

## Chapter 2

### Density, Specific Gravity, Specific Weight

1. What is the specific gravity of 38°API oil?

$$38^\circ\text{API oil sp.gr.} = \frac{141.5}{131.5 + ^\circ\text{API}} = \frac{141.5}{131.5 + 38}$$

$$\text{sp. gr.} = \frac{141.5}{169.5} = 0.835$$

2. The specific gravity of manometer gage oil is 0.826. What is its density and its °API rating?

$$\text{sp. gr.} = 0.826; \quad \rho = 1000(0.826) = 826 \text{ kg/m}^3$$

$$\rho = 62.4(0.826) = 51.54 \text{ lbm/ft}^3$$

$$\text{sp. gr.} = \frac{141.5}{131.5 + ^\circ\text{API}} \quad 131.5 + ^\circ\text{API} = \frac{141.5}{0.826}$$

$$^\circ\text{API} = 171.3 - 131.5; \quad ^\circ\text{API} = 39.8^\circ\text{API} \approx 40^\circ\text{API}$$

3. What is the difference in density between a 50°API oil and a 40°API oil?

$$\text{sp. gr.} = \frac{141.5}{131.5 + ^\circ\text{API}} = \frac{141.5}{131.5 + 50} = 0.7796 \text{ for a } 50^\circ \text{ oil}$$

$$\text{sp. gr.} = \frac{141.5}{131.5 + ^\circ\text{API}} = \frac{141.5}{131.5 + 40} = 0.826 \text{ for a } 40^\circ \text{ oil}$$

$$0.825 - 0.7796 = 0.0455 \text{ density difference}$$

4. A 35°API oil has a viscosity of 0.825 N·s/m<sup>2</sup>. Express its viscosity in Saybolt Universal Seconds (SUS).

$$35^\circ\text{API oil sp. gr.} = \frac{141.5}{131.5 + ^\circ\text{API}} = \frac{141.5}{131.5 + 35} = 0.850$$

$$\mu = 0.825 \text{ N·s/m}^2 \quad \nu = \frac{\mu g_c}{\rho} = \frac{0.825}{0.850(1000)} = 10 \times 10^{-4}$$

Highly viscous; try

$$\nu = 0.2158 \times 10^{-6} (\text{SUS}) \quad \text{if } \text{SUS} > 215$$

$$\text{SUS} = \frac{10 \times 10^{-4}}{0.2158 \times 10^{-6}} = 4633 \text{ SUS}$$

5. Air is collected in a  $1.2 \text{ m}^3$  container and weighed using a balance as indicated in Figure P2.5. On the other end of the balance arm is  $1.2 \text{ m}^3$  of  $\text{CO}_2$ . The air and the  $\text{CO}_2$  are at  $27^\circ\text{C}$  and atmospheric pressure. What is the difference in weight between these two volumes?

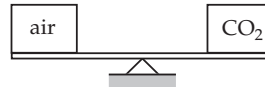


FIGURE P2.5.

Air at  $27^\circ\text{C} = 300 \text{ K}$  has  $\rho = 1.177 \text{ kg/m}^3$

$\text{CO}_2$  at  $27^\circ\text{C} = 300 \text{ K}$  has  $\rho = 1.797 \text{ kg/m}^3$

For a volume of  $1.2 \text{ m}^3$ , the weight of air is

$$(1.177 \text{ kg/m}^3)(1.2 \text{ m}^3)(9.81 \text{ m/s}^2) = 13.86 \text{ N}$$

For  $\text{CO}_2$

$$(1.797 \text{ kg/m}^3)(1.2 \text{ m}^3)(9.81 \text{ m/s}^2) = 21.15 \text{ N}$$

$$\text{Weight difference is } 21.15 - 13.86 = \boxed{7.29 \text{ N}}$$

6. A container of castor oil is used to measure the density of a solid. The solid is cubical in shape,  $30 \text{ mm} \times 30 \text{ mm} \times 30 \text{ mm}$ , and weighs  $9 \text{ N}$  in air. While submerged, the object weighs  $7 \text{ N}$ . What is the density of the liquid?

Castor Oil  $\rho = 960 \text{ kg/m}^3$

$$\frac{\text{buoyant force}}{\text{volume}} = \frac{mg_{\text{in air}} - mg_{\text{submerged}}}{V} = \rho g$$

$$\rho = \frac{9 - 7}{(0.03)^3} \frac{1}{9.81} = \boxed{7551 \text{ kg/m}^3}$$

7. A brass cylinder (Sp. Gr. = 8.5) has a diameter of  $25.4 \text{ mm}$  and a length of  $101.6 \text{ mm}$ . It is submerged in a liquid of unknown density, as indicated in Figure P2.7. While submerged, the weight of the cylinder is measured as  $3.56 \text{ N}$ . Determine the density of the liquid.

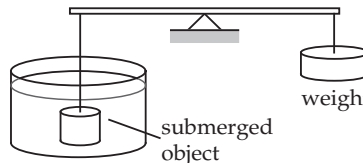


FIGURE P2.7.

$$\text{Buoyant force} = mg_{\text{in air}} - mg_{\text{submerged}} = mg - 0.8$$

$$\frac{\text{buoyant force}}{\text{volume}} = \frac{mg - 0.8}{V} = \rho g \quad V = \frac{\pi D^2}{4} h = \frac{\pi}{4} (0.0254)^2 (0.1016) = 5.15 \times 10^{-5} \text{ m}^3$$

$$mg = \rho_b V g = 8500 (5.15 \times 10^{-5}) (9.81) = 4.29 \text{ N}$$

$$\rho = \frac{mg - 0.8}{gV} = \frac{4.29 - 3.56}{9.81 (5.15 \times 10^{-5})}$$

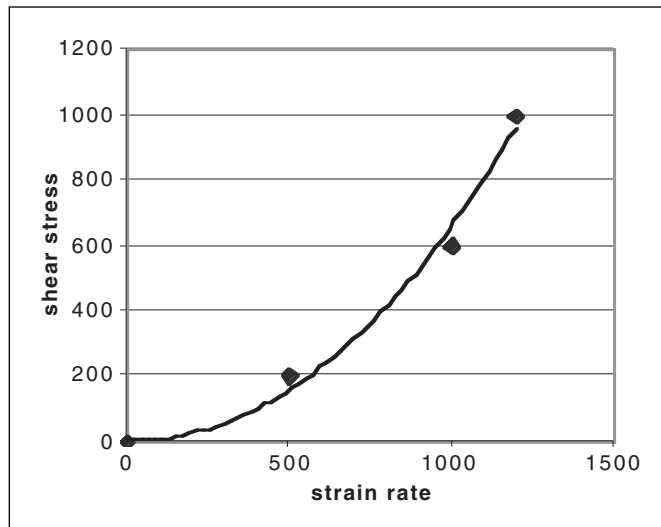
$$\boxed{\rho = 1454 \text{ kg/m}^3}$$

## Viscosity

8. Actual tests on Vaseline yielded the following data:

$\tau$ in $\text{N/m}^2$	0	200	600	1 000
$dV/dy$ in $1/\text{s}$	0	500	1 000	1 200

Determine the fluid type and the proper descriptive equation.



$$\tau = K \left( \frac{dV}{dy} \right)^n$$

Can be done instantly with spreadsheet; hand calculations follow for comparison purposes:

$dV/dy$	$\ln(dV/dy)$	$\tau$	$\ln \tau$	$\ln(\tau) \cdot \ln(dV/dy)$
0	—	0	—	.
500	6.215	200	5.298	32.93
1000	6.908	600	6.397	44.19
1200	7.090	1000	6.908	48.98
Sum	20.21		18.60	126.1

$$m = 3 \text{ Summation } (\ln(dV/dy))^2 = 136.6$$

$$b_1 = \frac{3(126.1) - 20.21(18.60)}{3(136.6) - 20.21^2} = 1.766$$

$$b_0 = \frac{18.60}{3} - 1.766 \frac{20.21}{3} = -5.697$$

$$K = \exp(b_0) = 0.00336; \quad n = b_1 = 1.766$$

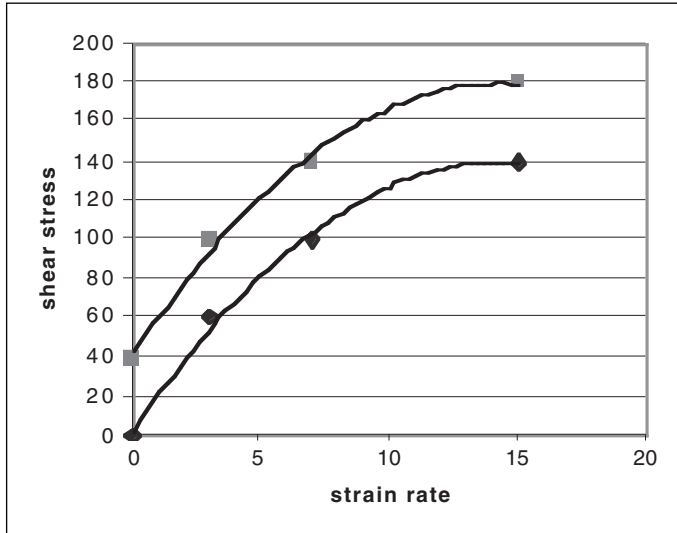
$$\tau = \tau_0 + K \left( \frac{dV}{dy} \right)^n = 0.00336 \left( \frac{dV}{dy} \right)^{1.766}$$

9. A popular mayonnaise is tested with a viscometer and the following data were obtained:

$\tau$ in g/cm <sup>2</sup>	40	100	140	180
$dV/dy$ in rev/s	0	3	7	15

Determine the fluid type and the proper descriptive equation.

The topmost line is the given data, but to curve fit, we subtract 40 from all shear stress readings.



$$\tau = \tau_0 + K \left( \frac{dV}{dy} \right)^n \quad \text{which becomes } \tau' = \tau - \tau_0 = K \left( \frac{dV}{dy} \right)^n$$

Can be done instantly with spreadsheet; hand calculations:

$dV/dy$	$\ln(dV/dy)$	$\tau$	$\tau'$	$\ln \tau'$	$\ln(\tau') \cdot \ln(dV/dy)$
0	—	40	0	—	—
3	1.099	100	60	4.094	4.499
7	1.946	140	100	4.605	8.961
15	2.708	180	140	4.942	13.38
Sum	5.753			13.64	26.84

$$m = 3 \quad \text{Summation } (\ln(dV/dy))^2 = 12.33$$

$$b_1 = \frac{3(26.84) - 5.753(13.64)}{3(12.33) - 5.753^2} = 0.526$$

$$b_0 = \frac{13.64}{3} - 0.526 \frac{5.753}{3} = 3.537$$

$$K = \exp(b_0) = 34.37; \quad n = b_1 = 0.526$$

$$\tau = \tau_0 + K \left( \frac{dV}{dy} \right)^n = 40 + 34.37 \left( \frac{dV}{dy} \right)^{0.526}$$

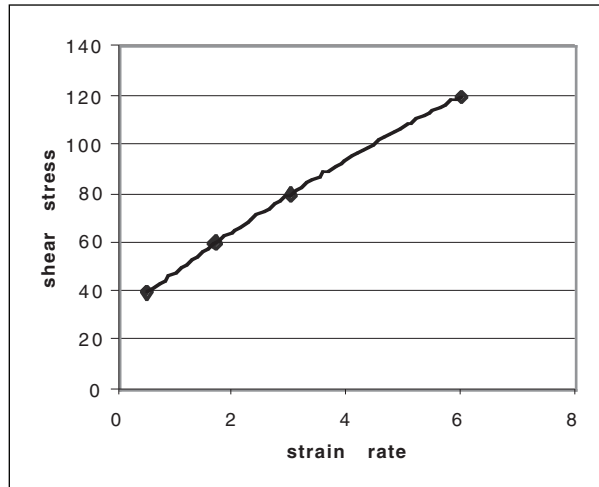
where  $dV/dy$  is in rev/s and  $\tau$  in g/cm<sup>2</sup>; these are not standard units.

10. A cod-liver oil emulsion is tested with a viscometer and the following data were obtained:

$\tau$ in	0	40	60	80	120
$dV/dy$ in rev/s	0	0.5	1.7	3	6

Graph the data and determine the fluid type. Derive the descriptive equation.

Cod liver oil; graph excludes the first data point.



$$\tau = K \left( \frac{dV}{dy} \right)^n$$

Can be done instantly with spreadsheet; hand calculations:

$dV/dy$	$\ln(dV/dy)$	$\tau$	$\ln \tau$	$\ln(\tau) \cdot \ln(dV/dy)$
0.5	-0.6931	40	3.689	-2.557
1.7	0.5306	60	4.094	2.172
3	1.099	80	4.382	4.816
6	1.792	120	4.787	8.578
Sum	2.729		16.95	13.01

$$m = 4 \text{ Summation } (\ln(dV/dy))^2 = 5.181$$

$$b_1 = \frac{4(13.01) - 2.729(16.95)}{4(5.181) - 2.729^2} = 0.4356$$

$$b_0 = \frac{16.95}{4} - 0.4356 \frac{2.729}{4} = 3.537$$

$$K = \exp(b_0) = 51.43; \quad n = b_1 = 0.4356$$

$$\tau = \tau_0 + K \left( \frac{dV}{dy} \right)^n = 51.43 \left( \frac{dV}{dy} \right)^{0.4356}$$

where  $dV/dy$  is in rev/s and  $\tau$  in lbf/ft<sup>2</sup>; these are not standard units.

11. A rotating cup viscometer has an inner cylinder diameter of 50.8 mm and the gap between cups is 5.08 mm. The inner cylinder length is 63.5 mm. The viscometer is used to obtain viscosity data on a Newtonian liquid. When the inner cylinder rotates at 10 rev/min, the torque on the inner cylinder is measured to be 0.01243 mN·m. Calculate the viscosity of the fluid. If the fluid density is 850 kg/m<sup>3</sup>, calculate the kinematic viscosity.

Rotating cup viscometer    R = 25.4 mm  
 $\delta = 5.08 \text{ mm}$     L = 63.5 mm

$$\omega = (10 \text{ rev/min}) \cdot (2\pi \text{ rad/rev}) (1 \text{ min} / 60 \text{ s}) = 1.047 \text{ rad/s} = \frac{dV}{dy}$$

$$T = 0.01243 \times 10^{-3} \text{ N}\cdot\text{m}$$

$$\rho = 850 \text{ kg/m}^3$$

$$\mu = \frac{T\delta}{2\pi R^2(R + \delta)L\omega}$$

$$\mu = \frac{1.243 \times 10^{-5} \times 5.08 \times 10^{-3}}{2\pi (0.0254)^2 (0.0254 + 5.08 \times 10^{-3}) (0.0635) (1.047)}$$

$$\mu = 7.7 \times 10^{-3} \text{ Pa}\cdot\text{s}$$

$$v = \frac{7.7 \times 10^{-3}}{850} = 9.762 \times 10^{-5} \text{ ft}^2/\text{s} \quad 9.06 \times 10^{-6} \text{ m}^2/\text{s}$$

12. A rotating cup viscometer has an inner cylinder whose diameter is 38 mm and whose length is 80 mm. The outer cylinder has a diameter of 42 mm. The viscometer is used to measure the viscosity of a liquid. When the outer cylinder rotates at 12 rev/min, the torque on the inner cylinder is measured to be  $4 \times 10^{-6} \text{ N}\cdot\text{m}$ . Determine the kinematic viscosity of the fluid if its density is 1 000 kg/m<sup>3</sup>.

$$R = 38/2 = 0.019 \text{ m}; \quad L = 0.08 \text{ m}$$

$$R_{\text{outside}} = 42/2 = 21 \text{ mm}$$

$$\delta = 21 - 19 = 2 \text{ mm} = 0.002 \text{ m}$$

$$\omega = (12 \text{ rev/min}) (2\pi/60) = 1.26 \text{ rad/s}$$

$$T = 3.8 \times 10^{-6} \text{ N}\cdot\text{m} \quad \rho = 1\,000 \text{ kg/m}^3$$

$$\mu = \frac{T\delta}{2\pi R^2(R + \delta)(L\omega)} = \frac{3.8 \times 10^{-6}(0.002)}{2\pi (0.019)^2 (0.019 + 0.002)(0.08)(1.26)}$$

$$\mu = 1.58 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$$

$$v = \frac{\mu}{\rho} = \frac{1.58 \times 10^{-3}}{1\,000} = 1.58 \times 10^{-6} \text{ m}^2/\text{s}$$

13. A rotating cup viscometer has an inner cylinder diameter of 57.15 mm and an outer cylinder diameter of 62.25 mm. The inner cylinder length is 76.2 mm. When the inner cylinder rotates at 15 rev/min, what is the expected torque reading if the fluid is propylene glycol?

$$D = 57.15 \text{ mm} \quad R = 28.58 \text{ mm} \quad 2(R + \delta) = 62.25 \text{ mm}$$

$$R + \delta = 31.125$$

$$\delta = 2.545 \text{ mm} \quad \rho = 968 \text{ kg/m}^3 \quad \mu = 0.0421 \text{ Pa}\cdot\text{s}$$

$$\omega = (15 \text{ rev/min})(2\pi/60) = 1.572 \text{ rad/s}$$

$$T = \frac{2\pi R^2(R + \delta)(L\omega)\mu}{\delta} = \frac{2\pi(0.02858)^2(0.031125)(0.0762)(1.571)(0.0421)}{0.002545}$$

$$T = 3.16 \times 10^{-4} \text{ N}\cdot\text{m}$$

14. A capillary tube viscometer is used to measure the viscosity of water (density is  $1000 \text{ kg/m}^3$ , viscosity is  $0.89 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$ ) for calibration purposes. The capillary tube inside diameter must be selected so that laminar flow conditions (i.e.,  $VD/v < 2100$ ) exist during the test. For values of  $L = 76.2 \text{ mm}$  and  $z = 254 \text{ mm}$ , determine the maximum tube size permissible.

$$\text{Capillary tube viscometer } \frac{V}{t} = \rho g \frac{z}{L} \frac{\pi R^4}{8\mu} \quad \rho = 1000 \text{ kg/m}^3$$

$$\mu = 0.89 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$$

$$z = 0.254 \text{ m} \quad L = 0.0762 \text{ m}$$

$$\frac{V}{t} = \text{Volume flow rate} = AV = \pi R^2 V; \text{ substituting into the equation,}$$

$$\pi R^2 V = \rho g \frac{z}{L} \frac{\pi R^4}{8\mu} \quad \text{Rearrange and solve for } V, V = \rho g \frac{z}{L} \frac{R^2}{8\mu}$$

The limiting value is  $Re < 2100$ ; using equality,

$$\frac{V(2R)}{v} = 2100; \quad \frac{\rho V(2R)}{\mu} = 2100 \text{ or}$$

$$V = \frac{2100\mu}{2\rho R} = \rho g \frac{z}{L} \frac{R^2}{8\mu} \quad \text{Rearrange and solve for } R^3$$

$$R^3 = \frac{2100\mu^2(8)(L)}{2\rho^2gz} = \frac{2100(0.89 \times 10^{-3})^2(8)(0.0762)}{2(1000)^2(9.81)(0.254)}$$

$$R^3 = 2.035 \times 10^{-10} \text{ or}$$

$$R = 5.88 \times 10^{-4} \text{ m} = 0.588 \text{ mm} \quad \text{Any larger, flow no longer laminar}$$

15. A Saybolt viscometer is used to measure oil viscosity and the time required for  $6 \times 10^{-5} \text{ m}^3$  of oil to pass through a standard orifice is 180 SUS. The specific gravity of the oil is found as  $44^\circ\text{API}$ . Determine the absolute viscosity of the oil.

For 180 SUS,

$$\nu = 0.223 \times 10^{-6}(180) - \frac{155 \times 10^{-6}}{180} = \boxed{3.928 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$44^\circ\text{API oil; sp.gr.} = \frac{141.5}{131.5 + 44} = 0.8063; \rho = 806.3 \text{ kg/m}^3$$

$$\mu = \rho\nu = 806.3(3.928 \times 10^{-5}) = \boxed{3.167 \times 10^{-2} \text{ N}\cdot\text{s/m}^2}$$

16. A  $10^4 \text{ m}^3$  capillary tube viscometer is used to measure the viscosity of a liquid. For values of  $L = 40 \text{ mm}$ ,  $z = 250 \text{ mm}$ , and  $D = 0.8 \text{ mm}$ , determine the viscosity of the liquid. The time recorded for the experiment is 12 seconds.

$$\nu = \left( \frac{z\pi R^4 g}{8LV} \right) t = \left( \frac{0.25\pi (0.0008/2)^4 (9.81)}{8(0.04)(10 \times 10^{-6})} \right) (12)$$

$$\boxed{\nu = 7.39 \times 10^{-7} \text{ m}^2/\text{s}}$$

17. A Saybolt viscometer is used to obtain oil viscosity data. The time required for 60 ml of oil to pass through the orifice is 70 SUS. Calculate the kinematic viscosity of the oil. If the specific gravity of the oil is  $35^\circ\text{API}$ , find also its absolute viscosity.

For 70 SUS,

$$\nu = 0.224 \times 10^{-6}(70) - \frac{185 \times 10^{-6}}{70}$$

$$\boxed{\nu = 1.304 \times 10^{-5} \text{ m}^2/\text{s}}$$

$35^\circ\text{API oil}$

$$\text{sp. gr.} = \frac{141.5}{131.5 + 35} = 0.8498 \quad \rho = 849.8 \text{ kg/m}^3$$

$$\mu = \frac{\rho\nu}{g_c} = 849.8(1.304 \times 10^{-5})$$

$$\boxed{\mu = 1.108 \times 10^{-2} \text{ N}\cdot\text{s/m}^2}$$



18. A 2-mm diameter ball bearing is dropped into a container of glycerine. How long will it take the bearing to fall a distance of 1 m?

$$\mu = \left( \frac{\rho_s}{\rho} - 1 \right) \rho g \frac{D^2}{18V} \quad V = \frac{L}{t} \quad L = 1 \text{ m} \quad D = 2 \text{ mm} = 0.002 \text{ m}$$

$$\rho_s = 7900 \text{ kg/m}^3 \quad \rho = 1263 \quad \mu = 950 \times 10^{-3} \text{ Pa}\cdot\text{s}$$

$$V = \left( \frac{\rho_s}{\rho} - 1 \right) \rho g \frac{D^2}{18\mu} = \left( \frac{7.9}{1.263} - 1 \right) (1263)(9.81)(0.002^2) \frac{1}{18(950 \times 10^{-3})}$$

$$V = 0.0152 \text{ m/s}$$

$$\text{Check on Re} = \frac{\rho VD}{\mu} = \frac{1263(0.0152)(0.002)}{950 \times 10^{-3}} = 0.04 < 1 \quad \text{OK}$$

$$\frac{L}{t} = 0.0152; \quad t = \frac{1}{0.0152}$$

$$t = 65.8 \text{ s}$$

19. A 3.175 mm diameter ball bearing is dropped into a viscous oil. The terminal velocity of the sphere is measured as 40.6 mm/s. What is the kinematic viscosity of the oil if its density is 800 kg/m<sup>3</sup>?

$$\mu = \left( \frac{\rho_s}{\rho} - 1 \right) \rho g \frac{D^2}{18V} \quad V = \frac{L}{t} = 40.6 \times 10^{-3} \text{ m/s} \quad D = 0.003175 \text{ m}$$

$$\rho_s = 7900 \text{ kg/m}^3$$

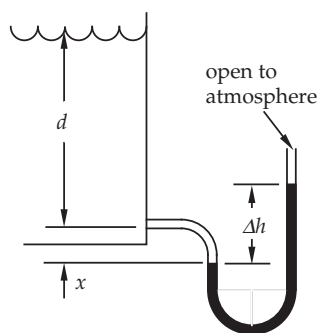
$$\nu = \frac{\mu}{\rho} = \left( \frac{\rho_s}{\rho} - 1 \right) \frac{gD^2}{18V} = \left( \frac{7900}{800} - 1 \right) \frac{(9.81)(0.003175)^2}{18(40.6 \times 10^{-3})}$$

$$\nu = 1.204 \times 10^{-3} \text{ m}^2/\text{s}$$

$$\text{Check on Re} = \frac{VD}{\nu} = \frac{40.6 \times 10^{-3}(0.003175)}{1.204 \times 10^{-3}} = 0.107 < 1 \quad \text{OK}$$

### Pressure and Its Measurement

20. A mercury manometer is used to measure pressure at the bottom of a tank containing acetone, as shown in Figure P2.20. The manometer is to be replaced with a gage. What is the expected reading in psig if  $\Delta h = 127 \text{ mm}$  and  $x = 50.8 \text{ mm}$ ?



$$\text{Acetone } \rho_a = 787 \text{ kg/m}^3$$

$$\text{Hg } \rho = 13600 \text{ kg/m}^3$$

$$p_A + \rho_a g x = p_{atm} + \rho g \Delta h$$

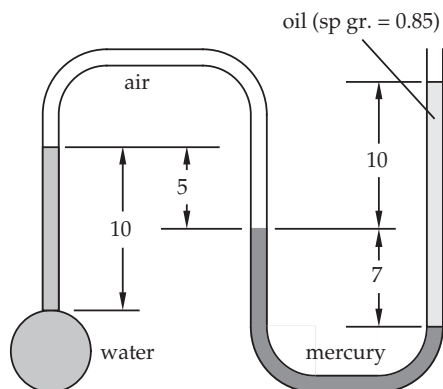
$$p_A + 787(9.81)(0.0508)(2/12) = 1.01325 \times 10^5 + 13600(9.81)(0.127)$$

$$p_A + 392.2 = 1.01325 \times 10^5 + 16943.8$$

$$p_A = 1.18 \times 10^5 \text{ Pa}$$

FIGURE P2.20.

21. Referring to Figure P2.21, determine the pressure of the water at the point where the manometer attaches to the vessel. All dimensions are in inches and the problem is to be worked using Engineering or British Gravitational units.



$$p_W = \frac{\rho_w g}{g_c} \frac{10}{12} + \frac{\rho_{air} g}{g_c} \frac{5}{12} + \frac{\rho_{Hg} g}{g_c} \frac{7}{12} - \frac{\rho_c g}{g_c} \frac{17}{12} = p_{atm}$$

$$p_W - 1.94(32.2)(10/12) + 13.6(1.94)(32.2)(7/12) - 0.85(1.94)(32.2)(17/12) = 14.7(144)$$

$$p_W - 52.06 + 495.6 - 75.22 = 2117$$

$$p_W = 1749 \text{ psf} = 12.14 \text{ psia}$$

FIGURE P2.21.

22. Figure P2.22 shows a portion of a pipeline that conveys benzene. A gage attached to the line reads 150 kPa. It is desired to check the gage reading with a benzene-over-mercury U-tube manometer. Determine the expected reading  $\Delta h$  on the manometer.

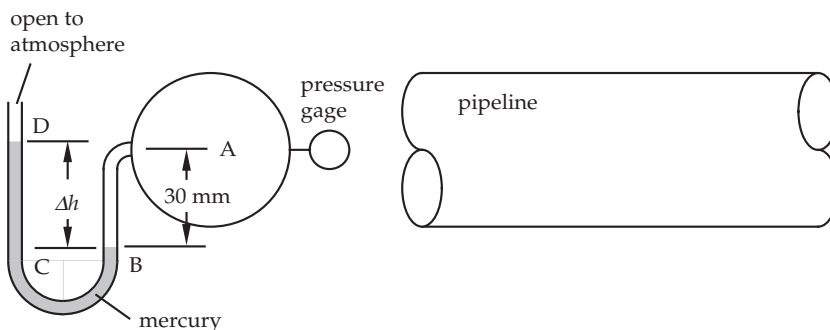


FIGURE P2.22.

$$p_D + \rho_{Hg}g\Delta h - \rho_B g(0.03) = p_A \quad p_D = p_{atm} = 0$$

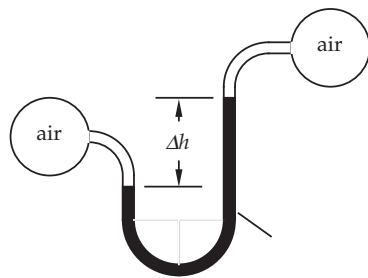
$$0 + 13.6(1\,000)(9.81)\Delta h - 876(9.81)(0.03) = 150\,000 \text{ (which is a gage reading)}$$

$$0 + 133\,400\Delta h - 257.8 = 150\,000$$

$$\Delta h = \frac{150\,000 + 257.8}{133\,400}$$

$$\Delta h = 1.126 \text{ m}$$

23. An unknown fluid is in the manometer of Figure P2.23. The pressure difference between the two air chambers is 700 kPa and the manometer reading  $\Delta h$  is 60 mm. Determine the density and specific gravity of the unknown fluid.



Because  $\rho_{air} \ll \rho_{liquid}$ , then

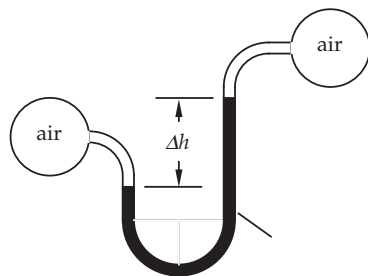
$$p_A - p_B = \rho g \Delta h; \quad \Delta h = 60 \text{ mm} = 0.06 \text{ m, and}$$

$$p_A - p_B = 700 \text{ N/m}^2 \text{ given; so}$$

$$\rho = \frac{p_A - p_B}{g \Delta h} = \frac{700}{9.81(0.06)} = \boxed{1\,190 \text{ kg/m}^3}$$

FIGURE P2.23.

24. A U-tube manometer is used to measure the pressure difference between two air chambers, as shown in Figure P2.24. If the reading  $\Delta h$  is 152.4 mm, determine the pressure difference.



Because  $\rho_{air} \ll \rho_{liquid}$ , then

$$p_A - p_B = \rho g \Delta h; \quad \Delta h = 152.4 \times 10^{-3} \text{ m}$$

$$p_A - p_B = 1000 \text{ kg/m}^3 (9.81)(0.1524)$$

$$\boxed{p_A - p_B = 1495 \text{ Pa}}$$

FIGURE P2.24.

25. A manometer containing mercury is used to measure the pressure increase experienced by a water pump as shown in Figure P2.25. Calculate the pressure rise if  $\Delta h$  is 70 mm of mercury (as shown). All dimensions are in mm.

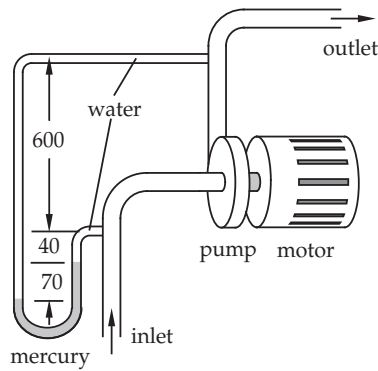


FIGURE P2.25.

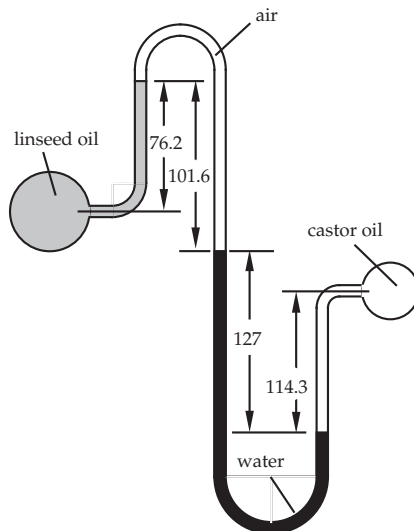
$$p_{outlet} + \rho g \left( \frac{600 + 40 + 70}{1000} \right) - \rho_{Hg} g (0.07) - \rho g (0.04) = p_{inlet}$$

$$p_{outlet} + 1000(9.81)(0.71) - 13.6(1000)(9.81)(0.07) - 1000(9.81)(0.04) = p_{inlet}$$

$$p_{outlet} + 6965 - 9339 - 392.4 = p_{inlet}$$

$$p_{outlet} - p_{inlet} = 2766 \text{ Pa} = 2.77 \text{ kPa}$$

26. Determine the pressure difference between the linseed and castor oils of Figure P2.26. (All dimensions are in mm.)



$$p_A - \rho_{LO} g (0.0762) + \rho_{air} g (0.1016) + \rho_{H_2O} g (0.127) - \rho_{CO} g (0.1143) = p_B$$

$$\rho_{LO} = 930 \text{ kg/m}^3; \quad \rho_{CO} = 960 \text{ kg/m}^3$$

$$\rho_{H_2O} = 1000 \text{ kg/m}^3 \quad \rho_{air} \text{ negligible}$$

$$p_A - p_B = \rho_{LO} g (0.0762) + \rho_{H_2O} g (0.127) - \rho_{CO} g (0.1143)$$

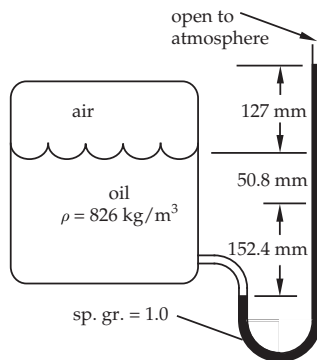
$$p_A - p_B = 930(9.81)(0.0762) - 1000(9.81)(0.127) - 960(9.81)(0.1143)$$

$$p_A - p_B = 695.2 - 1246.3 + 1076.8$$

$$p_A - p_B = 526 \text{ Pa}$$

FIGURE P2.26.

27. For the system of Figure P2.27, determine the pressure of the air in the tank.



$$p_{air} + \rho_{oil} g (0.0508 + 0.1524) - \rho g (0.127 + 0.0508 + 0.1524) = p_{atm}$$

$$p_{air} + 826(9.81)(0.2032) - 1000(9.81)(0.3302) = 1.01325 \times 10^5$$

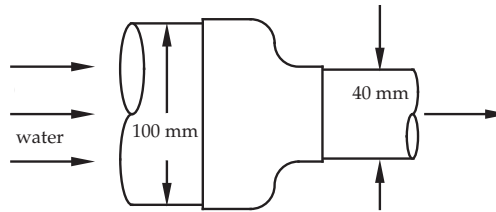
$$p_{air} + 1647 - 3240 = 1.01325 \times 10^5$$

$$p_{air} = 1.03 \times 10^5 \text{ Pa}$$

FIGURE P2.27.

## Continuity Equation

28. Figure P2.28 shows a reducing bushing. A liquid leaves the bushing at a velocity of 4 m/s. Calculate the inlet velocity. What effect does the fluid density have?



$$D_1 = 100 \text{ mm} = 0.1 \text{ m}; D_2 = 40 \text{ mm} = 0.04 \text{ m}$$

$$V_2 = 4 \text{ m/s}$$

Density has no effect

$$Q = A_1 V_1 = A_2 V_2$$

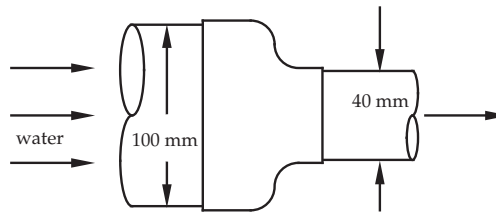
$$\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2$$

$$V_1 = V_2 \frac{D_2^2}{D_1^2} = 4 \left( \frac{0.04^2}{0.1^2} \right)$$

FIGURE P2.28, P2.29.

$$V_1 = 0.64 \text{ m/s}$$

29. Figure P2.29 shows a reducing bushing. Liquid enters the bushing at a velocity of 0.5 m/s. Calculate the outlet velocity.



$$D_1 = 100 \text{ mm} = 0.1 \text{ m}; D_2 = 40 \text{ mm} = 0.04 \text{ m}$$

$$V_1 = 0.5 \text{ m/s}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2$$

$$V_2 = V_1 \frac{D_1^2}{D_2^2} = 0.5 \left( \frac{0.1^2}{0.04^2} \right)$$

FIGURE P2.28, P2.29.

$$V_2 = 3.13 \text{ m/s}$$

30. Water enters the tank of Figure P2.30 @  $0.00189 \text{ m}^3/\text{s}$ . The inlet line is 63.5 mm in diameter. The air vent is 38 mm in diameter. Determine the air exit velocity at the instant shown.

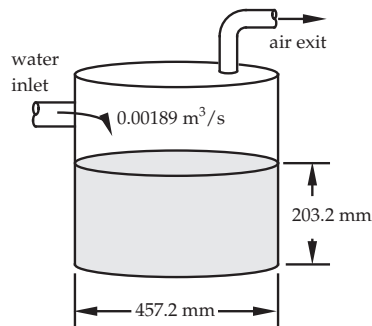


FIGURE P2.30.

For low pressures and temperatures, air can be treated as incompressible.

$$Q_{\text{H}_2\text{O in}} = Q_{\text{air out}}$$

$$Q_{\text{H}_2\text{O in}} = 0.00189 \text{ m}^3/\text{s}$$

$$\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3 \quad \rho_{\text{air}} = 1.19 \text{ kg/m}^3$$

$$Q_{\text{air out}} = AV = \frac{\pi D^2}{4} V = \frac{\pi}{4} [(0.038)]^2 = 1.14 \times 10^{-3} V$$

$$\text{So } 0.00189 = 1.14 \times 10^{-3} V$$

$$V_{\text{air}} = 1.66 \text{ m/s}$$

31. An air compressor is used to pressurize a tank of volume  $3 \text{ m}^3$ . Simultaneously, air leaves the tank and is used for some process downstream. At the inlet, the pressure is 350 kPa, the temperature is  $20^\circ\text{C}$ , and the velocity is 2 m/s. At the outlet, the temperature is  $20^\circ\text{C}$ , the velocity is 0.5 m/s, and the pressure is the same as that in the tank. Both flow lines (inlet and outlet) have internal diameters of 2.7 cm. The temperature of the air in the tank is a constant at  $20^\circ\text{C}$ . If the initial tank pressure is 200 kPa, what is the pressure in the tank after 5 minutes?

$$0 = \frac{\partial m}{\partial t} + (\rho AV)_{\text{out}} - (\rho AV)_{\text{in}} \quad m = \frac{pV}{RT} \quad \frac{\partial m}{\partial t} = \frac{V}{RT} \frac{dp}{dt}$$

$$(pAV)_{\text{out}} - (pAV)_{\text{in}} = \frac{p_{\text{out}}}{RT_{\text{out}}} A_{\text{out}} V_{\text{out}} - \frac{p_{\text{in}}}{RT_{\text{in}}} A_{\text{in}} V_{\text{in}}$$

Substituting,

$$0 = \frac{V}{RT} \frac{dp}{dt} + \frac{p_{\text{out}}}{RT_{\text{out}}} A_{\text{out}} V_{\text{out}} - \frac{p_{\text{in}}}{RT_{\text{in}}} A_{\text{in}} V_{\text{in}}$$

For constant  $T$ , all  $RT$  products cancel

$$V \frac{dp}{dt} = -p_{\text{out}} A_{\text{out}} V_{\text{out}} + p_{\text{in}} A_{\text{in}} V_{\text{in}} \quad p_{\text{out}} = p$$

$$A_{\text{in}} = \frac{\pi(0.027)^2}{4} = 5.726 \times 10^{-4} \text{ m}^2 = A_{\text{out}} \quad \text{Areas are equal}$$

$$3 \frac{dp}{dt} = -p(5.726 \times 10^{-4})(0.5) + 350\,000(5.726 \times 10^{-4})(2)$$

$$3 \frac{dp}{dt} = 400.8 - 2.863 \times 10^{-4} p \quad \text{or} \quad \frac{dp}{dt} = 133.6 - 9.543 \times 10^{-5} p$$

Separating variables,

$$\int_p^{200\,000} \frac{dp}{133.6 - 9.543 \times 10^{-5} p} = \int_0^{300} dt$$

$$\left. \frac{\ln(133.6 - 9.543 \times 10^{-5} p)}{-9.543 \times 10^{-5}} \right|_{200\,000}^p = 300 - 0$$

$$\ln(133.6 - 9.543 \times 10^{-5} p) - \ln(133.6 - 9.543 \times 10^{-5}(200\,000)) = 300(-9.543 \times 10^{-5})$$

$$\ln(133.6 - 9.543 \times 10^{-5} p) - 4.741 = -2.863 \times 10^{-2}$$

$$\ln(133.6 - 9.543 \times 10^{-5} p) = 4.712$$

Exponentiating,

$$133.6 - 9.543 \times 10^{-5} p = 1.113 \times 10^2 \quad \text{or} \quad -9.543 \times 10^{-5} p = -22.3$$

$$\boxed{p = 2.34 \text{ kPa}}$$

32. Figure P2.32 shows a cross-flow heat exchanger used to condense Freon-12. Freon-12 vapor enters the unit at a flow rate of 0.065 kg/s. Freon-12 leaves the exchanger as a liquid (Sp. Gr. = 1.915) at room temperature and pressure. Determine the exit velocity of the liquid.

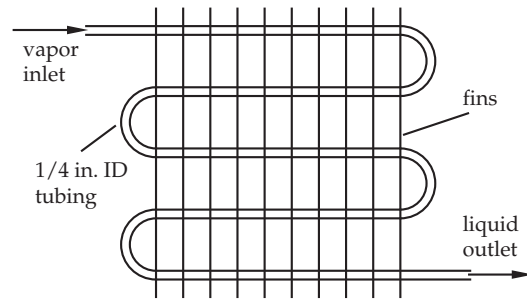


FIGURE P2.32.

$$\dot{m}_{in} = \rho_{out} A_{out} V_{out}$$

$$\dot{m}_{in} = 0.065 \text{ kg/s}$$

$$\rho = 1.915(1000) \text{ kg/m}^3$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.25/12)^2}{4} = 3.41 \times 10^{-4} \text{ ft}^2$$

$$A = 3.41 \times 10^{-4} (9.29 \times 10^{-2}) = 3.17 \times 10^{-5} \text{ m}^2$$

Substituting,

$$0.065 = 1.915(1000)3.17 \times 10^{-5} V_{out}$$

$$V_{out} = 1.07 \text{ m/s}$$

33. Nitrogen enters a pipe at a flow rate of 90.7 g/s. The pipe has an inside diameter of 101.6 mm. At the inlet, the nitrogen temperature is 26.7°C ( $\rho = 1.17 \text{ kg/m}^3$ ) and at the outlet, the nitrogen temperature is 727°C ( $\rho = 0.34 \text{ kg/m}^3$ ). Calculate the inlet and outlet velocities of the nitrogen. Are they equal? Should they be?

$$\dot{m} = 0.0907 \text{ kg} \quad D = 0.1016 \text{ m} \quad \rho_1 = 1.17 \text{ kg/m}^3$$

$$\rho_2 = 0.34 \text{ kg/m}^3$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.1016)^2}{4} = 8.11 \times 10^{-3} \text{ m}^2 \quad \dot{m} = \rho A V$$

$$V_1 = \frac{\dot{m}}{\rho_1 A} = \frac{0.0907}{1.17(8.11 \times 10^{-3})}$$

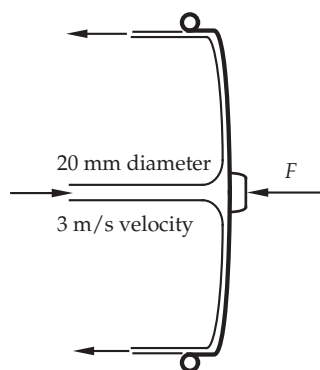
$$V_1 = 9.56 \text{ m/s}$$

$$V_2 = \frac{0.0907}{0.34(8.11 \times 10^{-3})}$$

$$V_2 = 32.8 \text{ m/s}$$

### Momentum Equation

34. A garden hose is used to squirt water at someone who is protecting herself with a garbage can lid. Figure P2.34 shows the jet in the vicinity of the lid. Determine the restraining force  $F$  for the conditions shown.



$$\Sigma F = \dot{m}(V_{out} - V_{in}) \quad \dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

$$\text{flow magnitude of } V_{in} = \text{magnitude of } V_{out}$$

$$F = [\rho AV]_{inlet}(-V_{in} - V_{in})$$

$$F = 2\rho AV^2 \quad \rho = 1000 \text{ kg/m}^3$$

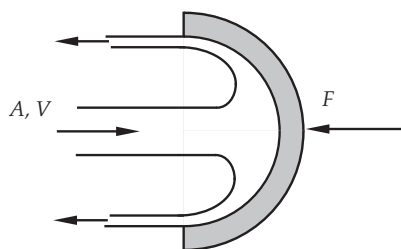
$$A = \frac{\pi(0.02)^2}{4} = 3.14 \times 10^{-4} \text{ m}^2 \quad V = 3 \text{ m/s}$$

$$F = 2(1000)(3.14 \times 10^{-4})(3)^2$$

$$F = 5.65 \text{ N}$$

FIGURE P2.34.

35. A two-dimensional, liquid jet strikes a concave semicircular object, as shown in Figure P2.35. Calculate the restraining force  $F$ .



$$\Sigma F = \dot{m}(V_{out} - V_{in})$$

$$\dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

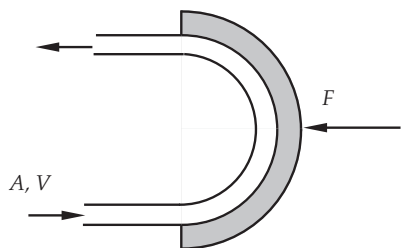
$$\text{magnitude of } V_{in} = \text{magnitude of } V_{out}$$

$$F = [\rho AV]_{inlet}(-V_{in} - V_{in})$$

$$F = 2\rho AV^2$$

FIGURE P2.35.

36. A two-dimensional, liquid jet strikes a concave semicircular object, as shown in Figure P2.36. Calculate the restraining force  $F$ .



$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in})$$

$$\dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

$$\text{magnitude of } V_{in} = \text{magnitude of } V_{out}$$

$$F = \frac{[\rho AV]_{inlet}}{g_c}(-V_{in} - V_{in})$$

$$g_c = 1 \text{ in SI units}$$

$$F = \frac{2\rho AV^2}{g_c}$$

FIGURE P2.36.

37. A two-dimensional liquid jet is turned through an angle  $\theta$  ( $0^\circ < \theta < 90^\circ$ ) by a curved vane, as shown in Figure P2.37. The forces are related by  $F_2 = 3F_1$ . Determine the angle  $\theta$  through which the liquid jet is turned.



$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in}); \dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

magnitude of  $V_{in}$  = magnitude of  $V_{out}$

$$-F_1 = \frac{[\rho AV]_{inlet}}{g_c}(V_{outx} - V_{inx})$$

$$V_{outx} = V \cos \theta; \quad V_{inx} = V$$

$$-F_1 = \frac{[\rho AV]_{inlet}}{g_c}(V \cos \theta - V) = \frac{\rho AV^2}{g_c}(\cos \theta - 1)$$

$$F_1 = \frac{\rho AV^2}{g_c}(1 - \cos \theta)$$

$$V_{outy} = V \sin \theta;$$

$$F_2 = \frac{[\rho AV]_{inlet}}{g_c}(V \sin \theta) = \frac{\rho AV^2}{g_c}(\sin \theta)$$

$$F_2 = \frac{[\rho AV]_{inlet}}{g_c}(V_{outy} - V_{iny})$$

$$V_{iny} = 0$$

$$F_2 = 3F_1; \quad \sin \theta = 3(1 - \cos \theta)$$

$$\frac{1}{3} \sin \theta = 1 - \cos \theta \quad \text{T\&E solution is quickest}$$

$\theta$	$(1/3) \sin \theta$	$1 - \cos \theta$
45°	0.2357	0.2929
50°	0.2553	0.3572
40°	0.2143	0.234
35°	0.1912	0.1808
37°	0.2006	0.2014
36.8°	0.1997	0.1993

$$\boxed{\theta = 36.8^\circ}$$

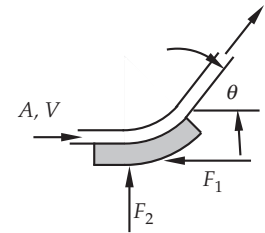


FIGURE P2.37.

38. A two-dimensional liquid jet is turned through an angle  $\theta$  ( $0^\circ < \theta < 90^\circ$ ) by a curved vane as shown in Figure P2.38. The forces are related by  $F_1 = 2F_2$ . Determine the angle  $\theta$  through which the liquid jet is turned.

$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in}); \dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

magnitude of  $V_{in}$  = magnitude of  $V_{out}$

$$-F_1 = \frac{[\rho AV]_{inlet}}{g_c}(V_{outx} - V_{inx})$$

$$V_{outx} = -V \cos \theta; \quad V_{inx} = V$$

$$-F_1 = \frac{[\rho AV]_{inlet}}{g_c}(-V \cos \theta - V) = -\frac{\rho AV^2}{g_c}(\cos \theta + 1)$$

$$F_1 = \frac{\rho AV^2}{g_c}(1 + \cos \theta)$$

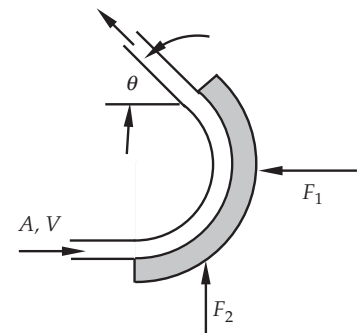


FIGURE P2.39.

$$F_2 = \frac{[\rho AV]_{\text{inlet}}}{g_c} (V_{\text{outy}} - V_{\text{iny}})$$

$$V_{\text{outy}} = V \sin \theta; \quad V_{\text{iny}} = 0$$

$$F_2 = \frac{[\rho AV]_{\text{inlet}}}{g_c} (V \sin \theta) = \frac{\rho AV^2}{g_c} (\sin \theta)$$

$$F_1 = 2F_2; \quad 1 + \cos \theta = 2 \sin \theta$$

T & E solution is quickest

$\theta$	$1 - \cos \theta$	$2 \sin \theta$
45°	1.707	1.414
50°	1.643	1.532
55°	1.574	1.638
53°	1.602	1.597
54°	1.588	1.618
53.5°	1.595	1.608
53.4°	1.596	1.606
53.2°	1.599	1.601
53.1°	1.600	1.599

$$\theta = 53.1^\circ$$

## Energy Equation

39. Figure P2.39 shows a water turbine located in a dam. The volume flow rate through the system is  $0.315 \text{ m}^3/\text{s}$ . The exit pipe diameter is 1.22 m. Calculate the work done by (or power received from) the water as it flows through the dam. (Compare to the results of the example problem in this chapter.)

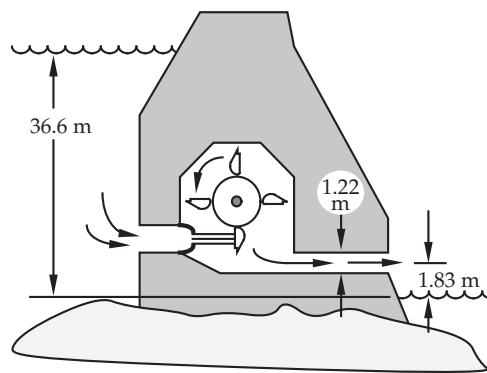


FIGURE P2.39.

We apply the energy equation between any two sections. Section 1 = the free surface upstream, and Section 2 = the outlet downstream.

$$p_2 = p_1 = p_{\text{atm}}$$

$$V_1 \approx 0 \text{ (reservoir surface velocity)}$$

$$z_2 = 1.83 \text{ m}; \quad z_1 = 36.6 \text{ m}$$

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi (1.22)^2}{4} = 1.169 \text{ m}^2$$

$$Q = 0.315 \text{ m}^3/\text{s}$$

$$V_2 = \frac{Q}{A} = \frac{0.315}{1.169} = 0.27 \text{ m/s} \quad \rho = 1000 \text{ kg/m}^3$$

$$\rho V A = \dot{m} = 1000(1.169)(0.27) = 315.25 \text{ kg/s} \text{ evaluated at outlet}$$

Substituting,

$$-\frac{\partial W}{\partial t} = \left\{ \left( \frac{p}{\rho} + \frac{v^2}{2} + gz \right) \Big|_2 - \left( \frac{p}{\rho} + \frac{v^2}{2} + gz \right) \Big|_1 \right\} \rho V A$$

$$-\frac{\partial W}{\partial t} = \left\{ \left( 0 + \frac{0.27}{2} + 9.81(1.83) - (0 + 0 + 9.81(36.6)) \right) \right\} 315.25$$

$$-\frac{\partial W}{\partial t} = \{ (0 + 0.03645 + 17.95 - 0 - 0 - 359.05) \} 315.25$$

$$\boxed{+\frac{\partial W}{\partial t} = 1.075 \times 10^5 \text{ W}}$$

40. Air flows through a compressor at a mass flow rate of 0.0438 kg/s. At the inlet, the air velocity is negligible. At the outlet, air leaves through an exit pipe of diameter 50.8 mm. The inlet properties are 101.3 kPa and 23.9°C. The outlet pressure is 827 kPa. For an isentropic (reversible and adiabatic) compression process, we have

$$\frac{T_2}{T_1} = \left\{ \frac{p_2}{p_1} \right\}^{(\gamma-1)/\gamma}$$

Determine the outlet temperature of the air and the power required. Assume that air behaves as an ideal gas ( $dh = c_p dT$ ,  $du = c_v dT$ , and  $\rho = p/RT$ ).

$$\frac{T_2}{T_1} = \left\{ \frac{p_2}{p_1} \right\}^{(\gamma-1)/\gamma}$$

Determine the outlet temperature of the air and the power required. Assume that air behaves as an ideal gas ( $dh = c_p dT$ ,  $du = c_v dT$ , and  $\rho = p/RT$ ).

Solution:

$$\dot{m} = 0.0438 \text{ kg} \quad V_{\text{in}} = 0 \quad V_{\text{out}} = \text{unknown}$$

$$p_{\text{in}} = 101.3 \times 10^3 \text{ Pa} \quad p_{\text{out}} = 827 \times 10^3 \text{ Pa}$$

$$D_{\text{out}} = 0.0508 \text{ m} \quad A_{\text{out}} = \pi D^2/4 = 0.00203 \text{ m}^2 \quad \gamma = 1.4$$

$$R_{\text{air}} = 8.314 \text{ J/K mole} \quad c_{p\text{air}} = 1004 \text{ J/kg K}$$

$$\frac{T_{\text{out}}}{T_{\text{in}}} = \left\{ \frac{p_{\text{out}}}{p_{\text{in}}} \right\}^{(\gamma-1)/\gamma}$$

$$\frac{T_{\text{out}}}{(273 + 23.9)} = \left\{ \frac{827 \times 10^3}{101.3 \times 10^3} \right\}^{(1.4-1)/1.4} = 1.822 \quad T_{\text{out}} = 296.9(1.822)$$

$$\boxed{T_{\text{out}} = 540.95 \text{ K} = 268^\circ\text{C}}$$

$$\rho_{out} \frac{p}{RT} = \frac{827 \times 10^3 \times 29}{8314(540.95)} = 5.33 \text{ kg/m}^3$$

$$V_{out} = \frac{\dot{m}}{\rho A_{out}} = \frac{0.0438}{5.33(0.00203)} = 4.05 \text{ m/s} \quad \frac{V_{out}^2}{2} = \frac{4.05^2}{2} = 8.2$$

$$-\frac{\partial W}{\partial t} = \left\{ \left( h + \frac{V^2}{2} + gz \right) \Big|_{out} - \left( h + \frac{V^2}{2} + gz \right) \Big|_{in} \right\} \rho V A$$

$$-\frac{\partial W}{\partial t} = (h_{out} - h_{in} + \frac{V_{out}^2}{2}) \rho V A \quad \Delta PE = 0$$

$$(h_{out} - h_{in}) = c_p(T_{out} - T_{in}) = 1004(268 - 23.9) = 2.45 \times 10^5 \text{ J/kg}$$

$$(h_{out} - h_{in}) = 2.45 \times 10^5 \text{ J/kg}$$

$$-\frac{\partial W}{\partial t} = (2.45 \times 10^5 + 8.2)(0.0438) = 10735 \text{ W}$$

$$\text{or } \boxed{-\frac{\partial W}{\partial t} = 14.4 \text{ HP}} \quad \text{Assuming no losses}$$

41. An air turbine is used with a generator to generate electricity. Air at the turbine inlet is at 700 kPa and 25°C. The turbine discharges air to the atmosphere at a temperature of 11°C. Inlet and outlet air velocities are 100 m/s and 2 m/s, respectively. Determine the work per unit mass delivered to the turbine from the air.

$$\begin{aligned} p_{in} &= 700 \text{ kPa} & p_{out} &= 101.3 \text{ kPa} \\ T_{in} &= 25^\circ\text{C} & T_{out} &= 11^\circ\text{C} \\ V_{in} &= 100 \text{ m/s} & V_{out} &= 2 \text{ m/s} \\ c_p &= 1005.7 \text{ J/(kg}\cdot\text{K)} \end{aligned}$$

$$-\frac{\partial W}{\partial t} = \left\{ \left( h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \Big|_{out} - \left( h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \Big|_{in} \right\} \rho V A$$

$$-\frac{\partial W/\partial t}{\dot{m}} = (h_{out} - h_{in}) + \left( \frac{V_{out}^2}{2g_c} + \frac{V_{in}^2}{2g_c} \right) + \frac{g}{g_c}(z_{out} - z_{in})$$

$$(h_{out} - h_{in}) = c_p(T_{out} - T_{in}) \quad z_{out} = z_{in}$$

$$-\frac{\partial W/\partial t}{\dot{m}} = 1005.7(25 - 11) + \left( \frac{2^2}{2} + \frac{100^2}{2} \right) = 1.4 \times 10^4 - 5 \times 10^3$$

$$\boxed{-\frac{\partial W/\partial t}{\dot{m}} = 9 \times 10^3 \text{ J/kg}}$$

42. A pump moving hexane is illustrated in Figure P2.42. The flow rate is  $0.02 \text{ m}^3/\text{s}$ ; inlet and outlet gage pressure readings are  $-4 \text{ kPa}$  and  $190 \text{ kPa}$ , respectively. Determine the required power input to the fluid as it flows through the pump.

We apply the energy equation between any two sections. Section 1 = inlet pressure gage (actually the centerline of the pipe where the pressure gage is attached), and Section 2 = outlet pressure gage.

$$p_2 = 190\,000 \text{ Pa} \quad z_2 = 1.5 \text{ m}$$

$$p_1 = -4000 \text{ Pa} \quad z_1 = 1.0 \text{ m}$$

$$AV = 0.02 \text{ m}^3/\text{s}$$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.10)^2}{4} = 7.854 \times 10^{-3} \text{ m}^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.075)^2}{4} = 4.42 \times 10^{-3} \text{ m}^2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.02}{7.854 \times 10^{-3}} = 2.55 \text{ m/s} \quad V_2 = \frac{Q}{A_2} = \frac{0.02}{4.42 \times 10^{-3}} = 4.52 \text{ m/s}$$

$$\rho = 0.657(1\,000) \text{ for hexane}$$

$$-\frac{\partial W}{\partial t} = \left\{ \left( \frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \Big|_2 - \left( \frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \Big|_1 \right\} \rho VA$$

$$-\frac{\partial W}{\partial t} = \left\{ \frac{190\,000}{657} + \frac{4.52^2}{2} + 1.5(9.81) - \left( \frac{-4\,000}{657} + \frac{2.55^2}{2} + 1.0(9.81) \right) \right\} 657(0.02)$$

$$-\frac{\partial W}{\partial t} = \{289.2 + 10.22 + 14.72 + 6.088 - 3.25 - 9.81\} (13.14)$$

$$\boxed{-\frac{\partial W}{\partial t} = 4.04 \times 10^3 \text{ N}\cdot\text{m/s} = 4.0 \text{ kW}}$$

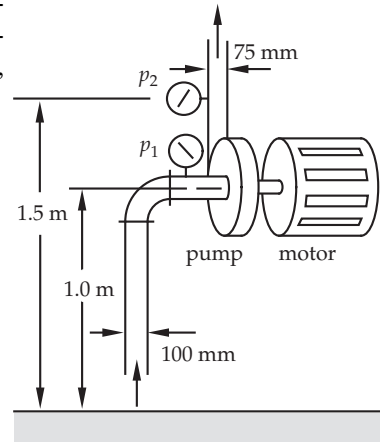


FIGURE P2.42.

### Bernoulli Equation

43. Figure 2.15 shows a venturi meter. Show that the Bernoulli and continuity equations when applied combine to become

$$Q = A_2 \sqrt{\frac{2g\Delta h}{1 - (D_2^4/D_1^4)}}$$

Hydrostatic equation for manometer; all measurements are from the centerline

$$p_1 - \rho_1 g x - \rho_1 g \Delta h = p_2 - \rho_1 g x - \rho_2 g \Delta h \quad \text{or} \quad p_1 - p_2 = -\rho_1 g \Delta h$$

$$\dot{m}_1 = \dot{m}_2 \quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{or} \quad A_1 V_1 = A_2 V_2$$

In terms of diameter,  $\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 = Q$

Bernoulli Equation

$$\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho_1 g} + \frac{V_2^2}{2g} + z_2 \quad \text{With } z_1 = z_2,$$

$$\frac{(p_1 - p_2)}{\rho_1 g} = \frac{1}{2g} (V_2^2 - V_1^2) \quad \text{Substitute for } V \text{ in terms of } Q$$

$$\frac{(p_1 - p_2)}{\rho_1 g} 2g = Q^2 (1/A_2^2 - 1/A_1^2)$$

$$\frac{2\rho_1 g \Delta h}{\rho_1 g c} = \frac{Q^2}{A_2^2} \left( 1 - \frac{A_2^2}{A_1^2} \right) = \frac{Q^2}{A_2^2} \left( 1 - \frac{D_2^4}{D_1^4} \right)$$

$$A_2 \sqrt{2g \Delta h} = Q \sqrt{1 - D_2^4/D_1^4} \quad \text{or finally,}$$

$$Q = A_2 \sqrt{\frac{2g \Delta h}{1 - D_2^4/D_1^4}}$$

44. A jet of water issues from a kitchen faucet and falls vertically downward at a flow rate of  $4.44 \times 10^{-5} \text{ m}^3/\text{s}$ . At the faucet, which is 355.6 mm above the sink bottom, the jet diameter is 15.88 mm. Determine the diameter of the jet where it strikes the sink.

$$Q = 4.44 \times 10^{-5} \text{ m}^3/\text{s}$$

$$D_1 = 0.01588 \text{ m} \quad A_1 = 1.98 \times 10^{-4} \text{ m}^2$$

$$V_1 = \frac{Q}{A_1} = 0.222 \text{ m/s} \quad h = z_1 = 0.3556 \text{ m}$$

Bernoulli Equation

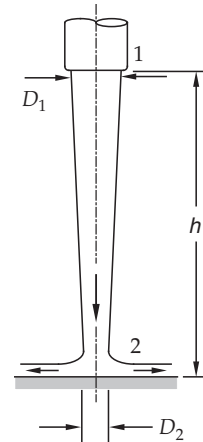
$$\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho_1 g} + \frac{V_2^2}{2g} + z_2$$

$$p_1 = p_2 \quad z_1 = 0.3556 \text{ m} \quad z_2 = 0$$

Substituting,

$$0 + \frac{0.222^2}{2(9.81)} + \frac{1}{9.81} 0.3556 = 0 + \frac{V_2^2}{2(9.81)} + 0$$

which becomes



$$(2.512 \times 10^{-3} + 0.3556)(2(9.81)) = V_2^2 \text{ or}$$

$$V_2 = 2.65 \text{ m/s}$$

$$A_2 = \frac{Q}{V_2} = \frac{4.44 \times 10^{-5}}{2.65} = 1.675 \times 10^{-5} \text{ m}^2$$

$$\frac{\pi D_2^2}{4} = 1.675 \times 10^{-5} \quad D_2 = \sqrt{\frac{4}{\pi}(1.675 \times 10^{-5})} \quad \text{or}$$

$$D_2 = 4.62 \times 10^{-3} \text{ m} = 4.62 \text{ mm}$$

45. A jet of water issues from a valve and falls vertically downward at a flow rate of  $3 \times 10^{-5} \text{ m}^3/\text{s}$ . The valve exit is 50 mm above the ground; the jet diameter at the ground is 5 mm. Determine the diameter of the jet at the valve exit.

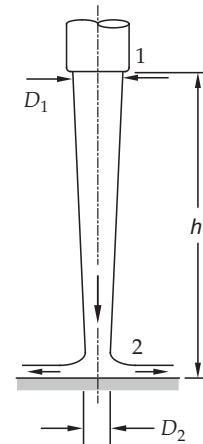
Section 1 is the exit; section 2 is the ground.

$$\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho_1 g} + \frac{V_2^2}{2g} + z_2$$

$$Q = 3 \times 10^{-5} \text{ m}^3/\text{s}; \quad p_1 = p_2 = p_{\text{atm}}; \quad z_2 = 0; \quad z_1 = 0.05 \text{ m}$$

$$D_2 = 5 \text{ mm}; \quad A_2 = \frac{\pi(0.005)^2}{4} = 1.963 \times 10^{-5} \text{ m}^2$$

$$V_2 = \frac{Q}{A_2} = \frac{30 \times 10^{-6}}{1.963 \times 10^{-5}} = 1.53 \text{ m/s};$$



Bernoulli Equation becomes

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{2(9.81)} = \frac{1.53^2}{2(9.81)} - 0.05 = 0.06931$$

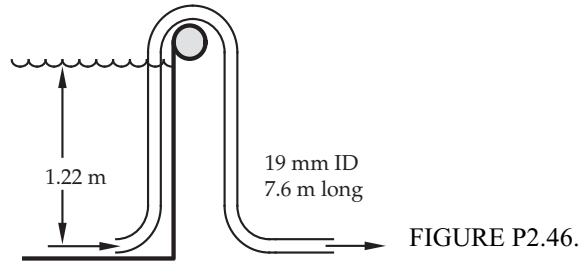
$$V_1^2 = 1.36; \quad V_1 = 1.17 \text{ m/s}$$

$$Q = A_1 V_1 = \frac{\pi D_1^2}{4} V_1 \quad D_1 = \sqrt{\frac{4Q}{\pi V_1}}$$

$$D_1 = \sqrt{\frac{4(30 \times 10^{-6})}{\pi(1.17)}} = 5.7 \times 10^{-3} \text{ m}$$

$$D_1 = 5.7 \text{ mm}$$

46. A garden hose is used as a siphon to drain a pool, as shown in Figure P2.46. The garden hose has a 19 mm inside diameter. Assuming no friction, calculate the flow rate of water through the hose if the hose is 25 ft long.



Section 1 is the free surface; section 2 is the hose outlet.

$$\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho_1 g} + \frac{V_2^2}{2g} + z_2 \quad p_1 = p_2 = p_{atm}; \quad V_1 = 0; \quad z_1 = 1.22 \text{ m}$$

Substituting,

$$0 + 0 + (1.22) = 0 + \frac{V_2^2}{2(9.81)} + 0$$

$$V_2 = \sqrt{2(9.81)(1.22)} = 4.89 \text{ m/s}$$

$$D = 19 \text{ mm} \quad A = \frac{\pi D^2}{4} = 2.835 \times 10^{-4} \text{ m}^2$$

$$Q = AV = 2.835 \times 10^{-4}(4.89);$$

$$Q = 1.386 \times 10^{-3} \text{ m}^3/\text{s}$$

### Miscellaneous Problems

47. A pump draws castor oil from a tank, as shown in Figure P2.47. A venturi meter with a throat diameter 50.8 mm is located in the discharge line. For the conditions shown, calculate the expected reading on the manometer of the meter. Assume that frictional effects are negligible and that the pump delivers 186.5 W to the liquid. If all that is available is a 1.83 m tall manometer, can it be used in the configuration shown? If not, suggest an alternative way to measure pressure difference. (All measurements are in mm.)



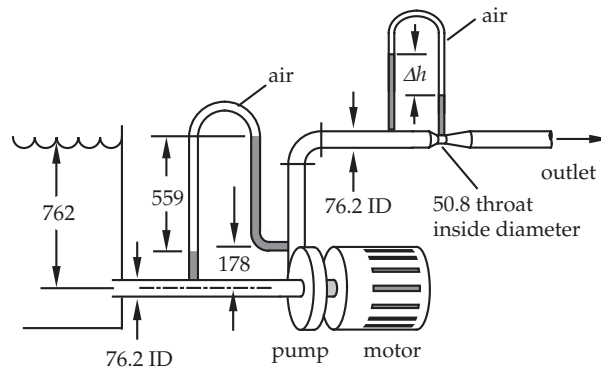


FIGURE P2.47.

$$p_1 - \rho g x - \rho_{\text{air}} g (0.559) + \rho g (0.559 + x - 0.178) = p_2$$

$$\rho_{\text{air}} \text{ is negligible } x \text{ terms cancel; } \rho = 960 \text{ kg/m}^3$$

$$p_2 - p_1 = \rho g (0.559 - 0.178) = 960(9.81)(0.381) = 3588 \text{ Pa}$$

Energy equation, 1 to 2:

$$-\frac{\partial W}{\partial t} = \left\{ \left( \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \right|_2 - \left( \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \right|_1 \} \rho V A$$

$$D_1 = D_2 = 0.0762 \text{ m } A_1 V_1 = A_2 V_2 \text{ so } V_1 = V_2$$

$$z_1 = 0 \quad z_2 = 0.178 \text{ m } \rho A V = \rho Q$$

The power was given as

$$-\frac{\partial W}{\partial t} = 186.5 \text{ W } \text{Substituting,}$$

$$186.5 = \rho Q \left( \frac{(p_2 - p_1)}{\rho} + g z_2 \right) = 960 Q \left( \frac{3588}{960} + 9.81(0.178) \right)$$

Solving for  $Q$

$$Q = 0.0354 \text{ m}^3/\text{s}$$

Now for the venturi meter, the throat diameter is  $D_{th} = 0.0508 \text{ m}$

$$D = 0.0762 \text{ m } A_{th} = \frac{\pi D_{th}^2}{4} = 2.03 \times 10^{-3} \text{ m}^2$$

$$Q = A_{th} \sqrt{\frac{2g \Delta h}{1 - D_{th}^4/D^4}}$$

$$0.0354 = 2.03 \times 10^{-3} \sqrt{\frac{2(9.81) \Delta h}{1 - (0.0508/0.0762)^4}}$$

$$\Delta h = 12.44 \text{ m of castor oil}$$

A 1.83 m tall air-over-oil manometer is not tall enough. A Hg manometer will work; pressure transducers will also work.

48. A 42 mm ID pipe is used to drain a tank, as shown in Figure P2.48. Simultaneously, a 52 mm ID inlet line fills the tank. The velocity in the inlet line is 1.5 m/s. Determine the equilibrium height  $h$  of the liquid in the tank if it is octane. How does the height change if the liquid is ethyl alcohol? Assume in both cases that frictional effects are negligible, and that  $z$  is 40 mm.

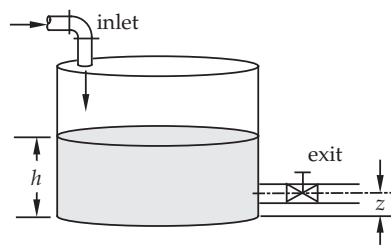


FIGURE P2.48.

$$Q_{in} = AV \quad A = \frac{\pi (0.052)^2}{4} = 2.124 \times 10^{-3} \text{ m}^2$$

$$Q_{in} = 2.124 \times 10^{-3} (1.5) = 3.19 \times 10^{-3} \text{ m}^3/\text{s}$$

Section 1 is the free surface in the tank, and 2 is at the exit of the pipe. Apply the Bernoulli equation, 1 to 2:

$$\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho_1 g} + \frac{V_2^2}{2g} + z_2$$

$$p_1 = p_2 = p_{atm}; \quad V_1 = 0; \quad z_1 = h; \quad z_2 = 0.04 \text{ m}; \text{ the Bernoulli equation becomes}$$

$$h = \frac{V_2^2}{2g} + z_2; \quad \text{At equilibrium, } Q_{out} = Q_{in} = 3.19 \times 10^{-3} \text{ m}^3/\text{s}$$

$$A_{out} = \frac{\pi (0.042)^2}{4} = 1.39 \times 10^{-3} \text{ m}^2; \quad \text{and} \quad V_2 = \frac{Q}{A_{out}} = \frac{3.19 \times 10^{-3}}{1.39 \times 10^{-3}} = 2.3 \text{ m/s}$$

$$h = \frac{V_2^2}{2g} + z_2 = \frac{2.3^2}{2(9.81)} + 0.04$$

$$h = 0.309 \text{ m} \quad \text{which is independent of fluid properties, and with no friction}$$

### Computer Problems

49. One of the examples in this chapter dealt with the following impact problem, with the result that the ratio of forces is given by:

$$\frac{F_x}{F_y} = \frac{(\cos \theta_1 - \cos \theta_2)}{(\sin \theta_2 + \sin \theta_1)}$$

For an angle of  $\theta_1 = 0$ , produce a graph of the force ratio as a function of the angle  $\theta_2$ .

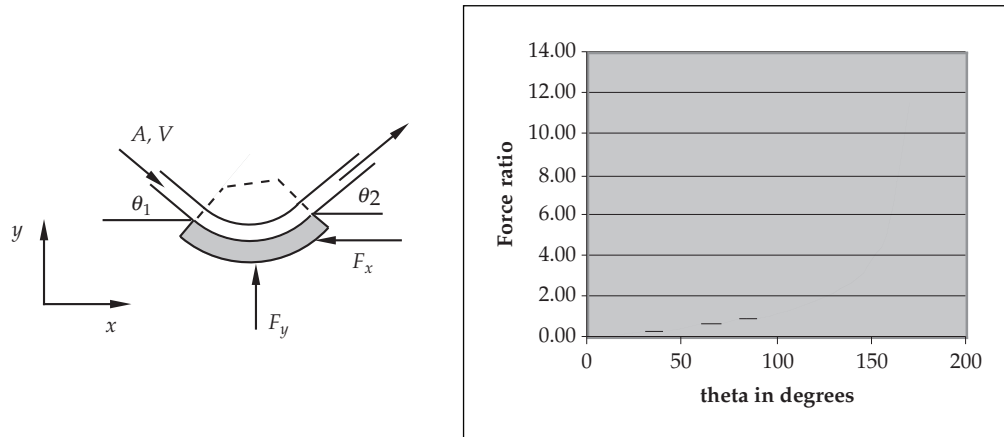


FIGURE P2.49.

50. One of the examples in this chapter involved calculations made to determine the power output of a turbine in a dam (see Figure P2.50). When the flow through the turbine was  $3.15 \text{ m}^3/\text{s}$ , and the upstream height is 36.6 m, the power was found to be 1.06 kW. The relationship between the flow through the turbine and the upstream height is linear. Calculate the work done by (or power received from) the water as it flows through the dam for upstream heights that range from 18.3 to 36.6 m.

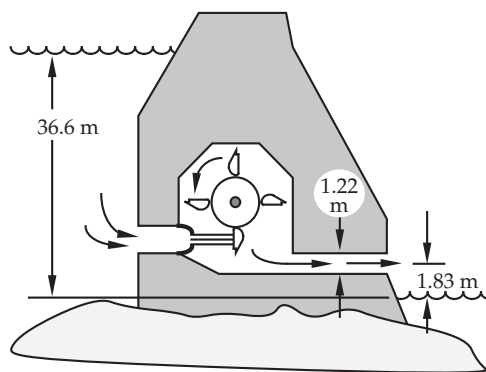


FIGURE P2.50.

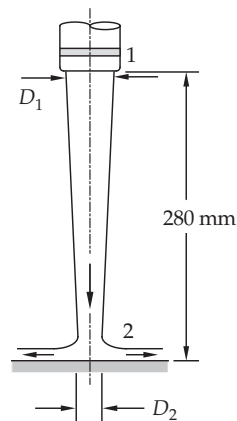
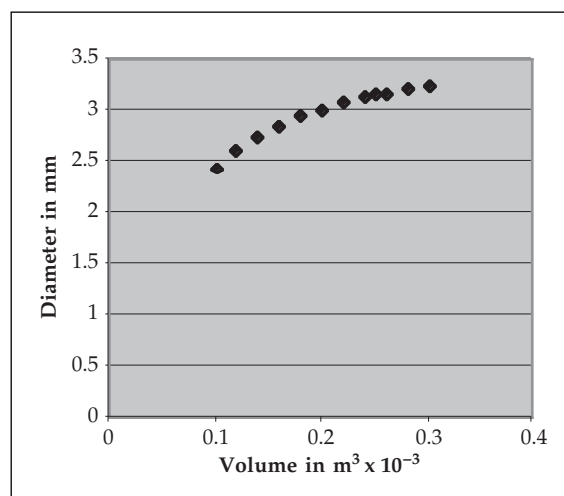
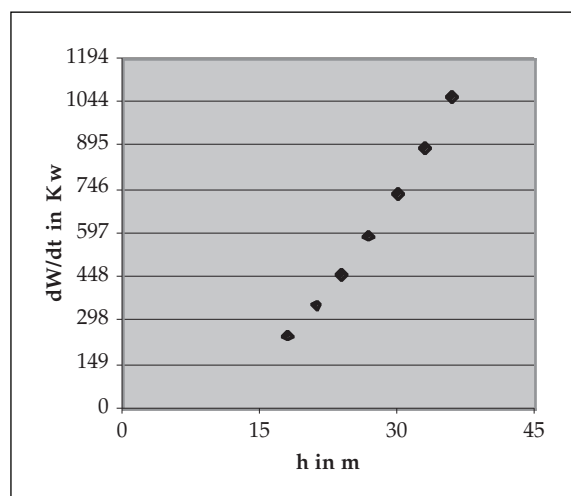


FIGURE P2.51.



51. One of the examples in this chapter dealt with a water jet issuing from a faucet. The water flow rate was  $3.125 \times 10^{-5} \text{ m}^3/\text{s}$ , the jet diameter at faucet exit is 3.5 mm, and the faucet is 280 mm above the sink. Calculations were made to find the jet diameter at impact on the sink surface. Repeat the calculations for volumes per time that range from  $1.25 \times 10^{-5} \text{ m}^3/\text{s}$  to  $6.25 \times 10^{-5} \text{ m}^3/\text{s}$ , and graph jet diameter at 2 as a function of the volume flow rate.