

INSTRUCTOR'S SOLUTIONS MANUAL

DISCRETE MATHEMATICS

FIFTH EDITION

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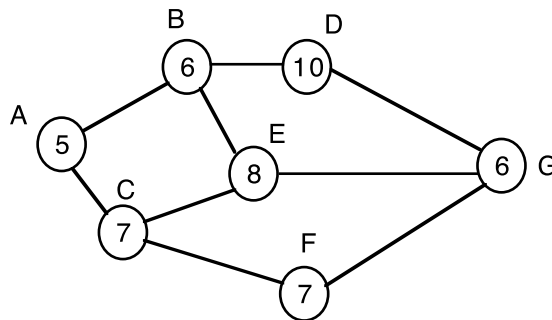
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Chapter 1

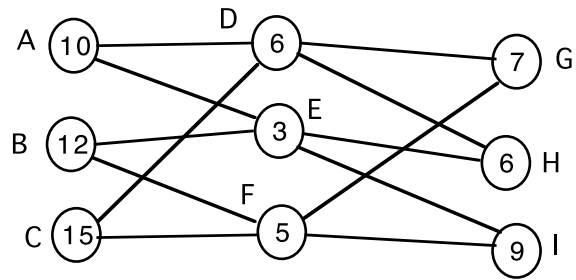
An Introduction to Combinatorial Problems and Techniques

1.1 THE TIME TO COMPLETE A PROJECT

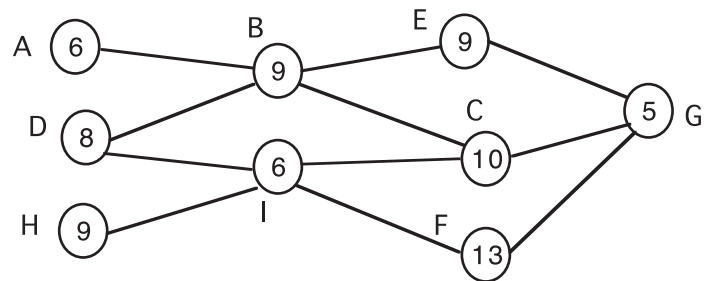
- 2. 31; A-B-E-G
- 4. 39; A-C-G-H
- 6. 16; B-D-F-H
- 8. 27; A-D-E-H
- 10. 27; A-B-D-G



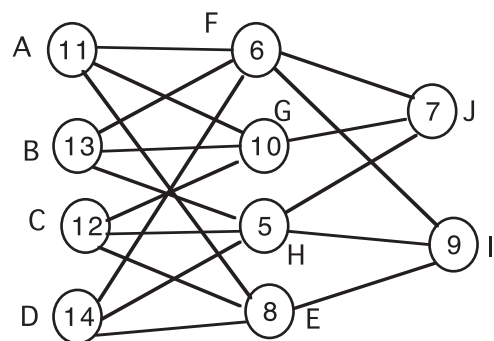
12. 29; C-F-I



14. 33; H-I-F-G



16. 31; D-E-I



18. 20 minutes

1.2 A MATCHING PROBLEM

- | | | | |
|----------------|-------------------|----------|----------|
| 2. 720 | 4. 210 | 6. 84 | 8. 1680 |
| 10. 19,958,400 | 12. $\frac{1}{8}$ | 14. 5040 | 16. 126 |
| 18. 210 | 20. 119 | 22. 1320 | 24. 5040 |
| 26. 240 | 28. 1200 | | |

1.3 A KNAPSACK PROBLEM

- | | | | |
|--|----------|---------|--------|
| 2. T | 4. F | 6. F | 8. F |
| 10. T | 12. T | 14. T | 16. no |
| 18. yes; 32 | | | |
| 20. $\emptyset$, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}; 16 | | | |
| 22. 128 | 24. 1024 | 26. 256 | 28. 26 |
| 30. {1, 4, 6, 7, 8, 9, 10, 11, 12} | | | |

1.4 ALGORITHMS AND THEIR EFFICIENCY

- | | |
|-----------------------------------|-------------------------|
| 2. yes; 0 | 4. no |
| 6. no | 8. -1, 9, 84; 3, 17, 84 |
| 10. -4, -4, 41, 95; 2, 11, 33, 95 | 12. 111000 |
| 14. 001010 | |

16.

k	j	a_1	a_2	a_3
3		1	1	1
2		1	1	1
1		1	1	1
0		1	1	1

18.

k	j	a_1	a_2	a_3	a_4
4		1	1	1	0
		1	1	1	1

20. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	j	k
23	5	(17)	(12)	1	3
23	(5)	(12)	17		2
(23)	(5)	12	17		1
5	23	(12)	(17)	2	3
5	(23)	(12)	17		2
5	12	(23)	(17)	3	3
5	12	17	23		

22. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	a_5	j	k
88	2	75	(10)	(48)	1	4
88	2	(75)	(10)	48		3
88	(2)	(10)	75	48		2
(88)	(2)	10	75	48		1
2	88	10	(75)	(48)	2	4
2	88	(10)	(48)	75		3
2	(88)	(10)	48	75		2
2	10	88	(48)	(75)	3	4
2	10	(88)	(48)	75		3
2	10	48	(88)	(75)	4	4
2	10	48	75	88		

24. 6.5 years, 2.7 seconds

26. 2.3×10^{10} years, 12.5 seconds

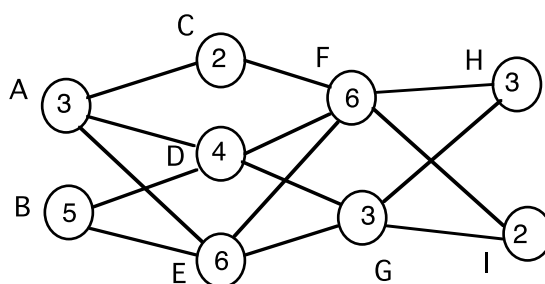
28. $4n - 3$

30. $3n - 2$

32. $-4, -4, 41, 95$

SUPPLEMENTARY EXERCISES

2. 20; B-E-F-H



4. 336

6. 40

8. 14040

10. T

12. F

14. T

16. T

18. 16

20. no

22. yes; 0

24. $-5, 7, 7, 88$

26. $\emptyset, \{4\}, \{3\}, \{3, 4\}, \{2\}, \{2, 4\}, \{2, 3\}, \{2, 3, 4\}, \{1\}, \{1, 4\}, \{1, 3\}, \{1, 3, 4\}, \{1, 2\}, \{1, 2, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}$

28. 4.92×10^8 years

30. 4

32. $4r - 3$

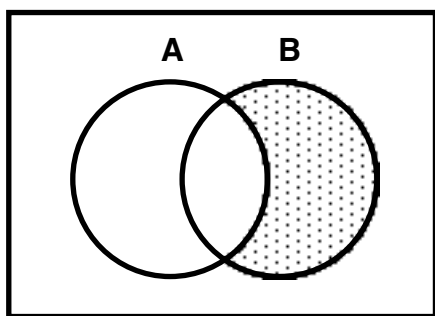
Chapter 2

Sets, Relations, and Functions

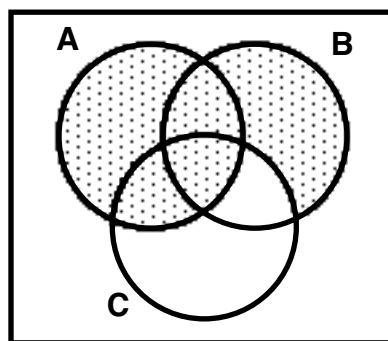
2.1 SET OPERATIONS

2. $A \cup B = \{1, 2, 4, 5, 6, 7, 9\}$, $A \cap B = \{1, 4, 6, 9\}$, $A - B = \emptyset$, $\overline{A} = \{2, 3, 5, 7, 8\}$, and $\overline{B} = \{3, 8\}$
4. $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$, $A \cap B = \{7, 9\}$, $A - B = \{3, 4, 6, 8\}$, $\overline{A} = \{1, 2, 5\}$, and $\overline{B} = \{1, 3, 4, 6, 8\}$
6. $\{(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$
8. $\{(p, a), (p, c), (p, e), (q, a), (q, c), (q, e), (r, a), (r, c), (r, e), (s, a), (s, c), (s, e)\}$

10.



12.



14. $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1\}$
16. $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1\}$

2. reflexive and symmetric
4. reflexive, symmetric, and transitive
6. reflexive and symmetric
8. none
10. reflexive and symmetric
12. reflexive and transitive

14. The equivalence class of R containing ABCD consists of the string ABC and the strings of 4 letters having A as their first letter and C as their third letter. There are $26^2 = 676$ distinct equivalence classes of R .

16. The equivalence class of R containing $\{1, 2, 3\}$ is the set containing the following four elements of S : $\{1, 3\}$, $\{1, 2, 3\}$, $\{1, 2, 3, 4\}$, and $\{1, 3, 4\}$. There are 8 different equivalence classes of R , namely the sets consisting of the elements S , $S \cup \{2\}$, $S \cup \{4\}$, and $S \cup \{2, 4\}$ for every $S \subseteq \{1, 3, 5\}$.

18. The equivalence class of R containing $(5, 8)$ is the set

There are infinitely many distinct equivalence classes of R , namely, the sets of the form

20. $\{(1, 1), (1, 3), (1, 6), (3, 1), (3, 3), (3, 6), (6, 1), (6, 3), (6, 6), (2, 2), (2, 5), (5, 2), (5, 5), (4, 4)\}$
24. The equivalence classes have the form $E_1 \times E_2$, where E_i is an equivalence class of R_i .
28. There are 5 partitions of a set with three elements.
32. Let S be a nonempty set and R an equivalence relation on S . Then there is a function f with domain S such that $s_1 R s_2$ if and only if $f(s_1) = f(s_2)$.

2.3 PARTIAL ORDERING RELATIONS

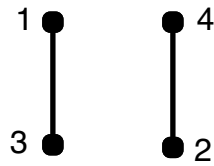
2. not antisymmetric

4. partial ordering

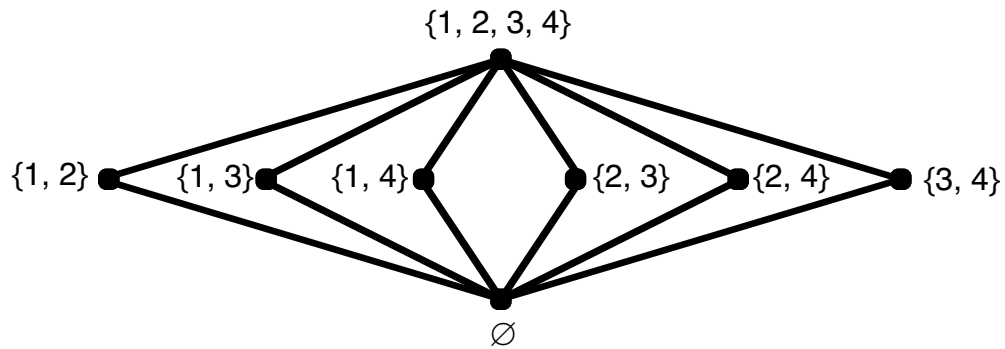
6. not antisymmetric

8. not antisymmetric

10.



12.



14. $R = \{(a, a), (b, b), (b, a), (c, c), (c, b), (c, a), (d, d), (d, a)\}$

16. $R = \{(1, 1), (2, 2), (2, 1), (2, 4), (4, 4), (8, 8), (8, 4)\}$

18. The maximal elements are $\{1\}$, $\{2\}$, and $\{3\}$; the only minimal element is $\{1, 2, 3\}$.

20. The only minimal element is 0; there are no maximal elements.

22. One possible sequence is 1, 3, 2, 4.

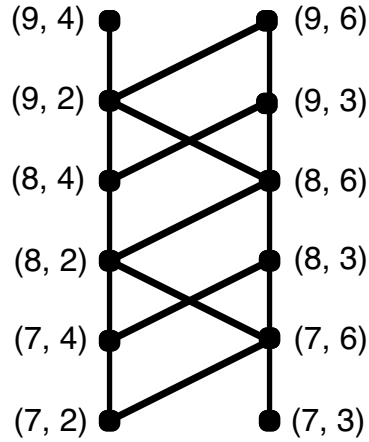
24. One possible sequence is 1, 3, 2, 6, 4, 12.

26. Let S denote the set of residents of Illinois and R be defined so that $x R y$ means that x is a sister of y .

28. Every prime integer is a minimal element; there are no maximal elements.

30. $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)$

32.



36. (a) Suppose that y is element of S such that $x R y$ is false. If there is no element y_1 in S such that $y_1 R y$, then y is a minimal element of S , contradicting that x is the unique minimal element of S . Thus there must be such an element y_1 . If there is no element y_2 in S such that $y_2 R y_1$, then y_1 is a minimal element of S , another contradiction. So there must be such an element y_2 . Because S is finite, continuing in this manner must produce a minimal element y_k of S different from x . Because x is the unique minimal element of S , the assumption that there is an element y of S such that $x R y$ is false must be incorrect. Thus $x R s$ is true for every $s \in S$.
- (b) Let Z denote the set of integers and $S = Z \cup \{\emptyset\}$. Let R be the relation defined on S by $x R y$ if and only if one of the following holds: (i) $x, y \in Z$ and $x \leq y$, or (ii) $x = y = \emptyset$. Then $\emptyset$ is the unique minimal element in S , but $\emptyset R z$ is false for every $z \in Z$.

40. 2^n 42. $2^n \cdot 3^{n(n-1)/2}$

2.4 FUNCTIONS

2. not a function with domain X 4. a function with domain X
6. a function with domain X 8. a function with domain X
10. a function with domain X 12. not a function with domain X
14. 4 16. 13 18. 8 20. 7
22. -1 24. 6 26. -2 28. 10
30. 0.78 32. 6.64 34. 3.22 36. -2.56
38. $gf(x) = g(f(x)) = \sqrt{x^2 + 1}$ and $fg(x) = f(g(x)) = x + 1$
40. $gf(x) = \frac{1}{3x}$ and $fg(x) = \frac{3}{x}$
42. $gf(x) = 5(2^x) - 2^{2x}$ and $fg(x) = 2^{5x-x^2}$
44. $gf(x) = \frac{4x-1}{2-x}$ and $fg(x) = \frac{3x-2}{3-x}$
46. one-to-one and onto 48. neither one-to-one nor onto
50. one-to-one but not onto 52. onto but not one-to-one
54. $f^{-1}(x) = \frac{x+2}{3}$ 56. $f^{-1}(x)$ does not exist.
58. $f^{-1}(x)$ does not exist. 60. $f^{-1}(x) = \sqrt[3]{x+1}$
62. For $Y = \{x \in X: x \neq 0\}$, we have $g^{-1}(x) = \frac{-1}{x}$.
64. If $n < m$, there are no one-to-one functions; and if $m \leq n$, there are
- $$P(n, m) = n(n-1)(n-2) \cdots (n-m+1)$$
- one-to-one functions.
68. Let $X = \{1\}$, $Y = \{2, 3\}$, and $Z = 4$. Define $f: X \rightarrow Y$ by $f(x) = 2$ for all $x \in X$ and $g: Y \rightarrow Z$ by $g(y) = 4$ for all $y \in Y$.

2.5 MATHEMATICAL INDUCTION

2. 7, 9, 13, 21, 37, 69
4. $x_n = \begin{cases} x & \text{if } n = 1 \\ x \cdot x^{n-1} & \text{if } n \geq 2 \end{cases}$
6. Let x_n denote the n th odd positive integer. Then $x_n = \begin{cases} 1 & \text{if } n = 1 \\ x_{n-1} + 2 & \text{if } n \geq 2. \end{cases}$
8. If $k = 1$, the sets $\{x_1, x_2, \dots, x_k\}$ and $\{x_2, x_3, \dots, x_{k+1}\}$ are disjoint.
10. If $k = 0$, the induction hypothesis cannot be applied to a^{k-1} .
28. $s_0 + s_1 + \dots + s_n = (n+1) \left(s_0 + \frac{nd}{2} \right)$

2.6 APPLICATIONS

- | | |
|--|----------|
| 2. 56 | 4. 924 |
| 6. 120 | 8. 715 |
| 10. n | 12. $r!$ |
| 14. 31 | 16. 4096 |
| 18. 128 | 20. 15 |
| 22. 36 | 24. 286 |
| 26. 126 | 28. 64 |
| 44. There are $\frac{n^2 + n + 2}{2}$ regions. | |

SUPPLEMENTARY EXERCISES

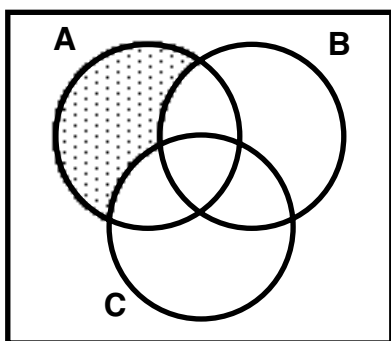
2. $\{1, 2, 3, 4, 5\}$

4. $\{1, 2, 4\}$.

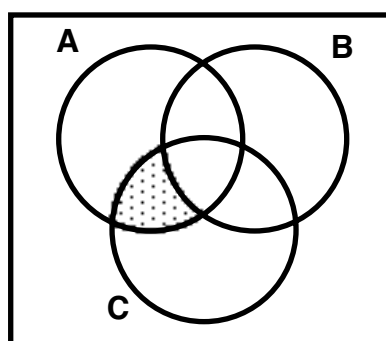
6. $\{1, 2, 4, 5, 6\}$

8. $\{2, 3\}$

10.



12.



14. $gf(x) = 2(x^3 + 1) - 5 = 2x^3 - 3$ and $fg(x) = (2x - 5)^3 + 1 = 8x^3 - 60x^2 + 150x - 124$

16. not a function with domain X

18. not a function with domain X

20. neither one-to-one nor onto

22. one-to-one and onto

24. f^{-1} does not exist.

26. $f^{-1} = \sqrt[3]{x-5}$

28. 84

30. 210

32. $\{1, 7\}, \{2, 6\}, \{3, 5\}, \{4\}, \{8\}$

34. the sets of positive and negative real numbers

36. 5

38. 6

40. (b) $a = \pm 1$

42. reflexive only

48. $x \vee x = x$ for all $x \in S$, $1 \vee y = y \vee 1 = y$ for all $y \in S$, $2 \vee 3 = 3 \vee 2 = 6$, $2 \vee 4 = 4 \vee 2 = 4$, $2 \vee 6 = 6 \vee 2 = 6$, $3 \vee 6 = 6 \vee 3 = 6$, $x \vee y$ is undefined in all other cases

Chapter 2 Supplementary Exercises

52. Let S denote the set of all subsets of $\{1, 2, 3\}$ that contain at most two elements, and let R be the relation “is a subset of.” If $A = \{1\}$, $B = \{2\}$, and $C = \{3\}$, then $A \vee B = \{1, 2\}$, $B \vee C = \{2, 3\}$, and $A \vee C = \{1, 3\}$, but $(A \vee B) \vee C$ does not exist.
54. $[n] = \{-n, n\}$
56.
$$f(x) = \begin{cases} x & \text{if } x \equiv 0 \pmod{3} \\ x - 1 & \text{if } x \equiv 1 \pmod{3} \\ x - 2 & \text{if } x \equiv 2 \pmod{3} \end{cases}$$