to the number of mesh currents. First, use KVL to write voltage
equations for meshes that do not contain current sources. Next, if any
current sources are present, write expressions for their currents in
terms of the mesh currents. Finally, if a current source is common to two
meshes, write a KVL equation for the supermesh.
3. If the circuit contains dependent sources, find expressions for the
controlling variables in terms of the mesh currents. Substitute into the
network equations, and obtain equations having only the mesh currents as
unknowns.
4. Put the equations into standard form. Solve for the mesh currents by
use of determinants or other means.

5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

P2.67* Writing KVL equations around each mesh, we have

$$5i_1 + 15(i_1 - i_2) = 20 \quad \text{and} \quad 15(i_2 - i_1) + 10i_2 = 10$$

Putting the equations into standard from we have

$$20i_1 - 15i_2 = 20 \quad \text{and} \quad -15i_1 + 25i_2 = 10$$

Solving, we obtain $i_1 = 2.364$ A and $i_2 = 1.818$ A.
Then, the power delivered to the 15-Ω resistor is $P = (i_1 - i_2)^2 15 = 4.471$ W.

P2.68* Writing and simplifying the mesh-current equations, we have:
$$28i_1 - 10i_2 = 12$$
$$-10i_1 + 40i_2 - 30i_3 = 0$$
$$-30i_2 + 60i_3 = 0$$
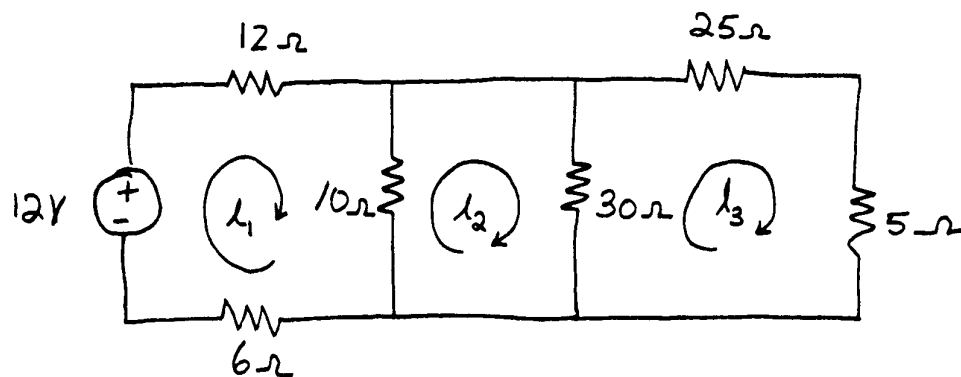
Solving, we obtain

$$i_1 = 0.500 \qquad i_2 = 0.200 \qquad i_3 = 0.100$$

Thus, $v_2 = 5i_3 = 0.500$ V and the power delivered by the source is $P = 12i_1 = 6$ W.

P2.69* Because of the current sources, two of the mesh currents are known.
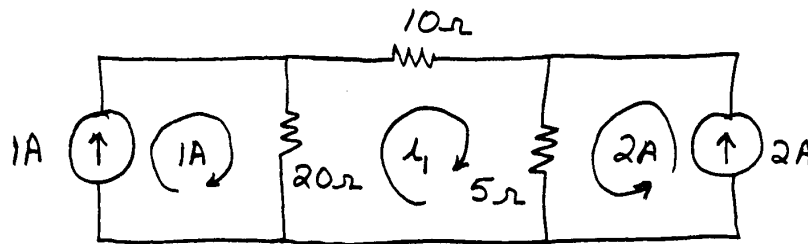
Writing a KVL equation around the middle loop we have

$$20(i_1 - 1) + 10i_1 + 5(i_1 + 2) = 0$$

Solving, we find $i_1 = 0.2857$ A.

P2.70 Writing KVL equations around each mesh, we have

$$5i_1 + 7(i_1 - i_3) - 62 = 0$$
$$11(i_2 - i_3) + 3i_2 + 62 = 0$$
$$i_3 + 11(i_3 - i_2) + 7(i_3 - i_1) = 0$$

Putting the equations into standard from, we have

$$12i_1 - 7i_3 = 62$$
$$14i_2 - 11i_3 = -62$$
$$-7i_1 - 11i_2 + 19i_3 = 0$$

Using Matlab to solve, we have
R = [12 0 -7; 0 14 -11; -7 -11 19];
V = [62; -62; 0];
 I = R\V
I =

   4.0000
  -6.0000
  -2.0000

Then, the power delivered by the source is $P = 62(i_1 - i_2) = 620$ W.

P2.71 Writing and simplifying the mesh equations, we obtain:

$$40i_1 - 20i_2 = 10 \qquad\qquad -20i_1 + 40i_2 = 0$$
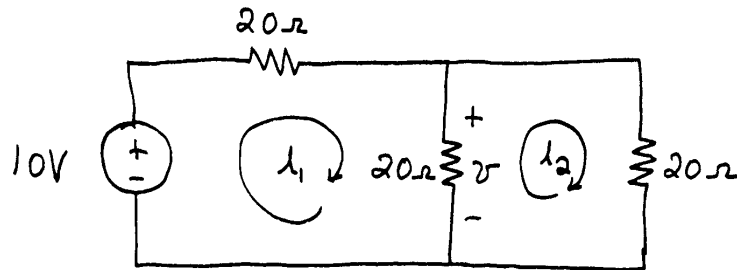
Solving, we find $i_1 = 0.3333$ and $i_2 = 0.1667$.

Thus, $v = 20(i_1 - i_2) = 3.333 \text{ V}$.

P2.72 The mesh currents and corresponding equations are:
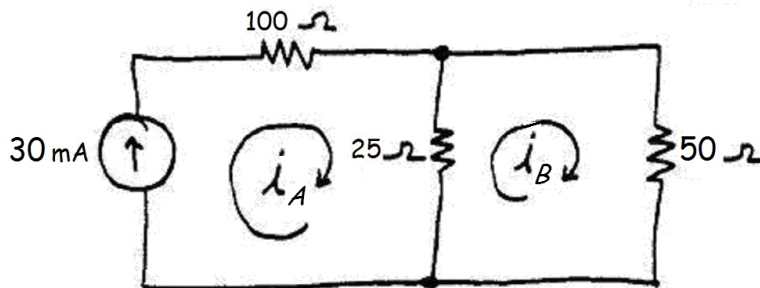
$$i_A = 30 \text{ mA} \qquad 25(i_B - i_A) + 50i_B = 0$$

Solving, we find $i_B = 10 \text{ mA}$.

However, i_3 shown in Figure P2.39 is the same as i_B, so the answer is $i_3 = 10 \text{ mA}$.

P2.73 First, we select the mesh currents and then write three equations.
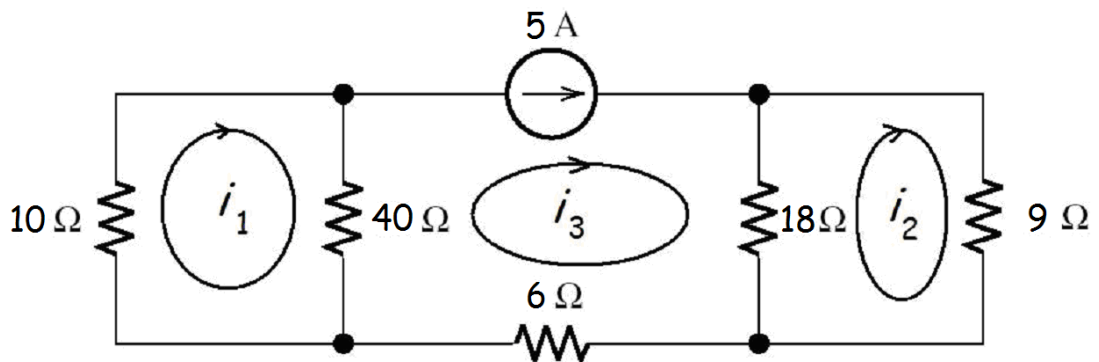
Mesh 1: $\quad 10i_1 + 40(i_1 - i_3) = 0$

Mesh 2: $9i_2 + 18(i_2 - i_3) = 0$

However by inspection, we have $i_3 = 5$. Solving, we obtain

$i_1 = 4$ A and $i_2 = 3.333$ A.

P2.74   We assume that i_1 is a mesh current flowing around the left-hand mesh and that i_2 flows around the right-hand mesh. Writing and simplifying the mesh equations yields:

$$14i_1 - 8i_2 = 10$$
$$-8i_1 + 16i_2 = 0$$

Solving, we find $i_1 = 1.000$ and $i_2 = 0.500$.

Finally, the power delivered by the source is $P = 10i_1 = 10$ W.

P2.75   The mesh (KVL) equations are:

$$7i_A + 2i_A + 18(i_A - i_B) = 0$$
$$18(i_B - i_A) + 4i_B = -30$$

Solving we find $i_A = -2$ A and $i_B = -3$ A. Then, we have $i_1 = -i_B = 3$ A and

$i_2 = i_A - i_B = 1.0$ A.

P2.76   By inspection: $i_A = 3$ A.

Mesh B: $25i_B + 5i_B + 20(i_B - i_A) = 0$

Solving, we find $i_B = 1.2$ A. Then, we have $i_1 = i_A - i_B = 1.8$ A and

$i_2 = i_B = 1.2$ A.

P2.77   (a) First, we select mesh-current variables as shown.
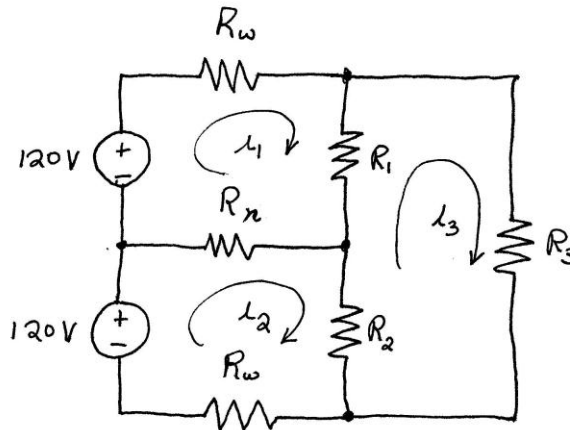
Then, we can write

$$(R_w + R_n + R_1)i_1 - R_n i_2 - R_1 i_3 = 120$$

$$-R_n i_1 + (R_w + R_n + R_2)i_2 - R_2 i_3 = 120$$
$$-R_1 i_1 - R_2 i_2 + (R_1 + R_2 + R_3)i_3 = 0$$

Alternatively, because the network consists of independent voltage sources and resistances, and all of the mesh currents flow clockwise, we can enter the matrices directly into MATLAB.

Rw = 0.1; Rn=0.1; R1 = 20; R2 = 10; R3 = 16;
R = [Rw+Rn+R1 -Rn -R1; -Rn Rw+Rn+R2 -R2; -R1 -R2 R1+R2+R3];
V = [120; 120; 0];
I = R\V;
% Finally, we compute the voltages across the loads.
Vr1 = R1*(I(1) - I(3)), Vr2 = R2*(I(2) - I(3)), Vr3 = R3*I(3)...
% which results in:
Vr1 =
  118.5121
Vr2 =
  116.7862
Vr3 =
  235.2983

These values are within the normal range for nearly all devices.

(b) Next, we change Rn to a very high value such as 10^9 which for practical calculations is equivalent to an open circuit, and again compute the voltages resulting in:

Vr1 =
  156.9910
Vr2 =
   78.4955
Vr3 =
  235.4865

The voltage across R_1 is certainly high enough to damage most loads designed to operate at 110 to 120 V.

P2.78
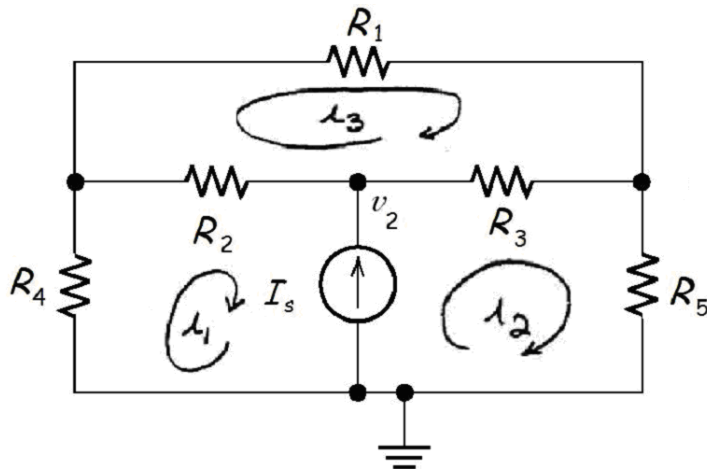
Current source in terms of mesh currents: $-i_1 + i_2 = I_s$

KVL for mesh 3: $-R_2 i_1 - R_3 i_2 + (R_1 + R_2 + R_3)i_3 = 0$

KVL around outside of network: $R_4 i_1 + R_5 i_2 + R_1 i_3 = 0$

Then using MATLAB:

```
R1 = 4; R2 = 5; R3 = 8; R4 = 6; R5 = 8; Is = 4;
R = [-1 1 0; -R2 -R3 (R1+R2+R3); R4 R5 R1];
V = [Is; 0; 0];
I = R\V;
V2 = R5*I(2) + R3*(I(2) - I(3))...
% and the answer is:
V2 =
    26.0414
```

P2.79
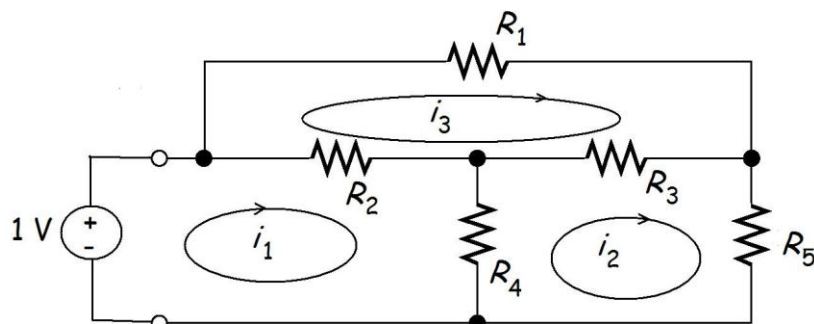
$(R_2 + R_4)i_1 - R_4 i_2 - R_2 i_3 = 1$

$-R_4 i_1 + (R_3 + R_4 + R_5)i_2 - R_3 i_3 = 0$

$-R_2 i_1 - R_3 i_2 + (R_1 + R_2 + R_3)i_3 = 0$

Now using MATLAB:

```
R1 = 15; R2 = 15; R3 = 15; R4 = 10; R5 = 10;
R = [(R2+R4) -R4 -R2; -R4 (R3+R4+R5) -R3; -R2 -R3 (R1+R2+R3)];
V = [1; 0; 0];
I = R\V;
Req = 1/I(1) % Gives answer in ohms.
Req =
   12.5
```

P2.80   Mesh 1: $3i_1 + 7i_1 + 30(i_1 - i_2) = 1$

Mesh 2: $3i_2 + 12(i_2 - i_3) + 4i_2 + 30(i_2 - i_1) = 0$

Mesh 3: $24i_3 + 12(i_3 - i_2) = 0$
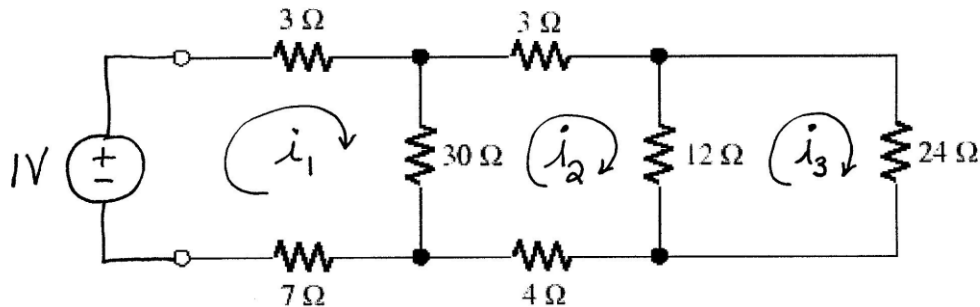
Solving, we find $i_1 = 0.05$ A. Then $R_{eq} = 1/i_1 = 20$ Ω.

P2.81   We write equations in which voltages are in volts, resistances are in kΩ, and currents are in mA.

KVL mesh 1: $4i_1 + 1(i_1 - i_2) + 3(i_1 - i_3) = 0$

KVL mesh 2: $1(i_2 - i_1) + 2(i_2 - i_4) = -20$

KVL supermesh: $2i_3 + 3(i_3 - i_1) + 2(i_4 - i_2) + 5i_4 = 0$

Current source: $i_4 - i_3 = 5$

Now, we proceed in Matlab.

```
R = [8 -1 -3 0; -1 3 0 -2; -3 -2 5 7; 0 0 -1 1];
V = [0; -20; 0; 5];
I = R\V % This yields the mesh currents in mA.
I =

  -2.7561
  -7.4878
  -4.8537
   0.1463
```

P2.82.   1. Perform two of these:

        a. Determine the open-circuit voltage $V_t = v_{oc}$.

        b. Determine the short-circuit current $I_n = i_{sc}$.

        c. Zero the independent sources and find the Thévenin resistance R_t looking back into the terminals. Do not zero dependent sources.

2. Use the equation $V_t = R_t I_n$ to compute the remaining value.

3. The Thévenin equivalent consists of a voltage source V_t in series with R_t.

4. The Norton equivalent consists of a current source I_n in parallel with R_t.

P2.83* First, we write a node voltage equation to solve for the open-circuit voltage:
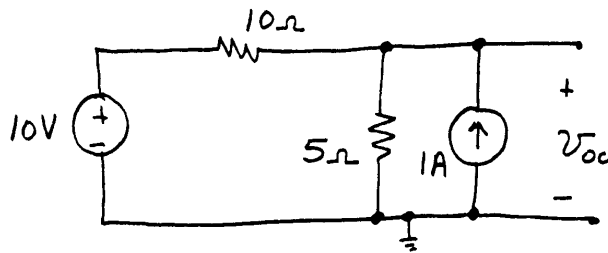
$$\frac{V_{oc} - 10}{10} + \frac{V_{oc}}{5} = 1$$

Solving, we find $v_{oc} = 6.667 \text{ V}$.

Then zeroing the sources, we have this circuit:
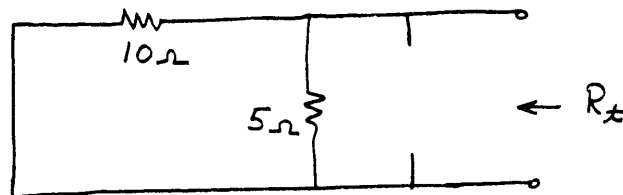
Thus, $R_t = \dfrac{1}{1/10 + 1/5} = 3.333 \, \Omega$. The Thévenin and Norton equivalents are:

$3.333\,\Omega$

$6.667V$

$2A$ $\uparrow$

$3.333\,\Omega$
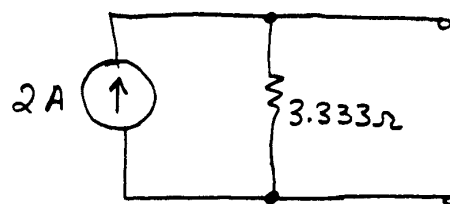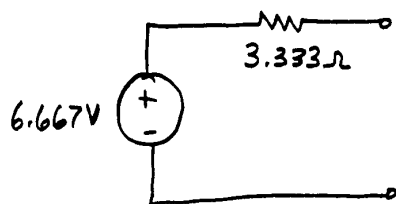
P2.84* The equivalent circuit of the battery with the resistance connected is

$9V$ R_t $\vec{i}$ $6V$ $100\,\Omega$
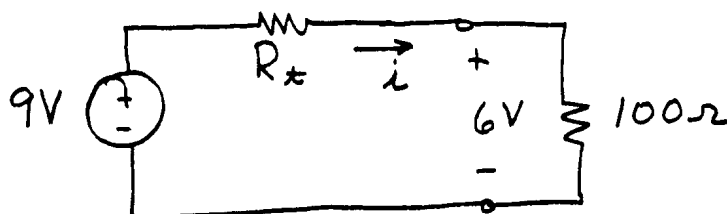
$$i = 6/100 = 0.06 \ A \qquad\qquad R_t = \frac{9-6}{0.06} = 50\ \Omega$$

P2.85 The 9-Ω resistor has no effect on the equivalent circuits because the voltage across the 12-V source is independent of the resistor value.

$R_t = 5\ \Omega$

a

$V_t = 15\ V$

$o\ b$

$I_n = 3\ A$

$R_t = 5\ \Omega$

a

$o\ b$
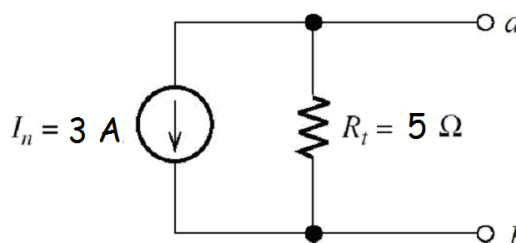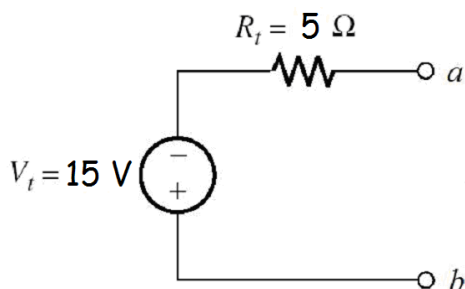
P2.86 With open-circuit conditions:

$9\,\Omega$

$36\,\Omega$

$3\ A$

$9\,\Omega$

$o\ a$

$o\ b$
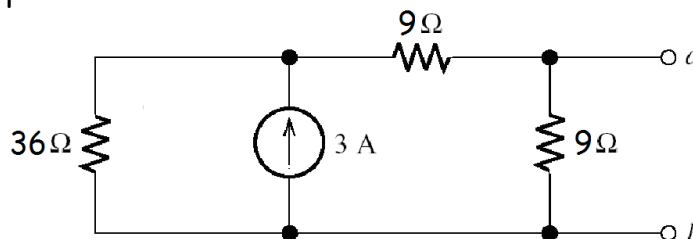
Solving, we find $v_{ab} = 18\ V$.

With the source zeroed:

$9\,\Omega$

$36\,\Omega$

$9\,\Omega$

a

b
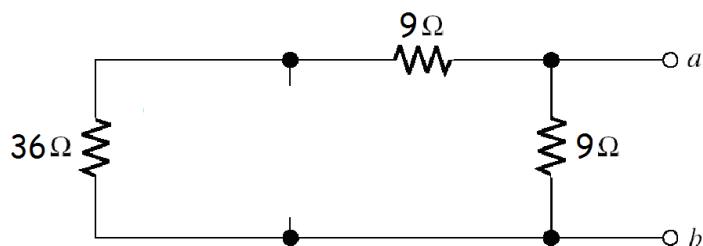
$$R_t = \dfrac{1}{1/9 + 1/(9+36)} = 7.5\,\Omega$$

The equivalent circuits are:

$R_t = 7.5\,\Omega$

$V_t = 18\,\text{V}$

a

b

$I_n = 2.4\,\text{A}$

$R_t = 7.5\,\Omega$

a

b
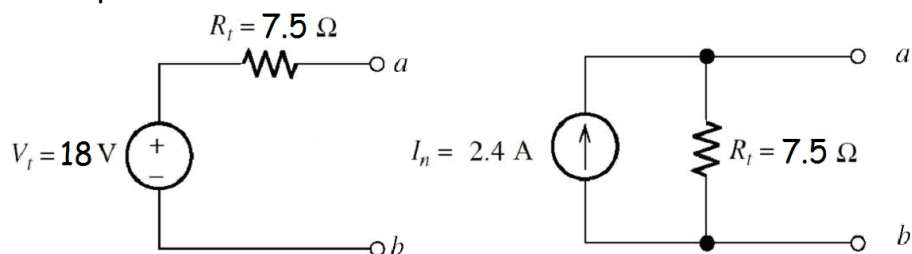
Notice the source polarity relative to terminals a and b.

P2.87   First, we combine the 30-Ω resistances that are in parallel replacing them with a 15-Ω resistance. Then, we solve the network with a short circuit:

$10\,\Omega$ $15\,\Omega$

i_{10} i_{15}

$64\,\text{V}$ $10\,\Omega$ $30\,\Omega$
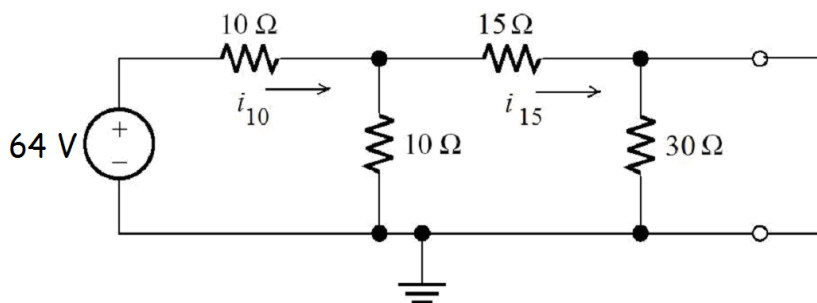
$$R_{eq} = 10 + \dfrac{1}{1/10 + 1/15} = 16\,\Omega$$

$$i_{10} = 64/R_{eq} = 4\,\text{A}$$

$$i_{15} = i_{10}\,\dfrac{10}{10+15} = 1.6\,\text{A}$$

$$i_{sc} = i_{15} = 1.6\,\text{A}$$

Zeroing the source, we have:
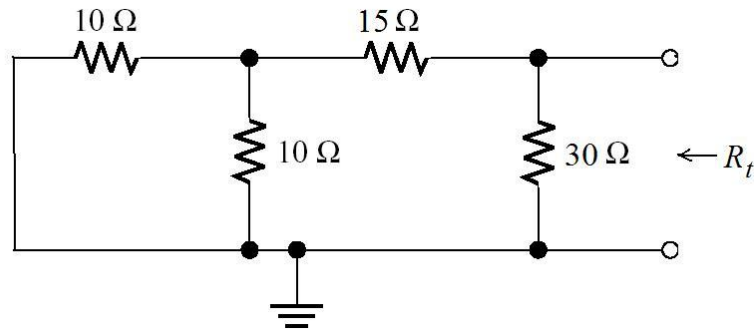
Combining resistances in series and parallel we find $R_t = 12\,\Omega$.

Then the Thévenin voltage is $v_t = i_{sc}R_t = 19.2\,\text{V}$. The Thévenin and Norton equivalents are:
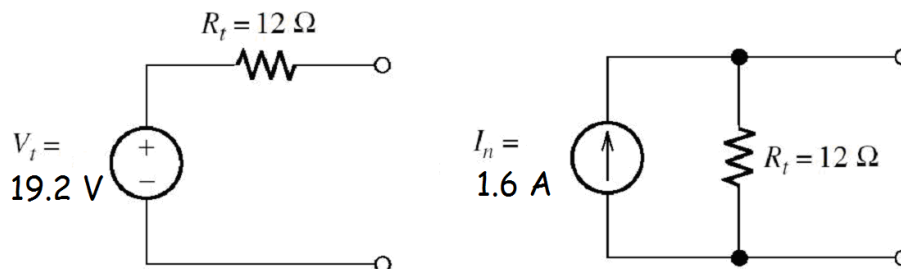
P2.88   The Thévenin voltage is equal to the open-circuit voltage which is 12.5 V. The equivalent circuit with the 0.1-Ω load connected is:
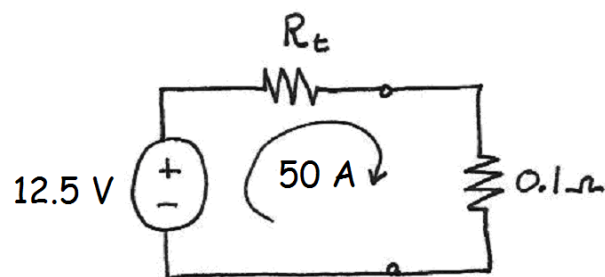
We have $12.5/(R_t + 0.1) = 500$ from which we find $R_t = 0.15\,\Omega$. The Thévenin and Norton equivalent circuits are:
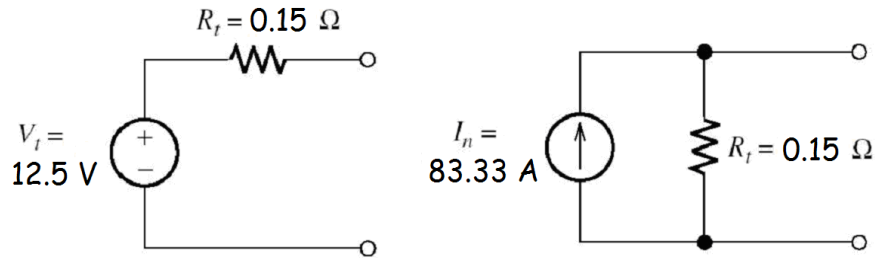
The short-circuit current is 83.33 A. Because no energy is converted from chemical form to heat in a battery under open-circuit conditions, the Thévenin equivalent seems more realistic from an energy conversion standpoint.

P2.89 The Thévenin voltage is equal to the open-circuit voltage, which is 9 V. The circuit with the load attached is:
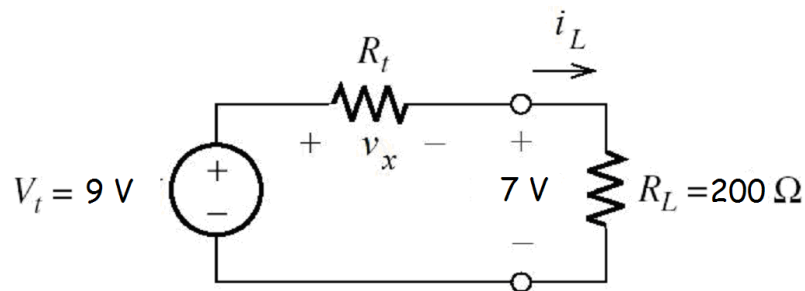
We have $i_L = \dfrac{7}{200} = 35 \text{ mA}$ and $v_x = V_t - 7 = 2 \text{ V}$. Thus, the Thévenin resistance is $R_t = \dfrac{2 \text{ V}}{35 \text{ mA}} = 57.14 \ \Omega$.

P2.90 The equivalent circuit with a load attached is:
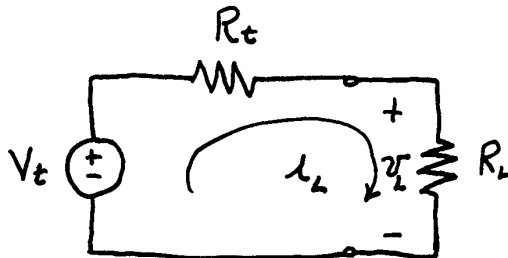
For a load of 1 kΩ, we have $i_L = 8/1000 = 8 \text{ mA}$, and we can write $v_L = V_t - R_t i_L$. Substituting values this becomes

$$8 = V_t - 0.008 R_t \quad (1)$$

Similarly, for the 2-kΩ load we obtain
$$10 = V_t - 0.005R_t \quad (2)$$
Solving Equations (1) and (2), we find $V_t = 13.33$ V and $R_t = 666.7\ \Omega$.

P2.91   Open-circuit conditions:
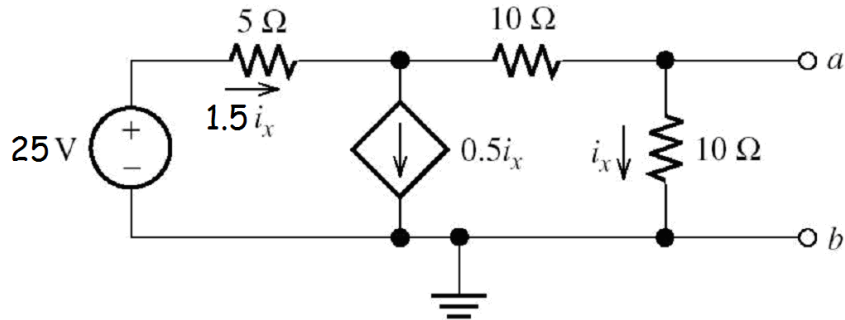
Using KVL, we have $25 = 5(1.5i_x) + 10i_x + 10i_x$ Solving, we find $i_x = 0.90909$ A and then we have $V_t = V_{oc} = 10i_x = 9.0909$ V.
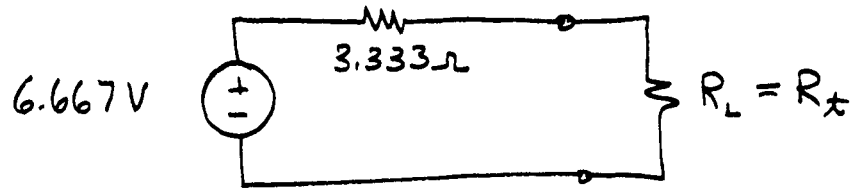
Under short-circuit conditions, we have $i_x = 0$ and the controlled source becomes an open circuit:
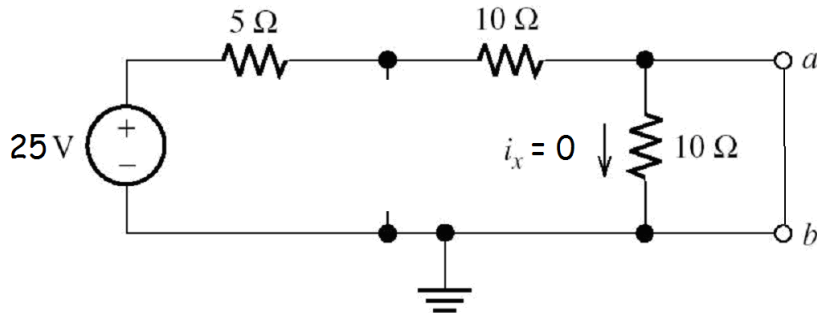
$i_{sc} = \dfrac{25}{15} = 1.667$ A. Then, we have $R_t = V_{oc}/i_{sc} = 5.45\ \Omega$. Thus, the equivalents are:
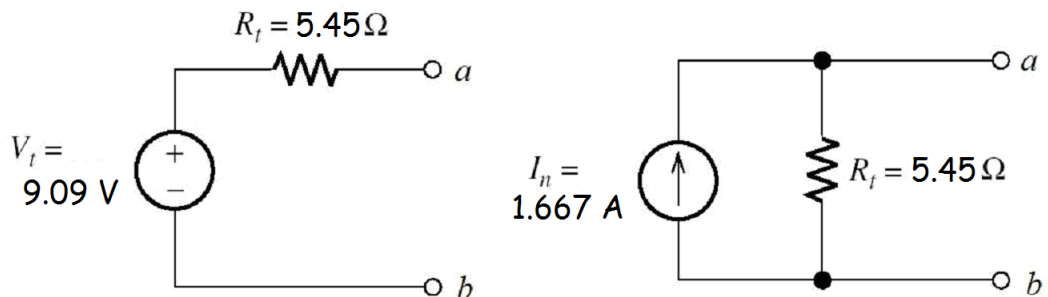
P2.92 As is Problem P2.83, we find the Thévenin equivalent:

Then maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{max} = \frac{(V_t/2)^2}{R_t} = 3.333 \text{ W}$$

P2.93 As in Problem P2.86, we find the Thévenin equivalent:
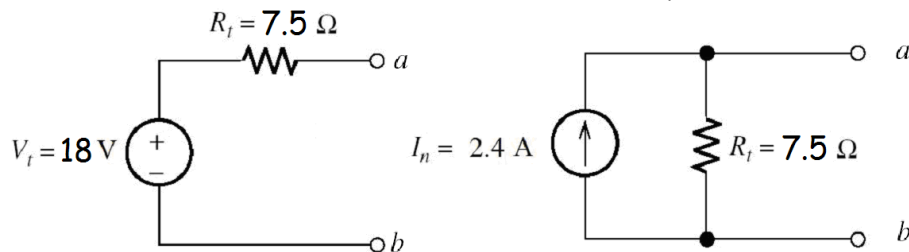
Then, maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{max} = \frac{(V_t/2)^2}{R_t} = 10.8 \text{ W}$$

P2.94 For maximum power conditions, we have $R_L = R_t$. The power taken from the voltage source is

$$P_s = \frac{(V_t)^2}{R_t + R_L} = \frac{(V_t)^2}{2R_t}$$

Then, half of V_t appears across the load and the power delivered to the load is

$$P_L = \frac{(0.5V_t)^2}{R_t}$$

Thus, the percentage of the power taken from the source that is delivered to the load is

$$\eta = \frac{P_L}{P_s} \times 100\% = 50\%$$

On the other hand, for $R_L = 9R_t$, we have

$$P_s = \frac{(V_t)^2}{R_t + R_L} = \frac{(V_t)^2}{10R_t}$$

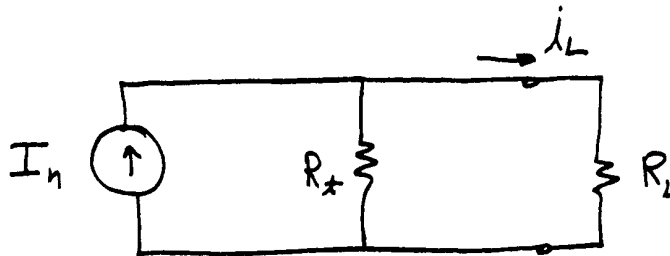$$P_L = \frac{(0.9V_t)^2}{9R_t}$$

$$\eta = \frac{P_L}{P_s} \times 100\% = 90\%$$

Design for maximum power transfer is relatively inefficient. Thus, systems in which power efficiency is important are almost never designed for maximum power transfer.

P2.95* To maximize the power to R_L, we must maximize the voltage across it. Thus, we need to have $R_t = 0$. The maximum power is

$$P_{max} = \frac{20^2}{5} = 80 \text{ W}$$

P2.96 The circuit is

By the current division principle:

$$i_L = I_n \frac{R_t}{R_L + R_t}$$

The power delivered to the load is

$$P_L = (i_L)^2 R_L = (I_n)^2 \frac{(R_t)^2 R_L}{(R_L + R_t)^2}$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dP_L}{dR_L} = 0 = (I_n)^2 \frac{(R_t)^2 (R_t + R_L)^2 - 2(R_t)^2 R_L (R_t + R_L)}{(R_t + R_L)^4}$$
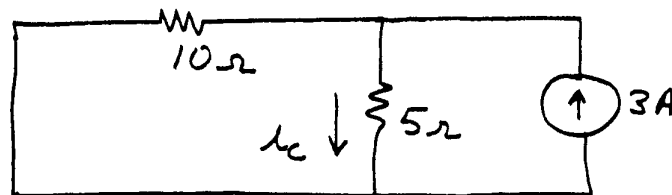
which yields $R_L = R_t$.

The maximum power is $P_{Lmax} = (I_n)^2 R_t / 4$.

P2.97* First, we zero the current source and find the current due to the voltage source.
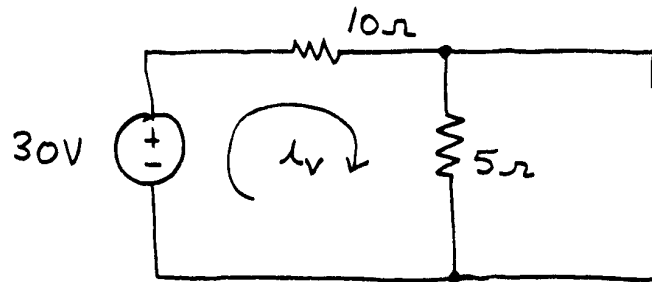
$i_v = 30/15 = 2$ A

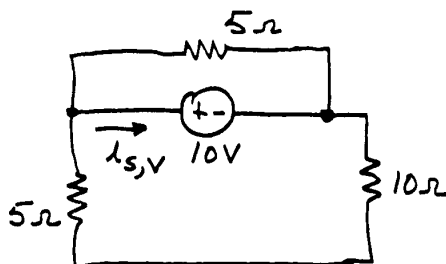Then, we zero the voltage source and use the current-division principle to find the current due to the current source.
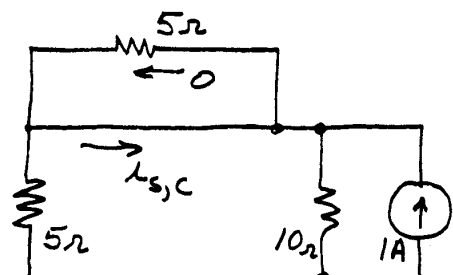

$$i_c = 3\frac{10}{5+10} = 2 \text{ A}$$

Finally, the total current is the sum of the contributions from each source.

$$i = i_v + i_c = 4 \text{ A}$$

P2.98* The circuits with only one source active at a time are:
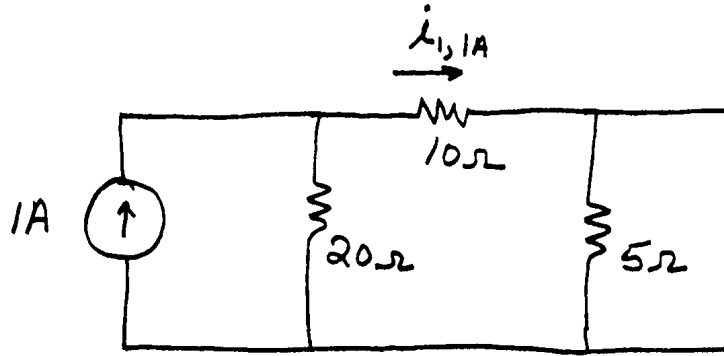

$$R_{eq} = \frac{1}{1/5 + 1/15} = 3.75 \ \Omega$$

$$i_{s,c} = -1\frac{10}{10+5} = -0.667 \text{ A}$$

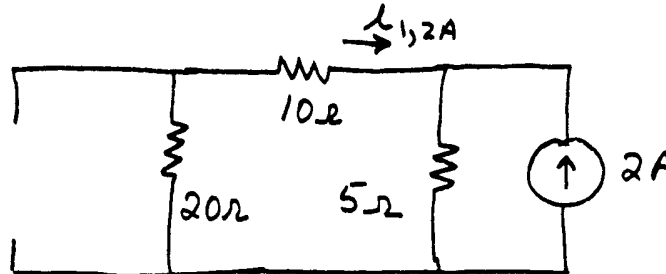$$i_{s,v} = -\frac{10 \text{ V}}{R_{eq}} = -2.667 \text{ A}$$

Then the total current due to both sources is $i_s = i_{s,v} + i_{s,c} = -3.333 \text{ A}$.

P2.99 Zero the 2-A source and use the current-division principle:

$$i_{1,1A} = 1\frac{20}{15+20} = 0.5714 \text{ A}$$

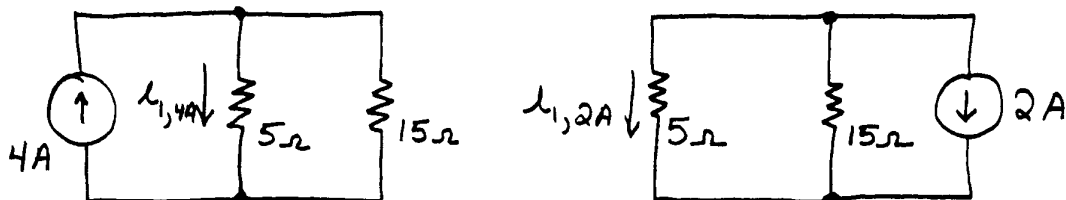Then zero the 1 A source and use the current-division principle:

$$i_{1,2A} = -2\frac{5}{5+30} = -0.2857 \text{ A}$$

Finally,
$$i_1 = i_{1,1A} + i_{1,2A} = 0.2857 \text{ A}$$

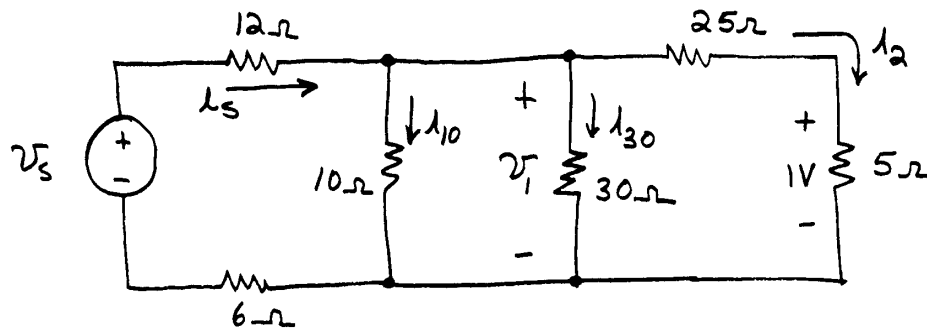**P2.100** The circuits with only one source active at a time are:

$$i_{1,4A} = 4 \times \frac{15}{15+5}$$
$$= 3\,A$$

$$i_{1,2A} = -2 \times \frac{15}{15+5}$$
$$= -1.5$$

Finally, we add the components to find the current with both sources active.

$$i_1 = i_{1,4A} + i_{1,2A} = 1.5\,A$$

P2.101 The circuit, assuming that $v_2 = 1\,V$ is:

$$i_2 = (v_2/5) = 0.2\,A$$
$$v_1 = 30i_2 = 6\,V$$
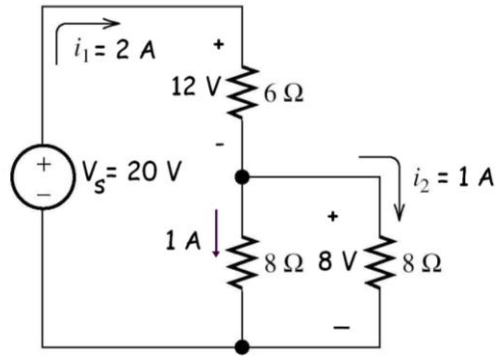$$i_{10} = v_1/10 = 0.6\,A$$
$$i_{30} = v_1/30 = 0.2\,A$$
$$i_s = i_2 + i_{10} + i_{30} = 1\,A$$
$$v_s = 12i_s + v_1 + 6i_s = 24\,V$$

We have established that for $v_s = 24\,V$, we have $v_2 = 1\,V$. Thus, for $v_s = 12\,V$, we have:

$$v_2 = 1 \times \frac{12}{24} = 0.5\,V$$

P2.102 We start by assuming $i_2 = 1$ A and work back through the circuit to determine the value of v_s. The results are shown on the circuit diagram.
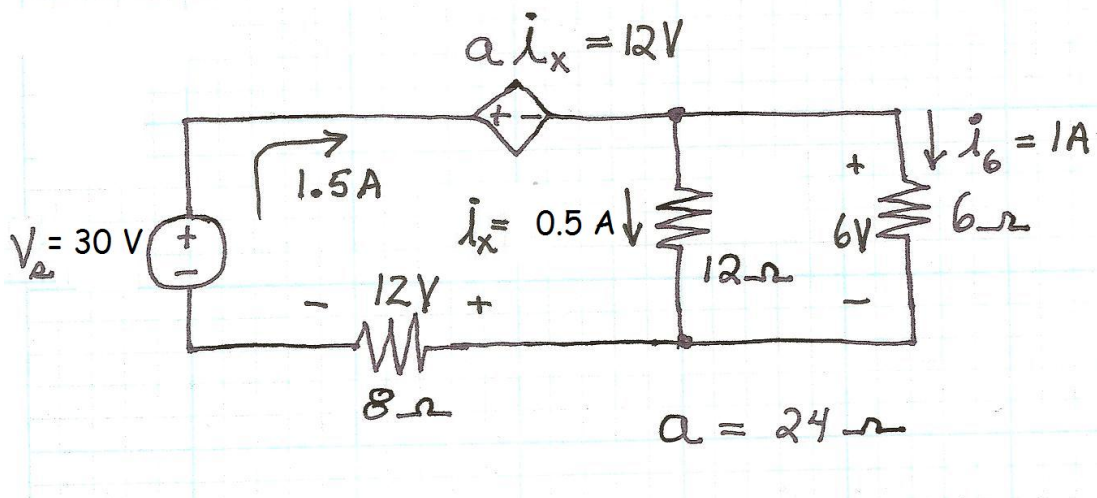
However, the circuit actually has $v_s = 10$ V, so the actual value of i_2 is

$$\frac{10}{20} \times (1 \text{ A}) = 0.5 \text{ A}.$$

P2.103 We start by assuming $i_6 = 1$ A and work back through the circuit to determine the value of V_s. This results in $V_s = 30$ V.

However, the circuit actually has $V_s = 10$ V, so the actual value of i_6 is

$$\frac{10}{30} \times (1 \text{ A}) = 0.3333 \text{ A}.$$

P2.104 (a) With only the 2-A source activated, we have
$$i_2 = 2 \text{ and } v_2 = 2(i_2)^3 = 16 \text{ V}$$
(b) With only the 1-A source activated, we have
$$i_1 = -1 \text{ A and } v_1 = 2(i_1)^3 = -2 \text{ V}$$
(c) With both sources activated, we have

$$i = 1 \text{ A and } v = 2(i)^3 = 2 \text{ V}$$

Notice that $i \neq i_1 + i_2$. Superposition does not apply because device A has a nonlinear relationship between v and i.
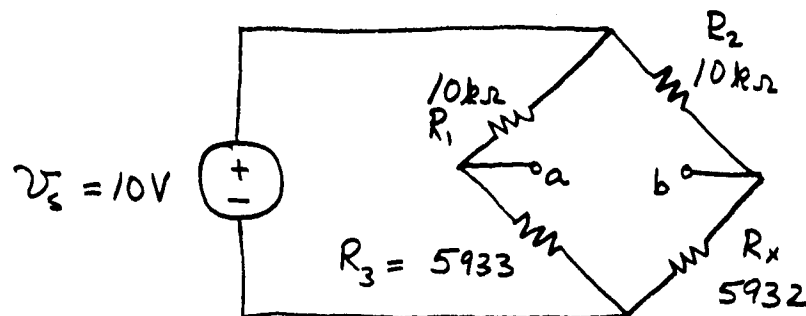
P2.105 From Equation 2.91, we have

(a) $R_x = \dfrac{R_2}{R_1} R_3 = \dfrac{1\,k\Omega}{1\,k\Omega} \times 3419 = 3419\ \Omega$

(b) $R_x = \dfrac{R_2}{R_1} R_3 = \dfrac{100\,k\Omega}{10\,k\Omega} \times 3419 = 34.19\ k\Omega$

P2.106* (a) Rearranging Equation 2.91, we have

$$R_3 = \dfrac{R_1}{R_2} R_x = \dfrac{10^4}{10^4} \times 5932 = 5932\ \Omega$$
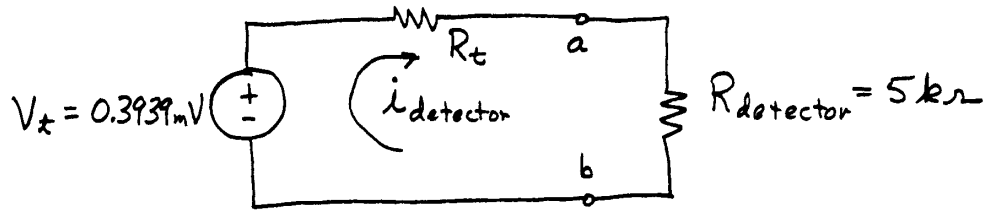
(b) The circuit is:

The Thévenin resistance is

$$R_t = \dfrac{1}{1/R_3 + 1/R_1} + \dfrac{1}{1/R_2 + 1/R_x} = 7447\ \Omega$$

The Thévenin voltage is

$$v_t = v_s \dfrac{R_3}{R_1 + R_3} - v_s \dfrac{R_x}{R_x + R_2}$$

$$= 0.3939 \text{ mV}$$

Thus, the equivalent circuit is:

$$i_{detector} = \frac{V_t}{R_t + R_{detector}} = 31.65 \times 10^{-9} \text{ A}$$

Thus, the detector must be sensitive to very small currents if the bridge is to be accurately balanced.

P2.107 If R_1 and R_3 are too small, large currents are drawn from the source. If the source were a battery, it would need to be replaced frequently. Large power dissipation could occur, leading to heating of the components and inaccuracy due to changes in resistance values with temperature.

If R_1 and R_3 are too large, we would have very small detector current when the bridge is not balanced, and it would be difficult to balance the bridge accurately.

P2.108 With the source replaced by a short circuit and the detector removed, the Wheatstone bridge circuit becomes
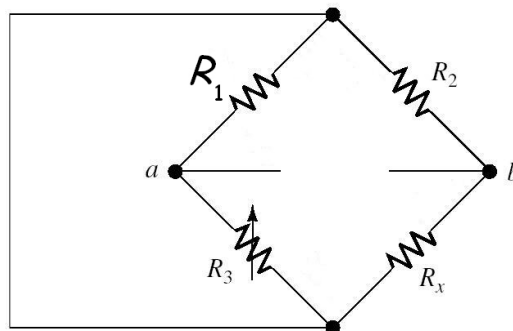
The Thévenin resistance seen looking back into the detector terminals is

$$R_t = \frac{1}{1/R_3 + 1/R_1} + \frac{1}{1/R_2 + 1/R_x}$$

The Thévenin voltage is zero when the bridge is balanced.

P2.109 Using the voltage-division principle, the voltage at node *a* is

$$v_a = V_s \frac{R_0 + \Delta R}{R_0 + \Delta R + R_0 - \Delta R} = V_s \frac{R_0 + \Delta R}{2R_0}$$

Similarly at node b, we have

$$v_b = V_s \frac{R_0 - \Delta R}{2R_0}$$

Then, the output voltage is

$$v_o = v_{ab} = v_a - v_b = V_s \frac{\Delta R}{R_0}$$

Finally using Equation 2.92 to substitute for ΔR, we have

$$v_o = V_s G \frac{\Delta L}{L}$$

P2.110   Before strain is applied, the resistance is

$$R_0 = \frac{\rho L}{A}$$

After strain is applied, the length becomes $L + \Delta L = L(1 + \Delta L / L)$, and the cross sectional area becomes $A /((1 + \Delta L / L)$ so the volume is constant. Thus, the resistance becomes

$$R_0 + \Delta R = R_0(1 + \frac{\Delta R}{R_0}) = \frac{\rho L(1 + \Delta L / L)}{A /(1 + \Delta L / L)} = R_0(1 + \Delta L / L)^2$$

$$R_0(1 + \frac{\Delta R}{R_0}) = R_0(1 + 2\Delta L / L + (\Delta L / L)^2)$$

However, we have $\Delta L / L \ll 1$ so we can neglect the $(\Delta L / L)^2$ term to a good approximation. This results in

$$R_0(1 + \frac{\Delta R}{R_0}) \cong R_0(1 + 2\Delta L / L)$$

$$G = \frac{\Delta R / R_0}{\Delta L / L} \cong 2$$

P2.111   In this case, the bridge would be balanced for any value of ΔR and the output voltage v_o would be zero regardless of the strain.

Practice Test

T2.1   (a) 6, (b) 10, (c) 2, (d) 7, (e) 10 or 13 (perhaps 13 is the better answer), (f) 1 or 4 (perhaps 4 is the better answer), (g) 11, (h) 3, (i) 8, (j) 15, (k) 17, (l) 14.

T2.2 The equivalent resistance seen by the voltage source is:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 16\ \Omega$$

$$i_s = \frac{V_s}{R_{eq}} = 6\ A$$

Then, using the current division principle, we have

$$i_4 = \frac{G_4}{G_2 + G_3 + G_4} i_s = \frac{1/60}{1/48 + 1/16 + 1/60} 6 = 1\ A$$

T2.3 Writing KCL equations at each node gives

$$\frac{v_1}{4} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{2} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} = 2$$

$$\frac{v_3}{1} + \frac{v_3 - v_1}{2} = -2$$

In standard form, we have:

$$0.95v_1 - 0.20v_2 - 0.50v_3 = 0$$
$$-0.20v_1 + 0.30v_2 = 2$$
$$-0.50v_1 + 1.50v_3 = -2$$

In matrix form, we have

$$\mathbf{GV = I}$$

$$\begin{bmatrix} 0.95 & -0.20 & -0.50 \\ -0.20 & 0.30 & 0 \\ -0.50 & 0 & 1.50 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$$

The MATLAB commands needed to obtain the column vector of the node voltages are

G = [0.95 -0.20 -0.50; -0.20 0.30 0; -0.50 0 1.50]
I = [0; 2; -2]
V = G\I   % As an alternative we could use V = inv(G)*I

Actually, because the circuit contains only resistances and independent current sources, we could have used the short-cut method to obtain the **G** and **I** matrices.

T2.4 We can write the following equations:

KVL mesh 1: $R_1 i_1 - V_s + R_3(i_1 - i_3) + R_2(i_1 - i_2) = 0$

KVL for the supermesh obtained by combining meshes 2 and 3:

$$R_4 i_2 + R_2(i_2 - i_1) + R_3(i_3 - i_1) + R_5 i_3 = 0$$
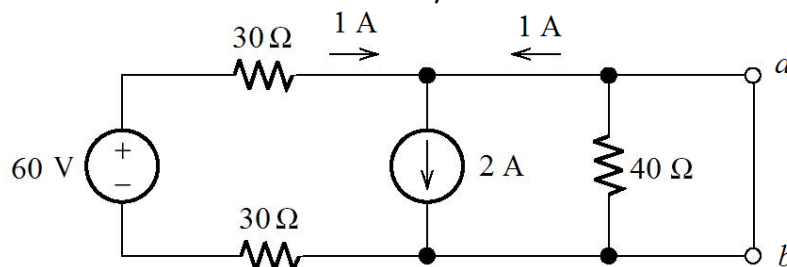
KVL around the periphery of the circuit:

$$R_1 i_1 - V_s + R_4 i_2 + R_5 i_3 = 0$$

Current source: $i_2 - i_3 = I_s$

A set of equations for solving the network must include the current source equation plus two of the mesh equations. The three mesh equations are dependent and will not provide a solution by themselves.

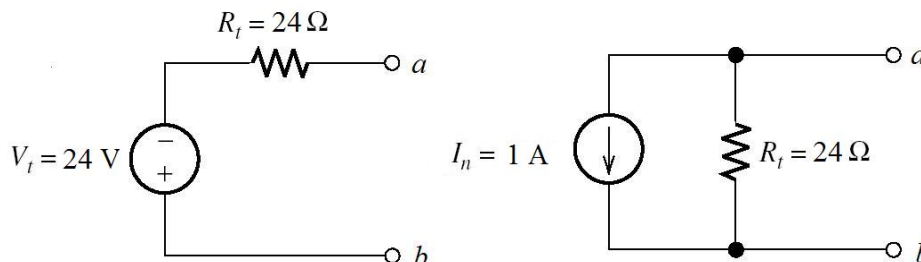**T2.5** Under short-circuit conditions, the circuit becomes

Thus, the short-circuit current is 1 A flowing out of *b* and into *a*.
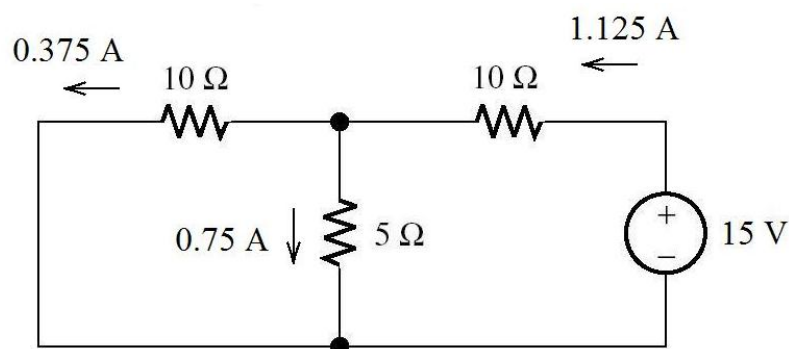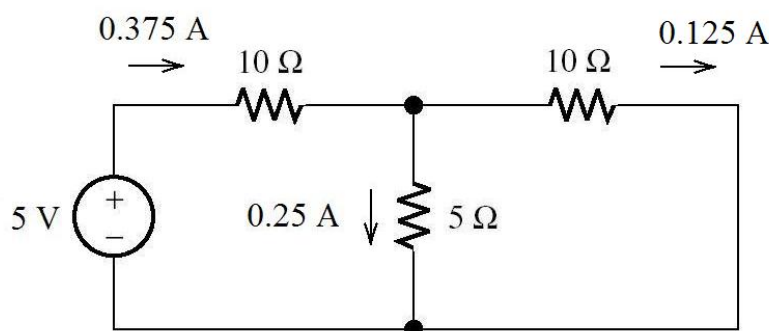Zeroing the sources, we have
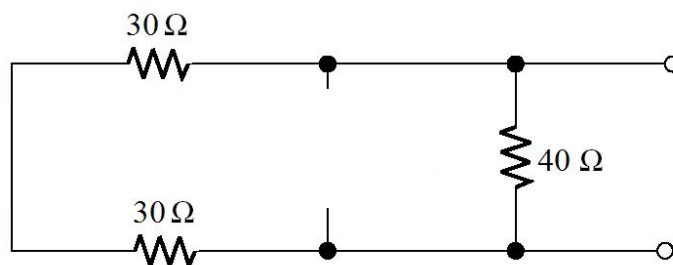
Thus, the Thévenin resistance is

$$R_t = \frac{1}{1/40 + 1/(30 + 30)} = 24 \ \Omega$$

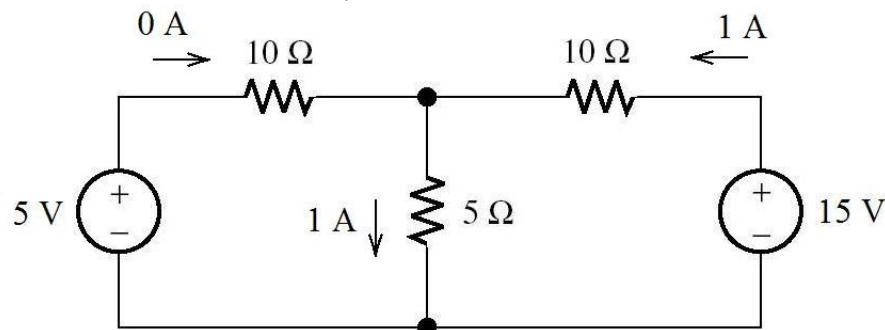and the Thévenin voltage is $V_t = I_{sc} R_t = 24 \ \text{V}$. The equivalent circuits are:

Because the short-circuit current flows out of terminal *b*, we have oriented the voltage polarity positive toward *b* and pointed the current source reference toward *b*.

T2.6   With one source active at a time, we have

Then, with both sources active, we have

We see that the 5-V source produces 25% of the total current through the 5-Ω resistance. However, the power produced by the 5-V source with both sources active is zero. Thus, the 5-V source produces 0% of the power delivered to the 5-Ω resistance. Strange, but true! Because power is a nonlinear function of current (i.e., $P = Ri^2$), the superposition principle does not apply to power.