Chapter 1: Speaking Mathematically

Many college students appear to have difficulty using and interpreting language involving if-then statements and quantification. Section 1.1 is a gentle introduction to the relation between informal and formal ways of expressing important kinds of mathematical statements. Experience with the exercises in the section is meant as a warm-up to prepare students to master the linguistic aspects of learning mathematics to help them come to understand the meaning of mathematical statements and evaluate their truth or falsity in later chapters. Sections 1.2 and 1.3 form a brief introduction to the language of sets, relations, and functions. Covering them at the beginning of the course can help students relate discrete mathematics to the pre-calculus or calculus they have studied previously while broadening their perspective to include a larger proportion of discrete examples.

Proofs of set properties, such as the distributive laws, and proofs of properties of relations and functions, such as transitivity and surjectivity, are considerably more complex than the examples used in this book to introduce students to the idea of mathematical proof. Thus set theory as a theory is left to Chapter 6, properties of functions to Chapter 7, and properties of relations to Chapter 8. Instructors who wish to do so could cover Section 1.2 just before starting Chapter 6 and Section 1.3 just before starting Chapter 7.

An aspect of students' backgrounds that may surprise college and university mathematics instructors concerns their understanding of the meaning of "real number." When asked to evaluate the truth or falsity of a statement about real numbers, it is not unusual for students to think only of integers. Thus an informal description of the relationship between real numbers and points on a number line is given in Section 1.2 on page 8 to illustrate that there are many real numbers between any pair of consecutive integers, Examples 3.3.5 and 3.3.6 on page 121 show that while there is a smallest positive integer there is no smallest positive real number, and the discussion on pages 433 and 434 (preceding the proof of the uncountability of the real numbers between 0 and 1) describes a procedure for approximating the (possibly infinite) decimal expansion for an arbitrarily chosen point on a number line.