

## Chapter 2. Discovering the Universe for Yourself

This chapter introduces major phenomena of the sky, with emphasis on:

- The concept of the celestial sphere.
- The basic daily motion of the sky, and how it varies with latitude.
- The cause of seasons.
- Phases of the Moon and eclipses.
- The apparent retrograde motion of the planets, and how it posed a problem for ancient observers.

As always, when you prepare to teach this chapter, be sure you are familiar with the online quizzes, interactive figures and tutorials, assignable homework, and other resources available on the MasteringAstronomy Web site.

**Key Changes for the 7th Edition:** For those who have used earlier editions of our textbook, please note the following significant changes in this chapter:

- We have edited throughout the chapter to improve clarity for students, including changes to several of the annotated figures.
- We have reworked the introduction to Section 2.3 and Figure 2.21 to focus more clearly on the scale of the Moon's orbit and why sunlight reaching it is essentially coming in with parallel rays.
- We have added two new See It for Yourself activities, one each in Sections 2.3 and 2.4, designed to encourage students to make naked eye sky observations.
- We have updated the discussion of eclipses, including revising the Table 2.1 and Figure 2.30 of upcoming eclipses.

### Teaching Notes (By Section)

#### Section 2.1 Patterns in the Night Sky

This section introduces the concepts of constellations and of the celestial sphere, and introduces horizon-based coordinates and daily and annual sky motions.

- Stars in the daytime: You may be surprised at how many of your students actually believe that stars disappear in the daytime. If you have a campus observatory or can set up a small telescope, it's well worth offering a daytime opportunity to point the telescope at some bright stars, showing the students that they are still there.
- In class, you may wish to go further in explaining the correspondence between the Milky Way Galaxy and the Milky Way in our night sky. Tell your students to imagine being a tiny grain of flour inside a very thin pancake (or crepe!) that bulges in the middle and a little more than halfway toward the outer edge. Ask, "What will you see if you look toward the middle?" The answer should be "dough." Then ask what they will see if they look toward the far edge, and they'll give the same answer. Proceeding similarly, they should soon realize that they'll

see a band of dough encircling their location, but that if they look away from the plane, the pancake is thin enough that they can see to the distant universe.

- Sky variation with latitude: Here, the intention is only to give students an overview of the idea and the most basic rules (e.g., latitude = altitude of NCP or SCP). Those instructors who want their students to be able to describe the sky in detail should cover Chapter S1, which covers this same material, but in much more depth.
- Note that in our jargon-reduction efforts, we do not introduce the term *asterism*, instead speaking of patterns of stars in the constellations. We also avoid the term *azimuth* when discussing horizon-based coordinates. Instead, we simply refer to *direction* along the horizon (e.g., south, northwest). The distinction of “along the horizon” should remove potential ambiguity with direction on the celestial sphere (where “north” would mean toward the north celestial pole rather than toward the horizon).

## Section 2.2 The Reason for Seasons

This section focuses on seasons and why they occur.

- In combating misconceptions about the cause of the seasons, we recommend that you follow the logic in the Common Misconceptions box. That is, begin by asking your students what they think causes the seasons. When many of them suggest it is linked to distance from the Sun, ask how seasons differ between the two hemispheres. They should then see for themselves that it can’t be distance from the Sun, or seasons would be the same globally rather than opposite in the two hemispheres.
- As a follow-up on the above note: Some students get confused by the fact that season diagrams (such as our Figure 2.15) cannot show the Sun-Earth distance and size of Earth to scale. Thus, unless you emphasize this point (as we do in the figure), it might actually look like the two hemispheres are at significantly different distances from the Sun. This is another reason why we believe it is critical to emphasize ideas of scale throughout your course. In this case, use the scale model solar system as introduced in Section 1.2, and students will quickly see that the two hemispheres are effectively at the same distance from the Sun at all times.
- Note that we do not go deeply into the physics that causes precession, as even a basic treatment of this topic requires discussing the vector nature of angular momentum. Instead, we include a brief motivation for the cause of precession by analogy to a spinning top.
- FYI regarding Sun signs: Most astrologers have “delinked” the constellations from the Sun signs. Thus, most astrologers would say that the vernal equinox still is in Aries—it’s just that Aries is no longer associated with the same pattern of stars as it was in A.D. 150. For a fuller treatment of issues associated with the scientific validity (or, rather, the lack thereof) of astrology, see Section 3.5.

## Section 2.3 The Moon, Our Constant Companion

This section discusses the Moon's motion and its observational consequences, including the lunar phases and eclipses.

- For what appears to be an easy concept, many students find it remarkably difficult to understand the phases of the Moon. You may want to do an in-class demonstration of phases by darkening the room, using a lamp to represent the Sun, and giving each student a Styrofoam ball to represent the Moon. If your lamp is bright enough, the students can remain in their seats and watch the phases as they move the ball around their heads.
- Going along with the above note, it is virtually impossible for students to understand phases from a flat figure on a flat page in a book. Thus, we have opted to eliminate the “standard” Moon phases figure that you’ll find in almost every other text, which shows the Moon in eight different positions around Earth—students just don’t get it, and the multiple moons confuse them. Instead, our Figure 2.22 shows how students can conduct a demonstration that will help them understand the phases. The Phases of the Moon tutorial on the MasteringAstronomy Web site has also proved very successful at helping students understand phases.
- *Note about the appearance of lunar phases:* We have often heard instructors describe the appearance of the lunar phases in terms of, e.g., the illuminated portion of the moon progressing from “right to left” during the cycle of phases. However, please remember that this is true only of the Northern Hemisphere; it appears reversed in the Southern Hemisphere, and in equatorial regions looks more like a bottom to top. For that reason, we recommend not focusing on left/right and instead focusing on time of visibility: waxing moons in the afternoon/evening and waning moons in the morning.
- When covering the causes of eclipses, it helps to demonstrate the Moon's orbit. Keep a model “Sun” on a table in the center of the lecture area; have your left fist represent Earth, and hold a ball in the other hand to represent the Moon. Then you can show how the Moon orbits your “fist” at an inclination to the ecliptic plane, explaining the meaning of the nodes. You can also show eclipse seasons by demonstrating the Moon's orbit (with fixed nodes) as you walk around your model Sun: The students will see that eclipses are possible only during two periods each year. If you then add in precession of the nodes, students can see why eclipse seasons occur slightly more often than every 6 months.
- The *Moon Pond* painting in Figure 2.24 should also be an effective way to explain what we mean by *nodes* of the Moon's orbit.
- FYI: We've found that even many astronomers are unfamiliar with the saros cycle of eclipses. Hopefully our discussion is clear, but some additional information may help you as an instructor: The nodes of the Moon's orbit precess with an 18.6-year period; note that the close correspondence of this number to the 18-year 11-day saros has no special meaning (it essentially is a mathematical coincidence). The reason that the same type of eclipse (e.g., partial versus total) does not recur in

each cycle is because the Moon's line of apsides (i.e., a line connecting perigee and apogee) also precesses—but with a different period (8.85 years).

- FYI: The actual saros period is 6585.32 days, which usually means 18 years, 11.32 days, but instead is 18 years 10.32 days if 5 leap years occur during this period.

## Section 2.4 The Ancient Mystery of the Planets

This section covers the ancient mystery of planetary motion, explaining the motion, how we now understand it, and how the mystery helped lead to the development of modern science.

- We have chosen to refer to the westward movement of planets in our sky as *apparent* retrograde motion, in order to emphasize that planets only appear to go backward but never really reverse their direction of travel in their orbits. This makes it easy to use analogies—for example, when students try the demonstration in Figure 2.33, they never say that their friend really moves backward as they pass by, only that the friend appears to move backward against the background.
- You should emphasize that apparent retrograde motion of planets is noticeable only by comparing planetary positions over many nights. In the past, we've found a tendency for students to misinterpret diagrams of retrograde motion and thereby expect to see planets moving about during the course of a single night.
- It is somewhat rare among astronomy texts to introduce stellar parallax so early. However, it played such an important role in the historical debate over a geocentric universe that we feel it must be included at this point. Note that we do *not* give the formula for finding stellar distances at this point; that comes in Chapter 15.

## Answers/Discussion Points for Think About It/See It for Yourself Questions

The Think About It and See It for Yourself questions are not numbered in the book, so we list them in the order in which they appear, keyed by section number.

### Section 2.1

- (p. 27) The simple answer is no, because a galaxy located in the direction of the galactic center will be obscured from view by the dust and gas of the Milky Way. Note, however, that this question can help you root out some student misconceptions. For example, some students might wonder if you could see the galaxy “sticking up” above our own galaxy's disk—illustrating a misconception about how angular size declines with distance. They might also wonder if a telescope would make a difference, illustrating a misconception about telescopes being able to “see through” things that our eyes cannot see through. Building on this idea, you can also foreshadow later discussions of nonvisible light by pointing out that while no telescope can help the problem in visible light, we *can* penetrate the interstellar gas and dust in some other wavelengths.

- (p. 29) No. We can only describe angular sizes and distances in the sky, so physical measurements do not make sense. This is a difficult idea for many children to understand, but hopefully comes easily for college students!
- (p. 30) Yes, because it is Earth's rotation that causes the rising and setting of all the objects in the sky. *Note:* Many instructors are surprised that this question often gives students trouble, but the trouble arises from at least a couple misconceptions harbored by many students. First, even though students can recite the fact that the motion of the stars is caused by the rotation of Earth, they haven't always absorbed the idea and therefore don't automatically apply it to less familiar objects like galaxies. Second, many students have trouble visualizing galaxies as fixed objects on the celestial sphere like stars, perhaps because they try to see them as being "big" and therefore have trouble fitting them onto the sphere in their minds. Thus, this simple question can help you address these misconceptions and thereby make it easier for students to continue their progress in the course.
- (p. 31 SIFY) This activity is designed to help students become familiar with their local sky by learning their latitude and then checking to see that the north or south celestial pole is indeed at the altitude it should be.
- (p. 32 SIFY) This activity checks that students can properly interpret Figure 2.14 and then asks that they go outside to check their answers in the sky. Sample answer for September 21: The Sun appears to be in Virgo, which means you'll see the opposite zodiac constellation—Pisces—on your horizon at midnight. After sunset, you'll see Libra setting in the western sky, since it is east of Virgo and therefore follows it around the sky.

## Section 2.2

- (p. 33) Jupiter does not have seasons because of its lack of appreciable axis tilt. Saturn, with an axis tilt similar to Earth, does have seasons.
- (p. 38) In 2000 years, the summer solstice will have moved about the length of one constellation along the ecliptic. Since the summer solstice was in Cancer a couple thousand years ago (as you can remember from the Tropic of Cancer) and is in Gemini now, it will be in Taurus in about 2000 years.

## Section 2.3

- (p. 39 SIFY) This activity asks students to observe the change in the Moon's position among the stars over the course of the night, making it another good way to help students connect their in-class learning to the real sky.
- (p. 40) A "half light and half dark" moon visible in the morning must be third-quarter, since third-quarter moon rises around midnight and sets around noon.
- (p. 41) About 2 weeks each. Because the Moon takes about a month to rotate, your "day" would last about a month. Thus, you'd have about 2 weeks of daylight followed by about 2 weeks of darkness as you watched Earth hanging in your sky and going through its cycle of phases.

- (p. 45) Remember that each eclipse season lasts a few weeks. Thus, if the timing of the eclipse season is just right, it is possible for two full moons to occur during the same eclipse season, giving us two lunar eclipses just a month apart. In such cases at least one of the eclipses will almost always be penumbral, because the penumbral shadow is much larger than the umbral shadow and therefore it is more likely that the Moon passes through it than through the smaller umbral shadow.

## Section 2.4

- (p. 46 SIFY) This activity asks students to learn which planets are visible in tonight's sky, and then to go out and look for them.
- (p. 48) Opposite ends of Earth's orbit are about 300 million kilometers apart, or about 30 meters on the 1-to-10-billion scale used in Chapter 1. The nearest stars are tens of trillions of kilometers away, or thousands of kilometers on the 1-to-10-billion scale, and are typically the size of grapefruits or smaller. The challenge of detecting stellar parallax should now be clear.

## Solutions to End-of-Chapter Problems (Chapter 2)

### Visual Skills Check

1. B
2. D
3. C
4. d
5. b
6. d
7. c
8. c

### Review Questions

1. A constellation is a section of the sky, like a state within the United States. They are based on groups of stars that form patterns that suggested shapes to the cultures of the people who named them. The official names of most of the constellations in the Northern Hemisphere came from ancient cultures of the Middle East and the Mediterranean, while the constellations of the Southern Hemisphere got their official names from 17th-century Europeans.
2. If we were making a model of the celestial sphere on a ball, we would definitely need to mark the north and south celestial poles, which are the points directly above Earth's poles. Halfway between the two poles we would mark the great circle of the celestial equator, which is the projection of Earth's equator out into space. And we definitely would need to mark the circle of the ecliptic, which is the path that the Sun appears to make across the sky. Then we could add stars and borders of constellations.

3. No, space is not really full of stars. Because the distance to the stars is very large and because stars lie at different distances from Earth, stars are not really crowded together.
4. The local sky looks like a dome because we see half of the full celestial sphere at any one time.  
Horizon—The boundary line dividing the ground and the sky.  
Zenith—The highest point in the sky, directly overhead.  
Meridian—The semicircle extending from the horizon due north to the zenith to the horizon due south.  
We can locate an object in the sky by specifying its altitude and its direction along the horizon.
5. We can measure only angular size or angular distance on the sky because we lack a simple way to measure distance to objects just by looking at them. It is therefore usually impossible to tell if we are looking at a smaller object that's near us or a more distant object that's much larger.  
Arcminutes and arcseconds are subdivisions of degrees. There are 60 arcminutes in 1 degree, and there are 60 arcseconds in 1 arcminute.
6. Circumpolar stars are stars that never appear to rise or set from a given location, but are always visible on any clear night. From the North Pole, every visible star is circumpolar, as all circle the horizon at constant altitudes. In contrast, a much smaller portion of the sky is circumpolar from the United States, as most stars follow paths that make them rise and set.
7. Latitude measures angular distance north or south of Earth's equator. Longitude measures angular distance east or west of the Prime Meridian. The night sky changes with latitude, because it changes the portion of the celestial sphere that can be above your horizon at any time. The sky does not change with changing longitude, however, because as Earth rotates, all points on the same latitude line will come under the same set of stars, regardless of their longitude.
8. The zodiac is the set of constellations in which the Sun can be found at some point during the year. We see different parts of the zodiac at different times of the year because the Sun is always somewhere in the zodiac and so we cannot see that constellation at night at that time of the year.
9. If Earth's axis had no tilt, Earth would not have significant seasons because the intensity of sunlight at any particular latitude would not vary with the time of year.
10. The summer solstice is the day when the Northern Hemisphere gets the most direct sunlight and the Southern Hemisphere the least direct. Also, on the summer solstice the Sun is as far north as it ever appears on the celestial sphere. On the winter solstice, the situation is exactly reversed: The Sun appears as far south as it will get in the year, and the Northern Hemisphere gets its least direct sunlight while the Southern Hemisphere gets its most direct sunlight.

On the equinoxes, the two hemispheres get the same amount of sunlight, and the day and night are the same length (12 hours) in both hemispheres. The Sun is found directly overhead at the equator on these days, and it rises due east and sets due west.

11. The direction in which Earth's rotation axis points in space changes slowly over the centuries, and we call this change "precession." Because of this movement, the celestial poles and therefore the pole star changes slowly in time. So while Polaris is the pole star now, in 13,000 years the star Vega will be the pole star instead.
12. The Moon's phases start with the new phase when the Moon is nearest the Sun in our sky; we cannot see the new moon, both because the Moon's night side is facing us and because the dim light we might otherwise see from the night side (reflected light from Earth) is overwhelmed by the bright daytime sky. The waxing phases — in which we see a gradually increasing amount of the Moon's visible face illuminated — then progress with one side of the Moon's visible face slowly becoming sunlit, moving to crescent, then to first-quarter (when we see a half-lit moon), to gibbous and then to full. Full moon is when the entire visible face of the Moon is sunlit and the Moon is visible all night long. The waning phases then occur in reverse as the Moon's sunlit fraction decreases, through gibbous, third-quarter, crescent, and back to new again.

We can never see a full moon at noon because for the Moon to be full, it and the Sun must be on opposite sides of Earth. So as the full moon rises, the Sun must be setting and when the Moon is setting, the Sun is rising. (*Exception:* At very high latitudes, there may be times when the full moon is circumpolar, in which case it could be seen at noon—but would still be 180° away from the Sun's position.)

13. We always see the same face of the Moon because the Moon displays synchronous rotation, meaning that the Moon's rotation period and its orbital period around Earth are the same.
14. While the Moon must be in its new phase for a solar eclipse or in its full phase for a lunar eclipse, we do not see eclipses every month. This is because the Moon usually passes to the north or south of the Sun during these times, because its orbit is tilted relative to the ecliptic plane.
15. The apparent retrograde motion of the planets refers to the planets' behaviors when they sometimes stop moving eastward relative to the stars and move westward for a few weeks or months. While the ancients had to resort to complex systems to explain this behavior, our Sun-centered model makes this motion a natural consequence of the fact that the different planets move at different speeds as they go around the Sun. We see the planets appear to move backward because we are overtaking them in our orbit (if they orbit farther from the Sun than Earth) or they are overtaking us (if they orbit closer to the Sun than Earth).
16. Stellar parallax is the apparent movement of some of the nearest stars relative to more distant ones as Earth goes around the Sun. This is caused by our slightly changing perspective on these stars through the year. The shift due to parallax is very small because Earth's orbit is much smaller than the distances to even the closest stars. Because the effect is so small, the ancients were unable to observe it. However, they correctly realized that if Earth is going around the Sun, they should see stellar parallax. Since they could not see the stars shift, they concluded that Earth does not move.



## Does It Make Sense?

17. *The constellation of Orion didn't exist when my grandfather was a child.* This statement does not make sense, because the constellations don't appear to change on the time scales of human lifetimes.
18. *When I looked into the dark lanes of the Milky Way with my binoculars, I saw what must have been a cluster of distant galaxies.* This statement does not make sense, because we cannot see through the band of light we call the Milky Way to external galaxies; the dark fissure is gas and dust blocking our view.
19. *Last night the Moon was so big that it stretched for a mile across the sky.* This statement does not make sense, because a mile is a physical distance, and we can measure only angular sizes or distances when we observe objects in the sky.
20. *I live in the United States, and during my first trip to Argentina I saw many constellations that I'd never seen before.* This statement makes sense, because the constellations visible in the sky depend on latitude. Since Argentina is in the Southern Hemisphere, the constellations visible there include many that are not visible from the United States.
21. *Last night I saw Jupiter right in the middle of the Big Dipper. (Hint: Is the Big Dipper part of the zodiac?)* This statement does not make sense, because Jupiter, like all the planets, is always found very close to the ecliptic in the sky. The ecliptic passes through the constellations of the zodiac, so Jupiter can appear to be only in one of the zodiac constellations—and the Big Dipper (part of the constellation Ursa Major) is not part of the zodiac.
22. *Last night I saw Mars move westward through the sky in its apparent retrograde motion.* This statement does not make sense, because apparent retrograde motion is noticeable only over many nights, not during a single night. (Earth's rotation means that all celestial objects, including Mars, move from east to west over the course of each single night.)
23. *Although all the known stars rise in the east and set in the west, we might someday discover a star that will rise in the west and set in the east.* This statement does not make sense. The stars aren't really moving around us; they only appear to rise in the east and set in the west because *Earth* rotates.
24. *If Earth's orbit were a perfect circle, we would not have seasons.* This statement does not make sense. As long as Earth still has its axis tilt, we'll still have seasons.
25. *Because of precession, someday it will be summer everywhere on Earth at the same time.* This statement does not make sense. Precession does not change the tilt of the axis, only its orientation in space. As long as the tilt remains, we will continue to have opposite seasons in the two hemispheres.
26. *This morning I saw the full moon setting at about the same time the Sun was rising.* This statement makes sense, because a full moon is opposite the Sun in the sky.

## Quick Quiz

27. c
28. a
29. a

- 30. a
- 31. a
- 32. b
- 33. b
- 34. b
- 35. a
- 36. b

## Process of Science

- 37. (a) Consistent with Earth-centered view, simply by having the stars rotate around Earth. (b) Consistent with Earth-centered view by having the Sun actually move slowly among the constellations on the path of the ecliptic, so that its position north or south of the celestial equator is thought of as “real” rather than as a consequence of the tilt of Earth’s axis. (c) Consistent with Earth-centered view, since phases are caused by relative positions of Sun, Earth, and Moon—which are about the same with either viewpoint, since the Moon really does orbit Earth. (d) Consistent with Earth-centered view; as with (c), eclipses depend only on the Sun-Earth-Moon geometry. (e) In terms of just having the “heavens” revolve around Earth, apparent retrograde motion is inconsistent with the Earth-centered view. However, this view was not immediately rejected because the absence of parallax (and other beliefs) caused the ancients to go to great lengths to find a way to preserve the Earth-centered system. As we’ll see in the next chapter, Ptolemy succeeded well enough for the system to remain in use for another 1500 years. Ultimately, however, the inconsistencies in predictions of planetary motion led to the downfall of the Earth-centered model.
- 38. The shadow shapes are wrong. For example, during gibbous phase the dark portion of the Moon has the shape of a crescent, and a round object could not cast a shadow in that shape. You could also show that the crescent moon, for example, is nearly between Earth and the Sun, so Earth can’t possibly cast a shadow on it.

## Group Work Exercise (no solution provided)

### Short Answer/Essay Questions

- 40. The planet will have seasons because of its axis tilt, even though its orbit is circular. Because its  $35^\circ$  axis tilt is greater than Earth’s  $23.5^\circ$  axis tilt, we’d expect this planet to have more extreme seasonal variations than Earth.
- 41. Answers will vary with location; the following is a sample answer for Boulder, CO.
  - a. The latitude in Boulder is  $40^\circ\text{N}$  and the longitude is about  $105^\circ\text{E}$ .
  - b. The north celestial pole appears in Boulder’s sky at an altitude of  $40^\circ$ , in the direction due north.
  - c. Polaris is circumpolar because it never rises or sets in Boulder’s sky. It makes a daily circle, less than  $1^\circ$  in radius, around the north celestial pole.

42.
  - a. When you see a full Earth, people on Earth must have a new moon.
  - b. At full moon, you would see new Earth from your home on the Moon. It would be daylight at your home, with the Sun on your meridian and about a week until sunset.
  - c. When people on Earth see a waxing gibbous moon, you would see a waning crescent Earth.
  - d. If you were on the Moon during a total lunar eclipse (as seen from Earth), you would see a total eclipse of the Sun.
43. You would not see the Moon go through phases if you were viewing it from the Sun. You would always see the sunlit side of the Moon, so it would always be “full.” In fact, the same would be true of Earth and all the other planets as well.
44. If the Moon were twice as far from Earth, its angular size would be too small to create a total solar eclipse. It would still be possible to have annular eclipses, although the Moon would cover only a small portion of the solar disk.
45. If Earth were smaller in size, solar eclipses would still occur in about the same way, since they are determined by the Moon’s shadow on Earth.
46. This is an observing project that will stretch over several weeks.
47. This is a literary essay that requires reading the Mark Twain novel.

## Quantitative Problems

48.
  - a. There are  $360 \times 60 = 21,600$  arcminutes in a full circle.
  - b. There are  $360 \times 60 \times 60 = 1,296,000$  arcseconds in a full circle.
  - c. The Moon’s angular size of  $0.5^\circ$  is equivalent to 30 arcminutes or  $30 \times 60 = 1800$  arcseconds.
49.
  - a. We know that  $\text{circumference} = 2 \times \pi \times \text{radius}$ , so we can compute the circumference of Earth:
 
$$\begin{aligned}\text{circumference} &= 2 \times \pi \times (6370 \text{ km}) \\ &= 40,000 \text{ km}\end{aligned}$$
  - b. There are  $90^\circ$  of latitude between the North Pole and the equator. This distance is also one-quarter of Earth’s circumference. Using the circumference from part (a), this distance is

$$\begin{aligned}\text{equator to pole distance} &= \frac{\text{circumference}}{4} \\ &= \frac{40,000 \text{ km}}{4} \\ &= 10,000 \text{ km}\end{aligned}$$

So if 10,000 kilometers is the same as  $90^\circ$  of latitude, then we can convert  $1^\circ$  into kilometers:

$$1^\circ \times \frac{10,000 \text{ km}}{90^\circ} = 111 \text{ km}$$

So  $1^\circ$  of latitude is the same as 111 kilometers on Earth.

- c. There are 60 arcminutes in a degree. So we can find how many arcminutes are in a quarter-circle:

$$90^\circ \times \frac{60 \text{ arcminutes}}{1^\circ} = 5400 \text{ arcminutes}$$

Doing the same thing as in part (b):

$$1 \text{ arcminute} \times \frac{10,000 \text{ km}}{5400 \text{ arcminutes}} = 1.85 \text{ km}$$

Each arcminute of latitude represents 1.85 kilometers.

- d. We cannot provide similar answers for longitude, because lines of longitude get closer together as we near the poles, eventually meeting at the poles themselves. So there is no single distance that can represent  $1^\circ$  of longitude everywhere on Earth.
50. a. We start by recognizing that there are 24 whole degrees in this number. So we just need to convert the  $0.3^\circ$  into arcminutes and arcseconds. So first converting to arcminutes:

$$0.3^\circ \times \frac{60 \text{ arcminutes}}{1^\circ} = 18 \text{ arcminutes}$$

Since there is no fractional part left to convert into arcseconds, we are done. So  $24.3^\circ$  is the same as  $24^\circ 18' 0''$ .

- b. Leaving off the whole degree, we convert the  $0.59^\circ$  to arcminutes:

$$0.59^\circ \times \frac{60 \text{ arcminutes}}{1^\circ} = 35.4 \text{ arcminutes}$$

So we have 35 whole arcminutes and a fractional part of 0.4 arcminute that we need to convert into arcseconds:

$$0.4 \text{ arcminute} \times \frac{60 \text{ arcseconds}}{1 \text{ arcminute}} = 24 \text{ arcseconds}$$

So  $1.59^\circ$  is the same as  $1^\circ 35' 24''$ .

- c. We have 0 whole degrees, so we convert the fractional degree into arcminutes:

$$0.1^\circ \times \frac{60 \text{ arcminutes}}{1^\circ} = 6 \text{ arcminutes}$$

Since there is no fractional part to this, we do not need any arcseconds to represent this number. So  $0.1^\circ$  is the same as  $0^\circ 6' 0''$ .

- d. We again have no whole degrees, so we start by converting  $0.01^\circ$  to arcminutes:

$$0.01^\circ \times \frac{60 \text{ arcminutes}}{1^\circ} = 0.6 \text{ arcminute}$$

There are no whole arcminutes here, either, so we have to convert 0.6 arcminute into arcseconds:

$$0.6 \cancel{\text{arcminute}} \times \frac{60 \text{ arcseconds}}{1 \cancel{\text{arcminute}}} = 36 \text{ arcseconds}$$

So  $0.01^\circ$  is the same as  $0^\circ 0' 36''$ .

- e. We again have no whole degrees, so we start by converting  $0.001^\circ$  to arcminutes:

$$0.001^\circ \times \frac{60 \text{ arcminutes}}{1^\circ} = 0.06 \text{ arcminute}$$

There are no whole arcminutes here, either, so we have to convert 0.06 arcminute into arcseconds:

$$0.06 \cancel{\text{arcminute}} \times \frac{60 \text{ arcseconds}}{1 \cancel{\text{arcminute}}} = 3.6 \text{ arcseconds}$$

So  $0.01^\circ$  is the same as  $0^\circ 0' 3.6''$ .

51. a. We will start by converting the 42 arcseconds into arcminutes:

$$42 \cancel{\text{arcseconds}} \times \frac{1 \text{ arcminute}}{60 \cancel{\text{arcseconds}}} = 0.7 \text{ arcsecond}$$

So now we have  $7^\circ 38.7'$ . Converting the 38.7 arcminutes to degrees:

$$38.7 \cancel{\text{arcminutes}} \times \frac{1^\circ}{60 \cancel{\text{arcminutes}}} = 0.645^\circ$$

So  $7^\circ 38' 42''$  is the same as  $7.645^\circ$ .

- b. We will start by converting the 54 arcseconds into arcminutes:

$$54 \cancel{\text{arcseconds}} \times \frac{1 \text{ arcminute}}{60 \cancel{\text{arcseconds}}} = 0.9 \text{ arcminute}$$

So now we have 12.9 arcminutes. Converting this to degrees:

$$12.9 \cancel{\text{arcminutes}} \times \frac{1^\circ}{60 \cancel{\text{arcminutes}}} = 0.215^\circ$$

So  $12' 54''$  is the same as  $0.215^\circ$ .

- c. We will start by converting the 59 arcseconds into arcminutes:

$$59 \cancel{\text{arcseconds}} \times \frac{1 \text{ arcminute}}{60 \cancel{\text{arcseconds}}} = 0.9833 \text{ arcminute}$$

So now we have  $1^{\circ} 59.9833'$  arcminutes. Converting this to degrees:

$$59.9833 \cancel{\text{ arcminutes}} \times \frac{1^{\circ}}{60 \cancel{\text{ arcminutes}}} = 0.9997^{\circ}$$

So  $1^{\circ} 59' 59''$  is the same as  $1.9997^{\circ}$ , very close to  $2^{\circ}$ .

- d. In this case, we need only convert 1 arcminute to degrees:

$$1 \cancel{\text{ arcminute}} \times \frac{1^{\circ}}{60 \cancel{\text{ arcminutes}}} = 0.017^{\circ}$$

So  $1'$  is the same as  $0.017^{\circ}$ .

- e. We can convert this from arcseconds to degrees in one step since there are no arcminutes to add in:

$$1 \cancel{\text{ arcsecond}} \times \frac{1 \cancel{\text{ arcminute}}}{60 \cancel{\text{ arcseconds}}} \times \frac{1^{\circ}}{60 \cancel{\text{ arcminutes}}} = 2.78 \times 10^{-4}^{\circ}$$

So  $1''$  is the same as  $2.78 \times 10^{-4}^{\circ}$ .

52. Answers will vary for individual students based on size of their finger and arm length.
53. To solve this problem, we turn to Mathematical Insight 2.1, where we learn that the physical size of an object, its distance, and its angular size are related by the equation:

$$\text{physical size} = \frac{2\pi \times (\text{distance}) \times (\text{angular size})}{360^{\circ}}$$

We are told that the Sun is  $0.5^{\circ}$  in angular diameter and is about 150,000,000 kilometers away. So we put those values in:

$$\begin{aligned} \text{physical size} &= \frac{2\pi \times (150,000,000 \text{ km}) \times (0.5^{\circ})}{360^{\circ}} \\ &= 1,310,000 \text{ km} \end{aligned}$$

For the values given, we estimate the size to be about 1,310,000 kilometers. We are told that the actual value is about 1,390,000 kilometers. The two values are pretty close and the difference can be explained by the Sun's actual diameter not being exactly  $0.5^{\circ}$  and the distance to the Sun not being exactly 150,000,000 kilometers.

54. To solve this problem, we use the equation relating distance, physical size, and angular size given in Mathematical Insight 2.1:

$$\text{physical size} = \frac{2\pi \times (\text{distance}) \times (\text{angular size})}{360^{\circ}}$$

In this case, we are given the distance to Betelgeuse as 600 light-years and the angular size as 0.05 arcsecond. We have to convert this number to degrees (so that the units in the numerator and denominator cancel), so:

$$0.05 \cancel{\text{arcsecond}} \times \frac{1 \cancel{\text{arcminute}}}{60 \cancel{\text{arcseconds}}} \times \frac{1^\circ}{60 \cancel{\text{arcminutes}}} = (1.39 \times 10^{-5})^\circ$$

We can leave the distance in light-years for now. So we can calculate the size of Betelgeuse:

$$\begin{aligned} \text{physical size} &= \frac{2\pi \times (600 \text{ light-years}) \times (1.39 \times 10^{-5})^\circ}{360^\circ} \\ &= 1.5 \times 10^{-4} \text{ light-years} \end{aligned}$$

Clearly, we've chosen to express this in the wrong units: light-years are too large to be convenient for expressing the size of stars. So we convert to kilometers using the conversion factor found in Appendix A:

$$1.5 \times 10^{-4} \cancel{\text{light-years}} \times \frac{9.46 \times 10^{12} \text{ km}}{1 \cancel{\text{light-year}}} = 1.4 \times 10^9 \text{ km}$$

(Note that we could have converted the distance to Betelgeuse to kilometers before we calculated Betelgeuse's size and gotten the diameter in kilometers out of our formula for physical size.)

The diameter of Betelgeuse is about 1.4 billion kilometers, which is more than 1000 times the Sun's diameter of  $1.39 \times 10^6$  kilometers. It is also almost ten times the distance between Earth and Sun ( $1.5 \times 10^8$  kilometers).

55. a. Using the small-angle formula given in Mathematical Insight 2.1, we know that:

$$\text{angular size} = \text{physical size} \times \frac{360^\circ}{2\pi \times \text{distance}}$$

We are given the physical size of the Moon (3476 kilometers) and the minimum orbital distance (356,400 kilometers), so we can compute the angular size:

$$\text{angular size} = (3476 \cancel{\text{km}}) \times \frac{360^\circ}{2\pi \times (356,400 \cancel{\text{km}})} = 0.559^\circ$$

When the Moon is at its most distant, it is 406,700 kilometers, so we can repeat the calculation for this distance:

$$\text{angular size} = (3476 \cancel{\text{km}}) \times \frac{360^\circ}{2\pi \times (406,700 \cancel{\text{km}})} = 0.426^\circ$$

The Moon's angular diameter varies from  $0.426^\circ$  to  $0.559^\circ$  (at its farthest point from Earth and at its closest, respectively).

- b. We can do the same thing as in part (a), except we use the Sun's diameter (1,390,000 kilometers) and minimum and maximum distances (147,500,000 kilometers and 152,600,000 kilometers) from Earth. At its closest, the Sun's angular diameter is:

$$\text{angular size} = (1,390,000 \text{ km}) \times \frac{360^\circ}{2\pi \times (147,500,000 \text{ km})} = 0.540^\circ$$

At its farthest from Earth, the Sun's angular diameter is:

$$\text{angular size} = (1,390,000 \text{ km}) \times \frac{360^\circ}{2\pi \times (152,600,000 \text{ km})} = 0.522^\circ$$

The Sun's angular diameter varies from  $0.522^\circ$  to  $0.540^\circ$ .

- c. When both objects are at their maximum distances from Earth, both objects appear with their smallest angular diameters. At this time, the Sun's angular diameter is  $0.522^\circ$  and the Moon's angular diameter is  $0.426^\circ$ . The Moon's angular diameter under these conditions is significantly smaller than the Sun's, so it could *not* fully cover the Sun's disk. Since it cannot completely cover the Sun, there can be no total eclipse under these conditions. There can be only an annular or partial eclipse under these conditions.