

2 Transformers

2.1 Solutions To Exercises

EXERCISE 2-1

$$Z_L = \frac{(120)^2}{5000} = 2.88 \, \Omega \qquad Z_1 = 288 \left(\frac{240}{120} \right)^2 = \underline{\underline{11.52 \, \Omega}}$$

EXERCISE 2-2

$$\text{a) } Z_{eH} = 0.5 + j1.2 + (2)^2 (0.125 + j0.30) = \underline{\underline{1 + j2.4 \, \Omega}}$$

$$\text{b) } Z_{eL} = \frac{1}{4} (1 + j2.4) = \underline{\underline{0.25 + j0.6 \, \Omega}}$$

$$\text{c) } Z_{epu} = (0.25 + j0.6) \frac{5000}{(240)^2} = \underline{\underline{0.022 + j0.052 \, \text{p.u.}}}$$

$$\begin{aligned} \text{d) } V_{2n.t} &= V_L + I Z_e \\ &= 1 \angle 0^\circ + 1 \angle -36.9^\circ (0.022 + j0.052) \\ &= 1.049 \angle 1.56^\circ \end{aligned}$$

$$\begin{aligned} V_s &= 1.049 (480) \angle 1.56^\circ \\ &= \underline{\underline{503.52 \angle 1.56^\circ \text{V}}} \end{aligned}$$

EXERCISE 2-3

$$V_{Z_{pu}} = I_{r_{pu}} Z_{e_{pu}} = 1.0 Z_{e_{pu}} = Z_{e_{pu}}$$

EXERCISE 2-4

1. a) Primary:

$$I_L = \frac{1500}{\sqrt{3}(4.16)} = \underline{208.2 \text{ A}}$$

$$I_P = 208.2/\sqrt{3} = \underline{120.2 \text{ A}}$$

Secondary:

$$I_P = I_L = \frac{1500}{\sqrt{3}(0.48)} = \underline{1804.2 \text{ A}}$$

$$\text{b) } Z_{bL} = \frac{(0.48)^2}{1.5} = 0.154 \text{ } \Omega/\text{Phase}$$

$$X_e = 0.06 (0.154) = \underline{0.0092 \text{ } \Omega/\text{Phase}}$$

$$Z_{eH} = \frac{(4.16)^2}{1.5} = 11.537 \text{ } \Omega/\text{Phase}$$

$$X_{eH} = 0.06 (11.537) = 0.69 \text{ } \Omega/\text{Phase, star connected}$$

$$= 3 (0.69) = \underline{2.08 \text{ } \Omega}, \text{ delta connected}$$

c) Secondary:

$$I_L = I_P = \frac{1500}{\sqrt{3}(0.48)} \left(\frac{1}{0.06} \right) = \underline{30.07 \text{ kA}}$$

Primary:

$$I_L = \left(\frac{1}{0.06} \right) \frac{1500}{\sqrt{3}(4.16)} = \underline{3.47 \text{ kA}}$$

$$I_P = 3.47/\sqrt{3} = 2.0 \text{ kA}$$

2. a) Primary

$$I_L = 208.2 (1.333) = \underline{277.6 \text{ A}}$$

$$I_P = 277.6/\sqrt{3} = \underline{160.3 \text{ A}}$$

Secondary:

$$I_L = \frac{2000}{\sqrt{3}(0.48)} = \underline{\underline{2405.6 \text{ A}}}$$

b) No change.

c) No change.

EXERCISE 2-5

From

$$\begin{aligned} I_A + I_C &= I_B; & I_C &= I_T \\ I_A \pm I_T - I_S &= 0, & I_B \pm I_T - I_S &= 0 \\ I_T &= \underline{\underline{0 \text{ A}}}, & I_A = I_B &= \underline{\underline{25 \text{ A}}} \end{aligned}$$

EXERCISE 2-6

Parameter	Δ - Δ	Δ -Y
Insulation level of secondary winding	nominal	lower for star winding
Exciting current	non-linear	non-linear
Output voltage waveforms	sinusoidal	sinusoidal

EXERCISE 2-7

c) 120 V coil: $\frac{500}{120} = 4.166 \text{ A}$

240 V coil: 2.08 A

$$\begin{aligned} \text{Load: } S &= \sqrt{3} V_{L-L} I \\ &= \sqrt{3} (0.48) (10.4167) \\ &= \underline{\underline{8.66 \text{ kVA}}} \end{aligned}$$

EXERCISE 2-8

$$S_b = 500 \text{ kVA} \quad V_b = 480 \text{ V} \quad I_b = \frac{500}{0.48} = 1041.67 \text{ A}$$

$$Z_{e1} = (0.02 + j0.035) \frac{5}{3} = 0.0672 \angle 60.3^\circ \text{ p u}$$

$$Z_{e2} = (0.018 + j0.04) \frac{5}{2.5} = 0.0877 \angle 65.8^\circ \text{ p u}$$

$$V_1 = V_2$$

$$V_1 = 1 \angle 0^\circ + I_1 Z_{e1}$$

$$V_2 = 1 \angle 0^\circ + I_2 Z_{e2}$$

Thus,

$$I_1 = I_2 \frac{Z_{e2}}{Z_{e1}}$$

$$\begin{aligned} I_1 &= \left(\frac{0.0877}{0.0672} \right) \angle 65.8^\circ - 60.3^\circ I_2 \\ &= 1.31 \angle 5.5^\circ I_2 \end{aligned}$$

Also

$$I_1 + I_2 = 1 \angle -25.8^\circ$$

or

$$(1.31 \angle 5.5^\circ) I_2 + I_2 = 1 \angle -25.8^\circ$$

From which

$$\begin{aligned} I_2 &= 0.434 \angle -29^\circ \text{ p u} \\ &= 0.434 (1041.67) = \underline{\underline{452.29 \text{ A}}} \end{aligned}$$

and

$$I_1 = 0.567 \angle -23.4^\circ = 0.567 (1041.67) = \underline{590.4 \text{ A}}$$

$$S_2 = 452.29 (0.48) = \underline{217.1 \angle 29^\circ \text{ kVA}}$$

$$S_1 = 590.4 (0.48) = \underline{283.4 \angle 23.4^\circ \text{ kVA}}$$

EXERCISE 2-9

Shorting the load will increase by many folds the current through the primary winding of the CT. As a result, the CT will be damaged—if no precaution is taken—due to excessive copper losses. The PT's primary current will remain at its nominal level.

EXERCISE 2-10

- a) PT. When the fuse is blown out the secondary winding becomes open circuited.

There is no danger of fire.

CT. When the fuse is blown out the secondary winding becomes open circuited.

There is a danger of fire.

- b) Voltage and current are phasors while the meters read only scalar quantities.

2.2 Solutions To Problems

PROBLEM 2-1

a) $Z_{eH} = 0.5 + j2.6 + 10^2 (0.005 + j0.026) = 1 + j5.2 \Omega$

$$Z_{eL} = 0.005 + j0.026 + \frac{1}{100} (0.5 + j2.6) = 0.01 + j0.052$$

$$= 0.053 \angle 79.11^\circ \Omega$$

$$b) \quad I_2 = \frac{50000}{(230)} = 217.39 \angle -25.8^\circ \text{ A}$$

$$V_L = \angle 0^\circ = 230 \angle \beta - 217.39 (0.053) \angle 79.11^\circ - 25.8^\circ$$

$$\beta = 2.3^\circ, \quad V_L = \underline{\underline{222.93 \text{ V}}}$$

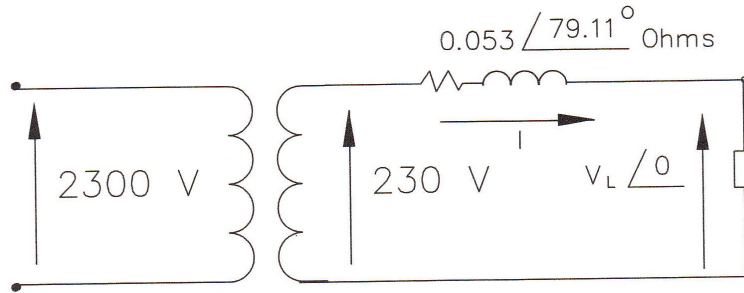


FIG. SP2-1

PROBLEM 2-2

$$a) \quad Z_{mH} = \frac{2400}{0.35} = 6857.1 \Omega$$

$$R_m = \frac{150}{(0.35)^2} = 1224.49 \Omega$$

$$X_m = \sqrt{6857.1^2 - 1224.49^2} = 6746.93 \Omega$$

$$Z_{mH} = 6857.1 \angle 79.7^\circ \Omega, \quad Z_{mL} = 68.57 \angle 79.71^\circ \Omega$$

$$Z_{eL} = \frac{12}{41.67} = 0.288 \Omega, \quad R_{eL} = \frac{320}{(41.67)^2} = 0.184 \Omega$$

$$X_{eL} = (0.288^2 - 0.184^2)^{1/2} = 0.2213 \Omega$$

$$Z_{eL} = 0.288 \angle 50.2^\circ$$

$$Z_{eH} = 28.8 \angle 50.2^\circ$$

$$b) \quad V = 10 (240 + 0.288 \angle 50.2^\circ (41.67 \angle -25.8^\circ))$$

$$= \underline{\underline{2509.8 \angle 1.1^\circ \text{ V}}}$$

$$c) \quad \text{Reg} = \left(\frac{2509.8}{10} - 240 \right) \frac{1}{240} (100) = \underline{4.57\%}$$

$$\eta = \frac{10(0.9)}{10(0.9) + 0.320 + 0.15 \left(\frac{2509.8}{2400} \right)^2} = \underline{0.949}$$

$$d) \quad Z_{\text{Bl}} = \frac{(240)^2}{10000} = 5.76 \, \Omega$$

$$Z_{\text{epu}} = \frac{0.288 \angle 50.2^\circ}{5.76} = \underline{0.05 \angle 50.2^\circ \text{ pu}}$$

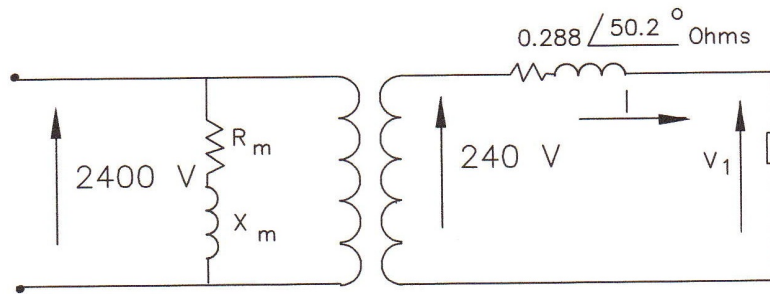


FIG. SP2-2

PROBLEM 2-3

$$a) \quad Z_{\text{Bl}} = \frac{120^2}{50000} = 0.288 \, \Omega, \quad R_a = 0.023(0.288) = 0.0066 \, \Omega$$

$$I^2 (0.0066) = 600, \quad I = \underline{300.96 \text{ A}}$$

$$b) \quad \eta = \frac{50}{50 + 0.6 + 0.6} = \underline{97.66\%}$$

$$\eta = \frac{50}{50 + 0.6 + (416.67)^2 (0.0066)} = \underline{96.6\%}$$

$$c) \quad Z_e = 0.023 + j0.05 = 0.055 \angle 65.3^\circ \text{ p u}$$

$$V_s = V_L + I Z$$

$$1 \angle \beta = 1 \angle 0^\circ + 1 \angle \Theta (0.055 \angle 65.3^\circ)$$

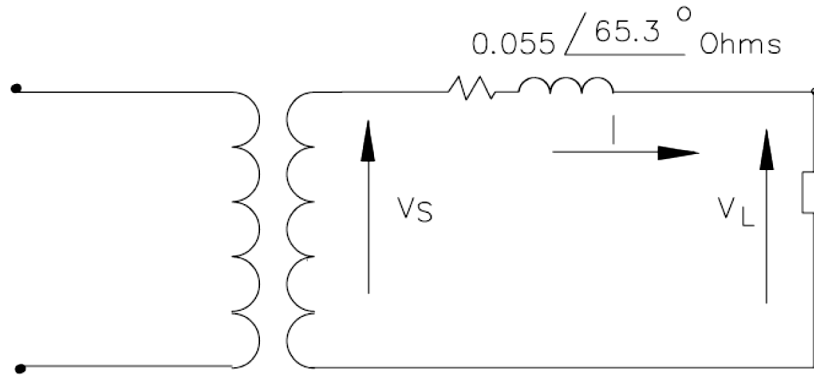


FIG. SP2-3

$$1 \angle \beta = 1 \angle 0^\circ + 0.055 \angle \mu; \quad \mu = \Theta + 65.3^\circ$$

$$\cos \beta = 1 + 0.055 \cos \mu$$

$$\sin \beta = 0.055 \sin \mu$$

From the last two relationships

$$\cos^2 \beta + \sin^2 \beta = 1 + (0.055)^2 (\cos^2 \mu + \sin^2 \mu) + 2(0.055) \cos \mu$$

or

$$\cos \mu = -\frac{0.055}{2} = -0.0275$$

$$\mu = 91.5^\circ$$

and

$$\Theta = 26.3^\circ; \quad \cos \Theta = \underline{0.90 \text{ leading}}$$

PROBLEM 2-4

$$P_z = \frac{2.5}{100} (100) = 2.5 \text{ kW}, \quad P_m = P_T - P_z = 100,000 \frac{1 - 0.9575}{0.9575} - 2500$$

$$P_m = 4438.6 - 2500 = 1938.6 \text{ W}$$

$$1938.6 = K_1 (120)^2 + K_2 60 \left(\frac{120}{60} \right)^h \quad \dots(1)$$

$$1400 = K_1 (100)^2 + K_2 50 \left(\frac{100}{50} \right)^h \quad \dots(2)$$

From (1) and (2),

$$K_1 = 107.75 \times 10^{-3}$$

a) $P_e = 107.75 \times 10^{-3} (120)^2 = \underline{1551.6 \text{ W}}$

$$P_h = 1938.6 - 1551.6 = \underline{387 \text{ W}}$$

b) $P_h = 1400 - 107.75 \times 10^{-3} (100)^2 = \underline{322.5 \text{ W}}$

c) The hysteresis losses will be increased but not substantially.

Original: $P_h = K_2 (60) 2^h$

New: $P_h = K_2 (50)(2.4)^h$

PROBLEM 2-5

a) 240/120 V

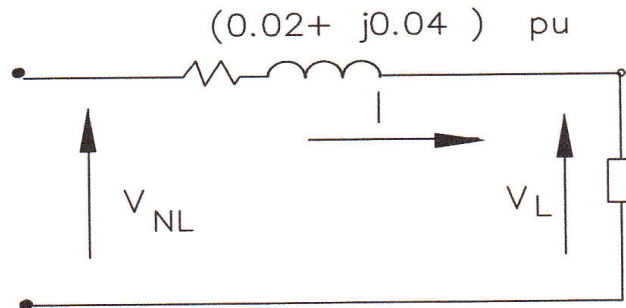


FIG. SP2-5

b) I) $V_L = V_{NL} - I Z$

$$V_L \angle 0^\circ = 1 \angle \beta - 1 \angle 0^\circ (0.02 + j0.04)$$

$$\beta = 2.3^\circ, \quad V_L = 0.98 \text{ p u}$$

$$\text{Reg} = \frac{1 - 0.98}{0.98} (100) = \underline{\underline{2.04\%}}$$

$$\text{II) } V_L \angle 0^\circ = 1 \angle \beta - 1 \angle 36.9^\circ (0.045 \angle 63.4^\circ)$$

$$V_L \angle 0^\circ = 1 \angle \beta - 0.045 \angle 100.3^\circ$$

$$\beta = 2.52^\circ, \quad V_L = 1.007 \text{ p u}$$

$$\text{Reg} = \frac{1 - 1.007}{1.007} (100) = \underline{\underline{-0.698\%}}$$

$$\text{III) } V_L \angle 0^\circ = 1 \angle \beta - 1 \angle -36.9^\circ (0.045 \angle 63.4^\circ)$$

$$V_L \angle 0^\circ = 1 \angle \beta - 0.045 \angle 26.6^\circ$$

$$\beta = 1.1^\circ, \quad V_L = 0.96 \text{ p u}$$

$$\text{Reg} = \frac{1 - 0.96}{0.96} (100) = \underline{\underline{4.2\%}}$$

PROBLEM 2-6

$$\text{a) } V_{nl} = 1 \angle 0^\circ + 1.0 \angle -25.8^\circ \times (0.015 + j0.06)$$

$$= 1.0407 \angle 2.6^\circ \text{ p u}$$

$$\text{Regulation} = \frac{1.0407 - 1.0}{1.0} (100) = \underline{\underline{4.07\%}}$$

$$P_L = 0.9 \left(\frac{1 - 0.97}{0.97} \right) = 0.0278 \text{ p u}$$

Actual:

$$P_{cl} = 0.0278 - 0.015 = 0.0128 \text{ p u}$$

X: nominal core loss

$$X (1.0407)^2 = 0.0128$$

$$X = \underline{\underline{0.013 \text{ p u}}}$$

- b) Set the no-load voltage taps to 1.05 p u

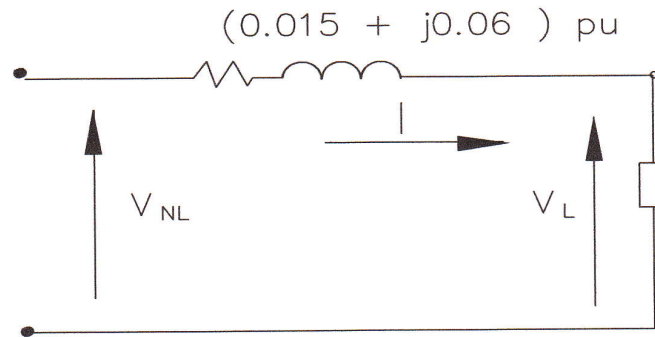


FIG. SP2-6

PROBLEM 2-7

$$\begin{aligned} \text{a)} \quad I_{AB} &= 60 \left(\frac{480}{\sqrt{3}} \right) \frac{1}{4160} \\ &= \underline{4.0 \text{ A}} \end{aligned}$$

$$\text{b)} \quad I_{AB} = I_{CB} = \underline{4.0 \text{ A}}$$

$$\text{c)} \quad I_a = \frac{100}{\sqrt{3}(0.48)(0.80)(0.9)} = \underline{167.06 \text{ A}}$$

$$I_{AB} = I_{BC} = I_{CA} = 167.06 \left(\frac{480}{\sqrt{3}} \right) \frac{1}{4160} = \underline{11.1 \text{ A}}$$

PROBLEM 2-8

$$\text{a)} \quad Z_{bL} = \frac{(0.48)^2}{2} = 0.1152 \, \Omega/\emptyset, \quad Z_{bH} = \frac{(25)^2}{2} = 312.5 \, \Omega/\emptyset$$

$$X_s = \frac{(25)^2}{500} \left(\frac{1}{312.5} \right) = 0.004 \text{ p u}$$

$$Z_A = \frac{5 + j8}{312.5} = \underline{(0.016 + j0.0256) \text{ p u}}$$

$$Z_B = \frac{0.005 + j0.01}{0.1125} = \underline{(0.0434 + j0.0867) \text{ p u}}$$

$$\text{b) } I_1 = \frac{500}{\sqrt{3}(0.48)(0.95)} = 633.1 \angle -36.9^\circ \text{ A}$$

$$I_2 = \frac{600}{\sqrt{3}(0.48)(0.9)(0.9)} = 891 \angle -25.8^\circ \text{ A}$$

$$I_t = I_1 + I_2 = 1517.2 \angle -30.4^\circ \text{ A}$$

$$I_{bL} = \frac{2000}{\sqrt{3}(0.48)} = 2405.6 \text{ A}$$

$$|I_t| = \frac{1517.2}{2405.6} = 0.6307 \text{ p u}$$

$$R_t = 0.016 + 0.01 + 0.0434 = 0.0694 \text{ p u}$$

$$X_t = 0.004 + 0.0256 + 0.062 + 0.0867 = 0.1784$$

$$\text{Loss} = (0.6307)^2(0.0694) = 0.0276 \text{ p u}$$

$$= 0.0276 (2000) = \underline{55.2 \text{ kW}}$$

$$\text{c) } V_s = 1 \angle 0^\circ + 0.6307 \angle -30.4^\circ (0.069 + j0.1777)$$

$$= 1 \angle 0^\circ + 0.121 \angle 38.5^\circ; \quad |V_s| = 1.1 \text{ p u} = \underline{27.4 \text{ kV}}$$

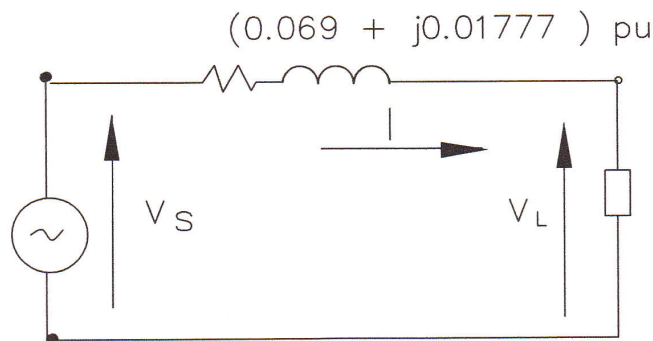


FIG. SP2-8

PROBLEM 2-9

b) The motor current is:

$$I_m = \frac{25}{\sqrt{3}(0.55)(0.8)(0.95)} = 34.53 \text{ A}$$

Take as reference V_{ab} , Phase sequence ABC.

$$I_{m_a} = 34.53 \angle 6.9^\circ \text{ A}$$

$$I_{m_b} = 34.53 \angle -113.1^\circ \text{ A}$$

$$I_{m_c} = 34.53 \angle 126.9^\circ \text{ A}$$

Load P₃

$$I_{ab} = \frac{20}{0.55} \angle 0^\circ = 36.36 \angle 0^\circ \text{ A}$$

Load P₂

$$I_{bn} = \frac{10\sqrt{3}}{0.55(0.9)(0.9)} = 38.88 \angle -145.8^\circ - 30 = 38.88 \angle -175.8^\circ \text{ A}$$

Load P₁

$$I_{cn} = \frac{12\sqrt{3}}{0.55(0.8)(0.95)} = 49.72 \angle 53.1^\circ \text{ A}$$

The line currents through the secondary of the transformer are:

$$\begin{aligned} I_{a-a} &= I_{ma} + I_{ab} \\ &= 34.53 \angle 6.9^\circ + 36.36 \angle 0^\circ = \underline{\underline{70.77 \angle 3.3^\circ \text{ A}}} \end{aligned}$$

$$\begin{aligned} I_{b-b} &= I_{mb} + I_{bn} - I_{ab} \\ &= 34.53 \angle -113.1^\circ + 38.88 \angle -175.8^\circ - 36.36 \angle 0^\circ \\ &= \underline{\underline{95.2 \angle -158.7^\circ \text{ A}}} \end{aligned}$$

$$\begin{aligned}
 I_{c-ca} &= I_{mc} + I_{cn} \\
 &= 34.53 \angle 126.9^\circ + 49.72 \angle 53.1^\circ \\
 &= \underline{\underline{68 \angle 82.3^\circ \text{ A}}}
 \end{aligned}$$

Primary of transformer

The per-phase turn's ratio is:

$$\frac{4800}{550/\sqrt{3}} = 15.1$$

Then

$$I_A = \frac{70.77}{15.1} = \underline{\underline{4.68 \text{ A}}}$$

$$I_B = \frac{95.2}{15.1} = \underline{\underline{6.30 \text{ A}}}$$

$$I_C = \frac{68}{15.1} = \underline{\underline{4.5 \text{ A}}}$$

- a) The rating of the transformer must be based on the highest winding current requirement.

$$I_{b-b} = 95.2 \text{ A}$$

$$|S| = \sqrt{3} (0.55) (95.2) = \underline{\underline{90.70 \text{ kVA}}}$$

Use a commercially available transformer whose capacity is

$$3-\emptyset, \quad 4800\text{-}550/317 \text{ V}, \quad 112.5 \text{ kVA}$$

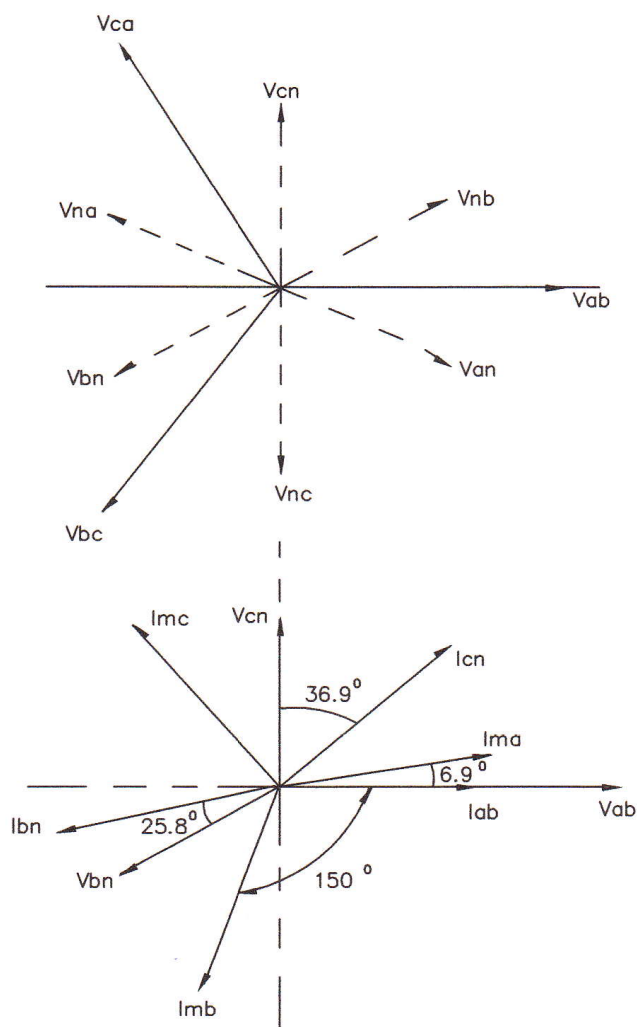


FIG. SP2-9

PROBLEM 2-10

a) $V_1 = 3 I_a + 2 I_b + I_c \quad \dots(1)$

$V_2 = 2 I_a + 3 I_b + I_c \quad \dots(2)$

$V_3 = I_a + I_b + I_c \quad \dots(3)$

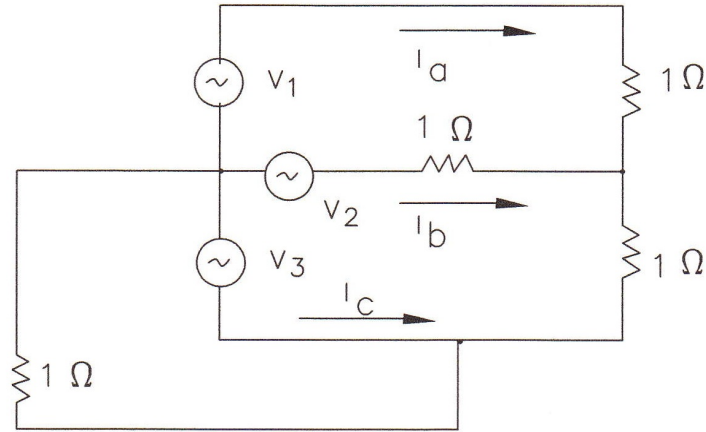


FIG. SP2-10(a)

$$D = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3$$

$$\begin{aligned} N_1 &= \begin{vmatrix} V_1 & 2 & 1 \\ V_2 & 3 & 1 \\ V_3 & 1 & 1 \end{vmatrix} = 2V_1 - V_3 - V_2 \\ &= \frac{480}{\sqrt{3}} (2\angle 0^\circ - 1\angle 120^\circ - 1\angle -120^\circ) \\ &= 3 \left(\frac{480}{\sqrt{3}} \right) V \end{aligned}$$

$$I_a = \frac{N_1}{D} = 3 \frac{480}{\sqrt{3}} \left(\frac{1}{3} \right) = \underline{\underline{277.1 \text{ A}}}$$

Similarly

$$N_2 = -3 \left(\frac{480}{\sqrt{3}} \right) \angle 60^\circ$$

$$I_b = \underline{\underline{-277.1 \angle 60^\circ \text{ A}}}$$

$$N_3 = \frac{480}{\sqrt{3}} (6) \angle 120^\circ$$

$$I_c = \frac{6}{3} \frac{480}{\sqrt{3}} \angle 120^\circ$$

$$= \underline{\underline{554.3 \angle 120^\circ \text{ A}}}$$

To check

$$V_3 = I_a + I_b + I_c$$

$$= \frac{480}{\sqrt{3}} \angle 120^\circ = 277.1 \angle 0^\circ - 277.1 \angle 60^\circ + 554.3 \angle 120^\circ$$

$$\text{R H S} = \frac{480}{\sqrt{3}} \angle 120^\circ, \quad \text{OK}$$

$V_{\text{mf-g}} = 0$, no danger to personnel.

Fuse in phase “c” will blow.

b)

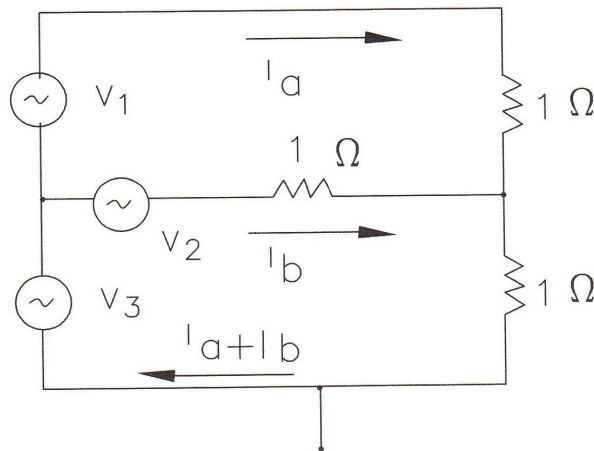


FIG. SP2-10(b)

$$V_1 - I_a - (I_a + I_b) - V_3 = 0 \quad \dots(4)$$

$$V_2 - I_b - (I_a + I_b) - V_3 = 0 \quad \dots(5)$$

From Eqs. (4) and (5):

$$I_a = -\frac{1}{3}[V_2 + V_3 - 2V_1] = -\frac{1}{3}\left(\frac{480}{\sqrt{3}}\right)[1\angle -120^\circ + 1\angle 120^\circ - 2\angle 0^\circ]$$

$$= \underline{277.1 \text{ A}}$$

$$I_b = V_1 - V_3 - 2\left(\frac{480}{\sqrt{3}}\right)$$

$$= \left(\frac{480}{\sqrt{3}}\right)[1\angle 0^\circ - 1\angle 120^\circ - 2] = 277.1\angle 120^\circ \text{ A}$$

$$I_c = -(I_b + I_a) = 277.1 (1\angle 0^\circ + 1\angle 120^\circ) = 277.1\angle 120^\circ \text{ A}$$

$$V_{mf-g} = 277.1 \text{ V}$$

Fuses will not blow.

c) $I_c = \infty$

$$V_{mf-g} = 0$$

Fuse in line “c” will blow.

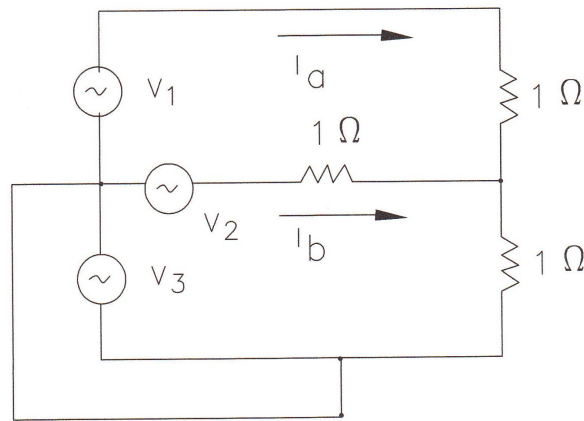


FIG. SP2-10(c)

d) Nominal three-phase operation.

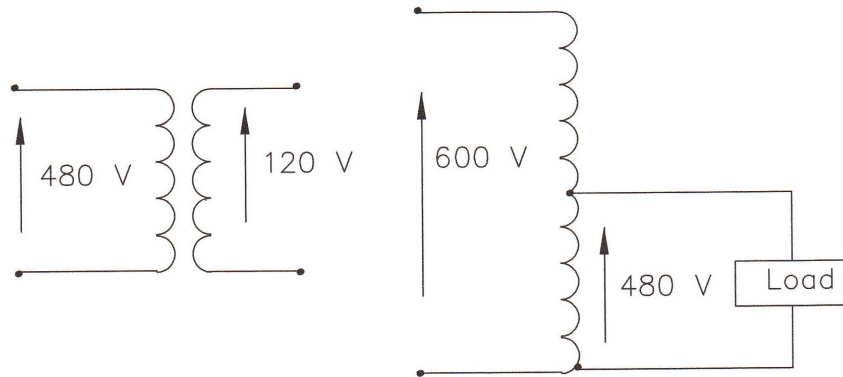
$$V_{mf-g} \approx \underline{277.1 \text{ V}}$$

None of the fuses will blow.

PROBLEM 2-11

a) $S = 600 (41.67) = \underline{25 \text{ kVA}}$

b) $25 - 5 = \underline{20 \text{ kVA}}$

**FIG. SP2-11****PROBLEM 2-12**

a) $S = 3.4 (200) = \underline{680 \text{ kVA}}$

$680 - 200 = 480 \text{ kVA conducted}$

$\frac{480}{680} (100) = \underline{70.6\%}$

b) $P_L = P_T \frac{(1-\eta)}{\eta} = 200(0.85) \left(\frac{1-0.96}{0.96} \right) = 7.08 \text{ kW}$

$\eta = \frac{680 (0.85)}{680 (0.85) + 7.08} = \underline{0.99}$

c) $Z_b = \frac{3400}{(0.02 + j0.06)5} = \underline{10.75 \text{ kA}}$

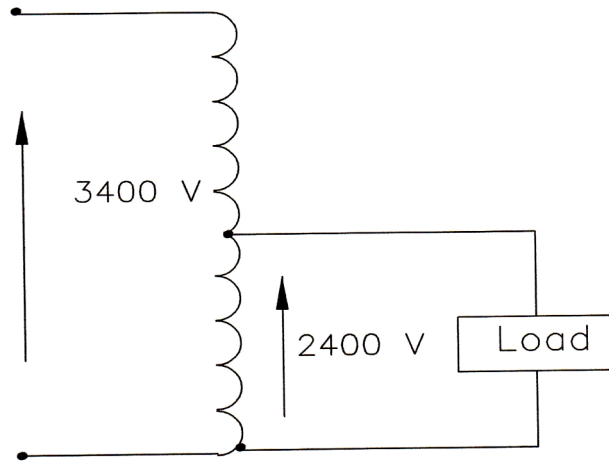


FIG. SP2-12

PROBLEM 2-13

$$S_b = 1000 \text{ kVA} \quad V_b = 480 \text{ V} \quad I_b = \frac{1000}{0.480} = 2.08 \text{ kA}$$

$$\text{T-1} \quad Z_{e1} = (0.02 + j0.04) \frac{1000}{550} = 0.0364 + j0.0727 = 0.0813 \angle 63.4^\circ \Omega$$

$$\begin{aligned} \text{T-2} \quad Z_{e2} &= (0.02 + j0.05) \left(\frac{1000}{550} \right) \left(\frac{468}{480} \right)^2 = 0.0346 + j0.0864 \\ &= 0.0931 \angle 68.2^\circ \Omega \end{aligned}$$

$$\frac{V_1}{V_2} = \frac{480}{468} = 1.0256$$

$$V_1 \angle \beta = 1 \angle 0^\circ + I_1 Z_{e1} \quad \dots(1)$$

$$V_2 \angle \beta = 1 \angle 0^\circ + I_2 Z_{e2} \quad \dots(2)$$

$$\frac{V_1}{V_2} = 1.0256 = \frac{1 \angle 0^\circ + I_1 Z_{e1}}{1 \angle 0^\circ + I_2 Z_{e2}} \quad \dots(3)$$

$$I = I_1 + I_2 \quad \dots(4)$$

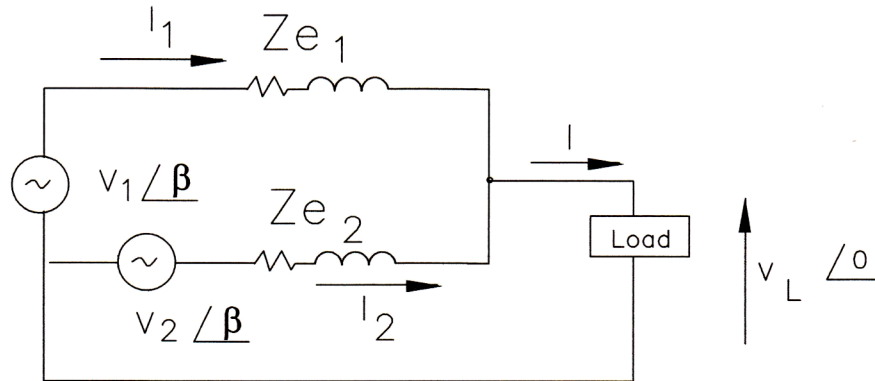


FIG. SP2-13

From (3) and (4)

$$I_1 = 0.655 \angle -32.2^\circ \text{ p.u.}$$

$$I_2 = 0.36 \angle -14^\circ \text{ p.u.}$$

$$\begin{aligned} \text{a) } V_1 \angle \beta &= 1 \angle 0^\circ + 0.655 \angle -32.2^\circ (0.0813 \angle 63.4^\circ) \\ &= 1.0455 + j0.0276 = 1.0459 \angle 1.5^\circ \\ &= 1.0459 (4160) = \underline{4350.8 \text{ V, L-L}} \end{aligned}$$

$$\text{b) T-1 } I_1 = 0.655(2.08) = \underline{1.36 \text{ kA}}$$

$$\text{T-2 } I_2 = 0.36(2.08) = \underline{750 \text{ A}}$$

$$\text{c) T-1 } S_1 = 0.655 (1) [1000] = \underline{655 \text{ kVA}}$$

$$\text{T-2 } S_2 = 0.36 (1000) = \underline{360 \text{ kVA}}$$

$$\text{d) } V_2 = \frac{V_1}{1.0256} = \frac{1.0459}{1.0256} = 1.0198$$

$$\begin{aligned} I &= \frac{V_1 - V_2}{Z_1 + Z_2} = \frac{1.0459 \angle 1.5^\circ - 1.0198 \angle 1.5^\circ}{0.0364 + 0.0346 + j(0.0727 + 0.0864)} \\ &= \frac{0.0261 + j0.0007}{()} = \frac{0.0261 \angle 1.5^\circ}{0.1742 \angle 66^\circ} \\ &= 0.1498 \angle -64.5^\circ \\ &= 0.1498 (2.08) = \underline{312 \text{ A}} \end{aligned}$$

$$\begin{aligned}
 \text{e)} \quad I &= I_1 + I_2 = \frac{1.0459 \angle 1.5^\circ}{0.0813 \angle 63.4^\circ} + \frac{1.0198 \angle 1.5^\circ}{0.0931 \angle 68.2^\circ} \\
 &= 12.8647 \angle -61.9^\circ + 10.9538 \angle -66.7^\circ \\
 &= 10.3922 - j21.4088 = 23.799 \angle -64.1^\circ \\
 &= 23.799 (138.8) = \underline{\underline{3.3 \text{ kA}}}
 \end{aligned}$$

PROBLEM 2-14

$$\text{a)} \quad (1) \quad Z_{bH} = \frac{(25)^2}{1} = 625 \, \Omega/\emptyset$$

$$Z_f = \frac{20 + j30}{625} = (0.032 + j0.048) \text{ p u}$$

$$Z_t = 0.032 + j0.048 + j0.06 = 0.032 + j0.108 = 0.1126 \angle 73.5^\circ \text{ p u}$$

$$V_L = \angle 0^\circ = 1 \angle \beta - 0.1126 \angle 73.5^\circ (1 \angle -25.8^\circ)$$

$$\beta = 4.77^\circ, \quad V_L = 0.92 \text{ p u}$$

$$V_L = \underline{\underline{441.9 \text{ V, L-L}}}$$

$$I = \frac{1000}{\sqrt{3}(0.48)} = \underline{\underline{1202.8 \text{ A}}}$$

$$I_m = 1202.8 \left(\frac{5}{1500} \right) = \underline{\underline{4 \text{ A}}}$$

$$(2) \quad I = \frac{1}{0.1126} = 8.88 \text{ p u}$$

$$I = 0.88 (1202.8) = \underline{\underline{10.7 \text{ kA}}}$$

$$I_m = 10.7 \left(\frac{5}{1.5} \right) = \underline{\underline{35.6 \text{ A}}}, \quad V = 0 \text{ Volts}$$

$$(3) \quad X_s = \frac{(25)^2}{500} = 1.25 \, \Omega/\emptyset \quad X_{s \text{ p u}} = \frac{1.25}{625} = 0.002 \text{ p u}$$

$$X_t = 0.002 + 0.048 + 0.06 = 0.11$$

$$V_L = \angle 0^\circ = 1 \angle \beta - 1.0 \angle 25.84^\circ (0.032 + j0.11)$$

$$\beta = 6.5^\circ, \quad V_L = 1.02 \text{ p.u.}, \quad V_L = \underline{489.2 \text{ V}}$$

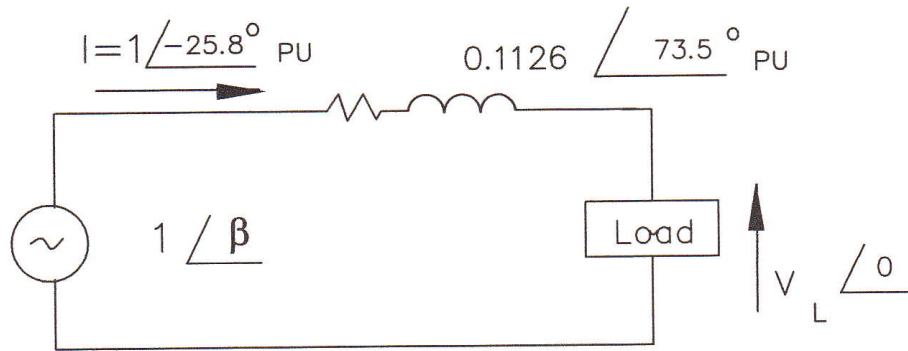


FIG. SP2-14

PROBLEM 2-15

$$\begin{aligned} \text{Factor: } \frac{(1 + r_x)^n - 1}{r_x (1 + r_x)^n} &= \frac{(1 + 0.0185)^5 - 1}{0.0185 (1.0185)^5}, & r_x &= \frac{1.1}{1.08} - 1 \\ &= 4.77338, & &= 1.0185 - 1 \\ & & &= 0.0185 \end{aligned}$$

Note: In some publications the exponent n in the denominator is replaced by $\left(n - \frac{1}{2}\right)$.

a) Manufacturer (A):

$$\text{Losses } 3 + \left(\frac{3}{4}\right)^2 (12) = 9.75 \text{ kW}$$

$$A = 9.75 (365)(24)(0.06) = \$5124.6$$

$$P = 5124.6 (4.7338) = \$24,258.8$$

$$P_T = 30000 + 24258.8 = \underline{\$54,259}$$

Manufacturer (B):

$$\text{Losses } 2.5 + \left(\frac{3}{4}\right)^2 9.5 = 7.8438 \text{ kW}$$

$$A = 7.8438 (365)(24)(0.06) = \$4122.675$$

$$P = 4122.675 (4.7338) = \$19,515.9$$

$$P_T = 35000 + 19515.9 = \underline{\underline{\$54,516}}$$

b) Similarly,

Manufacturer (A): \$44,928

Manufacturer (B): \$47,129

c) Manufacturer (A): \$48,660

Manufacturer (B): \$49,928

PROBLEM 2-16

The rms value (I) of the line current is

$$I = \sqrt{\left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{30}{\sqrt{2}}\right)^2 + \left(\frac{15}{\sqrt{2}}\right)^2 + \left(\frac{10}{\sqrt{2}}\right)^2} = 74.9166 \text{ A}$$

The per unit values of the component line current are

$$I_1 = \left(\frac{100/\sqrt{2}}{74.9166}\right) = 0.9439$$

$$I_3 = \left(\frac{30/\sqrt{2}}{74.9166}\right) = 0.2832$$

$$I_5 = \left(\frac{15/\sqrt{2}}{74.9166}\right) = 0.1416$$

$$I_7 = \left(\frac{10/\sqrt{2}}{74.9166} \right) = 0.0944$$

and

$$k = (0.9439)(1)]^2 + [(0.2832 \times 3)^2 + (0.1416 \times 5)^2 + (0.0944 \times 7)^2 \\ = \underline{\underline{2.55}}$$

A k-4 type of transformer is required.