

## Chapter 2 Solutions

### Section 2.1 Introduction

2.1 Current source

2.2 Voltage source

2.3 Resistor

2.4 Capacitor

2.5 Inductor

### Section 2.2 Charge and Current

2.6 b)

The current direction is designated as the direction of the movement of positive charges.

2.7 The relationship of charge and current is

$$q(t) = \int_{t_0}^t i(t) dt + q(t_0)$$

so

$$q(t) = \int_{t_0}^t 2 \sin(10\pi t) dt + q(t_0)$$

$$q(t) = \left[ \frac{-2}{10\pi} \cos(10\pi t) \right] + q(t_0)$$

2.8 The coulomb of one electron is denoted by  $e$  and

$$q(t) = \int_{t_0}^t i(t) dt + q(t_0)$$

So

$$n(t) = q(t) / e = \frac{1}{e} \int_{t_0}^t 12t dt + q(t_0)$$

If  $t_0 = 0$  and  $q(t_0) = 0$ ,

$$n(t) = \frac{6}{e} t^2$$

2.9

$$q(t) = \int i dt$$

$$q(t) = \int_0^t 5 dt$$

$$q(t) = 5t$$

2.10

$$q(t) = 5t \Big|_0^5 = 5(5) - 5(0) = 25 \text{ Coulombs}$$

2.11 **Using** the definition of current-charge relationship, the equation can be rewritten as:

$$i = \frac{dq}{dt} = \frac{\Delta n}{\Delta t} e$$

Thus, the current flow within  $t_1$  and  $t_2$  time interval is,

$$i = \frac{(5.75 - 2) \times 10^{19}}{2} (-1.6 \times 10^{-19}) = -3A$$

The negative sign shows the current flow in the opposite direction with respect to the electric charge.

2.12 **Assuming** the area of the metal surface is  $S$ , The mass of the nickel with depth  $d = 0.15\text{mm}$  is

$$m = \rho \times d \times S$$

Meanwhile, using the electro-chemical equivalent, the mass of the nickel can be expressed as

$$m = k \times I \times t$$

where  $I = \sigma \times S$ .

Equating the two expressions of the mass, the coating time is found:

$$t = \rho \times d / \sigma = 1.24 \times 10^5 \text{ s} \approx 34.4 \text{ hour}$$

## Section 2.3 Voltage

2.13 **By** the definition of voltage, when a positive charge moves from high voltage to low voltage, its potential energy decreases.

So a is “+”, b is “-”. In other words,  $u_{ab} = 1V$ .

2.14 The current  $i(t)$  is defined as:

$$i(t) = \begin{cases} 3 & 0 < t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Therefore, the charge is

$$g = \int_0^1 3 dt = 3 \text{ C}$$

The energy in Joules is given by:

$$J = V \times C = 5 \times 3 = 15 \text{ J}$$

2.15

$$1 \text{ electron} = -1.6 \times 10^{-19} \text{ Coulombs.}$$

Therefore, there are  $6.25 \times 10^{18}$  electrons in a coulomb.

$$\text{Coulombs of } 5 \times 10^{16} \text{ electrons} = \frac{5 \times 10^{16}}{6.25 \times 10^{18}} = 8 \times 10^{-3}$$

Therefore, the voltage is

$$V = \frac{J}{C} = \frac{15}{8 \times 10^{-3}} = 1875 \text{ V}$$

2.16

$$\begin{aligned} q(t) &= \int_0^1 2 \sin\left(\frac{3}{2}\pi t\right) dt \\ &= \left[ \frac{-4}{3\pi} \cos\left(\frac{3}{2}\pi t\right) \right]_0^1 = 0.4244 \text{ C} \\ J &= V \times C = 5 \times 0.4244 = 2.122 \text{ J} \end{aligned}$$

2.17

$$\begin{aligned} q &= \frac{20J}{2V} = 10 \text{ C} \\ i &= \frac{dq}{dt} = \frac{10 \text{ C}}{4 \text{ s}} = 2.5 \text{ A} \end{aligned}$$

## Section 2.4 Respective Direction of Voltage and Current

2.18 True

2.19 False

2.20 True

## Section 2.5 Kirchoff's Current Law

2.21 According to KCL

$$0 = 7 - i_2 - 3$$

Therefore,

$$i_2 = 4A$$

2.22 Notice that

$$0 = 3 - 2 + i_4$$

Therefore,

$$i_4 = -1A$$

It can be seen that

$$i_5 = -2A$$

$i_6$  can now be found.

$$0 = 5 + 2 + 1 - i_6$$

$$i_6 = 8A$$

2.23

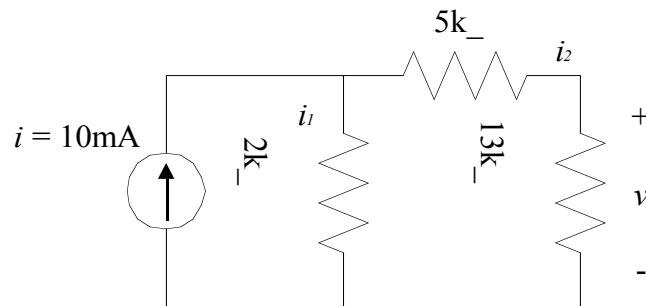


Figure S2.23: Circuit for Problem 2.23.

According to KCL,

$$10mA = i_1 + i_2$$

and

$$2ki_1 = 18ki_2$$

Therefore,

$$i_1 = 9mA, i_2 = 1mA$$

$$V = I \times R = 13k \times 1mA = 13V$$

2.24 Using the KCL method, we have,

$$i_1 = i_2 + i_3$$

Also, at the other end of the node, we have,

$$i_4 = i_2 + i_3$$

Then, combine these two equations we obtain,

$$i_1 = i_4$$

2.25 **The** algebraic sum of currents flowing into the node in Figure 2.11 is

$$i_1 + i_2 - i_3 - i_4$$

where  $i_1$  and  $i_2$  flow in, and  $i_3$  and  $i_4$  flow out

As current is defined as the rate of variation of the charge,

$$i_1 + i_2 - i_3 - i_4 = dq/dt$$

By law of the conservation of charge, charges cannot be stored in the node and charges can neither be destroyed nor created. Therefore the variation rate of the charge is zero and further the KCL law holds,

$$i_1 + i_2 - i_3 - i_4 = 0$$

2.26 The resistance across the voltage source is

$$R = 20 + 40 // 120 = 20 + \frac{40 \times 120}{40 + 120} = 50\Omega$$

So the current flowing through the voltage source is

$$I = \frac{V}{R} = \frac{5V}{50\Omega} = 0.1A$$

Therefore,

$$i = \frac{40}{160} \times 0.1 = 25mA$$

2.27 By KCL,

$$0 = \frac{V-3}{5} + \frac{V}{3} - 2I_x$$

Note that,

$$I_x = \frac{V}{3} \rightarrow V = 3I_x$$

Therefore,

$$\begin{aligned} 0 &= \frac{3I_x - 3}{5} + I_x - 2I_x \\ 0 &= 3I_x - 3 - 5I_x \end{aligned}$$

$$3 = -2I_x \rightarrow I_x = -1.5\text{A}$$

2.28 The voltage across node C and D (node C is positive) is  $U_2 - U_1$ , the voltage across node C and B is  $U_{AB} - U_1$ , and the voltage across node D and B is  $U_{AB} - U_2$ . The KCL equation for node C is

$$U_1/R = (U_2 - U_1)/R + (U_{AB} - U_1)/2R$$

which is

$$5U_1 - 2U_2 = U_{AB}$$

The KCL equation for node D is

$$U_2/2R + (U_2 - U_1)/R = (U_{AB} - U_2)/R$$

which is

$$5U_2 - 2U_1 = 2U_{AB}$$

Solving  $U_2$  and  $U_1$ ,  $U_1 = 9U_{AB}/21$ ,  $U_2 = 12U_{AB}/21$

Thus, by the KCL equation at node A

$$I_{AB} = U_1/R + U_2/2R$$

$$R_{AB} = U_{AB}/I_{AB} = 7R/5$$

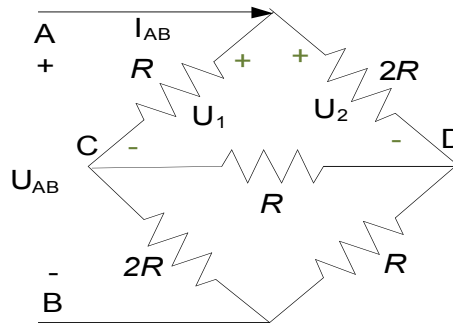


Figure S2.28: Circuit for Problem 2.28.

## Section 2.6 Kirchoff's Voltage Law

2.29 By KVL,

$$0 = -v_A - v_B - v_C + v_D$$

$$0 = -v_A - 2 + 6 + 6$$

Then

$$v_A = 10\text{V}$$

2.30 Using KVL,

$$0 = -12 + v_R - 6 + 14$$

Then

$$v_R = 4\text{V}$$

2.31 Using KVL,

$$0 = -v_D + 3 + 5$$

Then

$$v_D = 8 \text{ V}$$

Using KVL,

$$0 = -8 - 2 + v_C + 8$$

Then

$$v_C = 2 \text{ V}$$

2.32 Using KVL,

$$0 = -9 + 4 - v_C$$

Then

$$v_C = -5 \text{ V}$$

Using KVL,

$$0 = -5 + 3 - v_E - 5 - 6$$

Then

$$v_E = -13 \text{ V}$$

2.33 Using KVL,

$$0 = -v_D + 4 - 3$$

Then

$$v_D = 1 \text{ V}$$

Using KVL,

$$0 = 1 + 3 + 1 + v_C$$

Then

$$v_C = -5 \text{ V}$$

Using KVL,

$$5 + 3 - v_E = 0$$

Then

$$v_E = 8 \text{ V}$$

2.34 Using KVL,

$$0 = 7 - v_E + 1$$

Then

$$v_E = 8 \text{ V}$$

Using KVL,

$$0 = -v_B + 5 + 8$$

Then

$$v_B = 13 \text{ V}$$

Using KVL,

$$0 = 5 - v_D - 7$$

Then

$$v_D = -2 \text{ V}$$

Using KVL

$$0 = v_A + 13 - 2$$

Then

$$v_A = -11 \text{ V}$$

2.35 Using KVL,

$$10 = V_H + 2 \rightarrow V_H = 8 \text{ V}$$

$$2 + V_H = 5 + V_I \rightarrow V_I = 5 \text{ V}$$

$$V_C + 4 = 6 \rightarrow V_C = 2 \text{ V}$$

$$5 + V_D = 4 + V_C \rightarrow V_D = 1 \text{ V}$$

2.36

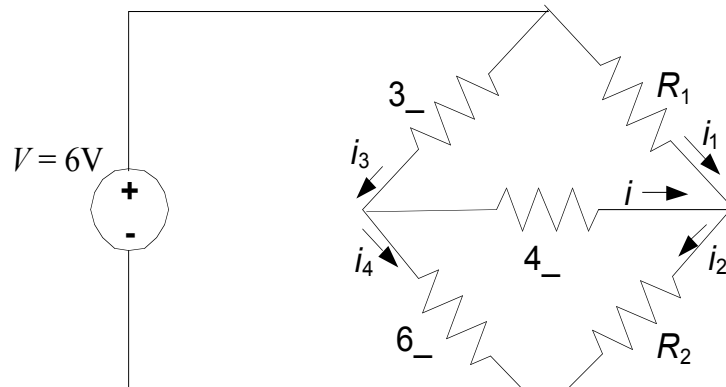


Figure S2.36: Circuit for Problem 2.36.

- a)  $R_1 = 6\Omega$ ,  $R_2 = 3\Omega$   
By KCL,

$$i_1 + i = i_2$$

$$i_4 + i = i_3$$

Applying KVL on the loop made by 6V source,  $R_1$  and  $R_2$ ,

$$6i_1 + 3i_2 = 6$$

Applying KVL on the loop made by 6V source,  $3\Omega$  resistor and  $6\Omega$  resistor,

$$3i_3 + 6i_4 = 6$$

Applying KVL on the loop made by  $3\Omega$  resistor,  $4\Omega$  resistor and  $R_1$ ,

$$3i_3 + 4i = 6i_1$$

Now we have 5 equations for 5 variables and  $i$  can be solved using Cramer's rule or variable elimination. Then we get

$$i = 1/4 \text{ A}, i_1 = 7/12 \text{ A}, i_2 = 5/6 \text{ A}, i_3 = 5/6 \text{ A}, i_4 = 7/12 \text{ A}$$

- b)  $i = 0 \Rightarrow i_1 = i_2, i_3 = i_4$ .



Moreover, the voltage across the  $4\Omega$  resistor is zero. This also leads to the voltage across  $R_1$  equals to the voltage across the  $3\Omega$  resistor and the voltage across  $R_2$  equals to the voltage across the  $6\Omega$  resistor,

$$R_1 i_1 = 3i_3$$

$$R_2 i_2 = 6i_4$$

Therefore,

$$R_1 / R_2 = 3/6 = 1/2$$

2.37 **The** current from node A to node C is  $I_{AB} - I_1$ , the current from node D to node B is  $I_1 - I_2$ , and the current from node C to node B is  $I_{AB} - I_1 + I_2$ .

Then the KVL equation for loop ADC is

$$I_1 2R + I_2 R = (I_{AB} - I_1)R$$

which is

$$3I_1 + I_2 = I_{AB}$$

The KVL equation for loop DBC is

$$(I_1 - I_2)R = I_2 R + (I_{AB} - I_1 + I_2)2R$$

which is

$$3I_1 - 4I_2 = 2I_{AB}$$

Solving  $I_1$  and  $I_2$ ,  $I_1 = 2I_{AB}/5$ ,  $I_2 = -I_{AB}/5$

Thus,

$$U_{AB} = I_1 2R + (I_1 - I_2)R = 7I_{AB}R / 5$$

$$R_{AB} = U_{AB} / I_{AB} = 7R / 5$$

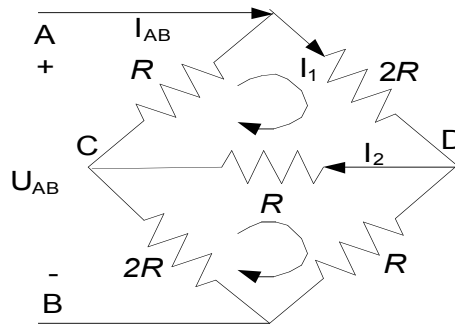


Figure S2.37: Circuit for Problem 2.37.

## Section 2.7 Ohm's Law

2.38 By KCL, the current through the  $4\Omega$  resistor is 3A. Using Ohm's law,

$$V = I \times R = 4 \times 3 = 12 \text{ V}$$

2.39 By KVL, the voltage across the resistor is 5V.

Using Ohm's law:

$$R = \frac{V}{I} = \frac{5}{2} = 2.5\Omega$$

2.40 Using Ohm's law:

$$I = \frac{V}{R} = \frac{15}{5} = 3A$$

2.41 The resistivity of copper is,

$$\rho = 1.72 \times 10^{-8} \Omega - m$$

The resistance of the copper wire is,

$$R = \frac{\rho L}{A} = \frac{1.72 \times 10^{-8} \times 0.5}{1.2286 \times 10^{-10}} = 70\Omega$$

Using the definition of Ohm's Law, we can obtain the maximum allowable voltage, which is:

$$V_{\max} = I_{\max} R = 1 \cdot 70 = 70V$$

2.42 Using Ohm's law, the resistance of the tissue is

$$R = \frac{10}{12.43 \times 10^{-6}} = 804.5k\Omega$$

Because,

$$R = \frac{\tilde{n}L}{A}$$

Therefore,

$$804.5k\Omega = \frac{175\tilde{U} \cdot m \times L}{3.468 \times 10^{-3} m^2}$$

$$L = 15.94 m$$

2.43 The resistance across the voltage source is

$$R = R_1 + R_2 // R_3 = R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

Using Ohm's law  $I = \frac{V}{R}$ , and

$$\begin{aligned} i &= \frac{R_2}{R_2 + R_3} \cdot I = \frac{R_2}{R_2 + R_3} \cdot \frac{V_s \cdot (R_2 + R_3)}{R_1(R_2 + R_3) + R_2 R_3} \\ &= \frac{V_s \cdot R_2}{R_1(R_2 + R_3) + R_2 R_3} \end{aligned}$$

2.44 By KCL:

$$I_{20} = 5A$$

Therefore, by Ohm's Law:

$$V_A = 20 \cdot 5 = 100V$$

$$V_B = 3 \cdot 10 = 30V$$

$$V_C = 2 \cdot 15 = 30V$$

By KVL:

$$V_S = 130V$$

2.45 Using Ohm's law:

$$I = \frac{240}{200 \times 10^3} = 1.2 \text{ mA}$$

2.46 For short circuit,

$$v(t) = 0$$

Using ohm's law

$$R_{\text{short}} = v(t)/i(t) = 0$$

For open circuit

$$i(t) = 0$$

Using ohm's law

$$R_{\text{open}} = v(t)/i(t) = \text{infinity}$$

2.47

$$V_{\text{out}} = V_S(R_S/(R_S + R_C)) - V_S(R_B/(R_A + R_B))$$

By product rule of derivation

$$dV_{\text{out}}/dR_S = V_S R_C / (R_S + R_C)^2$$

When  $R_S = 0$ , the above expression is minimum. However, you cannot control over sensor resistance, but you can choose  $R_C$  to achieve the maximum slope.

We differentiate  $dV_{\text{out}}/dR_S$  with respect to  $R_C$ ,

$$d(dV_{\text{out}}/dR_S)/dR_C = V_S(R_S - R_C)/(R_S + R_C)^3$$

The maximum slope is achieved when  $R_C = R_S$ .

$R_C$  can be chosen to have the same value as the nominal resistance of the sensor.

2.48 Using data pairs in the table, we get three equations in  $A$ ,  $B$ ,  $C$

$$(1/273) = A + B \ln(16330) + C (\ln(16330))^3$$

$$(1/298) = A + B \ln(5000) + C (\ln(5000))^3$$

$$(1/323) = A + B \ln(1801) + C (\ln(1801))^3$$

Solving those equations, we get

$$A = 0.001284$$

$$B = 2.364 \times 10^{-4}$$

$$C = 9.304 \times 10^{-8}$$

## Section 2.8 Power and Energy

2.49

- a) The voltage and the current are in associated direction,  
 $P = u \times i = -3 \times 1 = -3\text{W} \Rightarrow$  active element
- b) The voltage and the current are not in associated direction,  
 $P = -u \times i = 2 \times 2 = 4\text{W} \Rightarrow$  passive element
- c) The voltage and the current are not in associated direction,  
 $P = -u \times i = -2 \times 3 = -6\text{W} \Rightarrow$  active element

2.50

- a) The voltage and the current are in associated direction,  
 $P = u \times i = 20 \times 2.5e^{-2t} = 50e^{-2t}\text{W} \Rightarrow$  active element
- b) The voltage and the current are not in associated direction,  
 $P = -u \times i = -20 \times 2\sin t = -40\sin t\text{W} \Rightarrow t > 0$ , active element;  $t < 0$ , passive element

2.51 The current can be found using the power formula:

$$I = \frac{P}{V} = \frac{5}{10} = 0.5\text{ A}$$

And the resistance can then be found using Ohm's law:

$$R = \frac{V}{I} = \frac{10}{0.5} = 20\Omega$$

2.52

$$\begin{aligned} P &= V \times I = R \times I^2 \\ &= 20 \times 5^2 = 500\text{ W} \end{aligned}$$

2.53 Applying KCL, the current through each resistor is 4mA.

Given  $P = R \times I^2$ ,

$$P_1 = 5 \times 10^3 \times (4 \times 10^{-3})^2 = 80\text{mW}$$

$$P_2 = 14 \times 10^3 \times (4 \times 10^{-3})^2 = 224\text{mW}$$

$$P_3 = 32 \times 10^3 \times (4 \times 10^{-3})^2 = 512\text{mW}$$

2.54 According to KVL:

$$1.5ki_1 = 3ki_2$$

By KCL:

$$3\text{mA} = i_1 + i_2$$

$$i_2 = 1\text{mA}$$

$$V = 1mA \times 1k\Omega = 1V$$

2.55 From the problem 2.54, we have  $i_1 = 2mA$  and  $i_2 = 1mA$ . Recall  $P = I^2 R$ . Therefore,

$$P_{1.5k} = (2 \times 10^{-3})^2 \cdot 1.5 \times 10^3 = 6mW$$

$$P_{2k} = (1 \times 10^{-3})^2 \cdot 2 \times 10^3 = 2mW$$

$$P_{1k} = (1 \times 10^{-3})^2 \cdot 1 \times 10^3 = 1mW$$

2.56

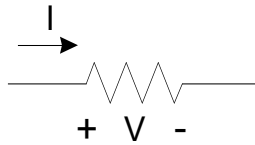


Figure S2.56: Resistor with current direction and voltage polarity.

**Consider** Figure S2.56, the power absorbed by the resistor is

$$P = VI$$

By Ohm's law,

$$V = RI$$

Thus

$$P = I^2 R = V^2 / R$$

Therefore  $P$  is always a nonnegative number when  $R$  is positive.

2.57 **The** voltage and the current are in associated direction, so

$$P = u \times i$$

and the waveform of the power is shown below

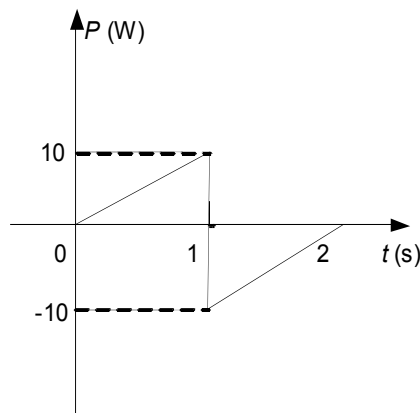


Figure S2.57: Power curve.

The consumed energy is

$$W = \int_0^2 P dt = 0$$

which is resulted from the negative symmetry of  $P$  around  $t = 1$ .

2.58

- a) **Let** the current through the  $-5V$  voltage source be in associated direction with the voltage, which is from the positive side to the negative side.

By KCL at the node below the  $-5V$  voltage source, the current through it equals  $2A + 4A = 6A$ .

Therefore the power of the  $-5V$  voltage source is

$$P = -5V \times 6A = -30W$$

- b) To zero the power above, the current through the  $-5V$  voltage has to be zero. So the  $4A$  current source needs to be changed to  $-2A$  by KCL.

### Section 2.9 Independent and Dependent Sources

- 2.59 **In** Figure S2.59,  $i$  axis is the current through the source and  $u$  axis is the voltage across the source.  $u_s$  denotes the voltage provided by the ideal voltage source and  $i_s$  denotes the current provided by the ideal current source. It is shown that for ideal voltage source the voltage does not change with the current through it; for ideal current source the current does not change with the voltage.

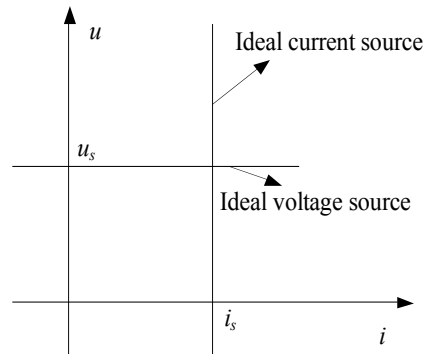


Figure S2.59: Voltage VS current curves.

- 2.60 **Because** the two current sources are in series, the currents through them are the same,

$$i = 1A$$

- 2.61 **Because** the  $3\Omega$  resistor is in series with the current source, the current through the resistor is  $1A$  and, by Ohm's law, its voltage is

$$u = 3\Omega \times 1A = 3V$$

By KVL, the voltage across the current source is

$$u + 2V = 5V$$

The power supplied by the current source is (the current and the voltage are not in associated direction)

$$P = -5V \times 1A = -5W$$

2.62 Using KVL:

$$0 = -10 + V_x + 3V_x$$

Therefore,

$$V_x = 2.5$$

2.63 Using KVL and Ohm's law,

$$0 = -6 - (I_x \times 3.5 \times 10^3) + 20I_x$$

$$6 = -3.48 \times 10^3 I_x$$

$$I_x = -1.724mA$$

2.64 **The** voltage across the  $2\Omega$  resistor is  $-1V$ , where the voltage polarity is associated with the current direction. So by Ohm's law

$$I = -1V/2\Omega = -0.5A$$

Therefore, the current of the dependent source is

$$2I = -1A$$

2.65 According to KCL, the current through  $R_3$  is  $\beta i_x$

According to Ohm's law,

$$v_0 = \beta i_x R_3$$

$i_x$  is given by:

$$i_x = \frac{v_s}{R_1 + R_2}$$

Therefore,

$$v_0 = \beta R_3 \frac{v_s}{R_1 + R_2}$$

2.66 **According** to Ohm's law, the current flow (in associated direction with the voltage) through the  $5\Omega$  resistor is

$$i_1 = 4.9V/5\Omega = 0.98A.$$

Because the CCCS is in series with the  $5\Omega$  resistor,

$$i_1 = 0.98i \Rightarrow i = 1A$$

which is the current through the  $6\Omega$ .

By KCL, the current through the  $0.1\Omega$  resistor is

$$i - 0.98i = 0.02A.$$

So the voltage across the  $0.1\Omega$  resistor is

$$u_1 = 0.1\Omega \times 0.02A = 0.002V.$$

Applying KVL in the loop made by  $u_s$ ,  $0.1\Omega$  and  $6\Omega$

$$u_s = u_1 + 6i = 0.002 + 6 = 6.002V$$

The power supplied by the dependent source is

$$\begin{aligned} P &= (0.98i) \times u = 0.98 \times (0.002 - 4.9) \\ &= 0.98 \times (-4.898) \\ &= -4.8W \end{aligned}$$

## Section 2.10 Analysis of Circuits using PSpice

2.67 A PSpice circuit is shown in Figure S2.67. Choose the Bias Point for the simulation analysis.

$$R_{TOTAL} = 20 + \left( \frac{1}{40} + \frac{1}{120} \right)^{-1} = 50\Omega$$

$$I_{TOTAL} = \frac{V_{TOTAL}}{R_{TOTAL}} = 0.1A$$

Using Ohm's law, we obtain:

$$R_1 i_1 = R_2 i_2$$

$$i_1 = \frac{R_2}{R_1} i_2$$

Apply the KCL:

$$I = i_1 + i_2$$

The  $i$  can be calculated as:

$$0.1 = i + \frac{120}{40} i$$

$$i = 0.025A$$

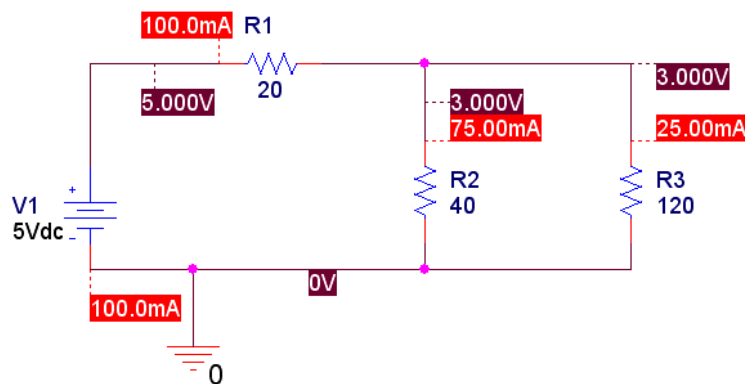


Figure S2.67: PSpice Circuit for Problem 2.67.



2.68 Applying KCL:

$$i_1 = i_2 + i_3 = 10A$$

For parallel voltage, the current relationship is given by:

$$i_2 = \frac{R_3}{R_2} i_3 = \frac{2}{3} i_3$$

Therefore

$$i_2 = 4A$$

$$i_3 = 6A$$

The total voltage across the circuit is:

$$V_{TOTAL} = i_1 R_1 + i_2 R_2 = 52V$$

The PSpice setup with current source is shown in Figure S2.68 a. Figure S2.68 b shows the current source is replaced by voltage source that produces the same current and voltage across each resistance as those in Figure S2.68 a.

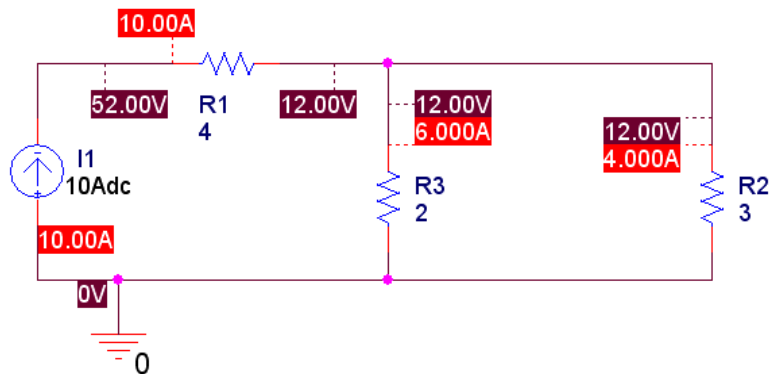


Figure S2.68 a: PSpice Circuit with Current source for Problem 2.68.

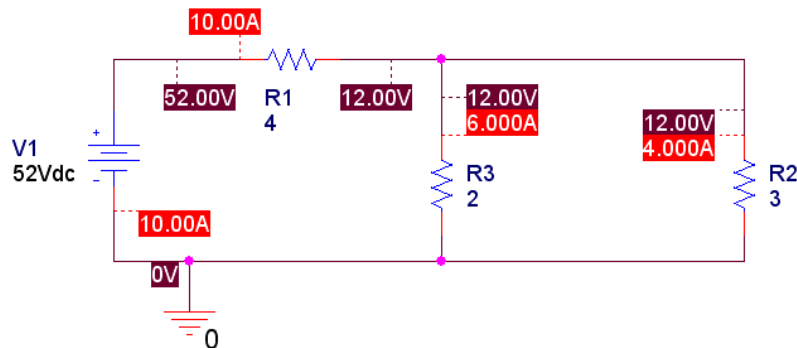


Figure S2.68 b: PSpice Circuit with Current source for Problem 2.68.

2.69 Applying KVL equation, we have:

$$V - R_2 i_3 - R_7 i_1 = 0 \quad (2.1)$$

$$R_1 i_2 + R_3 i_4 + R_4 i_4 + R_6 i_6 = R_2 i_3 \quad (2.2)$$

$$R_5 i_5 - i_4 R_3 - R_4 i_4 = 0 \quad (2.3)$$

Applying KCL equation, we have:

$$i_1 = i_2 + i_3 \quad (2.4)$$

$$i_2 = i_4 + i_5 \quad (2.5)$$

$$i_6 = i_4 + i_5 = i_2 \quad (2.6)$$

From equation (2.3), the current relationship can be written as:

$$2i_5 = 2i_4$$

$$i_5 = i_4$$

$$i_2 = 2i_4 = 2i_5$$

$$i_4 = i_5 = 0.5A$$

Using the relationship obtained above and substitute into equation (2.2), we obtain:

$$\begin{aligned} 2i_2 + i_2 + R_6 i_2 &= 4i_3 \\ 3 + R_6 &= 4i_3 \end{aligned} \quad (2.7)$$

Expressing the equation (2.1) for  $i_1$  in term of  $i_3$ :

$$\begin{aligned} 10 - 4i_3 - 3i_1 &= 0 \\ i_1 &= (10 - 4i_3) \frac{1}{3} \end{aligned} \quad (2.8)$$

Substitute equation (2.8) into equation (2.4),  $i_3$  is calculated:

$$\begin{aligned} \frac{10}{3} - \frac{4}{3}i_3 &= i_2 + i_3 \\ i_3 &= 1A \end{aligned} \quad (2.9)$$

Substitute equation (2.9) into equation (2.7),  $R_6$  and  $i_5$  can be determined:

$$\begin{aligned} 3 + R_6 &= 4 \\ R_6 &= 1\Omega \end{aligned} \quad (2.10)$$

$$i_5 = 0.5A \quad (2.11)$$

A PSpice schematic is shown in Figure S2.69 to verify the answers obtained:

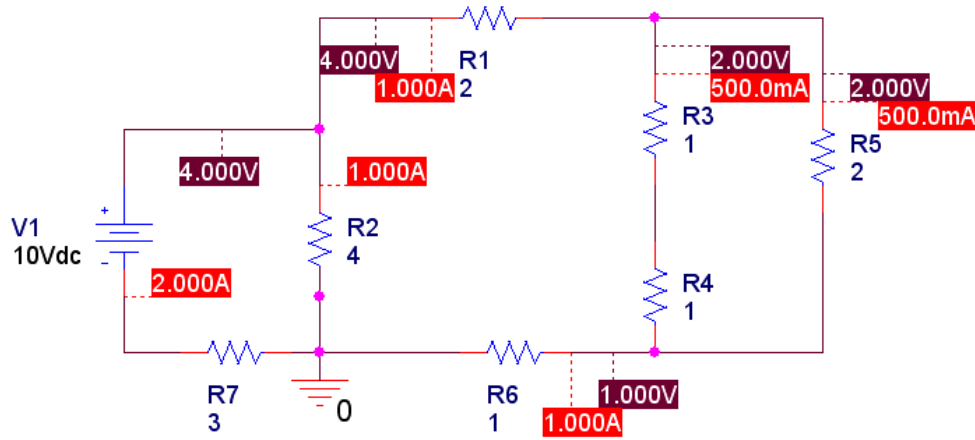


Figure S2.69: PSpice Circuit with Current source for Problem 2.69.

2.70 Applying the KCL equation:

$$i_1 = i_2 + i_3 + i_4 \quad (2.12)$$

$$i_2 = i_5 + i_6 \quad (2.13)$$

$$i_7 = i_4 + i_5 \quad (2.14)$$

$$i_8 = i_6 + i_7 = i_4 + i_5 + i_6 \quad (2.15)$$

Applying the KVL equation:

$$V_1 - R_2 i_3 - R_7 i_1 = 0 \quad (2.16)$$

$$V_2 - R_3 i_5 - R_1 i_2 + R_8 i_4 = 0 \quad (2.17)$$

$$V_3 - R_5 i_6 + R_4 i_7 + R_3 i_5 = 0 \quad (2.18)$$

$$R_2 i_3 - R_8 i_4 - R_4 i_7 - R_6 i_8 = 0 \quad (2.19)$$

Substituting equation (2.13) into equation (2.12), we obtain:

$$i_1 = i_3 + i_4 + i_5 + i_6 \quad (2.20)$$

Substituting equation (2.20) into equation (2.16), we obtain:

$$\begin{aligned} 10 - 2i_3 - 2i_1 &= 0 \\ 10 &= 2i_3 + 2(i_3 + i_4 + i_5 + i_6) \\ 10 &= 4i_3 + 2i_4 + 2i_5 + 2i_6 \end{aligned} \quad (2.21)$$

Using the current relationship in equation (2.13) and substituting it into equation (2.17), we get:

$$\begin{aligned} 2 - 2i_5 - 3i_2 + 8i_4 &= 0 \\ 2 &= 2i_5 + 3(i_4 + i_5) - 8i_4 \\ 2 &= -5i_4 + 5i_5 \end{aligned} \quad (2.22)$$

Substituting equation (2.14) and equation (2.18), we obtain:

$$\begin{aligned} 2 - 20i_6 + i_7 + 2i_5 \\ 2 = 20i_6 - i_4 - i_5 - 2i_5 \\ 2 = -i_4 - 3i_5 + 20i_6 \end{aligned} \quad (2.23)$$

Applying the current relationship in equation (2.14) and (2.15) into equation (2.19), and it can be rewritten as:

$$\begin{aligned} 2i_3 - 8i_4 - i_7 - 1.6i_8 &= 0 \\ 2i_3 - 10.6i_4 - 2.6i_5 - 1.6i_6 &= 0 \end{aligned} \quad (2.24)$$

We can express the equations (2.21) to (2.24) by using a matrix equation of  $Y = AX$ , which is given below:

$$\begin{aligned} X &= \begin{bmatrix} i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} \\ Y &= \begin{bmatrix} 10 \\ 2 \\ 2 \\ 0 \end{bmatrix} \\ A &= \begin{bmatrix} 4 & 2 & 2 & 2 \\ 0 & -5 & 5 & 0 \\ 0 & -1 & -3 & 20 \\ 2 & -10.6 & -2.6 & -1.6 \end{bmatrix} \end{aligned}$$

Matrix X can obtained,

$$\begin{aligned} X &= A^{-1}Y \\ i_3 &= 2A \\ i_4 &= 0.2A \\ i_5 &= 0.6A \\ i_6 &= 0.2A \end{aligned}$$

A PSpice schematic is shown in Figure S2.70:

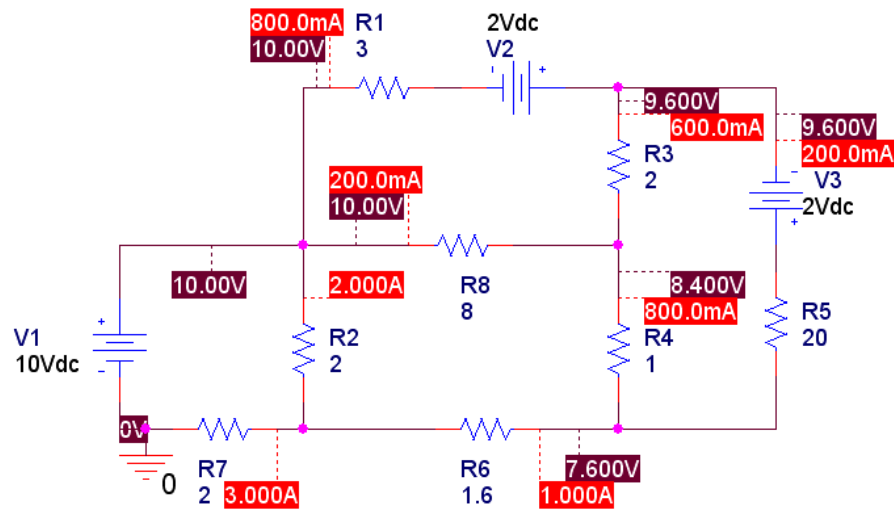


Figure S2.70: PSpice Circuit with Current source for Problem 2.70.