Chapter 2 Review of Basic Algebra

Exercise 2.1

A. 1. $\boxed{19a}$

2. $\boxed{3m}$

3. $\boxed{-a-10}$

4. $\boxed{-3a-14}$

5. $\boxed{-2x-4y}$

6. $\boxed{3p+q}$

7. $\boxed{14f-4v}$

8. $\boxed{2c-3d}$

9. $\boxed{0.8x}$

10. $\boxed{1.06x}$

11. $\boxed{1.4x}$

12. $\boxed{0.98x}$

13. $\boxed{2.79x}$

14. $\boxed{4.05y}$

15. $\boxed{-x^2-x-8}$

16. $\boxed{-ax+x-2}$

17. $2x-3y-x-4y=\boxed{x-7y}$

18. $-4+5a+2-3a=\boxed{2a-2}$

19. $12b+4c+9+8-8b-2c-15=\boxed{4b+2c+2}$

20. $a^2-ab+b^2-3a^2-5ab+4b^2=\boxed{-2a^2-6ab+5b^2}$

21. $-3m^2+4m+5-4+2m+2m^2=\boxed{-m^2+6m+1}$

22. $6-4x+3y-1-5x-2y+9=\boxed{14-9x+y}$

23. $7a-5b+3a-4b-5b=\boxed{10a-14b}$

24. $3f-f^2+fg-f+3f^2+2fg=\boxed{2f+2f^2+3fg}$

B. 1. $\boxed{-12x}$

2. $\boxed{-56a}$

3. $\boxed{-10ax}$

4. $\boxed{27ab}$

5. $\boxed{-2x^2}$

6. $\boxed{24m^2}$

7. $\boxed{60xy}$

8. $\boxed{-24abc}$

9. $\boxed{-2x+4y}$

10. $\boxed{10x-20}$

11. $\boxed{2ax^2-3ax-a}$

12. $\boxed{-24x+12bx+6b^2x}$

13. $20x-24-6+15x=\boxed{35x-30}$

14. $-24a+3b+14a-18b=\boxed{-10a-15b}$

15. $-15ax+3a+5a-2ax-3ax-3a=\boxed{-20ax+5a}$

16. $24y-32-4y+2-1+y=\boxed{21y-31}$

17. $3x^2-x+6x-2=\boxed{3x^2+5x-2}$

18. $5m^2-2mn-15mn+6n^2=\boxed{5m^2-17mn+6n^2}$

19. $x^3-x^2y+xy^2+x^2y-xy^2+y^3=\boxed{x^3+y^3}$

20. $a^3-2a^2+a-a^2+2a-1=\boxed{a^3-3a^2+3a-1}$

21. $10x^2-8x-5x+4-3x^2+21x-5x+35=\boxed{7x^2+3x+39}$

22. $2(2a^2-2a-3a+3)-3(3a^2-2a+3a-2)$

$=4a^2-10a+6-9a^2-3a+6$

$=\boxed{-5a^2-13a+12}$

23. $\boxed{4ab}$

24. $\boxed{-5y}$

25. $\boxed{4x}$

26. $\boxed{-6}$

27. $\boxed{10m-4}$

28. $\boxed{-2x+3}$

29. $\boxed{-2x^2+3x+6}$

30. $\boxed{a^2+4a+3}$

C. 1. $3x-2y-3=3(-4)-2(-5)-3=-12+10-3=\boxed{-5}$

2. $\frac{1}{2}(3x^2-x-1)-\frac{1}{4}(5-2x-x^2)$

$=\frac{1}{2}[3(-3)^2-(-3)-1]-\frac{1}{4}[5-2(-3)-(-3)^2]$

$=\frac{1}{2}(27+3-1)-\frac{1}{4}(5+6-9)$

$=\frac{1}{2}(29)-\frac{1}{4}(2)$

$=14.5-0.5$

$=\boxed{14}$

3. $(pq-vq)-f=(p-v)q-f=(12-7)2000-4500=10\ 000-4500=\boxed{5500}$

4. $F/C=13\ 000/0.65=\boxed{20\ 000}$

5. $(1-d1)(1-d2)(1-d3)=(1-0.35)(1-0.08)(1-0.02)=(0.65)(0.92)(0.98)=\boxed{0.58604}$

6. $C+0.38C=0.24C=(1+0.38+0.24)C=1.62C=1.62(\$25.00)=\boxed{\$40.50}$

7. $\frac{RP(n+1)}{2\text{N}}=\frac{0.21\times\$1200\times(77+1)}{2\times 26}=\boxed{\$378}$

8. $\dfrac{\text{I}}{\text{Pt}} = \dfrac{63}{840\times\frac{219}{365}} = \dfrac{63}{840\times 0.60} = \boxed{0.125}$

9. $\dfrac{\text{I}}{\text{rt}} = \dfrac{\$198}{0.165\times\frac{146}{365}} = \dfrac{\$198}{0.165\times 0.40} = \boxed{\$3000}$

10. $\dfrac{2\text{NC}}{\text{P}(n+1)} = \dfrac{2\times 52\times 60}{1800(25+1)} = \dfrac{2\times 52\times 60}{1800\times 26} = \boxed{0.1\dot{3}}$

11. $\text{P}(1+rt) = \$880\left(1+0.12\times\dfrac{76}{365}\right)$

$= \$880(1+0.024986) = \$880(1.024986) = \boxed{\$901.99}$

12. $\text{FV}(1-rt) = \$1200\left(1-0.175\times\dfrac{256}{365}\right)$

$= \$1200(1-0.122740) = \$1200(0.877260) = \boxed{\$1052.71}$

13. $\dfrac{\text{P}}{1-dt} = \dfrac{\$1253}{1-0.135\times\frac{284}{365}} = \dfrac{\$1253}{1-0.083219} = \dfrac{\$1253}{0.916781} = \boxed{\$1400.06}$

14. $\dfrac{\text{S}}{1+rt} = \dfrac{\$1752}{1+0.152\times\frac{228}{365}} = \dfrac{\$1752}{1+0.094948} = \dfrac{\$1752}{1.094948} = \boxed{\$1600.08}$

Exercise 2.2

A. 1. $\boxed{81}$

2. $\boxed{1}$

3. $\boxed{16}$

4. $\boxed{1}$

5. $\boxed{\dfrac{16}{81}}$

6. $\boxed{\dfrac{625}{1296}}$

7. $\boxed{-\dfrac{1}{64}}$

8. $\boxed{-\frac{8}{27}}$

9. $\boxed{0.25}$

10. $\boxed{113.379904}$

11. $\boxed{-0.001}$

12. $\boxed{-335.544320}$

13. $\boxed{1}$

14. $\boxed{1}$

15. $\boxed{\frac{1}{9}}$

16. $\boxed{512}$

17. $\boxed{-\frac{1}{125}}$

18. $\boxed{\frac{1}{167.9616}}$

19. $\boxed{125}$

20. $\boxed{\frac{81}{16}}$

21. $\boxed{\frac{1}{1.01}}$

22. $\boxed{1}$

B. 1. $2^5 \times 2^3 = 2^{5+3} = \boxed{2^8}$

2. $(-4)^3 \times (-4) = (-4)^{3+1} = \boxed{(-4)^4}$

3. $4^7 \div 4^4 = 4^{7-4} = \boxed{4^3}$

4. $(-3)^9 \div (-3)^7 = (-3)^{9-7} = \boxed{(-3)^2}$

5. $(2^3)^5 = 2^{3\times 5} = \boxed{2^{15}}$

6. $\left[(-4)^3\right]^6 = (-4)^{3\times6} = \boxed{(-4)^{18}}$

7. $a^4 \times a^{10} = a^{4+10} = \boxed{a^{14}}$

8. $m^{12} \div m^7 = m^{12-7} = \boxed{m^5}$

9. $3^4 \times 3^6 \times 3 = 3^{4+6+1} = \boxed{3^{11}}$

10. $(-1)^3(-1)^7(-1)^5 = (-1)^{3+7+5} = \boxed{(-1)^{15}}$

11. $\dfrac{6^7 \times 6^3}{6^9} = 6^{7+3-9} = \boxed{6}$

12. $\dfrac{(x^4)(x^5)}{x^7} = x^{4+5-7} = \boxed{x^2}$

13. $\left(\dfrac{3}{5}\right)^4\left(\dfrac{3}{5}\right)^7 = \left(\dfrac{3}{5}\right)^{4+7} = \boxed{\dfrac{3^{11}}{5^{11}}}$

14. $\left(\dfrac{1}{6}\right)^5 \div \left(\dfrac{1}{6}\right)^3 = \left(\dfrac{1}{6}\right)^{5-3} = \boxed{\dfrac{1}{6^2}}$

15. $\left(-\dfrac{3}{2}\right)\left(-\dfrac{3}{2}\right)^6\left(-\dfrac{3}{2}\right)^4 = \left(-\dfrac{3}{2}\right)^{1+6+4} = \boxed{\dfrac{(-3)^{11}}{2^{11}}}$

16. $\left(-\dfrac{3}{4}\right)^8 \div \left(-\dfrac{3}{4}\right)^7 = \left(-\dfrac{3}{4}\right)^{8-7} = \boxed{-\dfrac{3}{4}}$

17. $(1.025)^{80}(1.025)^{70} = (1.025)^{80+70} = \boxed{1.025^{150}}$

18. $1.005^{240} \div 1.005^{150} = 1.005^{240-150} = \boxed{1.005^{90}}$

19. $\left[1.04^{20}\right]^4 = 1.04^{20\times4} = \boxed{1.04^{80}}$

20. $\left[\left(-\dfrac{3}{7}\right)^5\right]^3 = \left(-\dfrac{3}{7}\right)^{5\times3} = \boxed{\dfrac{-3^{15}}{7^{15}}}$

21. $(1+i)^{100}(1+i)^{100} = (1+i)^{100+100} = \boxed{(1+i)^{200}}$

22. $(1-r)^2(1-r)^2(1-r)^2 = (1-r)^{2+2+2} = \boxed{(1-r)^6}$

23. $\left[(1+i)^{80}\right]^2 = (1+i)^{80\times2} = \boxed{(1+i)^{160}}$

24. $\left[(1-r)^{40}\right]^3 = (1-r)^{40\times3} = \boxed{(1-r)^{120}}$

25. $(ab)^5 = \boxed{a^5b^5}$

26. $(2xy)^4 = \boxed{16x^4y^4}$

27. $(m^3n)^8 = \boxed{m^{24}n^8}$

28. $\left(\dfrac{a^3b^2}{x}\right)^4 = \boxed{\dfrac{a^{12}b^8}{x^4}}$

29. $2^3 \times 2^5 \times 2^{-4} = 2^{3+5-4} = \boxed{2^4}$

30. $5^2 \div 5^{-3} = 5^{2-(-3)} = \boxed{5^5}$

31. $\left(\dfrac{a}{b}\right)^{-8} = \boxed{\dfrac{b^8}{a^8}}$

32. $\left(\dfrac{1+i}{i}\right)^{-n} = \boxed{\dfrac{i^n}{(1+i)^n}}$

***Exercise* 2.3**

A. 1. $\sqrt{5184} = \boxed{72.0000}$

2. $\sqrt{205.9225} = \boxed{14.3500}$

3. $\sqrt[7]{2187} = \boxed{3.0000}$

4. $\sqrt[10]{1.1046221} = \boxed{1.0100}$

5. $\sqrt[20]{4.3184} = 1.075886 = \boxed{1.0759}$

6. $\sqrt[16]{0.00001526} = 0.500002 = \boxed{0.5000}$

7. $\sqrt[6]{1.0825} = \boxed{1.0133}$

8. $\sqrt[12]{1.15} = 1.011715 = \boxed{1.0117}$

B. 1. $3025^{\frac{1}{2}} = \boxed{55}$

2. $2401^{\frac{1}{4}} = \boxed{7}$

3. $525.21875^{\frac{2}{5}} = \boxed{12.25}$

4. $21.6^{\frac{4}{3}} = \boxed{60.154991}$

5. $\sqrt[12]{1.125^7} = \boxed{1.071122}$

6. $\sqrt[6]{1.095} = \boxed{1.015241}$

7. $4^{\left(-\frac{1}{3}\right)} = \dfrac{1}{4^{\frac{1}{3}}} = \dfrac{1}{1.587401} = \boxed{0.629961}$

8. $1.06^{\left(-\frac{1}{12}\right)} = \dfrac{1}{1.06^{\frac{1}{12}}} = \dfrac{1}{1.004868} = \boxed{0.995156}$

9. $\dfrac{1.03^{60}-1}{0.03} = \dfrac{5.891603-1}{0.03} = \boxed{163.053437}$

10. $\dfrac{1-1.05^{-36}}{0.05} = \dfrac{1-0.172657}{0.05} = \boxed{16.546852}$

11. $\boxed{2.158925}$

12. $\boxed{0.589664}$

13. $26.50(1.043)\left(\dfrac{3.536138-1}{0.043}\right) = 26.50(1.043)(58.979962) = \boxed{1630.176673}$

14. $350.00(1.05)\left(\dfrac{2.653298-1}{0.05}\right) = 350.00(1.05)(33.065954) = \boxed{12\ 151.73813}$

15. $133.00\left(\dfrac{1-0.520035}{0.056}\right) = 133.00(8.570795) = \boxed{1139.915716}$

16. $270.00\left(\dfrac{1-0.759412}{0.035}\right) = 270.00(6.873956) = \boxed{1855.967995}$

17. $5000.00(0.581251) + 137.50\left(\dfrac{1-0.581251}{0.0275}\right)$

$= 2906.252832 + 137.50(15.227252) = 2906.252832 + 2093.747168 = \boxed{5000.00}$

18. $1000.00(0.623167) + 300.00\left(\dfrac{1-0.623167}{0.03}\right)$

$= 623.166939 + 300.00(12.561102) = 623.166939 + 3768.330608 = \boxed{4391.497547}$

19. $112.55 = 100.00(1+i)^4$

$$(1+i)^4 = 1.1255$$
$$(1+i) = 1.1255^{0.25}$$
$$(1+i) = 1.029998$$
$$i = \boxed{0.029998}$$

20. $380.47 = 300.00(1+i)^{12}$

$$(1+i)^{12} = 1.268233$$
$$(1+i) = 1.268233^{0.08\dot{3}}$$
$$(1+i) = 1.019999$$
$$i = \boxed{0.019999}$$

21. $3036.77 = 2400.00(1+i)^6$

$$(1+i)^6 = 1.265321$$
$$(1+i) = 1.265321^{0.16}$$
$$(1+i) = 1.04$$
$$i = \boxed{0.04}$$

22. $1453.36 = 800.00(1+i)^{60}$

$$(1+i)^{60} = 1.8167$$
$$(1+i) = 1.8167^{0.01\dot{6}}$$
$$(1+i) = 1.01$$
$$i = \boxed{0.01}$$

Exercise 2.4

A. 1. $2^9 = 512$

$$\boxed{9 = \log_2 512}$$

2. $3^7 = 2187$

$$\boxed{7 = \log_3 2187}$$

3. $5^{-3} = \frac{1}{125}$

$\boxed{-3 = \log_5 \frac{1}{125}}$

4. $10^{-5} = 0.00001$

$\boxed{-5 = \log_{10} 0.00001}$

5. $e^{2j} = 18$

$2j = \log_e 18$

or $\boxed{2j = \ln 18}$

6. $e^{-3x} = 12$

$-3x = \log_e 12$

or $\boxed{-3x = \ln 12}$

B. 1. $\log_2 32 = 5$

$\boxed{2^5 = 32}$

2. $\log_3 \frac{1}{81} = -4$

$\boxed{3^{-4} = \frac{1}{81}}$

3. $\log_{10} 10 = 1$

$\boxed{10^1 = 10}$

4. $\ln e^2 = 2$

$\boxed{e^2 = e^2}$

C. 1. $\ln 2 = \boxed{0.693147}$

2. $\ln 200 = \boxed{5.298317}$

3. $\ln 0.105 = \boxed{-2.253795}$

4. $\ln\left[300(1.10^{15})\right] = \ln 300 + \ln 1.10^{15}$

$$= \ln 300 + 15(\ln 1.10)$$
$$= 5.703782 + 15(0.095310)$$
$$= 5.703782 + 1.429653$$
$$= \boxed{7.133435}$$

5. $\ln\left[\dfrac{2000}{1.09^{9}}\right] = \ln 2000 - \ln 1.09^{9}$

$$= \ln 2000 - 9(\ln 1.09)$$
$$= 7.600902 - 9(0.086178)$$
$$= 7.600902 - 0.775599$$
$$= \boxed{6.825303}$$

6. $\ln 850\left[\dfrac{1.01^{-120}}{0.01}\right] = \ln 850 + \ln 1.01^{-120} - \ln 0.01$

$$= \ln 850 - 120(\ln 1.01) - \ln 0.01$$
$$= 6.745236 - 120(0.009950) - (-4.605170)$$
$$= 6.745236 - 1.194040 + 4.605170$$
$$= \boxed{10.156367}$$

Business Math News Box

1. The IOC distributes 90% of broadcast revenue to other organizations in the Olympic Movement. It retains 10% to cover operational and administrative costs.

   IOC retains 0.10(1 339 000 000) = $\boxed{\$133\ 900\ 000\ (\text{C}\$133.9\ \text{million})}$

2. IOC distribution (C$ millions) = $\boxed{\text{C}\$133.9 \times 10^{6}}$

3. Estimate for total Winter Games revenue = Revenue from broadcast partnerships / 0.50 = 1 339 000 000 / 0.50 = $2 678 000 000

   Answer expressed in exponent form = $\boxed{\text{C}\$2678 \times 10^{6}\ \text{million}}$

4. Annual growth rate in broadcast revenue since Lake Placid = (1.339 billion/21.7 million)$^{1/30}$ –1 = 0.147301 = $\boxed{14.73\%\ \text{per year}}$

Exercise 2.5

A. 1. $15x = 45$

$\boxed{x = 3}$

2. $-7x = 35$

$\boxed{x = -5}$

3. $0.9x = 72$

$\boxed{x = 80}$

4. $0.02x = 13$

$\boxed{x = 650}$

5. $\frac{1}{6}x = 3$

$\boxed{x = 18}$

6. $-\frac{1}{8}x = 7$

$\boxed{x = -56}$

7. $\frac{3}{5}x = -21$

$\frac{1}{5}x = -7$

$\boxed{x = -35}$

8. $-\frac{4}{3}x = -32$

$\frac{1}{3}x = 8$

$\boxed{x = 24}$

9. $x - 3 = -7$

$\boxed{x = -4}$

10. $-2x = 7 - 3x$

$\boxed{x = 7}$

11. $x+6=-2$

$$\boxed{x=-8}$$

12. $3x=9+2x$

$$\boxed{x=9}$$

13. $4-x=9-2x$

$$\boxed{x=5}$$

14. $2x+7=x-5$

$$\boxed{x=-12}$$

15. $x+0.6x=32$

$$1.6x=32$$

$$\boxed{x=20}$$

16. $x-0.3x=210$

$$0.7x=210$$

$$\boxed{x=300}$$

17. $x-0.04x=192$

$$0.96x=192$$

$$\boxed{x=200}$$

18. $x+0.07x=64.20$

$$1.07x=64.20$$

$$\boxed{x=60}$$

B. 1. $3x+5=7x-11$

$$-4x=-16$$

$$\boxed{x=4}$$

$$\begin{aligned}\text{LS}: 3x+5 &= 3(4)+5\\ &= 12+5\\ &= 17\\ \text{RS}: 7x-11 &= 7(4)-11\\ &= 28-11\\ &= 17\end{aligned}$$

2. $5-4x=-4-x$

$-3x=-9$

$\boxed{x=3}$

LS: $5-4x=5-(4)(3)$

$=5-12$

$=-7$

RS: $=-4-x$

$=-4-3$

$=-7$

3. $2-3x-9=2x-7+3x$

$-3x-7=5x-7$

$-8x=0$

$\boxed{x=0}$

LS: $=2-3x-9$

$=2-3(0)-9$

$=-7$

RS: $=2x-7+3x$

$=2(0)-7+3(0)$

$=-7$

4. $4x-8-9x=10+2x-4$

$-5x-8=6+2x$

$-7x=14$

$\boxed{x=-2}$

LS: $=4x-8-9x$

$=4(-2)-8-9(-2)$

$=-8-8+18=2$

RS: $=10+2x-4$

$=10+2(-2)-4$

$=10-4-4=2$

5. $3x+14=4x+9$

$-x=-5$

$\boxed{x=5}$

$\text{LS:}=3x+14$

$=3(5)+14$

$=15+14=29$

$\text{RS:}=4x+9$

$=4(5)+9$

$=20+9=29$

6. $16x-12=6x-32$

$10x=-20$

$\boxed{x=-2}$

$\text{LS:}=16x-12$

$=16(-2)-12$

$=-32-12=-44$

$\text{RS:}=6x-32$

$=6(-2)-32$

$=-12-32=-44$

7. $5+3+4x=5x+12-25$

$+4x-5x=+12-25-5-3$

$-x=-21$

$\boxed{x=21}$

$\text{LS:}=5+3+4x$

$=8+4(21)$

$=8+84=92$

$\text{RS:}=5x+12-25$

$=5(21)-13$

$=105-13=92$

8. $-3+2x+5=5x-36+14$

$$+2x-5x=-36+14+3-5$$
$$-3x=-24$$
$$\boxed{x=8}$$

$$\text{LS:}=-3+2x+5$$
$$=-3+2(8)+5$$
$$=-3+16+5$$
$$=18$$
$$\text{RS:}=5x-36+14$$
$$=5(8)-36+14$$
$$=40-36+14=18$$

Exercise **2.6**

A. 1. $12x-4(9x-20)=320$

$$12x-36x+80=320$$
$$-24x=240$$
$$\boxed{x=-10}$$
$$\text{LS}=12(-10)-4[9(-10)-20]$$
$$=-120-4[-90-20]$$
$$=-120+440$$
$$=320$$
$$\text{RS}=320$$

2. $5(x-4)-3(2-3x)=-54$

$$5x-20-6+9x=-54$$
$$14x-26=-54$$
$$14x=-28$$
$$\boxed{x=-2}$$
$$\text{LS}=5[-2-4]-3[2-3(-2)]$$
$$=5(-6)-3(2+6)$$
$$=-30-24$$
$$=-54$$
$$\text{RS}=-54$$

3. $3(2x-5)-2(2x-3)=-15$

$$6x-15-4x+6=-15$$
$$2x-9=-15$$
$$2x=-6$$
$$\boxed{x=-3}$$

$$\begin{aligned}\text{LS}&=3[2(-3)-5]-2[2(-3)-3]\\&=3[-65]-2[-6-3]\\&=3(-11)-2(-9)\\&=-33+18\\&=-15\\\text{RS}&=-15\end{aligned}$$

4. $17-3(2x-7)=7x-3(2x-1)$

$$17-6x+21=7x-6x+3$$
$$-6x+38=x+3$$
$$-7x=-35$$
$$\boxed{x=5}$$

$$\begin{aligned}\text{LS}&=17-3[2(5)-7]\\&=17-3[10-7]\\&=17-9=8\\\text{RS}&=7(5)-3[2(5)-1]\\&=35-3[10-1]\\&=35-27=8\end{aligned}$$

5. $4x+2(2x-3)=18$

$$4x+4x-6=18$$
$$8x=24$$
$$\boxed{x=3}$$

$$\begin{aligned}\text{LS}&=4(3)+2[2(3)-3]\\&=12+2[6-3]\\&=12+6\\&=18\\\text{RS}&=18\end{aligned}$$

6. $-3(1-11x)+(8x-15)=187$

$$-3+33x+8x-15=187$$
$$33x+8x=187+3+15$$
$$41x=205$$
$$\boxed{x=5}$$

$$\text{LS}=-3[(1-11(5)]+[8(5)-15]$$
$$=-3[-54]+25$$
$$=162+25$$
$$=187$$
$$\text{RS}=187$$

7. $10x-4(2x-1)=32$

$$10x-8x+4=32$$
$$2x=28$$
$$\boxed{x=14}$$

$$\text{LS}=10(14)-4[2(14)-1]$$
$$=140-4[27]$$
$$=140-108$$
$$=32$$
$$\text{RS}=32$$

8. $-2(x-4)+12(3-2x)=-8$

$$-2x+8+36-24x=-8$$
$$-2x-24x=-8-8-36$$
$$-26x=-52$$
$$\boxed{x=2}$$

$$\text{LS}=-2(2-4)+12[3-2(2)]$$
$$=-4+8+36-48$$
$$=-8$$
$$\text{RS}=-8$$

B. 1. $x-\frac{1}{4}x=15$

$$4x-x=60$$
$$3x=60$$
$$\boxed{x=20}$$

2. $x+\frac{5}{8}x=26$

$$8x+5x=208$$
$$13x=208$$
$$\boxed{x=16}$$

3. $\frac{2}{3}x-\frac{1}{4}=-\frac{7}{4}-\frac{5}{6}x$

$$8x-3=-21-10x$$
$$18x=-18$$
$$\boxed{x=-1}$$

4. $\frac{5}{3}-\frac{2}{5}x=\frac{1}{6}x-\frac{1}{30}$

$$50-12x=5x-1$$
$$-17x=-51$$
$$\boxed{x=3}$$

5. $\frac{3}{4}x+4=\frac{113}{24}-\frac{2}{3}x$

$$18x+96=113-16x$$
$$34x=17$$
$$\boxed{x=\frac{1}{2}}$$

6. $2-\frac{3}{2}x=\frac{2}{3}x+\frac{31}{9}$

$$36-27x=12x+62$$
$$-39x=26$$
$$\boxed{x=-\frac{2}{3}}$$

C. 1. $\frac{3}{4}(2x-1)-\frac{1}{3}(5-2x)=-\frac{55}{12}$

$$9(2x-1)-4(5-2x)=-55$$
$$18x-9-20+8x=-55$$
$$26x-29=-55$$
$$26x=-26$$
$$\boxed{x=-1}$$

2. $\frac{4}{5}(4-3x)+\frac{53}{40}=\frac{3}{10}x-\frac{7}{8}(2x-3)$

$$32(4-3x)+53=12x-35(2x-3)$$
$$128-96x+53=12x-70x+105$$
$$-96x+181=-58x+105$$
$$-38x=-76$$
$$\boxed{x=2}$$

3. $\frac{2}{3}(2x-1)-\frac{3}{4}(3-2x)=2x-\frac{20}{9}$

$$24(2x-1)-27(3-2x)=72x-80$$
$$48x-24-81+54x=72x-80$$
$$102x-105=72x-80$$
$$30x=25$$
$$\boxed{x=\frac{5}{6}}$$

4. $\frac{4}{3}(3x-2)-\frac{3}{5}(4x-3)=\frac{11}{60}+3x$

$$80(3x-2)-36(4x-3)=11+180x$$
$$240x-160-144x+108=11+180x$$
$$96x-52=11+180x$$
$$-84x=63$$
$$\boxed{x=-\frac{3}{4}}$$

D. 1. $y=mx+b$

$$y-b=mx$$
$$\boxed{x=\frac{y-b}{m}}$$

2. $r = \frac{M}{S}$

$Sr = M$

$$\boxed{S = \frac{M}{r}}$$

3. $\text{PV} = \frac{\text{PMT}}{i}$

$$\boxed{\text{PMT} = \text{PV}i}$$

4. $\text{I} = \text{P}\,rt$

$$\boxed{t = \frac{\text{I}}{\text{P}\,r}}$$

5. $\text{S} = \text{P}(1 + rt)$

$$\frac{\text{S}}{\text{P}} = 1 + rt$$

$$\frac{\text{S}}{\text{P}} - 1 = rt$$

$$r = \frac{\frac{\text{S}}{\text{P}} - 1}{t}$$

$$r = \frac{\frac{\text{S} - \text{P}}{\text{P}}}{t}$$

$$\boxed{r = \frac{\text{S} - \text{P}}{\text{P}t}}$$

6. $\text{PV} = \text{FV}(1 + i)^{-n}$

$$\frac{\text{PV}}{\text{FV}} = (1 + i)^{-n}$$

$$\left[\frac{\text{PV}}{\text{FV}}\right]^{-\frac{1}{n}} = 1 + i$$

$$\left[\frac{\text{FV}}{\text{PV}}\right]^{\frac{1}{n}} = 1 + i$$

$$\boxed{i = \left[\frac{\text{FV}}{\text{PV}}\right]^{\frac{1}{n}} - 1}$$

Exercise 2.7

A. 1. Let the cost be \$$x$.

Selling price $= \$\left(x+\frac{3}{4}x\right)$

$$\therefore x+\frac{3}{4}x=49.49$$
$$4x+3x=197.96$$
$$7x=197.96$$
$$x=28.28$$

The cost was $\boxed{\$28.28.}$

2. Let the regular selling price be \$$x$.

Sale price $= \$\left(x-\frac{1}{3}x\right)$

$$\therefore x-\frac{1}{3}x=576$$
$$3x-x=1728$$
$$2x=1728$$
$$x=864$$

The regular selling price was $\boxed{\$864.}$

3. Let the price be \$$x$.

Total $= \$x+0.05x$

$$\therefore x+0.05x=\$36.75$$
$$1.05x=\$36.75$$
$$x=35.00$$

The price was $\boxed{\$35.00.}$

4. Let the regular price be \$$x$.

Sale price $= \$(x-0.40x)$

$$\therefore x-0.40x=11.34$$
$$0.60x=11.34$$
$$x=18.90$$

The regular selling price was $\boxed{\$18.90.}$

5. Let the last month's index be x.

This month's index $= x - \frac{1}{12}x$

$$\therefore x - \frac{1}{12}x = 176$$
$$12x - x = 2112$$
$$11x = 2112$$
$$x = 192$$

Last month the index was $\boxed{192.}$

6. Let the original hourly wage be $\$x$.

New hourly wage $= \$\left(x + \frac{1}{8}x\right)$

$$\therefore x + \frac{1}{8}x = 12.78$$
$$8x + x = 102.24$$
$$9x = 102.24$$
$$x = 11.36$$

The hourly wage before the increase was $\boxed{\$11.36.}$

7. Let Vera's sales be $\$x$.

Tai's sales $= \$(3x - 140)$

Total sales $= \$(x + 3x - 140)$

$$\therefore x + 3x - 140 = 940$$
$$4x = 1080$$
$$x = 270$$

Tai's sales $= 3(270) - 140 = \boxed{\$670.}$

8. Let the shorter piece be x cm.

Length of longer piece $= (2x + 15)$ cm.

Total length $= (x + 2x + 15)$ cm

$$\therefore x + 2x + 15 = 90$$
$$3x = 75$$
$$x = 25$$

The longer piece is 2(25) cm + 15 cm $= \boxed{65 \text{ cm.}}$

9. Let the cost of a ticket be $\$x$.

Total $= \$(x+5.00)\times 1.05\times 2$

$$\therefore (x+5.00)\times 1.05\times 2 = 197.40$$
$$(x+5.00)\times 2.10 = 197.40$$
$$(x+5.00) = 94.00$$
$$x = 89.00$$

The cost per ticket is $\boxed{\$89.00.}$

10. Let Ken's investment be $\$x$.

Martina's investment $= \$\left(\frac{2}{3}x+2500\right)$

Total investment $= \$\left(x+\frac{2}{3}x+2500\right)$

$$\therefore x+\frac{2}{3}x+2500 = 55\,000$$
$$\frac{5x}{3} = 52\,500$$
$$x = 31\,500$$

Martina's investment is $\frac{2}{3}\times 31\,500+2500 = \boxed{\$23\,500.}$

11. Let the number of chairs produced by the first shift be x.

Number of chairs produced by the second shift $= \frac{4}{3}x-60.$

Total production $= x+\frac{4}{3}x-60 = 2320.$

$$\therefore x+\frac{4}{3}x-60 = 2320$$
$$\frac{7}{3}x = 2380$$
$$x = 1020$$

Production by the second shift is $\frac{4}{3}\times 1020-60 = \boxed{1300.}$

12. Let the number of type A lights be x.

Number of type B lights $= 60-x.$

Value of type A lights $= \$40x.$

Value of type B lights $= \$(60-x)50.$

$$\therefore 40x+50(60-x)=2580$$
$$40x+3000-50x=2580$$
$$-10x=-420$$
$$x=42$$

The number of type B lights is $\boxed{18.}$

13. Let the number of units of Product A be x;

then the number of units of Product B is $60-x$.

The number of hours for Product A is $4x$;

The number of hours for Product B is $3(60-x)$.

$$\therefore 4x+3(60-x)=200$$
$$4x+180-3x=200$$
$$x=20$$

Production of Product A is $\boxed{\text{20 units.}}$

14. Let the number of dimes be x.

Number of nickels $=3x-4$

Number of quarters $=\dfrac{3}{4}x+1$

Value of the dimes $=10x$ cents

Value of nickels $=5(3x-4)$ cents

Value of quarters $=25\left(\dfrac{3}{4}x+1\right)$ cents

$$\therefore 10x+5(3x-4)+25\left(\frac{3}{4}x+1\right)=880$$
$$10x+15x-20+\frac{75}{4}x+25=880$$
$$25x+\frac{75}{4}x=875$$
$$175x=3500$$
$$x=20$$

Alick has $\boxed{\text{20 dimes, 56 nickels, and 16 quarters.}}$

15. Let the number of \$12 tickets be x.

Number of \$8 tickets $= 3x + 10$

Number of \$15 tickets $= \frac{4}{5}x - 3$

Value of the \$12 tickets $= \$12x$

Value of the \$8 tickets $= \$8(3x + 10)$

Value of the \$15 tickets $= \$15\left(\frac{4}{5}x - 3\right)$

$$\therefore 12x + 8(3x + 10) + 15\left(\frac{4}{5}x - 3\right) = 1475$$
$$12x + 24x + 80 + 12x - 45 = 1475$$
$$48x = 1440$$
$$x = 30$$

Sales were

30	\$12 tickets,
100	\$ 8 tickets,
and 21	\$15 tickets.

16. Let the number of medium pizzas be x.

Number of large pizzas $= 3x - 1$

Number of small pizzas $= 2x + 1$

Value of medium pizzas $= \$15x$

Value of large pizzas $= \$18(3x - 1)$

Value of small pizzas $= \$11(2x + 1)$

$$\therefore 15x + 18(3x - 1) + 11(2x + 1) = 539$$
$$15x + 54x - 18 + 22x + 11 = 539$$
$$91x = 546$$
$$x = 6$$

Sales were

6	medium pizzas,
17	large pizzas,
and 13	small pizzas.

Review Exercise

1. (a) $3x-4y-3y-5x=\boxed{-2x-7y}$

   (b) $2x-0.03x=\boxed{1.97x}$

   (c) $(5a-4)-(3-a)=5a-4-3+a=\boxed{6a-7}$

   (d) $-(2x-3y)-(-4x+y)+(y-x)=-2x+3y+4x-y+y-x=\boxed{x+3y}$

   (e) $(5a^2-2b-c)-(3c+2b-4a^2)$

   $=5a^2-2b-c-3c-2b+4a^2=\boxed{9a^2-4b-4c}$

   (f) $-(2x-3)-(x^2-5x+2)=-2x+3-x^2+5x-2=\boxed{-x^2+3x+1}$

2. (a) $3(-5a)=\boxed{-15a}$

   (b) $-7m(-4x)=\boxed{28mx}$

   (c) $14m\div(-2m)=\boxed{-7}$

   (d) $(-15a^2b)\div(5a)=\boxed{-3ab}$

   (e) $-6(-3x)(2y)=\boxed{36xy}$

   (f) $4(-3a)(b)(-2c)=\boxed{24abc}$

   (g) $-4(3x-5y-1)=\boxed{-12x+20y+4}$

   (h) $x(1-2x-x^2)=\boxed{x-2x^2-x^3}$

   (i) $(24x-16)\div(-4)=\boxed{-6x+4}$

   (j) $(21a^2-12a)\div 3a=\boxed{7a-4}$

   (k) $4(2a-5)-3(3-6a)$

   $=8a-20-9+18a$

   $=\boxed{26a-29}$

   (l) $2a(x-a)-a(3x+2)-3a(-5x-4)$

   $=2ax-2a^2-3ax-2a+15ax+12a$

   $=\boxed{14ax-2a^2+10a}$

(m) $(m-1)(2m-5)$

$= 2m^2 - 2m - 5m + 5$

$= \boxed{2m^2 - 7m + 5}$

(n) $(3a-2)(a^2-2a-3)$

$= 3a^3 - 2a^2 - 6a^2 + 4a - 9a + 6$

$= \boxed{3a^3 - 8a^2 - 5a + 6}$

(o) $3(2x-4)(x-1) - 4(x-3)(5x+2)$

$= 3(2x^2 - 4x - 2x + 4) - 4(5x^2 - 15x + 2x - 6)$

$= 6x^2 - 18x + 12 - 20x^2 + 52x + 24$

$= \boxed{-14x^2 + 34x + 36}$

(p) $-2a(3m-1)(m-4) - 5a(2m+3)(2m-3)$

$= -2a(3m^2 - m - 12m + 4) - 5a(4m^2 + 6m - 6m - 9)$

$= -6am^2 + 26am - 8a - 20am^2 + 45a$

$= \boxed{-26am^2 + 26am + 37a}$

3. (a) for $x = -2,\ y = 5,$

$3xy - 4x - 5y = 3(-2)(5) - 4(-2) - 5(5) = -30 + 8 - 25 = \boxed{-47}$

(b) for $a = -\frac{1}{4},\ b = \frac{2}{3},$

$-5(2a-3b) - 2(a+5b)$

$= -10a + 15b - 2a - 10b$

$= -12a + 5b$

$= -12\left(-\frac{1}{4}\right) + 5\left(\frac{2}{3}\right) = 3 + 3\frac{1}{3} = \boxed{6\frac{1}{3}}$

(c) for $\text{N} = 12,\ \text{C} = 432,\ \text{P} = 1800,\ n = 35,$

$$\frac{2\text{NC}}{\text{P}(n+1)} = \frac{\overset{1}{2}\times 12\times \overset{48}{432}}{\underset{\substack{900\\100}}{1800}\times(35+1)} = \frac{\overset{1}{12}\times \overset{16}{48}}{100\times \underset{\substack{3\\1}}{36}} = \frac{16}{100} = \boxed{0.16}$$

(d) for $\text{I} = 600$, $r = 0.15$, $\text{P} = 7300$,

$$\frac{365\,\text{I}}{r\text{P}} = \frac{\overset{1}{365} \times \overset{\overset{2}{30}}{600}}{\underset{\underset{1}{15}}{0.15} \times \underset{0.01}{7300}} = \frac{2}{0.01} = \boxed{200}$$

(e) for $\text{A} = \$720$, $\text{d} = 0.135$, $\text{t} = \dfrac{280}{365}$,

$$\text{A}(1 - dt) = \$720\left(1 - 0.135 \times \frac{280}{365}\right) = \$720(1 - 0.103562) = 645.435616 = \boxed{\$645.44}$$

(f) for $\text{S} = 2755$, $r = 0.17$, $t = \dfrac{219}{365}$,

$$\frac{\text{S}}{1 + rt} = \frac{2755}{1 + 0.17 \times \frac{219}{365}} = \frac{2755}{1 + 0.034 \times 3} = \frac{2755}{1 + 0.102} = \boxed{2500}$$

4. (a) $(-3)^5 = \boxed{-243}$

(b) $\left(\dfrac{2}{3}\right)^4 = \boxed{\dfrac{16}{81}}$

(c) $(-5)^0 = \boxed{1}$

(d) $(-3)^{-1} = \boxed{-\dfrac{1}{3}}$

(e) $\left(\dfrac{2}{5}\right)^{-4} = \left(\dfrac{5}{2}\right)^4 = \boxed{\dfrac{625}{16}}$

(f) $(1.01)^0 = \boxed{1}$

(g) $(-3)^5(-3)^4 = (-3)^9 = \boxed{-19\,683}$

(h) $4^7 \div 4^2 = 4^5 = \boxed{1024}$

(i) $\left[(-3)^2\right]^5 = (-3)^{10} = \boxed{59\,049}$

(j) $(m^3)^4 = \boxed{m^{12}}$

(k) $\left(\dfrac{2}{3}\right)^3\left(\dfrac{2}{3}\right)^7\left(\dfrac{2}{3}\right)^{-6} = \left(\dfrac{2}{3}\right)^4 = \boxed{\dfrac{16}{81}}$

(l) $\left(-\frac{5}{4}\right)^5 \div \left(-\frac{5}{4}\right)^3 = \left(-\frac{5}{4}\right)^2 = \boxed{\frac{25}{16}}$

(m) $(1.03^{50})(1.03^{100}) = \boxed{1.03^{150}}$

(n) $(1+i)^{180} \div (1+i)^{100} = \boxed{(1+i)^{80}}$

(o) $\left[(1.05)^{30}\right]^5 = \boxed{1.05^{150}}$

(p) $(-2xy)^4 = \boxed{16x^4y^4}$

(q) $\left(\frac{a^2b}{3}\right)^{-4} = \left(\frac{3}{a^2b}\right)^4 = \boxed{\frac{81}{a^8b^4}}$

(r) $(1+i)^{-n} = \boxed{\frac{1}{(1+i)^n}}$

5. (a) $\sqrt{0.9216} = \boxed{0.96}$

(b) $\sqrt[6]{1.075} = \boxed{1.012126}$

(c) $14.974458^{1/40} = \boxed{1.07}$

(d) $1.08^{-5/12} = \frac{1}{1.08^{5/12}} = \boxed{0.968442}$

(e) $\ln 3 = \boxed{1.098612}$

(f) $\ln 0.05 = \boxed{-2.995732}$

(g) $\ln\left(\frac{5500}{1.10^{16}}\right) = \ln 5500 - \ln 1.10^{16}$

$$= \ln 5500 - 16 \ln 1.10$$
$$= 8.612503 - 16(0.095310)$$
$$= 8.612503 - 1.524963$$
$$= \boxed{7.087540}$$

(h) $\ln\left[375(1.01)\left(\frac{1-1.01^{-72}}{0.01}\right)\right] = \ln 375 + \ln 1.01 + \ln(1-1.01^{-72}) - \ln 0.01$

$$= \ln 375 + \ln 1.01 + \ln(1-0.488496) - \ln 0.01$$
$$= \ln 375 + \ln 1.01 + \ln 0.511504 - \ln 0.01$$
$$= 5.926926 + 0.009950 - 0.670400 - (-4.605170)$$
$$= \boxed{9.871647}$$

6. (a) $9x = -63$

$$\boxed{x = -7}$$

(b) $0.05x = 44$

$$5x = 4400$$
$$\boxed{x = 880}$$

(c) $-\frac{1}{7}x = 3$

$$-x = 21$$
$$\boxed{x = -21}$$

(d) $\frac{5}{6}x = -15$

$$\frac{1}{6}x = -3$$
$$\boxed{x = -18}$$

(e) $x - 8 = -5$

$$x - 8 + 8 = -5 + 8$$
$$\boxed{x = 3}$$

(f) $x + 9 = -2$

$$x + 9 - 9 = -2 - 9$$
$$\boxed{x = -11}$$

(g) $x + 0.02x = 255$

$$1.02x = 255$$
$$\boxed{x = 250}$$

(h) $x - 0.1x = 36$

$$0.9x = 36$$

$$9x = 360$$

$$\boxed{x = 40}$$

(i) $4x - 3 = 9x + 2$

$$-5x = 5$$

$$\boxed{x = -1}$$

(j) $9x - 6 - 3x = 15 + 4x - 7$

$$6x - 6 = 8 + 4x$$

$$2x = 14$$

$$\boxed{x = 7}$$

(k) $x - \frac{1}{3}x = 26$

$$\frac{2}{3}x = 26$$

$$\frac{1}{3}x = 13$$

$$\boxed{x = 39}$$

(l) $x + \frac{3}{8}x = 77$

$$\frac{11}{8}x = 77$$

$$\frac{1}{8}x = 7$$

$$\boxed{x = 56}$$

7. (a) $-9(3x - 8) - 8(9 - 7x) = 5 + 4(9x + 11)$

$$-27x + 72 - 72 + 56x = 5 + 36x + 44$$

$$29x = 49 + 36x$$

$$-7x = 49$$

$$\boxed{x = -7}$$

Check $\text{LS} = -9[3(-7) - 8] - 8[9 - 7(-7)] = -9(-29) - 8(58) = -203$

$\text{RS} = 5 + 4[9(-7) + 11] = 5 + 4(-52) = 5 - 208 = -203$

(b) $21x-4-7(5x-6)=8x-4(5x-7)$

$$21x-4-35x+42=8x-20x+28$$
$$-14x+38=-12x+28$$
$$-2x=-10$$
$$\boxed{x=5}$$

Check LS $=21(5)-4-7[5(5)-6]=105-4-7(19)=101-133=-32$

RS $=8(5)-4[5(5)-7]=40-4(18)=40-72=-32$

(c) $$\frac{5}{7}x+\frac{1}{2}=\frac{5}{14}+\frac{2}{3}x$$

$$42\left(\frac{5}{7}x\right)+42\left(\frac{1}{2}\right)=42\left(\frac{5}{14}\right)+42\left(\frac{2}{3}x\right)$$
$$6(5x)+21(1)=3(5)+14(2x)$$
$$30x+21=15+28x$$
$$2x=-6$$
$$\boxed{x=-3}$$

Check LS $=\frac{5}{7}(-3)+\frac{1}{2}=\frac{-30+7}{14}=-\frac{23}{14}$

RS $=\frac{5}{14}+\frac{2}{3}(-3)=\frac{5}{14}-2=-\frac{23}{14}$

(d) $$\frac{4x}{3}+2=\frac{9}{8}-\frac{x}{6}$$

$$8(4x)+24(2)=3(9)-4(x)$$
$$32x+48=27-4x$$
$$36x=-21$$
$$\boxed{x=-\frac{7}{12}}$$

Check LS $=\frac{4}{3}\left(-\frac{7}{12}\right)+2=-\frac{28}{36}+2=-\frac{7}{9}+\frac{18}{9}=\frac{11}{9}$

RS $=\frac{9}{8}-\frac{1}{6}\left(-\frac{7}{12}\right)=\frac{9}{8}+\frac{7}{72}=\frac{81+7}{72}=\frac{88}{72}=\frac{11}{9}$

(e) $\frac{7}{6}(6x-7)-\frac{3}{8}(7x+15)=25$

$$56(6x-7)-15(7x+15)=40(25)$$
$$336x-392-105x-225=1000$$
$$231x-617=1617$$
$$\boxed{x=7}$$

Check $\text{LS}=\frac{7}{5}[6(7)-7]-\frac{3}{8}[7(7)+15]$

$=\frac{7}{5}(35)-\frac{3}{8}(64)=7(7)-24=49-24=25$

$\text{RS}=25$

(f) $\frac{5}{9}(7-6x)-\frac{3}{4}(3-15x)=\frac{1}{12}(3x-5)-\frac{1}{2}$

$$20(7-6x)-27(3-15x)=3(3x-5)-18$$
$$140-120x-81+405x=9x-15-18$$
$$285x+59=9x-33$$
$$276x=-92$$
$$\boxed{x=-\frac{1}{3}}$$

Check $\text{LS}=\frac{5}{9}\left[7-6\left(-\frac{1}{3}\right)\right]-\frac{3}{4}\left[3-15\left(-\frac{1}{3}\right)\right]$

$=\frac{5}{9}(7+2)-\frac{3}{4}(3+5)$

$=5-6=-1$

$\text{RS}=\frac{1}{12}\left[3\left(-\frac{1}{3}\right)-5\right]-\frac{1}{2}$

$=\frac{1}{12}(-6)-\frac{1}{2}$

$=-\frac{1}{2}-\frac{1}{2}=-1$

(g) $\frac{5}{6}(4x-3)-\frac{2}{5}(3x+4)=5x-\frac{16}{15}(1-3x)$

$$25(4x-3)-12(3x+4)=150x-32(1-3x)$$
$$100x-75-36x-48=150x-32+96x$$
$$64x-123=246x-32$$
$$-182x=91$$
$$\boxed{x=-\frac{1}{2}}$$

$$\text{Check LS}=\frac{5}{6}\left[4\left(-\frac{1}{2}\right)-3\right]-\frac{2}{5}\left[3\left(-\frac{1}{2}\right)+4\right]$$
$$=\frac{5}{6}(-2-3)-\frac{2}{5}\left[-\frac{3}{2}+4\right]$$
$$=\frac{5}{6}(-5)-\frac{2}{5}\left(\frac{5}{2}\right)=-\frac{25}{6}-1=-\frac{31}{6}$$
$$\text{RS}=5\left(-\frac{1}{2}\right)-\frac{16}{15}\left[1-3\left(-\frac{1}{2}\right)\right]$$
$$=-\frac{5}{2}-\frac{16}{15}\left(1+\frac{3}{2}\right)=-\frac{5}{2}-\frac{16}{15}\left(\frac{5}{2}\right)$$
$$=-\frac{5}{2}-\frac{8}{3}=\frac{-15-16}{6}=-\frac{31}{6}$$

8. (a) $\text{I}=\text{P}rt$

$$\boxed{r=\frac{\text{I}}{\text{P}t}}$$

(b) $\text{S}=\text{P}(1+rt)$

$$\frac{S}{P} = 1 + rt$$

$$\frac{S}{P} - 1 = rt$$

$$t = \frac{\frac{S}{P} - 1}{r}$$

$$t = \frac{\frac{S - P}{P}}{r}$$

$$\boxed{t = \frac{S - P}{Pr}}$$

(c) $D = rL$

$$\boxed{r = \frac{D}{L}}$$

(d) $\text{FV} = \text{PMT}\left[\frac{(1+p)^n - 1}{p}\right]$

$$\boxed{\text{PMT} = \left[\frac{FVp}{(1+p)^n - 1}\right]}$$

9. (a) Let the size of the workforce be x.

Number laid off $= \frac{1}{6}x$

Number after the layoff $= x - \frac{1}{6}x$

$$\therefore x - \frac{1}{6}x = 690$$

$$\frac{5}{6}x = 690$$

$$5x = 4140$$

$$x = 828$$

$\therefore$ the number laid off is $\frac{1}{6} \times 828 = \boxed{138.}$

(b) Let last year's average property value be $\$x$.

Current average value $= \$\left(x + \frac{2}{7}x\right)$

$$\therefore x+\frac{2}{7}x=81\ 450$$

$$\frac{9}{7}x=81\ 450$$

$$\frac{1}{7}x=9050$$

$$x=63\ 350$$

$\therefore$ Last year's average value was $\boxed{\$63\ 350.}$

(c) Let the quoted price be \$$x$.

$$\therefore x+\frac{1}{20}x=2457$$

$$\frac{21}{20}x=2457$$

$$\frac{1}{20}x=117$$

$$x=2340$$

$\therefore$ The gratuities $=\frac{1}{20}$ of $2340=\boxed{\$117.}$

(d) Let the value of the building be \$$x$.

Value of the land $=\$\frac{1}{3}x-2000$

Total value of the property $=\$x+\frac{1}{3}x-2000$

$$\therefore x+\frac{1}{3}x-2000=184\ 000$$

$$\frac{4}{3}x=186\ 000$$

$$\frac{1}{3}x=46\ 500$$

$$x=139\ 500$$

The value assigned to land is $\$(184\ 000-139\ 500)=\boxed{\$44\ 500.}$

(e) Let the cost of power be \$$x$.

Cost of heat $=\$\left(\frac{3}{4}x+22\right)$

Cost of water $= \$\left(\frac{1}{3}x - 11\right)$

Total cost $= x + \frac{3}{4}x + 22 + \frac{1}{3}x - 11 = 2010 + 10\%$ of 2010.

$$12x + 9x + 4x = 12(2010 + 201 - 11)$$
$$25x = 26\,400$$
$$x = 1056$$

Cost of $\boxed{\text{heat}} = \frac{3}{4} \times 1056 + 22 = \boxed{\$814}$

Cost of $\boxed{\text{power}} = \boxed{\$1056}$

Cost of $\boxed{\text{water}} = \frac{1}{3} \times 1056 - 11 = \boxed{\$341}$

(f) Let the amount allocated to newspaper advertising be \$$x$.

Amount allocated to TV advertising $= \$(3x + 1000)$

Amount allocated to direct selling $= \frac{3}{4}[x + 3x + 1000]$

$$\therefore x + 3x + 1000 + \frac{3}{4}[4x + 1000] = 87\,500$$
$$4x + \frac{3}{4}[4x + 1000] = 86\,500$$
$$16x + 12x + 3000 = 346\,000$$
$$28x = 343\,000$$
$$x = 12\,250$$

The amount allocated to newspaper advertising is \$12 250; the amount allocated to TV advertising is \$37 750; the amount allocated to direct selling is $\boxed{\$37\,500}$

(g) Let the number of minutes on Machine B be x.

Time on Machine A $= \frac{4}{5}x - 3$ minutes

Time on Machine C $= \frac{5}{6}\left(x + \frac{4}{5}x - 3\right)$ minutes

Total time $= x + \frac{4}{5}x - 3 + \frac{5}{6}\left(x + \frac{4}{5}x - 3\right)$ minutes

$$\therefore x+\frac{4}{5}x-3+\frac{5}{6}\left(x+\frac{4}{5}x-3\right)=77$$

$$30x+24x-90+25\left(x+\frac{4}{5}x-3\right)=30(77)$$

$$54x-90+25x+20x-75=2310$$

$$99x-165=2310$$

$$99x=2475$$

$$x=25$$

Time on Machine B is 25 minutes; time on Machine A is $\frac{4}{5}(25)-3=17$ minutes; time on Machine C is $\frac{5}{6}(25+17)=$ $\boxed{\text{35 minutes.}}$

(h) Let the number of pairs of superlight poles be x.

Number of pairs of ordinary poles $=72-x$

Value of superlight poles $=\$30x$

Value of ordinary poles $=\$16(72-x)$

Total value of all poles $=\$30x+16(72-x)$

$$30x+16(72-x)=1530$$

$$30x+1152-16x=1530$$

$$14x=378$$

$$x=27$$

The number of pairs of $\boxed{\text{superlight poles is 27;}}$ the number of pairs of $\boxed{\text{ordinary poles is 45.}}$

(i) Let the number of \$2 coins be x.

Number of \$1 coins $=\frac{3}{5}x+1$

Number of quarters $=4\left(x+\frac{3}{5}x+1\right)$

Value of the \$2 coins $=\$2x$

Value of the \$1 coins $=\$\left[\frac{3}{5}x+1\right]$

Value of the quarters $=\$\frac{1}{4}(4)\left(x+\frac{3}{5}x+1\right)=x+\frac{3}{5}x+1$

$$\text{Total value} = 2x + \frac{3}{5}x + 1 + x + \frac{3}{5}x + 1 = 107$$
$$10x + 3x + 5 + 5x + 3x + 5 = 535$$
$$21x + 10 = 535$$
$$21x = 525$$
$$x = 25$$

The number of \$2 coins is 25; the number of \$1 coins is $\left(\frac{3}{5}\times 25+1\right)=16;$ the number of quarters is $4(25+16)=\boxed{164.}$

Self-Test

1. (a) $4-3x-6-5x=\boxed{-2-8x}$

   (b) $(5x-4)-(7x+5)=5x-4-7x-5=\boxed{-2x-9}$

   (c) $-2(3a-4)-5(2a+3)$

   $=-6a+8-10a-15$

   $=\boxed{-16a-7}$

   (d) $-6(x-2)(x+1)$

   $=-6(x^2-2x+x-2)$

   $=-6(x^2-x-2)$

   $=\boxed{-6x^2+6x+12}$

2. (a) For $x=-3,\ y=5$

   $2x^2-5xy-4y^2$

   $=2(-3)^2-5(-3)(5)-4(5)^2$

   $=18+75-100$

   $=\boxed{-7}$

   (b) For $a=\dfrac{2}{3},\ b=-\dfrac{3}{4}$

   $3(7a-4b)-4(5a+3b)$

   $=21a-12b-20a-12b$

   $=a-24b$

   $=\dfrac{2}{3}-24\left(-\dfrac{3}{4}\right)$

   $=\dfrac{2}{3}+18$

   $=\boxed{18\dfrac{2}{3}}$

   (c) For N = 12, C = 400, P = 2000, n = 24

   $$\frac{2\text{NC}}{\text{P}(n+1)}=\frac{(2)(12)(400)}{2000(24+1)}=\frac{2(12)(400)}{2000(25)}=\boxed{0.192}$$

(d) For I = 324, P = 5400, r = 0.15

$$\frac{\mathrm{I}}{\mathrm{P}r} = \frac{324}{5400 \times 0.15} = \boxed{0.4}$$

(e) For S = 1606, d = 0.125, $t = \dfrac{240}{365}$

$$\begin{aligned} \mathrm{S}(1-dt) &= 1606\left(1-0.125\times\frac{240}{365}\right) \\ &= 1606(1-0.082192) \\ &= 1606(0.917808) \\ &= \boxed{1474} \end{aligned}$$

(f) For S = 1566, r = 0.10, $t = \dfrac{292}{365}$

$$\begin{aligned} \frac{\mathrm{S}}{1+rt} &= \frac{1566}{1+0.10\times\frac{292}{365}} \\ &= \frac{1566}{1+0.08} \\ &= \boxed{1450} \end{aligned}$$

3. (a) $(-2)^3 = \boxed{-8}$

(b) $\left(-\dfrac{2}{3}\right)^2 = \boxed{\dfrac{4}{9}}$

(c) $(4)^0 = \boxed{1}$

(d) $(3)^2(3)^5 = (3)^7 = \boxed{2187}$

(e) $\left(\dfrac{4}{3}\right)^{-2} = \dfrac{1}{\left(\frac{4}{3}\right)^2} = \dfrac{1}{\frac{16}{9}} = \boxed{\dfrac{9}{16}}$

(f) $(-x^3)^5 = \boxed{-x^{15}}$

4. (a) $\sqrt[10]{1.35} = 1.35^{\frac{1}{10}} = 1.35^{0.10} = \boxed{1.030465}$

(b) $\dfrac{1-1.03^{-40}}{0.03} = \dfrac{1-0.306557}{0.03} = \dfrac{0.693443}{0.03} = \boxed{23.114772}$

(c) $\ln 1.025 = \boxed{0.024693}$

(d) $\ln(3.00e^{-0.2})$

$= \ln 3.00 + \ln e^{-0.2}$

$= \ln 3.00 - 0.2 \ln e$

$= 1.098612 - 0.2$

$= \boxed{0.898612}$

(e) $\ln\left(\frac{600}{1.06^{11}}\right)$

$= \ln 600 - \ln 1.06^{11}$

$= \ln 600 - 11 \ln 1.06$

$= 6.396930 - 11(0.058269)$

$= 6.396930 - 0.640958$

$= \boxed{5.755972}$

(f) $\ln\left[250\left(\frac{1.07^5 - 1}{0.07}\right)\right]$

$= \ln 250 + \ln(1.07^5 - 1) - \ln 0.07$

$= \ln 250 + \ln 0.402552 - \ln 0.07$

$= 5.521461 - 0.909932 - (-2.659260)$

$= 5.521461 - 0.909932 + 2.659260$

$= \boxed{7.270789}$

5. (a) $\frac{1}{81} = \left(\frac{1}{3}\right)^{n-2}$

$\frac{1}{3^4} = \left(\frac{1}{3}\right)^{n-2}$

$\left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^{n-2}$

Since the bases are common

$4 = n - 2$

$\boxed{n = 6}$

(b) $\frac{5}{2} = 40\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{2} = 8\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{16} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{n-1}$

$4 = n - 1$

$\boxed{n = 5}$

6. (a) $-\frac{2}{3}x = 24$

$x = 24\left(-\frac{3}{2}\right)$

$\boxed{x = -36}$

(b) $x - 0.06x = 8.46$

$0.94x = 8.46$

$\boxed{x = 9}$

(c) $0.2x - 4 = 6 - 0.3x$

$0.5x = 10$

$\boxed{x = 20}$

(d) $(3 - 5x) - (8x - 1) = 43$

$3 - 5x - 8x + 1 = 43$

$-13x = 39$

$\boxed{x = -3}$

(e) $4(8x - 2) - 5(3x + 5) = 18$

$32x - 8 - 15x - 25 = 18$

$17x - 33 = 18$

$17x = 51$

$\boxed{x = 3}$

(f) $x+\frac{3}{10}x+\frac{1}{2}+x+\frac{3}{5}x+1=103$

$$2x+\frac{3}{10}x+\frac{3}{5}x+\frac{3}{2}=103$$
$$20x+3x+6x+15=1030$$
$$29x=1015$$
$$\boxed{x=35}$$

(g) $x+\frac{4}{5}x-3+\frac{5}{6}\left(x+\frac{4}{5}x-3\right)=77$

$$30x+24x-90+25\left(x+\frac{4}{5}x-3\right)=30(77)$$
$$54x-90+25x+20x-75=2310$$
$$99x-165=2310$$
$$99x=2475$$
$$\boxed{x=25}$$

(h) $\frac{2}{3}(3x-1)-\frac{3}{4}(5x-3)=\frac{9}{8}x-\frac{5}{6}(7x-9)$

$$16(3x-1)-18(5x-3)=27x-20(7x-9)$$
$$48x-16-90x+54=27x-140x+180$$
$$-42x+38=-113x+180$$
$$71x=142$$
$$\boxed{x=2}$$

7. (a) $\mathrm{I}=\mathrm{P}rt$

$$\boxed{\mathrm{P}=\frac{\mathrm{I}}{rt}}$$

(b) $\mathrm{S}=\frac{P}{1-dt}$

$$\frac{\mathrm{S}}{P}=\frac{1}{1-dt}$$
$$\frac{P}{\mathrm{S}}=1-dt$$
$$dt=1-\frac{P}{\mathrm{S}}$$

$$d=\frac{1-\frac{P}{S}}{t}$$

$$d=\frac{\frac{S-P}{S}}{t}$$

$$\boxed{d=\frac{S-P}{St}}$$

8. (a) Let the regular selling price be $\$x$.

Reduction in price $=\$\frac{1}{5}x$

$$\therefore x-\frac{1}{5}x=192$$
$$\frac{4}{5}x=192$$
$$x=240$$

The regular selling price is $\boxed{\$240.}$

(b) Let the floor space occupied by shipping be x.

Floor space occupied by weaving $=2x+400$

Total floor space $=x+2x+400$

$$\therefore x+2x+400=6700$$
$$3x=6300$$
$$x=2100$$

The floor space occupied by weaving is $2(2100)+400=\boxed{\text{4600 square metres.}}$

(c) Let the number of units of Product A be x.

Number of units of Product B $=95-x$

Number of hours for Product A $=3x$

Number of hours for Product B $=5(95-x)$

$$\therefore 3x+5(95-x)=395$$
$$3x+475-5x=395$$
$$-2x=-80$$
$$x=40$$

The number of units of Product B is $95-40=\boxed{55.}$

(d) Let the sum of money invested in the bank be $\$x$.

Sum of money invested in the credit union $= \$\frac{2}{3}x + 500$

Yield on the bank investment $= \$\frac{1}{12}x$

Yield on the credit union investment $= \$\frac{1}{9}\left(\frac{2}{3}x + 500\right)$

$$\begin{aligned}
\therefore \frac{1}{12}x + \frac{1}{9}\left(\frac{2}{3}x + 500\right) &= 1000 \\
3x + 4\left(\frac{2}{3}x + 500\right) &= 36\,000 \\
3x + \frac{8}{3}x + 2000 &= 36\,000 \\
\frac{17x}{3} &= 34\,000 \\
17x &= 102\,000 \\
x &= 6000
\end{aligned}$$

The sum of money invested in the credit union certificate is

$\$\left(\frac{2}{3} \times 6000 + 500\right) = \boxed{\$4500.}$

Challenge Problems

1. Counting a nickel as a quarter overstates the total by \$0.20; for x nickels, the total must be reduced by \$0.20x.

   Counting a dime as a penny understates the total by \$0.09; for x dimes, the total must be increased by \$0.09x.

   The total adjustment $= -0.20x + 0.09x = -\$0.11x$

   $\boxed{\text{The clerk must reduce the total by \$0.11}x.}$

2. The number of rotations in 5;

The distance per rotation $= \frac{4000}{5} = 800$ km;

each tire will be used for four rotations for a total distance of **3200 km.** (See table below.)

Rotation	*Tire A*	*Tire B*	*Tire C*	*Tire D*	*Tire E*	*Distance travelled*
1	800	800	800	800	—	800
2	800	800	800	—	800	800
3	800	800	—	800	800	800
4	800	—	800	800	800	800
5	—	800	800	800	800	800
Total	3200	3200	3200	3200	3200	4000

3. The lowest possible two-digit number is 10;
the highest possible two-digit number is 99.

For a difference in value of \$17.82, the two-digit numbers must differ by 18, such as 10 and 28, 11 and 29, etc.

The lowest possible correct value of the cheque is \$10.28;
the largest possible correct value of the cheque is \$81.99.

In either case the difference between is \$17.82.

(a) FALSE In the possible correct cheque value \$81.99, the x-value 81 is greater than 70.

(b) TRUE In the possible correct cheque value \$18.36, the y-value 36 equals $2x$.

(c) TRUE A cheque cannot have zero cents.

(d) FALSE Let the correct amount be \$A;
then the incorrect amount is \$2A;
the difference is \$A;

$$A = 17.82$$

For the correct value \$17.82, the incorrect cheque value \$82.17 is unequal to 2(\$17.82).

(e) FALSE In the possible correct amount \$10.28, the sum of the digits is $1+0+2+8=11$, which is not divisible by 9.

Case Study

1. \$73 566 – \$49 355 = \$24 211
2. The contributions continue until the 65th year. Therefore, total contributions (65 – 45) × 12 months per year × \$100 = \$24 000.
3. The contributions continue until the 65th year.
   a. Total contributions (65 – 45) × 12 months per year × \$100 = \$24 000. Total value of TFSA = \$29 529. Therefore, interest earned is \$29 529 – 24 000 = \$5529.
   b. Total contributions (65 – 45) × 12 months per year × \$250 = \$60 000. Total value of TFSA = \$73 566. Therefore, interest earned is \$73 566 – 60 000 = \$13 566.
4. Annual salary of \$48 000/12 months = \$4000.00 per month.
   a. \$150/\$4000 = 0.0375 or 3.75% of salary
   b. \$250/\$4000 = 0.0625 = 6.25% of salary

CHAPTER 2

Review of Basic Algebra

Chapter Overview

Chapter 2 reviews the basics of algebra, including simplifying algebraic expressions, evaluating algebraic expressions by substituting numbers into the variables, solving algebraic equations, and creating and solving word problems. Expressions involving exponents appear in the study of compound interest developed later in the text. Examples are presented showing how positive, negative, fractional, and zero exponents are defined. The study of terms involving positive, negative, and zero exponents serves as a prelude to the introduction of logarithms. Logarithms are useful in solving equations in which the unknown is an exponent. Such equations appear in the area of finance and will be useful in solving financial problems. A review of the steps involved in solving an algebraic equation is presented as well as a four-step procedure for reading and solving word problems.

Learning Objectives

After studying Chapter 2, your students will be able to:

1. Simplify algebraic expressions using fundamental operations and substitution.
2. Simplify and evaluate powers with positive exponents, negative exponents, and exponent zero.
3. Use an electronic calculator to compute the numerical value of arithmetic expressions involving fractional exponents.
4. Write exponential equations in logarithmic form and use an electronic calculator equipped with a natural logarithm function to determine the value of natural logarithms.
5. Solve basic equations using addition, subtraction, multiplication, and division.
6. Solve equations involving algebraic simplification and formula rearrangement.
7. Solve word problems by creating and solving equations.

Suggested Priority of Chapter Topics

Must Cover

- Simplification of algebraic expressions
- Evaluation of exponential expressions
- Logarithmic functions
- Use of the electronic calculator to evaluate exponential and logarithmic expressions
- Solving algebraic equations
- Translating word problems into algebraic equations

Optional

- Case Study on investing in a tax-free savings account

<u>Chapter Outline</u>

Objective 1: Simplify algebraic expressions using fundamental operations and substitution.

A. The simplification of algebraic expressions is introduced by noting that only like terms may be combined together when performing addition or subtraction. Like terms are said to have the same **literal coefficients.**

Teaching Tip

Review the terminology "coefficient" because use of the word "coefficient" is helpful in explaining operations with algebraic expressions. Use an example such as 3*xy*. The numerical coefficient of *xy* is 3, and the literal coefficient of 3 is *xy*.

Consider the expression: $3x^3 - 2x + 4x^3 + 5x$. Note that $4x^3$ and $3x^3$ can be combined because they have the same literal coefficient of x^3. Likewise, -2*x* and 5*x* can be combined because they have the same literal coefficient of *x*. The original expression can be simplified to $7x^3 + 3x$.

B. Some algebraic expressions contain brackets, and the brackets must be removed in order to simplify the expression.

Teaching Tip

This is a good time to review the rules of signed numbers with the students. The four cases of addition, subtraction, multiplication, and division can be covered. Recall with multiplication and division that like signs produce a positive result, and unlike signs produce a negative result. Thus there is no change in sign if the brackets are preceded by a positive number and the signs are changed if the brackets are preceded by a negative number.

Present an example: $-(3x + 4y) + (2x - 3y) = -3x - 4y + 2x - 3y = -x - 7y$. Note the change of sign for each of the terms within the first set of brackets. It is important to point out to the student that <u>each term</u> within a set of brackets must be multiplied by the coefficient outside the brackets.

C. If you multiply two monomials together, multiply the numerical coefficients together, and then multiply the literal coefficients together. This can be expanded to the case of multiplying a polynomial by a monomial. In this case, each term in the polynomial is multiplied by the monomial, and any appropriate sign changes must be made.

D. Multiplying two polynomials together involves an extension of multiplying a polynomial by a monomial. Each term in the second polynomial is multiplied by each term in the first polynomial. Like terms are then collected. There is a lot of bookkeeping involved here because possible sign changes have to be taken into consideration as well.

Consider the example: $(2x+3y)(x-4y)$

Step 1: Multiply $x-4y$ by $2x$ to get $2x^2-8xy$.
Step 2: Multiply $x-4y$ by $3y$ to get $3xy-12y^2$.
Step 3: Add $2x^2-8xy+3xy-12y^2$ and simplify be combining like terms.
Step 4: The final result is $2x^2-5xy-12y^2$.

Teaching Tip

The **first, inner, outer, last** rule (FOIL) may help with expressions of the form $(ax+by)(cx+dy)$. The first terms *ax* and *cx* are multiplied together, the inner terms *by* and *cx* are multiplied together, the outer terms *ax* and *dy* are multiplied together, and the last terms *by* and *dy* are multiplied together. The expression is then simplified by combining like terms.

E. Monomials can be divided by finding the quotient of the numerical coefficients and finding the quotient of the literal coefficients. This can be expanded to the case of dividing a polynomial by a monomial, in which case each term of the polynomial is divided separately by the monomial.

F. Since simple interest will be formally covered in future chapters, expressions dealing with simple interest can be used to explain how to substitute numerical values into literal equations. Consider the expression $S=P(1+rt)$.

Let $P=5000$, $r=0.03$, and $t=30/365$. The order of operations can be recalled. The expression within the brackets is evaluated first. Next, *r* is multiplied by *t* and the number one is added to this result to give 1.002465753, which is then multiplied by 5000 to give a final answer of 5012.33 rounded.

Objective 2: Simplify and evaluate powers with positive exponents, negative exponents, and exponent zero.

A. Review base and exponent terminology. Convey to the student the various rules for interpreting integral, negative, positive, fractional, and zero exponents.

Related Exponent Rules:
Note: $a \neq 0$

$$a^n = (a)(a)(a)...(a) \quad \text{to } n \text{ factors of } a$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$
$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

B. Numerical examples can be presented for each case.

$$3^4 = 81$$
$$3^{-4} = \frac{1}{81}$$
$$3^0 = 1$$
$$81^{-1/4} = \frac{1}{81^{1/4}} = \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$$
$$81^{3/4} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$$

Teaching Tip

Distinguish between expressions like $-3^4 = -81$ and $(-3)^4 = 81$.

C. After discussing how to interpret exponential notation, the rules for operations with powers can be introduced.

Operations with Powers:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$\left(a^m\right)^n = a^{mn}$$

D. Numerical examples can be presented for each of the above rules.

$$3^2 \times 3^4 = 3^{2+4} = 3^6$$
$$3^8 \div 3^6 = 3^{8-6} = 3^2$$
$$\left(3^6\right)^2 = 3^{6\times 2} = 3^{12}$$

Teaching Tip

Distinguish between expressions like $3^2 \times 3^4$ and $\left(3^2\right)^4$. These two expressions look alike but are evaluated differently.

E. The definition of the zero exponent can be motivated through examples such as $\frac{2^5}{2^5} = 1$.

F. The definition of the negative exponent can be motivated through examples such as $\frac{6^3}{6^5} = \frac{1}{6^2} = 6^{-2}$.

Objective 3: Use an electronic calculator to compute the numerical value of arithmetic expressions involving fractional exponents.

A. Calculators evaluate exponents using keys that may include x^y, y^x, or ^.

Consider $(1.03)^{(1/4)}$.
On a calculator like the TI83 or TI84, we enter everything in the order it appears:
$1.03\wedge(1\div 4)$ ENTER
Calculators like the Sharp EL series follow a similar set of steps: $1.03\,x^y\left(1\div 4\right)=$
Finally, the BA II Plus, used in many financial texts, follows: $1.03\,y^x\left(1\div 4\right)=$
In each case, the result is 1.0074.

B. Calculators can be used to evaluate various expressions as illustrated below from the mathematics of finance. The student will be encountering such expressions in the chapters on compound interest and annuities.

a) $\frac{1.04^{20}-1}{0.04} = 29.778$

b) $(1.05)^3 = 1.158$

c) $(1.085)^{-10} = 0.442$

Objective 4: Write exponential equations in logarithmic form and use an electronic calculator equipped with a natural logarithm function to determine the value of natural logarithms.

A. Motivate the study of logarithms by pointing out that some expressions from the mathematics of finance involve solving an equation for the value of an unknown exponent. Logarithms will enable you to do that.

B. Emphasize that the exponential and logarithmic functions are related. The concept of a logarithm can be introduced by starting with an exponential expression such as:

$$10^3 = 1000$$

This expression can be written in logarithmic form as $\log_{10} 1000 = 3$. Note by definition that the logarithm is defined as the exponent to which a base must be raised to give a certain number. By comparing the logarithmic form with the exponential form, it can be easily seen that the logarithm is an exponent. Note that the base is the same in the exponential and logarithmic forms.

Teaching Tip

Give the students an exercise in which it is required to write exponential expressions in logarithmic form and logarithmic expressions in exponential form. See exercises 2.4 A and B on page 61 of the text for such problems.

C. The general interrelationship can be expressed as follows:

$$N = b^y$$
$$y = \log_b N$$

D. Describe the difference between the natural and common logarithmic systems. The common log has a base of 10 and is often designated as log N, whereas the natural log has a base of *e* and is often designated as ln N. It might be pointed out that the symbol *e* represents a special number in mathematics. It is an irrational number with a non-terminating decimal representation and is approximately equal to 2.718. I often liken *e* to π for the purpose of explanation as many students have seen π before. The log key on the calculator can be used to find the common logarithm of a number, and the ln key can be used to find the natural logarithm of a number.

E. The following points related to the ln function should be noted. The student can check these points on the calculator.

$\ln e = 1$

$\ln 1 = 0$

If $a > 1$, $\ln a$ is a positive number.

If $a < 1$, $\ln a$ is a negative number.

$\ln 0$ is not defined.

F. The logarithmic function has special properties that offer tools for evaluating expressions. It is important to note that these properties work for both log and ln.

$$\ln(ab) = \ln a + \ln b$$
$$\ln(abc) = \ln a + \ln b + \ln c$$
$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$
$$\ln\left(a^k\right) = k \ln a$$

Teaching Tip

The property that $\ln\left(a^k\right) = k \ln a$ is useful when solving financial equations of the form $600(1.09)^n = 1000$. The unknown in this equation is an exponent. In this case we can simplify the equation before using the properties of logarithms:

$(1.09)^n = 1.6667$

Now we apply logarithms:

$n \ln(1.09) = \ln(1.6667)$

This equation can be solved for n giving a result of $n = 5.93$. (You will not get an exact answer of 1000 upon substituting the result back into the original equation because of rounding, but it is very close.)

Objective 5: Solve basic equations using addition, subtraction, multiplication, and division.

A. State the formal definition of a linear equation as one in which the unknown appears to the power of 1.

B. Initial examples can involve solving equations that require only one step. You can illustrate during the solutions that addition and subtraction are inverse operations, and that multiplication and division are inverse operations.

Examples	To Isolate for *x*
$x+4=12$	Subtract 4 from each side – the inverse of addition
$x-3=9$	Add 3 to both sides – the inverse of subtraction
$2x=10$	Divide both sides by 2 – the inverse of multiplication
$\frac{x}{3}=12$	Multiply both sides by 3 = the inverse of division

Objective 6: Solve equations involving algebraic simplification and formula rearrangement.

A. If a linear equation contains common fractions, both sides of the equation can be multiplied by a common denominator to eliminate the fractions. It is important to follow through with such an operation on both sides of the equation.

B. An equation may involve brackets. Appropriate changes, including sign changes, must be made to each term within the brackets when the brackets are removed.

C. The term containing the unknown can be set up on the left hand side of the equation with all constant terms on the right hand side of the equation. This involves formula rearrangement. Recall how to "undo" operations by the use of inverse operations. If an operation is performed on one side of the equation, it must be performed on the other side of the equation as well to maintain the equality.

Objective 7: Solve word problems by creating and solving equations.

A. The text provides a four-step procedure for solving word problems.

To Start: Read the problem

1. Introduce the unknown variable
2. Translate the problem into an algebraic expression
3. Set up the equation
4. Solve the equation

To finish: Check you answer

The text has provided word problems in Exercise 2.7 to give students practice in applying the four steps. Select a few problems and go through the four steps for each case.

Students tend to struggle with word problems. It is often helpful to discuss some of the key words used to set up the equation, as in Pointers and Pitfalls on page 75.

Assignment Grid

Assignment	**Topic(s)**	**Learning Objective**	**Estimated Time in Minutes**	**Level of Difficulty**
Ex. 2.1	Simplifying and evaluating algebraic expressions	1	20-30	Easy
Ex. 2.2	Evaluating powers	2	20-30	Easy
Ex. 2.3B, 1-18	Evaluating expressions with exponents	2, 3	25-35	Medium
Ex. 2.3B, 19-22	Solving for an unknown in an exponential expression	2,3	20-30	Difficult
Ex. 2.4	Writing expressions in logarithmic form and evaluating logarithmic expressions	4	20-30	Easy
Ex. 2.5	Solving algebraic equations using addition, subtraction, multiplication, and division	5	20-30	Easy
Ex. 2.6	Solving equations using formula rearrangement	6	30-40	Medium
Ex. 2.7 (odd numbers)	Solving word problems	7	40-50	Medium/Difficult
Review Exercise	Preparation for exam	1-7	60-75	Medium

Name______________________________Date______________________Section_____

CHAPTER 2
TEN-MINUTE QUIZ

Circle the letter of the best response.

1. Consider the exponential expression: $2^6 = 64$. The logarithmic form of this expression is:
   a. $\log_2 6 = 64$
   b. $\log_6 2 = 64$
   c. $\log_2 64 = 6$
   d. $\log_{64} 6 = 2$

2. Simplify the following algebraic expression: $3x^2 + 4y - \left(x^3 + 7x^2 + 3y\right)$.
   a) $-4x^2 + y - x^3$
   b) $10x^2 + y - x^3$
   c) $-4x^2 + 7y - x^3$
   d) $-4x^2 - y + x^3$

3. Evaluate the following expression: $\dfrac{S}{1+rt}$.

   Note: $S = 1800,\ r = 0.125,\ t = 120/365$

   a) 1728.95
   b) 1615.89
   c) 4674.59
   d) 1699.50

4. Find the value of the following expression: $\ln\left[4000\left(1.0325^{-1}\right)\right]$
   a) 8.326
   b) 8.262
   c) 8.29
   d) 8.12

5. A logarithm is best defined by which of the following words:
   a) base
   b) radical
   c) radicand
   d) exponent

6. Consider the following algebraic equation and solve for *x*.

$$\frac{2(x-4)}{3}=(3x+6)$$

   a) -7.50
   b) -7.00
   c) -3.71
   d) 4.28

7. The base of the natural logarithm system is:
   a) 10
   b) e
   c) π
   d) 2

8. Jim, Sue, and Marion have set up a partnership. The total amount contributed to the partnership is $315 000. Sue's contribution is one-half of Jim's contribution, and Marion's contribution is two times Jim's contribution. Find the amount of Jim's contribution to the partnership.
   a) $90 000
   b) $45 000
   c) $180 000
   d) $60 000

9. Evaluate the following exponential expression: $1.05^{-3/2}$
   a) 1.076
   b) 1.575
   c) 0.929
   d) 0.968

10. Solve the following equation for k. (Hint: Note that k is an exponent.)

$$(1.06)^k = 2000$$

a) 130.45
b) 186.66
c) 7.17
d) 100.55

Answers:

1. c	**2. a**	**3. a**	**4. b**	**5. d**
6. c	**7. b**	**8. a**	**9. c**	**10. a**

Additional Questions:

1. Evaluate: $PMT\left[\frac{(1+i)^n - 1}{i}\right]$ for $PMT = 200,\ i = 0.02,\ n = 18$.
   a. 9714.35
   b. 14 232.46
   c. 4282.46
   d. 14 082.46

2. Simplify: $-(2a+4b-c)-(a+b+c)$.
   a. $-3a-3b+2c$
   b. $-3a+5b$
   c. $-3a-3b$
   d. $-3a-5b$

3. Evaluate using a calculator: $\frac{\ln(5.6)}{\ln(1.015)}$.
   a. 1.723
   b. 115.710
   c. 0.015
   d. 1.708

4. Solve: $x + 1.3x = 21.5$.
   a. 9.348
   b. 16.538
   c. 8.269
   d. 10.1

5. Simplify: $\left(16a^3 - 24a\right) \div 4a$.
   a. $16a^3 - 6$
   b. $-2a$
   c. $4a^2 - 6$
   d. $4a^2 - 6a$

6. Compute: $\ln\left[4e^{-0.5}\right]$.
   a. 0.8863
   b. 4
   c. -0.5
   d. 0.8408

7. Carla is framing in a square sand box in her back yard. She has set aside an area of 24 square feet for the sandbox. The sandbox is to be 1.5 times longer than it is wide. What is the length of the longer side for the sandbox?
   a. 16 ft
   b. 6 ft
   c. 4 ft
   d. 4.7 ft

8. Solve: $x + \frac{1}{2}x + \frac{3}{5}x = 27$.
   a. 47.25
   b. 12.86
   c. 15
   d. 19.29

9. Given: $S = P\left(1 + rt\right)$ solve for *t*.
   a. $t = \frac{S - P}{1 + r}$
   b. $t = \frac{S}{\text{Pr}} - 1$
   c. $t = \frac{S - P}{r}$
   d. $t = \frac{S - P}{\text{Pr}}$

10. Express in logarithmic form: $3^5 = 243$.
   a. $5 = \log_3 243$
   b. $5 = 3\log 243$
   c. $3 = \log_5 243$
   d. $3 = 5\log 243$

Answers:

1. c	**2. d**	**3. b**	**4. a**	**5. c**
6. a	**7. b**	**8. b**	**9. d**	**10. a**

Chapter 2 Review of Basic Algebra

Exercise 2.1

A. 1. $19a$

2. $3m$

3. $-a-10$

4. $-3a-14$

5. $-2x-4y$

6. $3p+q$

7. $14f-4v$

8. $2c-3d$

9. $0.8x$

10. $1.06x$

11. $1.4x$

12. $0.98x$

13. $2.79x$

14. $4.05y$

15. $-x^2-x-8$

16. $-ax+x-2$

17. $x-7y$

18. $2a-2$

19. $4b+2c+2$

20. $-2a^2-6ab+5b^2$

21. $-m^2+6m+1$

22. $14-9x+y$

23. $10a-14b$

24. $2f+2f^2+3fg$

B. 1. $-12x$

2. $-56a$

3. $-10ax$

4. $27ab$
5. $-2x^2$
6. $24m^2$
7. $60xy$
8. $-24abc$
9. $-2x+4y$
10. $10x-20$
11. $2ax^2-3ax-a$
12. $-24x+12bx+6b^2x$
13. $35x-30$
14. $-10a-15b$
15. $-20ax+5a$
16. $21y-31$
17. $3x^2+5x-2$
18. $5m^2-17mn+6n^2$
19. x^3+y^3
20. a^3-3a^2+3a-1
21. $7x^2+3x+39$
22. $-5a^2-13a+12$
23. $4ab$
24. $-5y$
25. $4x$
26. -6
27. $10m-4$
28. $-2x+3$
29. $-2x^2+3x+6$
30. a^2+4a+3

C. 1. -5
2. 14
3. 5500

4. 20 000
5. 0.58604
6. \$40.50
7. \$378
8. 0.125
9. \$3000
10. $0.1\dot{3}$
11. \$901.99
12. \$1052.71
13. \$1400.06
14. \$1600.08

***Exercise* 2.2**

A. 1. 81
2. 1
3. 16
4. 1
5. $\frac{16}{81}$
6. $\frac{625}{1296}$
7. $-\frac{1}{64}$
8. $-\frac{8}{27}$
9. 0.25
10. 113.379904
11. –0.001
12. –335.544320
13. 1
14. 1

15. $\frac{1}{9}$

16. 512

17. $-\frac{1}{125}$

18. $\frac{1}{167.9616}$

19. 125

20. $\frac{81}{16}$

21. $\frac{1}{1.01}$

22. 1

B. 1. 2^8

2. $(-4)^4$

3. 4^3

4. $(-3)^2$

5. 2^{15}

6. $(-4)^{18}$

7. a^{14}

8. m^5

9. 3^{11}

10. $(-1)^{15}$

11. 6

12. x^2

13. $\frac{3^{11}}{5^{11}}$

14. $\frac{1}{6^2}$

15. $\frac{(-3)^{11}}{2^{11}}$

16. $-\frac{3}{4}$

17. 1.025^{150}

18. 1.005^{90}

19. 1.04^{80}

20. $\frac{-3^{15}}{7^{15}}$

21. $(1+i)^{200}$

22. $(1-r)^{6}$

23. $(1+i)^{160}$

24. $(1-r)^{120}$

25. a^5b^5

26. $16x^4y^4$

27. $m^{24}n^8$

28. $\frac{a^{12}b^8}{x^4}$

29. 2^4

30. 5^5

31. $\frac{b^8}{a^8}$

32. $\frac{i^n}{(1+i)^n}$

***Exercise* 2.3**

A. 1. 72.0000

2. 14.3500

3. 3.0000

4. 1.0100

5. 1.0759

6. 0.5000

7. 1.0133
8. 1.0117

B. 1. 55
2. 7
3. 12.25
4. 60.154991
5. 1.071122
6. 1.015241
7. 0.629961
8. 0.995156
9. 163.053437
10. 16.546852
11. 2.158925
12. 0.589664
13. 1630.176673
14. 12 151.73813
15. 1139.915716
16. 1855.967995
17. 5000.00
18. 4391.497547
19. 0.029998
20. 0.019999
21. 0.04
22. 0.01

Exercise 2.4

A. 1. $9 = \log_2 512$
2. $7 = \log_3 2187$

3. $-3 = \log_5 \frac{1}{125}$
4. $-5 = \log_{10} 0.00001$
5. $2j = \ln 18$
6. $-3x = \ln 12$

B. 1. $2^5 = 32$

2. $3^{-4} = \frac{1}{81}$
3. $10^1 = 10$
4. $e^2 = e^2$

C. 1. 0.693147

2. 5.298317
3. −2.253795
4. 7.133435
5. 6.825303
6. 10.156367

Business Math News Box

1. $133 900 000 (C$133.9 million)
2. C$133.9 × 10^6
3. C$2678 × 10^6 million
4. 14.73% per year

***Exercise* 2.5**

A. 1. $x = 3$

2. $x = -5$
3. $x = 80$
4. $x = 650$
5. $x = 18$
6. $x = -56$

7. $x = -35$
8. $x = 24$
9. $x = -4$
10. $x = 7$
11. $x = -8$
12. $x = 9$
13. $x = 5$
14. $x = -12$
15. $x = 20$
16. $x = 300$
17. $x = 200$
18. $x = 60$

B. 1. $x = 4$
2. $x = 3$
3. $x = 0$
4. $x = -2$
5. $x = 5$
6. $x = -2$
7. $x = 21$
8. $x = 8$

***Exercise* 2.6**

A. 1. $x = -10$
2. $x = -2$
3. $x = -3$
4. $x = 5$
5. $x = 3$
6. $x = 5$
7. $x = 14$
8. $x = 2$

B. 1. $x = 20$

2. $x = 16$
3. $x = -1$
4. $x = 3$
5. $x = \frac{1}{2}$
6. $x = -\frac{2}{3}$

C. 1. $x = -1$
2. $x = 2$
3. $x = \frac{5}{6}$
4. $x = -\frac{3}{4}$

D. 1. $x = \frac{y - b}{m}$
2. $S = \frac{M}{r}$
3. $\text{PMT} = \text{PV}i$
4. $t = \frac{\text{I}}{\text{P}r}$
5. $r = \frac{\text{S} - \text{P}}{\text{P}t}$
6. $i = \left[\frac{\text{FV}}{\text{PV}}\right]^{\frac{1}{n}} - 1$

***Exercise* 2.7**

A. 1. $28.28.
2. $864.
3. $35.00.
4. $18.90.
5. 192.

6. $11.36.
7. $670.
8. 65 cm.
9. $89.00.
10. $23 500.
11. 1300.
12. 18.
13. 20 units.
14. 20 dimes, 56 nickels, and 16 quarters.
15. 30 $12 tickets,
    100 $ 8 tickets,
    and 21 $15 tickets.
16. 6 medium pizzas,
    17 large pizzas,
    and 13 small pizzas.

Review Exercise

1. (a) $-2x-7y$
   (b) $1.97x$
   (c) $6a-7$
   (d) $x+3y$
   (e) $9a^2-4b-4c$
   (f) $-x^2+3x+1$
2. (a) $-15a$
   (b) $28mx$
   (c) -7
   (d) $-3ab$
   (e) $36xy$
   (f) $24abc$
   (g) $-12x+20y+4$

(h) $x - 2x^2 - x^3$

(i) $-6x + 4$

(j) $7a - 4$

(k) $26a - 29$

(l) $14ax - 2a^2 + 10a$

(m) $2m^2 - 7m + 5$

(n) $3a^3 - 8a^2 - 5a + 6$

(o) $-14x^2 + 34x + 36$

(p) $-26am^2 + 26am + 37a$

3. (a) -47

(b) $6\frac{1}{3}$

(c) 0.16

(d) 200

(e) \$645.44

(f) 2500

4. (a) -243

(b) $\frac{16}{81}$

(c) 1

(d) $-\frac{1}{3}$

(e) $\frac{625}{16}$

(f) 1

(g) $-19\,683$

(h) 1024

(i) 59 049

(j) m^{12}

(k) $\frac{16}{81}$

(l) $\frac{25}{16}$

(m) 1.03^{150}

(n) $(1+i)^{80}$

(o) 1.05^{150}

(p) $16x^4y^4$

(q) $\frac{81}{a^8b^4}$

(r) $\frac{1}{(1+i)^n}$

5. (a) 0.96
   (b) 1.012126
   (c) 1.07
   (d) 0.968442
   (e) 1.098612
   (f) −2.995732
   (g) 7.087540
   (h) 9.871647
6. (a) $x = -7$
   (b) $x = 880$
   (c) $x = -21$
   (d) $x = -18$
   (e) $x = 3$
   (f) $x = -11$
   (g) $x = 250$
   (h) $x = 40$
   (i) $x = -1$
   (j) $x = 7$
   (k) $x = 39$
   (l) $x = 56$
7. (a) $x = -7$

(b) $x = 5$

(c) $x = -3$

(d) $x = -\frac{7}{12}$

(e) $x = 7$

(f) $x = -\frac{1}{3}$

(g) $x = -\frac{1}{2}$

8. (a) $r = \frac{I}{Pt}$

(b) $t = \frac{S - P}{Pr}$

(c) $r = \frac{D}{L}$

(d) $\text{PMT} = \left[\frac{FVp}{(1+p)^n - 1}\right]$

9. (a) 138.

(b) \$63 350.

(c) \$117.

(d) \$44 500.

(e) heat \$814;
power \$1056;
water \$341

(f) \$37 500

(g) 35 minutes.

(h) superlight poles is 27;
ordinary poles is 45.

(i) 164.

Self-Test

1. (a) $-2 - 8x$

(b) $-2x-9$

(c) $-16a-7$

(d) $-6x^2+6x+12$

2. (a) -7

(b) $18\frac{2}{3}$

(c) 0.192

(d) 0.4

(e) 1474

(f) 1450

3. (a) -8

(b) $\frac{4}{9}$

(c) 1

(d) 2187

(e) $\frac{9}{16}$

(f) $-x^{15}$

4. (a) 1.030465

(b) 23.114772

(c) 0.024693

(d) 0.898612

(e) 5.755972

(f) 7.270789

5. (a) $n=6$

(b) $n=5$

6. (a) $x=-36$

(b) $x=9$

(c) $x=20$

(d) $x=-3$

(e) $x=3$

(f) $x = 35$

(g) $x = 25$

(h) $x = 2$

7. (a) $\mathrm{P} = \frac{\mathrm{I}}{rt}$

   (b) $d = \frac{\mathrm{S} - P}{\mathrm{S}t}$

8. (a) $240.

   (b) 4600 square metres.

   (c) 55.

   (d) $4500.

Challenge Problems

1. The clerk must reduce the total by $\$0.11x$.
2. 3200 km.
3. (a) FALSE

   (b) TRUE

   (c) TRUE

   (d) FALSE

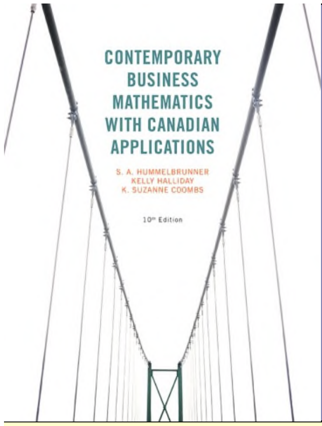
   (e) FALSE

Case Study

1. $24 211
2. $24 000
3. (a) $5529

   (b) $13 566
4. (a) 3.75% of salary

   (b) 6.25% of salary



Contemporary Business Mathematics With Canadian Applications

Tenth Edition

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PowerPoint Presentation by P. Au

Chapter 2

Review of Basic Algebra

Objectives

After completing chapter two, the student will be able to:

❶ Simplify algebraic expressions using fundamental operations and substitution

❷ Simplify and evaluate powers with positive exponents, negative exponents, and exponent zero

❸ Use an electronic calculator to compute the numerical value of arithmetic expressions involving fractional exponents

❹ Write exponential equations in logarithmic form and use an electronic calculator equipped with a natural logarithm function to determine the value of natural logarithms

Objectives

After completing chapter two, the student will be able to:

❺ Solve basic equations using addition, subtraction, multiplication, and division

❻ Solve equations involving algebraic simplification and formula rearrangement

❼ Solve word problems by creating and solving equations

Simplification of Algebraic Expressions

- Addition and subtraction
- In algebra, only like terms may be added or subtracted by *adding* or *subtracting* the numerical coefficients of the like terms
  - *retain* the common literal coefficient
    - 3x, -5x, 9x are like terms with the same literal coefficient "x"
    - $2y^2$, $-5y^2$ are like terms with the same literal coefficient "y^2"
- The process of adding and subtracting like terms is called combining like terms or collecting like terms

Addition and Subtraction Examples

- $3x + 2x + 7x = (3 + 2 + 7)x = 12x$
- $6x - 4y - 2x + 8y = (6 – 2)x + (-4 + 8)y = 4x + 4y$
- $7xy - 3xy - xy = (7 – 3 – 1)xy = 3xy$
- $4c - 5d - 3c + 4d = (4-3)c + (-5 + 4)d = 1c + (-1d)$

  $= c – d$
- $3x^2 + 2.5x^2 = (3 + 2.5) = 5.5x^2$

Simplification Involving Brackets

- If brackets are preceded by a + sign, do not change the sign of the terms inside the brackets
- (7a - 2b) + (4a -5b) = 11a -7b
- If brackets are preceded by a - sign, change the sign of each term inside the brackets
- (4c - 5d) - (2c -3d) = 2c -2d

Multiplication of Monomials

- The product of two or more monomials is the product of their numerical coefficients multiplied by the product of their literal coefficients
- $5(3a) = (5 \times 3)a = 15a$
- $(-7a)(4b) = (-7 \times 4)(a \times b) = -28ab$
- $(-3)(4x)(-5x) = (-3 \times 4 \times -5)(x \cdot x) = 60x^2$

The "dot" also means multiply

Multiplication of Monomials with Polynomials

- A polynomial is an expression with more than one term
- 5(a – 3) = 5(a) – (5)(3) = 5a – 15
- -4(3x^2 – 2x – 1 = -4(3x^2) + -4(-2x) + -4(-1)
  = (-12x^2) + (+8x) + (+4) = -12x2 + 8x + 4
- 3a(4a – 5b -2c) = (3a)(4a) + (3a)(-5b) + (3a)(-2c)
  = 12a^2 -15ab - 6c

Simplification Involving Brackets and Multiplication

- $3(x - 5) - 2(x - 7)$

  $= 3x - 15 - 2x + 14$

  $= x - 1$

- $a(3a - 1) - 4(2a + 3)$

  $= 3a2 - a - 8a - 12 = 3a2 - 9a - 12$

- $-4(5a - 3b - 2c) + 5(-2a - 4b + c)$

  $= -20a + 12b + 8c - 10a - 20b + 5c$

  $= -30a - 8b + 13c$

Multiplication of a Polynomial by a Polynomial

- Multiply each term of one polynomial by each term of the other polynomial
- $(3a + 2b)(4c - 3d) = 3a(4c - 3d) + 2b(4c - 3d)$

  $= 12ac - 9ad + 8bc - 6bd$
- $(5x - 2)(3x + 4)$

  $= 5x(3x + 4) - 2(3x + 4)$

  $= 15x2 + 20x - 6x - 8$

  $= 15x2 + 14x - 8$

Division of Monomials

- The quotient of two monomials is the quotient of their numerical coefficients multiplied by the quotient of their literal coefficients
- $24x^2 \div (-6x) = \left(\frac{24}{-6}\right)\left(\frac{x^2}{x}\right) = -4x$

Division of a Polynomial by a Monomial

- Divide each term of the polynomial by the monomial

$$(12a^3 - 15a^2 - 9a) \div (-3a)$$
$$= \frac{12a^3 - 15a^2 - 9a}{-3a} = \frac{12a^3}{-3a} + \frac{-15a^2}{-3a} + \frac{-9a}{-3a}$$
$$= -4a^2 + 5a = 3$$

Substitution and Evaluation

- The replacement or substitution of the variables takes place each time the variables appear in the expression

Evaluate $7x - 3y - 5$ for $x = -2$, $y = 3$

Solution $7x - 3y - 5$

$= 7(-2) - 3(3) - 5$ replace x by (-2) and y by 3

$= -14 - 9 - 5$

$= -28$

Exponents

- Power → a^n
- Base → **a**
- Exponent → **n**
- The factor a is multiplied by itself n times
- POWER = BASE $^{\text{TO THE EXPONENT}}$

Using Exponents

- $6^3 = 6 \times 6 \times 6$
- $(-2)^4 = (-2)(-2)(-2)(-2)$
- $(1+i)^5 = (1+i)(1+i)(1+i)(1+i)(1+i)$
- $(1/4)^2 = (0.25)(0.25)$

Operations with Powers

Rules	Examples
$a^m \times a^n = a^{m+n}$	$2^3 \times 2^2 = 2^{3+2} = 2^5$
$a^m \div a^n = a^{m-n}$	$2^5 \div 2^3 = 2^{5-3} = 2^2$
$(a^m)^n = a^{m \cdot n}$	$(2^3)^2 = 2^{3 \cdot 2} = 2^6$
$(ab)^m = a^m b^m$	$(4 \times 5)^2 = 4^2 \times 5^2$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$
$a^0 = 1 \ if \ a \neq 0$	$a^0 \ is \ undefined \ if \ a = 0$

Negative and Zero Exponents

Rules	Examples
$a^{-n} = \frac{1}{a^n}$	$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$
	$(1+i)^{-3} = \frac{1}{(1+i)^3}$
	$(1.05)^0 = 1$
	$(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$
	$\left(\frac{3}{4}\right)^{-3} = \frac{1}{\left({}^3/_4\right)^3} = \frac{1}{\left({}^{3^3}/_{4^3}\right)} = \frac{1}{\left({}^{27}/_{64}\right)} = \frac{64}{27} = 2.37$

Fractional Exponents

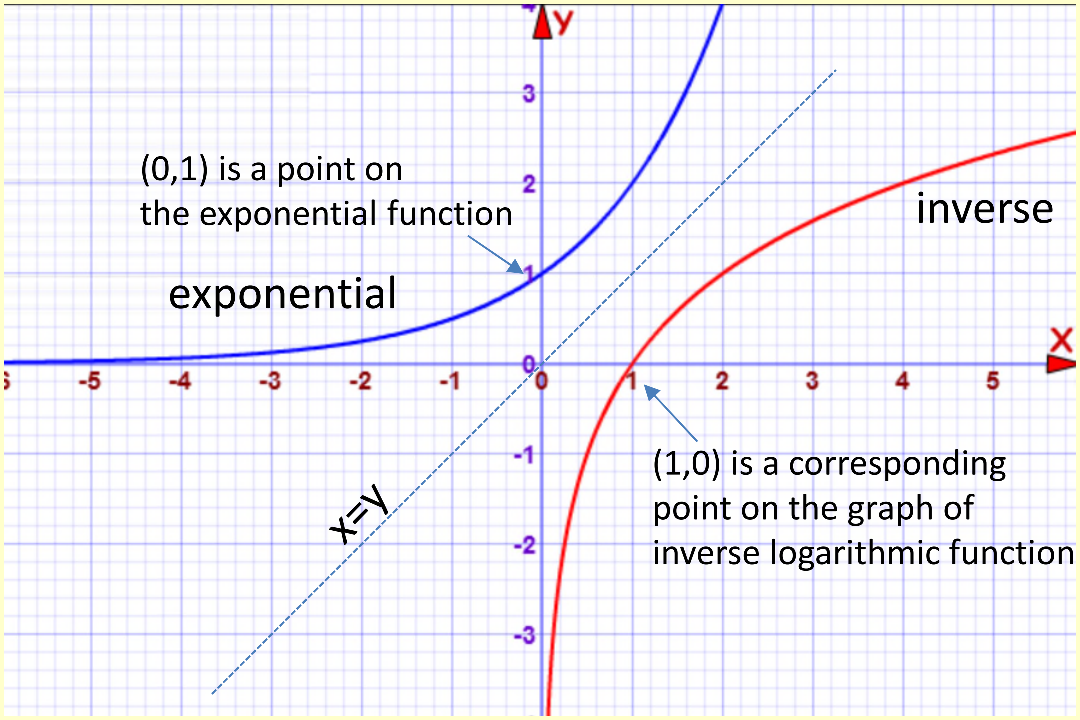
Rules	Examples
$a^{\frac{1}{n}} = \sqrt[n]{a}$	$49^{\frac{1}{2}} = \sqrt[2]{49} = \sqrt{49} = 7$
$a^{-\frac{1}{n}} = \frac{1}{a^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a}}$	$27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$
$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$36^{\frac{3}{2}} = \sqrt[2]{36^3} = \sqrt{46{,}656} = 216$
$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\sqrt[n]{a^m}}$	$64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{64^2}} = \frac{1}{\sqrt[3]{4096}} = \frac{1}{16}$

Logarithms

- In general, when a number is represented as a base raised to an exponent, the exponent is called a logarithm
- A logarithm is defined as the *exponent* to which a base must be raised to produce a given number
- Logarithmic functions are the inverse of exponential functions

Graphs of Logarithms

- Graph of an exponential function versus graph of its inverse (logarithmic function)



Exponents and Logarithms

Exponential Form	Logarithmic Form
$N = b^y$	$y = log_b N$
$8 = 2^3$	$3 = log_2 8$
$100 = 10^2$	$2 = log_{10} 100$

Properties of Logarithms

Rules (Properties) of Logarithms	Examples
$ln(ab) = \ln a + \ln b$	$ln[(3)(6)] = \ln 3 + \ln 6$
$ln\left(\frac{a}{b}\right) = \ln a - \ln b$	$ln\left(\frac{50}{3}\right) = \ln 50 \ - ln\ 3$
$ln\left(a^k\right) = k \ln a$	$ln(1.03)^6 = 6 \ln(1.03)$

Solving Linear Equations

- An equation is an expression of equality between two algebraic expressions

  $3x = 36$

  $2x + 4 = 60$

  $5x - 0.4 = 2.5$

- Linear equations are equations that when graphed look like a straight line

Solving Linear Equations

- The basics:
- Using Addition

  x - 3 = 9

  Add 3 to both sides

  x - 3 **+ 3** = 9 **+ 3**

  x = 12
- Using Subtraction

  x + 3 = 8

  Subtract 3 from both sides

  x + 3 **- 3** = 8 **- 3**

  x = 5

Solving Linear Equations

- The basics;
- Using Multiplication

$$\frac{x}{3} = 6$$

Multiply both sides by 3

$$3\left(\frac{x}{3}\right) = 3(6)$$

$$x = 18$$

- Using Division

$$4x = 24$$

Divide both sides by 4

$$\frac{4x}{4} = \frac{24}{4}$$

$$x = 6$$

Using Two or More Operations to Solve an Equation

- $3x - 5 = 2x + 6$

  $3x - 2x = 6 + 5$

  $x = 11$
- You can substitute your result back into the original equation to check your answer
- $3(11) - 5 = 2(11) + 6$

  $33 - 5 = 22 + 6$

  $28 = 28$
  - The left hand side equals the right hand side (LHS = RHS)

Solving Linear Equations

- Solving linear equations involving the product of integral constants and binomials
  - multiply first, then simplify
- $3(2x - 5) = -5(7 - 2x)$

  $6x - 15 = -35 + 10x$ 🡐 *expand*

  $6x - 10x = -35 + 15$ 🡐 *isolate the terms in x*

  $-4x = -20$

  $x = 5$

Solving Linear Equations

- Solving linear equations containing common fractions
  – Create an equivalent equation without common fractions by multiplying each term of the equation by the lowest common denominator (LCD) of the fractions

Solving Linear Equations

- Example
- $\frac{5}{8}x - 3 = \frac{3}{4} + \frac{5}{6}x \;\; the\ LCD\ is\ 24$
- $24\left(\frac{5x}{8}\right) - 24(3) = 24\left(\frac{3}{4}\right) + 24\left(\frac{5x}{6}\right)$
- $3(5x) - 72 = 6(3) + 4(5x)$
- $15x - 72 = 18 + 20x$
- $-5x = 90$
- $x = -18$

Formula Rearrangement

- The process of rearranging the terms of an equation to solve for a particular variable
- We want the variable to stand alone on the left side of the equation
- Developing your skill in rearranging formulas is very important because it saves a lot of time in memorization

Formula Rearrangement

- Given $FV = PV(1+i)^n \quad solve\ for\ PV$

  Divide both sides by $(1+i)^n$ and reverse members of the equation

- $\frac{FV}{(1+i)^n} = \frac{PV(1+i)^n}{(1+i)^n}$
- $\frac{FV}{(1+i)^n} = PV$
- $PV = \frac{FV}{(1+i)^n}$

Solving Word Problems

Before STEP 1 *Read the problem* to determine what type of question you are dealing with, determine the information given

- **STEP 1** *Introduce the variable* to be used by means of a complete sentence
  - this ensures a clear understanding and a record of what the variable is intended to represent
  - the variable is usually the item that you are asked to find
- **STEP 2** *Translate* the information in the problem statement in terms of the variable

Solving Word Problems

  - Determine what the words are telling you in relationship to the math
  - watch for key words such as “more than” or “less than,” “reduced by,” and “half of ” or “twice”

- **STEP 3** *Set up* an algebraic equation
  - this usually means matching the algebraic expressions developed in Step 2 to a specific number
  - often one side of the equation represents the total number of items described in the word problem

- **STEP 4** *Solve* the equation by rearranging the variables, state a conclusion, and check the conclusion

Solving Word Problems

- A TV set was sold during a sale for $575. What is the regular selling price of the set if the price of the set was reduced by 16 of the regular price?
- **STEP 1** *Introduce the variable*
  - let the regular selling price be represented by $*x*
- **STEP 2** *Translate*
  - the reduction in price is $\$\frac{1}{6}x$, and the reduced price is $\$\left(x - \frac{1}{6}x\right)$

Solving Word Problems

- **STEP 3** *Set up an equation*
  - the reduced price is given as $575

$$x - \frac{1}{6}x = 575$$

- **STEP 4** *Solve* the equation, state a conclusion, and check
- $6(x) - 6\left(\frac{1x}{6}\right) = 6(575)$
- $6x - x = 3{,}450$
- $5x = 3{,}450$
- $\frac{5x}{5} = \frac{3{,}450}{5}$
- $x = 690$
- The regular selling price is $690

Check

- Regular selling price is \$690
- Reduction: $\frac{1}{6}$ of 690 is 115
- Reduced price is \$575

Summary

- Exponential expressions, logarithms and algebraic equations are important tools in the solution of problems found in business mathematics and finance