7. Similarity: Both estimate proportions of the data contained within *k* standard deviations of the mean.
Difference: The Empirical Rule assumes the distribution is approximately symmetric and bell-shaped. Chebychev's Theorem makes no such assumption.

8. You must know that the distribution is approximately symmetric and bell-shaped.

9. Range = Max – Min = 34 – 24 = 10

10. Range = Max – Min = 98 – 74 = 24

11. (a) Range = Max – Min = 38.5 – 20.7 = 17.8
(b) Range = Max – Min = 60.5 – 20.7 = 39.8

12. Changing the maximum value of the data set greatly affects the range.

13. Range = Max – Min = 13 – 2 = 11

$$\mu = \frac{\sum x}{N} = \frac{121}{16} \approx 7.6$$

x	$x - \mu$	$(x-\mu)^2$
13	5.4	29.16
10	2.4	5.76
12	4.4	19.36
11	3.4	11.56
7	-0.6	0.36
8	0.4	0.16
6	-1.6	2.56
6	-1.6	2.56
10	2.4	5.76
7	-0.6	0.36
12	4.4	19.36
4	-3.6	12.96
6	-1.6	2.56
5	-2.6	6.76
2	-5.6	31.36
2	-5.6	31.36
$\sum x = 121$	$\sum(x-\mu) \approx 0$	$\sum(x-\mu)^2 = 181.96$

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{181.96}{16} \approx 11.4$$

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}} = \sqrt{\frac{181.96}{16}} \approx 3.4$$

14. Range = Max – Min = 230 – 160 = 70

$$\mu = \frac{\sum x}{N} = \frac{1861}{10} = 186.1$$

x	$x-\mu$	$(x-\mu)^2$
173	-13.1	171.61
175	-11.1	123.21
200	13.9	193.21
173	-13.1	171.61
160	-26.1	681.21
185	-1.1	1.21
195	8.9	79.21
230	43.9	1927.21
190	3.9	15.21
180	-6.1	37.21
$\sum x = 1861$	$\sum(x-\mu) = 0$	$\sum(x-\mu)^2 = 3400.9$

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{3400.9}{10} \approx 340.1$$

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}} = \sqrt{\frac{3400.9}{10}} \approx 18.4$$

15. Range = Max – Min = 24 – 14 = 10

$$\bar{x} = \frac{\sum x}{n} = \frac{340}{20} = 17$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
16	-1	1
18	1	1
19	2	4
17	0	0
14	-3	9
15	-2	4
17	0	0
17	0	0
17	0	0
16	-1	1
19	2	4
22	5	25
24	7	49
14	-3	9
16	-1	1
14	-3	9
17	0	0
16	-1	1
14	-3	9
18	1	1
$\sum x = 340$	$\sum(x-\bar{x}) = 0$	$\sum(x-\bar{x})^2 = 128$

$$s^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n-1} = \frac{128}{20-1} \approx 6.7$$

$$s = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n-1}} = \sqrt{\frac{128}{19}} \approx 2.6$$

16. Range = Max – Min = 299 – 264 = 35

$$\bar{x} = \frac{\sum x}{n} = \frac{5627}{20} \approx 281.4$$

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
277	-4.4	19.36
277	-4.4	19.36
267	-14.4	207.36
291	9.6	92.16
282	0.6	0.36
281	-0.4	0.16
295	13.6	184.96
279	-2.4	5.76
286	4.6	21.16
280	-1.4	1.96
296	14.6	213.16
269	-12.4	153.76
268	-13.4	179.56
285	3.6	12.96
264	-17.4	302.76
278	-3.4	11.56
269	-12.4	153.76
299	17.6	309.76
291	9.6	92.16
293	11.6	134.56
$\sum x = 5627$	$\sum\left(x-\bar{x}\right) \approx 0$	$\sum\left(x-\bar{x}\right)^2 = 2116.6$

$$s^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n-1} = \frac{2116.6}{20-1} \approx 111.4$$

$$s = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n-1}} = \sqrt{\frac{2116.6}{19}} \approx 10.6$$

17. The data set in (a) has a standard deviation of 24 and the data set in (b) has a standard deviation of 16 because the data in (a) have more variability.

18. The data set in (a) has a standard deviation of 2.4 and the data set in (b) has a standard deviation of 5 because the data in (b) have more variability.

19. Company B. An offer of $33,000 is two standard deviations from the mean of Company A’s starting salaries, which makes it unlikely. The same offer is within one standard deviation of the mean of Company B’s starting salaries, which makes the offer likely.

20. Company C. An offer of $42,000 is two standard deviations from the mean of Company D's starting salaries, which makes it unlikely. The same offer is within one standard deviation of the mean of Company C's starting salaries, which makes the offer likely.

21. (a) Greatest sample standard deviation: (ii)
Data set (ii) has more entries that are farther away from the mean.
Least sample standard deviation: (iii)
Data set (iii) has more entries that are close to the mean.
(b) The three data sets have the same mean but have different standard deviations.

22. (a) Greatest sample standard deviation: (i)
Data set (i) has more entries that are farther away from the mean.
Least sample standard deviation: (iii)
Data set (iii) has more entries that are close to the mean.
(b) The three data sets have the same mean, median, and mode, but have different standard deviations.

23. (a) Greatest sample standard deviation: (ii)
Data set (ii) has more entries that are farther away from the mean.
Least sample standard deviation: (iii)
Data set (iii) has more entries that are close to the mean.
(b) The three data sets have the same mean, median, and mode, but have different standard deviations.

24. (a) Greatest sample standard deviation: (iii)
Data set (iii) has more entries that are farther away from the mean.
Least sample standard deviation: (i)
Data set (i) has more entries that are close to the mean.
(b) The three data sets have the same mean and median but have different modes and standard deviations.

25. *Sample answer:* 3,3,3,7,7,7

26. *Sample answer:* 3,3,3,3,9,9,9,9

27. *Sample answer:* 9,9,9,9,9,9,9

28. *Sample answer:* 5,5,5,9,9,9

29. $(63,\ 71) \rightarrow (67-1(4),\ 67+1(4)) \rightarrow (\bar{x}-s,\ \bar{x}+s)$

68% of the vehicles have speeds between 63 and 71 mph.

30. 95% of the data falls between $\bar{x}-2s$ and $\bar{x}+2s$.

$\bar{x}-2s = 70-2(8) = 54$

$\bar{x}+2s = 70+2(8) = 86$

95% of the households have monthly utility bills between $54 and $86.

31. (a) $n = 75$; 68%(75) = (0.68)(75) ≈ 51 vehicles have speeds between 63 and 71 mph.
(b) $n = 25$; 68%(25) = (0.68)(25) ≈ 17 vehicles have speeds between 63 and 71 mph.

32. (a) $n = 40$; 95%(40) = (0.95)(40) ≈ 38 households have monthly utility bills between \$54 and \$86.

(b) $n = 20$; 95%(20) = (0.95)(20) ≈ 19 households have monthly utility bills between \$54 and \$86.

33. 78, 76, and 82 are unusual; 82 is very unusual because it is more than 3 standard deviations from the mean.

34. \$52 and \$98 are unusual; \$98 is very unusual because it is more than 3 standard deviations from the mean.

35. $\left(\bar{x} - 2s,\ \bar{x} + 2s\right) \to \left(0,\ 4\right)$ are 2 standard deviations from the mean.

$1 - \frac{1}{k^2} = 1 - \frac{1}{(2)^2} = 1 - \frac{1}{4} = 0.75 \Rightarrow$ At least 75% of the eruption times lie between 0 and 4.

If n = 40, at least (0.75)(40) = 30 households have between 0 and 4 pets.

36. $\left(\bar{x} - 2s,\ \bar{x} + 2s\right) \to \left(1.14,\ 5.5\right)$ are 2 standard deviations from the mean.

$1 - \frac{1}{k^2} = 1 - \frac{1}{(2)^2} = 1 - \frac{1}{4} = 0.75 \Rightarrow$ At least 75% of the eruption times lie between 1.14 and 5.5 minutes.

If n = 32, at least (0.75)(32) = 24 eruptions will lie between 1.14 and 5.5 minutes.

37. $\left(\bar{x} - 2s,\ \bar{x} + 2s\right) \to \left(80,\ 96\right)$ are 2 standard deviations from the mean.

At least 75% of the test scores are from 80 to 96.

38. $1 - \frac{1}{k^2} = 1 - \frac{1}{(2)^2} = 1 - \frac{1}{4} = 0.75$

At least 75% of the 800-meter freestyle times lie within 2 standard deviations of the mean.

$\left(\bar{x} - 2s,\ \bar{x} + 2s\right) \to \left(493.48,\ 512.20\right)$

At least 75% of the 800-meter freestyle times lie between 493.48 and 512.20 seconds.

39.

x	f	xf	$x - \bar{x}$	$\left(x - \bar{x}\right)^2$	$\left(x - \bar{x}\right)^2 f$
0	3	0	–1.74	3.0276	9.0828
1	15	15	–0.74	0.5476	8.2140
2	24	48	0.26	0.0676	1.6224
3	8	24	1.26	1.5876	12.7008
	$n = 50$	$\sum xf = 87$			$\sum\left(x - \bar{x}\right)^2 f = 31.62$

$\bar{x} = \frac{\sum xf}{n} = \frac{87}{50} \approx 1.7$

$$s=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2 f}{n-1}}=\sqrt{\frac{31.62}{49}}\approx 0.8$$

40.

Midpoint, x	f	xf
70.5	1	70.5
92.5	12	1110.0
114.5	25	2862.5
136.5	10	1365.0
158.5	2	317.0
	$n=50$	$\sum xf=5725$

$$\bar{x}=\frac{\sum xf}{n}=\frac{5725}{50}=114.5$$

$x-\bar{x}$	$\left(x-\bar{x}\right)^2$	$\left(x-\bar{x}\right)^2 f$
–44	1936	1936
–22	484	5808
0	0	0
22	484	4840
44	1936	3872
		$\sum\left(x-\bar{x}\right)^2 f=16{,}456$

$$s=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2 f}{n-1}}=\sqrt{\frac{16{,}456}{49}}\approx 18.33$$

41.

Class	**Midpoint, x**	f	xf
0-4	2	5	10
5-9	7	12	84
10-14	12	24	288
15-19	17	17	289
20-24	22	16	352
25-29	27	11	297
30+	32	5	160
		$n=90$	$\sum xf=1480$

$$\bar{x}=\frac{\sum xf}{n}=\frac{1480}{90}\approx 16.4$$

$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2 f$
-14.44	208.5136	1042.5680
-9.44	89.1136	1069.3632
-4.44	19.7136	473.1264
0.56	0.3136	5.3312
5.56	30.9136	494.6176
10.56	111.5136	1226.6496
15.56	242.1136	1210.5680
		$\sum(x-\bar{x})^2 f = 5522.2240$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{n-1}} = \sqrt{\frac{5522.2240}{89}} \approx 7.9$$

42.

Class	Midpoint, x	f	xf
0-1999	999.5	11	10,994.5
2000-3999	2999.5	12	35,994.0
4000-5999	4999.5	20	99,990.0
6000-7999	6999.5	10	69,995.0
8000-9999	8999.5	13	116,993.5
10,000+	10,999.5	10	109,995.0
		$n = 76$	$\sum xf = 443,962$

$$\bar{x} = \frac{\Sigma xf}{n} = \frac{443,962}{76} \approx \$5,841.61$$

$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2 f$
-4842.11	23,446,029.2521	257,906,321.7731
-2842.11	8,077,589.2521	96,931,071.0252
-842.11	709,149.2521	14,182,985.0420
1157.89	1,340,709.2521	13,407,092.5210
3157.89	9,972,269.2521	129,639,500.2773
5157.89	26,603,829.2521	266,038,292.5210
		$\sum(x-\bar{x})^2 f = 778,105,263.1596$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{n-1}} = \sqrt{\frac{778,105,263.1596}{75}} \approx \$3,220.98$$

43. Dallas: $\bar{x} = \frac{\sum x}{n} = \frac{442.8}{10} = 44.28$

x	$x - \bar{x}$	$(x - \bar{x})^2$
38.7	–5.58	31.1364
39.9	–4.38	19.1844
40.5	–3.78	14.2884
41.6	–2.68	7.1824
44.3	0.02	0.0004
44.7	0.42	0.1764
45.8	1.52	2.3104
47.8	3.52	12.3904
49.5	5.22	27.2484
50.0	5.72	32.7184
		$\sum(x - \bar{x})^2 = 146.636$

$$s^2 = \frac{\sum(x - \bar{x})^2}{n-1} = \frac{146.636}{9} \approx 16.29; \quad s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\frac{146.636}{9}} \approx 4.04$$

$$CV = \frac{s}{\bar{x}} = \frac{4.04}{44.28} \cdot 100\% \approx 9.1\%$$

New York City: $\bar{x} = \frac{\sum x}{n} = \frac{509.2}{10} = 50.92$

x	$x - \bar{x}$	$(x - \bar{x})^2$
41.5	–9.42	88.7364
42.3	–8.62	74.3044
45.6	–5.32	28.3024
47.2	–3.72	13.8384
50.6	–0.32	0.1024
51.0	0.08	0.0064
55.1	4.18	17.4724
57.6	6.68	44.6224
59.0	8.08	65.2864
59.3	8.38	70.2244
		$\sum(x - \bar{x})^2 = 402.896$

$$s^2 = \frac{\sum(x - \bar{x})^2}{n-1} = \frac{402.896}{9} \approx 44.77; \quad s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\frac{402.896}{9}} \approx 6.69$$

$$CV = \frac{s}{\bar{x}} = \frac{6.69}{50.92} \cdot 100\% \approx 13.1\%$$

Salaries for entry level accountants are more variable in New York City than in Dallas.

44. Boston: $\bar{x}=\frac{\sum x}{n}=\frac{667.4}{9}\approx 74.16$

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
58.5	–15.66	245.2356
64.5	–9.66	93.3156
69.9	–4.26	18.1476
70.4	–3.76	14.1376
71.6	–2.56	6.5536
79.9	5.74	32.9476
80.1	5.94	35.2836
84.2	10.04	100.8016
88.3	14.14	199.9396
		$\sum\left(x-\bar{x}\right)^2=746.3624$

$$s^2=\frac{\Sigma\left(x-\bar{x}\right)^2}{n-1}=\frac{746.3624}{9-1}\approx 93.30\ ;\quad s=\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n-1}}=\sqrt{\frac{746.3624}{8}}\approx 9.66$$

$$CV=\frac{s}{\bar{x}}=\frac{9.66}{74.16}\cdot 100\%\approx 13.0\%$$

Chicago: $\bar{x}=\frac{\sum x}{n}=\frac{599.5}{9}\approx 66.61$

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
59.9	–6.71	45.0241
60.9	–5.71	32.6041
62.9	–3.71	13.7641
65.4	–1.21	1.4641
68.5	1.89	3.5721
69.4	2.79	7.7841
70.1	3.49	12.1801
70.9	4.29	18.4041
71.5	4.89	23.9121
		$\sum\left(x-\bar{x}\right)^2=158.7089$

$$s^2=\frac{\Sigma\left(x-\bar{x}\right)^2}{n-1}=\frac{158.7089}{9-1}\approx 19.84\ ;\quad s=\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n-1}}=\sqrt{\frac{158.7089}{8}}\approx 4.45$$

$$CV=\frac{s}{\bar{x}}=\frac{4.45}{66.61}\cdot 100\%\approx 6.7\%$$

Salaries for entry level electrical engineers are more variable in Boston than in Chicago.

45. Ages: $\mu = \frac{\sum x}{N} = \frac{326}{12} \approx 27.2$

x	$x-\mu$	$(x-\mu)^2$
24	-3.17	10.0489
29	1.83	3.3489
37	9.83	96.6289
24	-3.17	10.0489
26	-1.17	1.3689
25	-2.17	4.7089
24	-3.17	10.0489
32	4.83	23.3289
22	-5.17	26.7289
29	1.83	3.3489
23	-4.17	17.3889
31	3.83	14.6689
		$\sum(x-\mu)^2 = 221.6668$

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{221.6668}{12} \approx 18.5\,;\ \ \sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}} = \sqrt{\frac{221.6668}{12}} \approx 4.3$$

$$CV = \frac{\sigma}{\mu} = \frac{4.3}{27.2} \cdot 100\% \approx 15.8\%$$

Heights: $\mu = \frac{\sum x}{N} = \frac{896}{12} \approx 74.7$

x	$x-\mu$	$(x-\mu)^2$
72	-2.67	7.1289
76	1.33	1.7689
73	-1.67	2.7889
73	-1.67	2.7889
77	2.33	5.4289
76	1.33	1.7689
72	-2.67	7.1289
74	-0.67	0.4489
75	0.33	0.1089
75	0.33	0.1089
74	-0.67	0.4489
79	4.33	18.7489
		$\sum(x-\mu)^2 = 48.6668$

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{48.6668}{12} \approx 4.1;\ \ \sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}} = \sqrt{\frac{48.6668}{12}} \approx 2.0$$

$$CV = \frac{\sigma}{\mu} = \frac{2.0}{74.7} \cdot 100\% \approx 2.7\%$$

Ages are more variable than heights for all pitchers on the St. Louis Cardinals.

46. Male: $\bar{x} = \dfrac{\sum x}{n} = \dfrac{13{,}140}{8} = 1642.5$

x	$x - \bar{x}$	$(x - \bar{x})^2$
1520	-122.5	15,006.25
1750	107.5	11,556.25
2120	477.5	228,006.25
1380	-262.5	68,906.25
1980	337.5	113,906.25
1650	7.5	56.25
1030	-612.5	375,156.25
1710	67.5	4,556.25
		$\sum (x - \bar{x})^2 = 817{,}150$

$s^2 = \dfrac{\sum (x - \bar{x})^2}{n-1} = \dfrac{817{,}150}{8-1} \approx 116{,}375.7$; $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{817{,}150}{7}} \approx 341.7$

$CV = \dfrac{s}{\bar{x}} = \dfrac{341.7}{1642.5} \cdot 100\% \approx 20.8\%$

Female: $\bar{x} = \dfrac{\sum x}{n} = \dfrac{13{,}680}{8} = 1710$

x	$x - \bar{x}$	$(x - \bar{x})^2$
1790	80	6,400
1510	-200	40,000
1500	-210	44,100
1950	240	57,600
2210	500	250,000
1870	160	25,600
1260	-450	202,500
1590	-120	14,400
		$\sum (x - \bar{x})^2 = 640{,}600$

$s^2 = \dfrac{\sum (x - \bar{x})^2}{n-1} = \dfrac{640{,}600}{8-1} \approx 91{,}514.3$; $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{640600}{7}} = 302.5$

$CV = \dfrac{s}{\bar{x}} = \dfrac{302.5}{1710} \cdot 100\% \approx 17.7\%$

SAT scores are more variable for males than for females.

47. Team A: $\bar{x} = \frac{\sum x}{n} = \frac{2.991}{10} = 0.2991$

x	$x - \bar{x}$	$(x - \bar{x})^2$
0.235	–0.0641	0.00410881
0.256	–0.0431	0.00185761
0.272	–0.0271	0.00073441
0.295	–0.0041	0.00001681
0.297	–0.0021	0.00000441
0.297	–0.0021	0.00000441
0.310	0.0109	0.00011881
0.320	0.0209	0.00043681
0.325	0.0259	0.00067081
0.384	0.0849	0.00720801
		$\sum(x - \bar{x})^2 = 0.0151609$

$s^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{0.0151609}{9} \approx 0.0017$; $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{0.0151609}{9}} = 0.0410$

$CV = \frac{s}{\bar{x}} = \frac{.0410}{.2991} \cdot 100\% \approx 13.7\%$

Team B: $\bar{x} = \frac{\sum x}{n} = \frac{2.610}{10} = 0.261$

x	$x - \bar{x}$	$(x - \bar{x})^2$
.204	-0.057	0.003249
.223	-0.038	0.001444
.226	-0.035	0.001225
.238	-0.023	0.000529
.256	-0.005	0.000025
.260	-0.001	0.000001
.292	0.031	0.000961
.299	0.038	0.001444
.300	0.039	0.001521
.312	0.051	0.002601
		$\sum(x - \bar{x})^2 = 0.013$

$s^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{0.013}{9} \approx 0.0014$; $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{0.013}{9}} \approx 0.0380$

$CV = \frac{s}{\bar{x}} = \frac{.0380}{.2610} \cdot 100\% \approx 14.6\%$

Batting averages are slightly more variable on Team B than on Team A.

48. Ages: $\mu = \frac{\sum x}{N} = \frac{235}{9} \approx 26.1$

x	$x-\mu$	$(x-\mu)^2$
25	-1.11	1.2321
24	-2.11	4.4521
24	-2.11	4.4521
31	4.89	23.9121
25	-1.11	1.2321
28	1.89	3.5721
26	-0.11	0.0121
30	3.89	15.1321
22	-4.11	16.8921
		$\sum(x-\mu)^2 = 70.8889$

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{70.8889}{9} \approx 7.9\text{; } \sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}} = \sqrt{\frac{70.8889}{9}} \approx 2.8$$

$$CV = \frac{\sigma}{\mu} = \frac{2.8}{26.1} \cdot 100\% \approx 10.7\%$$

Heights: $\mu = \frac{\sum x}{N} = \frac{1869}{9} \approx 207.7$

x	$x-\mu$	$(x-\mu)^2$
215	7.33	53.7289
217	9.33	87.0489
190	-17.67	312.2289
225	17.33	300.3289
192	-15.67	245.5489
215	7.33	53.7289
185	-22.67	513.9289
210	2.33	5.4289
220	12.33	152.0289
		$\sum(x-\mu)^2 = 1724.0001$

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{1724.0001}{9} \approx 191.6\text{; } \sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}} = \sqrt{\frac{1724.0001}{9}} \approx 13.8$$

$$CV = \frac{\sigma}{\mu} = \frac{13.8}{207.7} \cdot 100\% \approx 6.7\%$$

Ages are more variable than heights for all wide receivers on the San Diego Chargers.

49. Ages:

x	x^2
16	256
18	324
19	361
17	289
14	196
15	225
17	289
17	289
17	289
16	256
19	361
22	484
24	576
14	196
16	256
14	196
17	289
16	256
14	196
18	324
$\sum x = 340$	$\sum x^2 = 5908$

$$s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}} = \sqrt{\frac{5908 - \frac{(340)^2}{20}}{20-1}} = \sqrt{\frac{128}{19}} \approx 2.6$$

(b) The answer is the same as from Exercise 15.

50. (a) $\bar{x} \approx 41.5$ $\quad s \approx 5.3$

(b) $\bar{x} \approx 43.6$ $\quad s \approx 5.6$

(c) $\bar{x} \approx 3.5$ $\quad s \approx 0.4$

(d) By multiplying each entry by a constant *k*, the new sample mean is $k \cdot \bar{x}$ and the new sample standard deviation is $k \cdot s$.

51. (a) $\bar{x} \approx 41.7$ $\quad s \approx 6.0$

(b) $\bar{x} \approx 42.7$ $\quad s \approx 6.0$

(c) $\bar{x} \approx 39.7$ $\quad s \approx 6.0$

(d) By adding a constant *k* to, or subtracting it from, each entry, the new sample mean will be $\bar{x} + k$, or $\bar{x} - k$, with the sample standard being unaffected.

52. (a) Ages $\bar{x} = 17$

x	$\lvert x - \bar{x} \rvert$
16	1
18	1
19	2
17	0
14	3
15	2
17	0
17	0
17	0
16	1
19	2
22	5
24	7
14	3
16	1
14	3
17	0
16	1
14	3
18	1
$\sum x = 340$	$\sum \lvert x - \bar{x} \rvert = 36$

$$\frac{\sum \lvert x - \bar{x} \rvert}{n} = \frac{36}{20} = 1.8$$

$s \approx 2.6$

The mean absolute deviation is less than the sample standard deviation.

(b) Days: $\bar{x} \approx 281.4$

x	$\lvert x - \bar{x} \rvert$
277	4.4
277	4.4
267	14.4
291	9.6
282	0.6
281	0.4
295	13.6
279	2.4
286	4.6
280	1.4
296	14.6
269	12.4
268	13.4
285	3.6
264	17.4
278	3.4
269	12.4

299	17.6
291	9.6
293	11.6
$\sum x = 5627$	$\sum \lvert x - \bar{x} \rvert = 171.8$

$$\frac{\sum \lvert x - \bar{x} \rvert}{n} = \frac{171.8}{20} \approx 8.6$$

$s \approx 10.6$

The mean absolute deviation is less than the sample standard deviation.

53. $1 - \frac{1}{k^2} = 0.99 \Rightarrow 1 - 0.99 = \frac{1}{k^2} \Rightarrow k^2 = \frac{1}{0.01} \Rightarrow k = \sqrt{\frac{1}{0.01}} = 10$

At least 99% of the data in any data set lie within 10 standard deviations of the mean.

54. (a) $P = \frac{3(\bar{x} - \text{median})}{s} = \frac{3(17 - 19)}{2.3} \approx -2.61$; The data are skewed left.

(b) $P = \frac{3(\bar{x} - \text{median})}{s} = \frac{3(32 - 25)}{5.1} \approx 4.12$; The data are skewed right.

(c) $P = \frac{3(\bar{x} - \text{median})}{s} = \frac{3(9.2 - 9.2)}{1.8} = 0$; The data are symmetric.

(d) $P = \frac{3(\bar{x} - \text{median})}{s} = \frac{3(42 - 40)}{6.0} = 1$; The data are skewed right.

2.5 MEASURES OF POSITION

2.5 Try It Yourself Solutions

1a. 26, 31, 35, 37, 43, 43, 43, 44, 45, 47, 48, 48, 49, 50, 51, 51, 51, 51, 52, 54, 54, 54, 54, 55, 55, 55, 56, 57, 57, 57, 58, 58, 58, 58, 59, 59, 59, 62, 62, 63, 64, 65, 65, 65, 66, 66, 67, 67, 72, 86

b. $Q_2 = 55$

c. $Q_1 = 49,\ Q_3 = 62$

d. About one fourth of the 50 most powerful women are 49 years old or younger; about one half are 55 years old or younger; and about three fourths of the 50 most powerful women are 62 years old or younger.

2a. Enter data

b. $Q_1 = 23.5,\ Q_2 = 30,\ Q_3 = 45$

c. About one-quarter of these universities charge tuition of $23,500 or less; about one-half charge $30,000 or less; and about three-quarters charge $45,000 or less.

3a. $Q_1 = 49,\ Q_3 = 62$

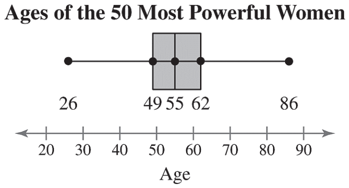
b. $\text{IQR} = Q_3 - Q_1 = 62 - 49 = 13$

c. $Q_1 - 1.5(IQR) = 49 - 1.5(13) = 29.5$; $Q_3 + 1.5(IQR) = 62 + 1.5(13) = 81.5$ The age 26 is less than $Q_1 - 1.5(IQR)$ and the age 86 is greater than $Q_3 + 1.5(IQR)$.

d. The ages of the 50 most powerful women in the middle portion of the data set vary by at most 13 years. The ages 62 and 86 are outliers.

4a. Min = 26, $Q_1 = 49$, $Q_2 = 55$, $Q_3 = 62$, Max = 86

bc.



d. About 50% of the ages are between 49 and 62 years old. About 25% of the ages are less than 49 years old. About 25% of the ages are older than 62 years old.

5a. about 62

b. About 75% of the most powerful women are 62 years old or younger.

6a. 17,18,19,20,20,23,24,26,29,29,29,30,30,34,35,36,38,39,39,43,44,44,44,45,45

b. 7 data entries are less than 26

c. $\text{Percentile of } 26 = \dfrac{\text{number of data entries less than } 26}{\text{total number of data entries}} \cdot 100 = \dfrac{7}{25} \cdot 100 = 28^{\text{th}} \text{ percentile}$

d. The tuition cost of $26,000 is greater than 28% of the other tuition costs.

7a. $\mu = 70,\ \sigma = 8$

$$x = 60:\ z = \frac{x-\mu}{\sigma} = \frac{60-70}{8} = -1.25$$

$$x = 71:\ z = \frac{x-\mu}{\sigma} = \frac{71-70}{8} = 0.125$$

$$x = 92:\ z = \frac{x-\mu}{\sigma} = \frac{92-70}{8} = 2.75$$

b. From the *z*-scores, the utility bill of $60 is 1.25 standard deviations below the mean, the bill of $71 is 0.125 standard deviation above the mean, and the bill of $92 is 2.75 standard deviations above the mean.

8a. 5 feet = 5(12) = 60 inches

b. Man: $z = \dfrac{x-\mu}{\sigma} = \dfrac{60-69.9}{3} = -3.3$; Woman: $z = \dfrac{x-\mu}{\sigma} = \dfrac{60-64.3}{2.6} \approx -1.7$

c. The *z*-score for the 5-foot-tall man is 3.3 standard deviations below the mean. This is an unusual height for a man. The *z*-score for the 5-foot-tall woman is 1.7 standard deviations below the mean. This is among the typical heights for a woman.

2.5 EXERCISE SOLUTIONS

1. The movie is shorter in length than 75% of the movies in the theater.

2. The car's fuel efficiency is higher than 90% of the other cars in its class.

3. The student scored higher than 83% of the students who took the actuarial exam.

4. The child's IQ is higher than 93% of the other children in the same age group.

5. The interquartile range of a data set can be used to identify outliers because data values that are greater than $Q_3 + 1.5(\text{IQR})$ or less than $Q_1 - 1.5(\text{IQR})$ are considered outliers.

6. Quartiles are special cases of percentiles. Q_1 is the 25th percentile, Q_2 is the 50th percentile, and Q_3 is the 75th percentile.

7. True

8. False. The second quartile is the median of an ordered data set.

9. False. An outlier is any number above $Q_3 + 1.5(\text{IQR})$ or below $Q_1 - 1.5(\text{IQR})$.

10. False. It is possible to have a *z*-score of zero when the *x*-value equals the mean.

11. (a) 51 54 56 **57** 59 60 60 **60** 60 62 63 **63** 63 65 80

↑ ↑ ↑

Q_1 Q_2 Q_3

(b) $\text{IQR} = Q_3 - Q_1 = 63 - 57 = 6$

(c) $Q_1 - 1.5(IQR) = 57 - 1.5(6) = 48$; $Q_3 + 1.5(IQR) = 63 + 1.5(6) = 72$. The date entry 80 is an outlier.

12. (a) 15 18 19 **25** 27 28 28 **34** 36 39 41 **47** 48 50 53

↑ ↑ ↑

Q_1 Q_2 Q_3

(b) $\text{IQR} = Q_3 - Q_1 = 47 - 25 = 22$

(c) $Q_1 - 1.5(IQR) = 25 - 1.5(22) = -8$; $Q_3 + 1.5(IQR) = 47 + 1.5(22) = 80$. There are no outliers.

13. (a) 19 26 28 34 **36 36** 37 38 38 **40 41** 42 43 43 **45 48** 50 52 53 56

↑ ↑ ↑

$Q_1 = 36$ $Q_2 = 40.5$ $Q_3 = 46.5$

(b) $\text{IQR} = Q_3 - Q_1 = 46.5 - 36 = 10.5$

(c) $Q_1 - 1.5(IQR) = 36 - 1.5(10.5) = 20.25$; $Q_3 + 1.5(IQR) = 46.5 + 1.5(10.5) = 62.25$. The data entry 19 is an outlier.

14. (a) 19 20 20 21 **21 21** 22 22 22 **22 23** 23 23 23 **24 24** 25 25 25 29

↑ ↑ ↑

$Q_1 = 21$ $Q_2 = 22.5$ $Q_3 = 24$

(b) $\text{IQR} = Q_3 - Q_1 = 24 - 21 = 3$

(c) $Q_1 - 1.5(IQR) = 21 - 1.5(3) = 16.5$; $Q_3 + 1.5(IQR) = 24 + 1.5(3) = 28.5$. The data entry 29 is an outlier.

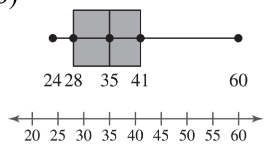
15. Min = 10, $Q_1 = 13$, $Q_2 = 15$, $Q_3 = 17$, Max = 20

16. Min = 100, $Q_1 = 130$, $Q_2 = 205$, $Q_3 = 270$, Max = 320

17. (a) $$24\ 26\ 27\ \underset{\overset{\uparrow}{Q_1}}{\mathbf{28}}\ 30\ 32\ 35\ \underset{\overset{\uparrow}{Q_2}}{\mathbf{35}}\ 36\ 39\ 39\ \underset{\overset{\uparrow}{Q_3}}{\mathbf{41}}\ 50\ 51\ 60$$

Min = 24, $Q_1 = 28$, $Q_2 = 35$, $Q_3 = 41$, Max = 60

(b)

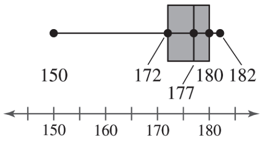


18. (a)

$$\overbrace{150\ 170\ 171\ \underset{\overset{\uparrow}{Q_1}}{}\ 173\ 174\ 176}^{\text{lower half}}\ \underset{\overset{\uparrow}{Q_2}}{}\ \overbrace{178\ 178\ 180\ \underset{\overset{\uparrow}{Q_3}}{}\ 180\ 181\ 182}^{\text{upper half}}$$

Min = 150, $Q_1 = 172$, $Q_2 = 177$, $Q_3 = 180$, Max = 182

(b)

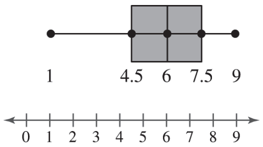


19. (a)

$$\overbrace{1\ 2\ 2\ 4\ 4\ \underset{\overset{\uparrow}{Q_1}}{}\ 5\ 5\ 5\ 6\ 6}^{\text{lower half}}\ \underset{\overset{\uparrow}{Q_2}}{}\ \overbrace{6\ 7\ 7\ 7\ 7\ \underset{\overset{\uparrow}{Q_3}}{}\ 8\ 8\ 8\ 9\ 9}^{\text{upper half}}$$

Min = 1, $Q_1 = 4.5$, $Q_2 = 6$, $Q_3 = 7.5$, Max = 9

(b)

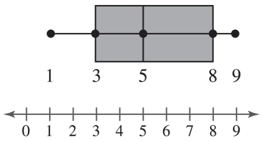


20. (a)

$$\overbrace{1\ 1\ 2\ 2\ 2\ 2\ 2\ 3\ \underset{\overset{\uparrow}{Q_1}}{3}\ 3\ 3\ 3\ 3\ 4\ 4\ 4\ 5}^{\text{lower half}}\ \underset{\overset{\uparrow}{Q_2}}{5}\ \overbrace{5\ 5\ 5\ 6\ 7\ 7\ 7\ 7\ \underset{\overset{\uparrow}{Q_3}}{8}\ 8\ 8\ 9\ 9\ 9\ 9\ 9\ 9}^{\text{upper half}}$$

Min = 1, $Q_1 = 3$, $Q_2 = 5$, $Q_3 = 8$, Max = 9

(b)



21. None. The Data are not skewed or symmetric.

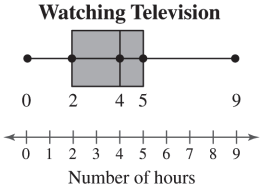
22. Skewed right. Most of the data lie to the left in the box-and-whisker plot.

23. Skewed left. Most of the data lie to the right in the box-and-whisker plot.

24. Symmetric. The data are evenly spaced to the left and to the right of the median.

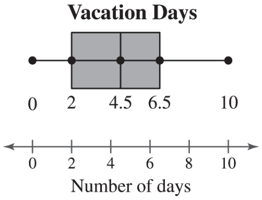
25. (a) $Q_1 = 2$, $Q_2 = 4$, $Q_3 = 5$

(b)



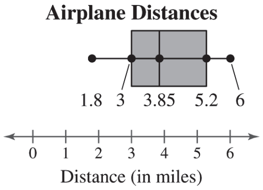
26. (a) $Q_1 = 2$, $Q_2 = 4.5$, $Q_3 = 6.5$

(b)



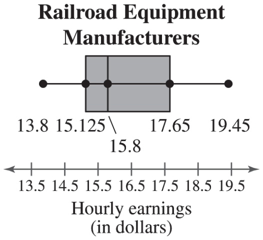
27. (a) $Q_1 = 3$, $Q_2 = 3.85$, $Q_3 = 5.2$

(b)



28. (a) $Q_1 = 15.125$, $Q_2 = 15.8$, $Q_3 = 17.65$

(b)



29. (a) 5 (b) 50% (c) 25%

30. (a) $17.65 (b) 50% (c) 50%

31. about 70 inches; About 60% of U.S. males ages 20-29 are shorter than 70 inches.

32. about 72 inches; About 80% of U.S. males ages 20-29 are shorter than 72 inches.

33. about 90th percentile; About 90% of U.S. males ages 20-29 are shorter than 73 inches.

34. about 20th percentile; About 20% of U.S. males ages 20-29 are shorter than 67 inches.

35. $\text{Percentile of } 40 = \dfrac{\text{number of data entries less than } 40}{\text{total number of data entries}} \cdot 100 = \dfrac{3}{30} \cdot 100 = 10^{\text{th}} \text{ percentile}$

36. $\text{Percentile of } 56 = \dfrac{\text{number of data entries less than } 56}{\text{total number of data entries}} \cdot 100 = \dfrac{21}{30} \cdot 100 = 70^{\text{th}} \text{ percentile}$

37. 75^{th} percentile $= Q_3 = 56$; Ages over 56 are 57,57,61,61,65,66

38. 25^{th} percentile $= Q_1 = 43$; Ages below 43 are 28,35,38,40,41,41,42

39. $\text{A} \Rightarrow z = -1.43$
$\text{B} \Rightarrow z = 0$
$\text{C} \Rightarrow z = 2.14$
The z-score 2.14 is unusual because it is so large.

40. $\text{A} \Rightarrow z = -1.54$
$\text{B} \Rightarrow z = 0.77$
$\text{C} \Rightarrow z = 1.54$
None of the z-scores are unusual.

41. (a) Bradley Wiggins: $x = 32 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{32-28.1}{3.4} \approx 1.15$
(b) An age of 32 is about 1.15 standard deviations above the mean.
(c) Not unusual.

42. (a) Jan Ullrich: $x = 24 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{24-28.1}{3.4} \approx -1.21$
(b) An age of 24 is about 1.21 standard deviations below the mean.
(c) Not unusual.

43. (a) Cadel Evans: $x = 34 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{34-28.1}{3.4} \approx 1.74$
(b) An age of 34 is about 1.74 standard deviations above the mean.
(c) Not unusual.

44. (a) Henri Cornet: $x = 20 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{20-28.1}{3.4} \approx -2.38$

(b) An age of 20 is about 2.38 standard deviations below the mean.
(c) Unusual.

45. (a) Firmin Lambot: $x = 36 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{36-28.1}{3.4} \approx 2.32$

(b) An age of 36 is about 2.32 standard deviations above the mean.
(c) Unusual.

46. (a) Philippe Thys: $x = 23 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{23-28.1}{3.4} = -1.5$

(b) An age of 23 is 1.5 standard deviations below the mean.
(c) Not unusual.

47. (a) $x = 34{,}000 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{34{,}000-35{,}000}{2{,}250} \approx -0.44$

$x = 37{,}000 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{37{,}000-35{,}000}{2{,}250} \approx 0.89$

$x = 30{,}000 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{30{,}000-35{,}000}{2{,}250} \approx -2.22$

The tire with a life span of 30,000 miles has an unusually short life span.

(b) $x = 30{,}500 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{30{,}500-35{,}000}{2{,}250} = -2 \Rightarrow$ about 2.5th percentile

$x = 37{,}250 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{37{,}250-35{,}000}{2{,}250} = 1 \Rightarrow$ about 84th percentile

$x = 35{,}000 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{35{,}000-35{,}000}{2{,}250} = 0 \Rightarrow$ about 50th percentile

48. (a) $x = 34 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{34-33}{4} = 0.25$

$x = 30 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{30-33}{4} = -0.75$

$x = 42 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{42-33}{4} = 2.25$

The fruit fly with a life span of 42 days has an unusually long life span.

(b) $x = 29 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{29-33}{4} = -1 \Rightarrow$ about 16th percentile

$x = 41 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{41-33}{4} = 2 \Rightarrow$ about 97.5th percentile

$x = 25 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{25-33}{4} = -2 \Rightarrow$ about 2.5th percentile

49. Robert Duvall: $x = 53 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{53-44}{8.8} \approx 1.02$

Jack Nicholson: $x = 46 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{46-50}{14.1} \approx -0.28$

The age of Robert Duvall was about a standard deviation above the mean age of Best Actor winners, and the age of Jack Nicholson was less than 1 standard deviation below the mean age of Best Supporting Actor winners. Neither actor's age is unusual.

50. Jamie Foxx: $x = 37 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{37-44}{8.8} \approx -0.80$

Morgan Freeman: $x = 67 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{67-50}{14.1} \approx 1.21$

The age of Jamie Foxx was less than 1standard deviation below the mean age of Best Actor winners, and the age of Morgan Freeman was between 1and 2 standard deviations above the mean age of Best Supporting Actor winners. Neither actor's age is unusual.

51. John Wayne: $x = 62 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{62-44}{8.8} \approx 2.05$

Gig Young: $x = 56 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{56-50}{14.1} \approx 0.43$

The age of John Wayne was more than 2 standard deviations above the mean age of Best Actor winners, which is unusual. The age of Gig Young was less than 1 standard deviation above the mean age of Best Supporting Actor winners, which is not unusual.

52. Henry Fonda: $x = 76 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{76-44}{8.8} \approx 3.64$

John Gielgud: $x = 77 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{77-50}{14.1} \approx 1.91$

The age of Henry Fonda was more than 3 standard deviations above the mean age of Best Actor winners, which is very unusual. The age of John Gielgud was less than 2 standard deviations above the mean age of Best Supporting Actor winners, which is not unusual.

53.

$$\begin{array}{ccccccccccc} 1 & 2 & 3 & 3 & 5 & & 5 & 7 & 7 & 8 & 10 \\ & & \uparrow & & & \uparrow & & & \uparrow & & \\ & & Q_1 & & & Q_2 & & & Q_3 & & \end{array}$$

$$\text{Midquartile} = \frac{Q_1 + Q_3}{2} = \frac{3+7}{2} = 5$$

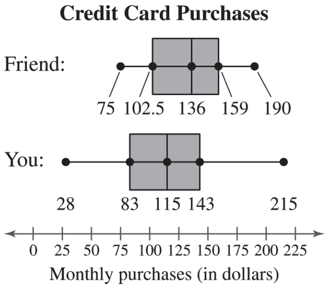
54.

$$\begin{array}{ccccccccccccccc} 22 & 23 & 24 & & 32 & 33 & 34 & & 36 & 38 & 39 & & 40 & 41 & 47 \\ & & & \uparrow & & & & \uparrow & & & & \uparrow & & & \\ & & & Q_1 & & & & Q_2 & & & & Q_3 & & & \end{array}$$

$$\text{Midquartile} = \frac{Q_1 + Q_3}{2} = \frac{28+39.5}{2} = 33.75$$

55. (a) The distribution of Concert 1 is symmetric. The distribution of Concert 2 is skewed right. Concert 1 has less variation.
(b) Concert 2 is more likely to have outliers because it has more variation.
(c) Concert 1, because 68% of the data should be between ± 16.3 of the mean.
(d) No, you do not know the number of songs played at either concert or the actual lengths of the songs.

56.



Your distribution is symmetric and your friend's distribution slightly skewed to the right.

57. (a)

$$\overbrace{2\ \ 7\ \ 8\ \ \underset{\underset{Q_1}{\uparrow}}{9}\ \ 9\ \ 10\ \ 10}^{\text{lower half}}\ \ \underset{\underset{Q_2}{\uparrow}}{11}\ \ \overbrace{11\ \ 12\ \ 12\ \ \underset{\underset{Q_3}{\uparrow}}{13}\ \ 15\ \ 16\ \ 24}^{\text{upper half}}$$

$Q_1 = 9$, $Q_2 = 11$, $Q_3 = 13$

$\text{IQR} = Q_3 - Q_1 = 13 - 9 = 4$

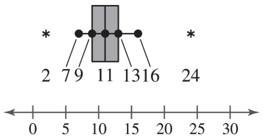
$1.5 \times \text{IQR} = 6$

$Q_1 - (1.5 \times \text{IQR}) = 9 - 6 = 3$

$Q_3 + (1.5 \times \text{IQR}) = 13 + 6 = 19$

Any values less than 6 or greater than 19 are outliers. So, 2 and 24 are outliers.

(b)



58. (a)

$$\overbrace{62\ \ 72\ \ 72\ \underset{Q_1}{\uparrow}\ 74\ \ 75\ \ 75}^{\text{lower half}}\ \underset{Q_2}{\uparrow}\ \overbrace{75\ \ 76\ \ 78\ \underset{Q_3}{\uparrow}\ 80\ \ 80\ \ 95}^{\text{upper half}}$$

$Q_1 = 73$, $Q_2 = 75$, $Q_3 = 79$

$\text{IQR} = Q_3 - Q_1 = 79 - 73 = 6$

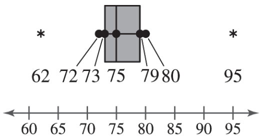
$1.5 \times \text{IQR} = 9$

$Q_1 - (1.5 \times \text{IQR}) = 73 - 9 = 64$

$Q_3 + (1.5 \times \text{IQR}) = 79 + 9 = 88$

Any values less than 64 or greater than 88 are outliers. So, 62 and 95 are outliers.

(b)



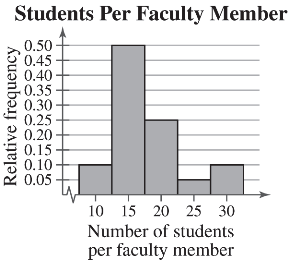
59. Answers will vary.

CHAPTER 2 REVIEW EXERCISE SOLUTIONS

1. $$\text{Class width} = \frac{\text{Max} - \text{Min}}{\text{Number of classes}} = \frac{30-8}{5} = 4.4 \Rightarrow 5$$

Class	Midpoint, *x*	Boundaries	Frequency, *f*	Relative frequency	Cumulative frequency
8-12	10	7.5-12.5	2	0.10	2
13-17	15	12.5-17.5	10	0.50	12
18-22	20	17.5-22.5	5	0.25	17
23-27	25	22.5-27.5	1	0.05	18
28-32	30	27.5-32.5	2	0.10	20
			$\sum f = 20$	$\sum \frac{f}{n} = 1$	

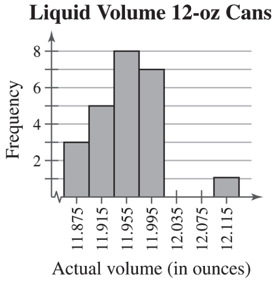
2.



Class with greatest relative frequency: 13-17
Class with least relative frequency: 23-27

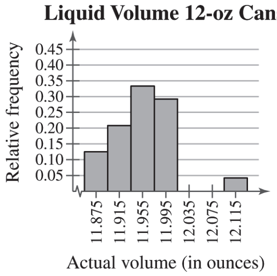
3. $$\text{Class width} = \frac{\text{Max} - \text{Min}}{\text{Number of classes}} = \frac{12.10-11.86}{7} \approx 0.03 \Rightarrow 0.04$$

Class	Midpoint	Frequency, *f*	Relative frequency
11.86-11.89	11.875	3	0.125
11.90-11.93	11.915	5	0.208
11.94-11.97	11.955	8	0.333
11.98-12.01	11.995	7	0.292
12.02-12.05	12.035	0	0.000
12.06-12.09	12.075	0	0.000
12.10-12.13	12.115	1	0.042
		$\sum f = 24$	$\sum \frac{f}{n} = 1$



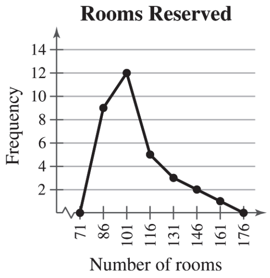
4. Class width $= \dfrac{\text{Max} - \text{Min}}{\text{Number of classes}} = \dfrac{12.10 - 11.86}{7} \approx 0.03 \Rightarrow 0.04$

Class	Midpoint	Frequency, f	Relative frequency
11.86-11.89	11.875	3	0.125
11.90-11.93	11.915	5	0.208
11.94-11.97	11.955	8	0.333
11.98-12.01	11.995	7	0.292
12.02-12.05	12.035	0	0.000
12.06-12.09	12.075	0	0.000
12.10-12.13	12.115	1	0.042
		$\sum f = 24$	$\sum \frac{f}{n} = 1$

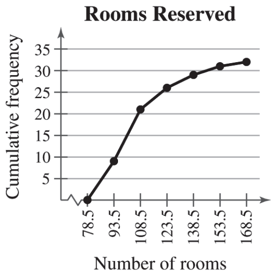


5. Class width $= \dfrac{\text{Max} - \text{Min}}{\text{Number of classes}} = \dfrac{166 - 79}{6} = 14.5 \Rightarrow 15$

Class	Midpoint	Frequency, f	Cumulative frequency
79-93	86	9	9
94-108	101	12	21
109-123	116	5	26
124-138	131	3	29
139-153	146	2	31
154-168	161	1	32
		$\sum f = 32$	



6.



7. 1 | 0 0 Key: $1|0 = 10$

2 | 0 0 2 5 5

3 | 0 3 4 5 5 8

4 | 1 2 4 4 7 8

5 | 2 3 3 7 9

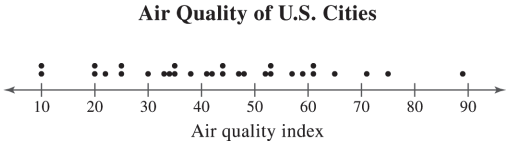
6 | 1 1 5

7 | 1 5

8 | 9

Sample answer: Most cities have an air quality index from 20 to 59.

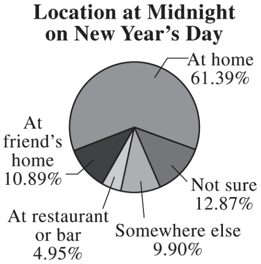
8.



Sample answer: Most cities have an air quality index from 20 to 59.

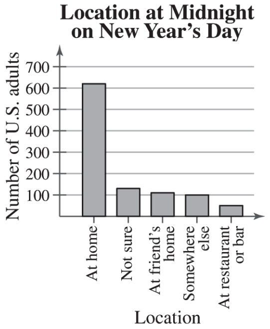
9.

Location	Frequency	Relative frequency	Degrees
At home	620	0.6139	221°
At friend's home	110	0.1089	39°
At restaurant or bar	50	0.0495	18°
Somewhere else	100	0.0990	36°
Not sure	130	0.1287	46°
	$\sum f = 1010$	$\sum \frac{f}{n} = 1$	



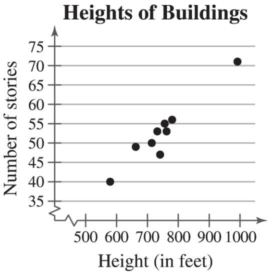
Sample answer: Over half of the people surveyed will be at home on New Year's Day at midnight.

10.



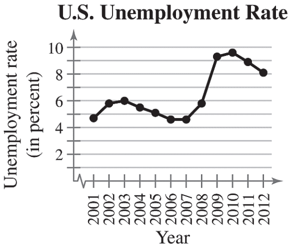
Sample answer: Most of the people surveyed will be at home on New Year's Day at midnight.

11.



Sample answer: The number of stories appears to increase with height.

12.



Sample answer: The unemployment rate varied by a couple of percentage points from 2001 to 2008 and then increased dramatically from 2008 to 2009.

13. $\bar{x} = \dfrac{\sum x}{n} = \dfrac{288}{10} = 28.8$

24.5 24.5 25.5 28.0 **28.5** **29.5** 30.0 31.0 32.5 34.0

$\uparrow$

median = 29

Mode = 24.5 (occurs 2 times)

The mode does not represent the center of the data because 24.5 is the smallest number in the data set.

14. $\bar{x}$ is not possible

median is not possible

mode = "Vote for"

The mean and median cannot be found because the data are at the nominal level of measurement.

15.

Source	**Score, *x***	**Weight, *w***	***x* · *w***
Test 1	78	0.15	11.7
Test 2	72	0.15	10.8
Test 3	86	0.15	12.9
Test 4	91	0.15	13.65
Test 5	87	0.15	13.05
Test 6	80	0.25	20
		$\sum w = 1$	$\sum (x \cdot w) = 82.1$

$\bar{x} = \dfrac{\sum (x \cdot w)}{\sum w} = \dfrac{82.1}{1} = 82.1$

16.

Source	**Score, *x***	**Weight, *w***	***x* · *w***
Test 1	96	0.2	19.2
Test 2	85	0.2	17
Test 3	91	0.2	18.2
Test 4	86	0.4	34.4
		$\sum w = 1$	$\sum (x \cdot w) = 88.8$

$\bar{x} = \dfrac{\sum (x \cdot w)}{\sum w} = \dfrac{88.8}{1} = 88.8$

17.

Midpoint, *x*	**Frequency, *f***	***x* · *f***
10	2	20
15	10	150
20	5	100
25	1	25
30	2	60
	$n = 20$	$\sum (x \cdot f) = 355$

$\bar{x} = \dfrac{\sum (x \cdot f)}{n} = \dfrac{355}{20} \approx 17.8$

18.

x	f	$x \cdot f$
0	13	0
1	9	9
2	19	38
3	8	24
4	5	20
5	2	10
6	4	24
	$n = 60$	$\sum(x \cdot f) = 125$

$$\bar{x} = \frac{\sum(x \cdot f)}{n} = \frac{125}{60} \approx 2.1$$

19. Skewed right **20.** Skewed right **21.** Skewed left **22.** Skewed right

23. Median, because the mean is to the left of the median in a skewed left distribution.

24. Mean, because the mean is to the right of the median in a skewed right distribution.

25. Range = Max – Min = 15 – 1 = 14

$$\mu = \frac{\sum x}{N} = \frac{96}{14} \approx 6.9$$

x	$x - \mu$	$(x - \mu)^2$
4	–2.9	8.41
2	–4.9	24.01
9	2.1	4.41
12	5.1	26.01
15	8.1	65.61
3	–3.9	15.21
6	–0.9	0.81
8	1.1	1.21
1	–5.9	34.81
4	–2.9	8.41
14	7.1	50.41
12	5.1	26.01
3	–3.9	15.21
3	–3.9	15.21
$\sum x = 96$	$\sum(x - \mu) \approx 0$	$\sum(x - \mu)^2 = 295.74$

$$\sigma^2 = \frac{\sum(x - \mu)^2}{N} = \frac{295.74}{14} \approx 21.1$$

$$\sigma = \sqrt{\frac{\sum(x - \mu)^2}{N}} = \sqrt{\frac{295.74}{14}} \approx 4.6$$

26. Range = Max – Min = 79 – 52 = 27

$$\mu = \frac{\sum x}{N} = \frac{599}{9} \approx 66.6$$

x	$x-\mu$	$(x-\mu)^2$
58	-8.56	73.2736
52	-14.56	211.9936
76	9.44	89.1136
76	9.44	89.1136
64	-2.56	6.5536
79	12.44	154.7536
74	7.44	55.3536
62	-4.56	20.7936
58	-8.56	73.2736
$\sum x = 599$	$\sum(x-\mu) \approx 0$	$\sum(x-\mu)^2 = 774.2224$

$$\sigma^2 = \frac{\Sigma(x-\mu)^2}{N} = \frac{774.2224}{9} \approx 86$$

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}} = \sqrt{\frac{774.2224}{9}} \approx 9.3$$

27. Range = Max – Min = \$6444 – \$4218 = \$2226

$$\bar{x} = \frac{\sum x}{n} = \frac{80,501}{15} \approx \$5366.73$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
5306	-60.73	3,688.1329
6444	1077.27	1,160,510.6529
5304	-62.73	3,935.0529
4218	-1148.73	1,319,580.6129
5159	-207.73	43,151.7529
6342	975.27	951,151.5729
5713	346.27	119,902.9129
4859	-507.73	257,789.7529
5365	-1.73	2.9929
5078	-288.73	83,365.0129
4334	-1032.73	1,066,531.2529
5262	-104.73	10,968.3729
5905	538.27	289,734.5929
6099	732.27	536,219.3529
5113	-253.73	64,378.9129
$\sum x = 80,501$	$\sum(x-\bar{x}) \approx 0$	$\sum(x-\bar{x})^2 = 5,910,910.9335$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{5,910,910.9335}{14} \approx 422,207.92$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{5,910,910.9335}{14}} \approx \$649.78$$

28. Range = Max – Min = 58,298 – 48,250 = \$10,048

$$\bar{x} = \frac{\sum x}{n} = \frac{416,659}{8} \approx \$52,082.38$$

x	$x - \bar{x}$	$\left(x - \bar{x}\right)^2$
49,632	–2450.4	6,004,460.16
54,619	2536.6	6,434,339.56
58,298	6215.6	38,633,683.36
48,250	–3832.4	14,687,289.76
51,842	–240.4	57,792.16
50,875	–1207.4	1,457,814.76
53,219	1136.6	1,291,859.56
49,924	–2158.4	4,658,690.56
$\sum x = 416,659$	$\sum\left(x - \bar{x}\right) \approx 0$	$\sum\left(x - \bar{x}\right)^2 = 73,225,929.88$

$$s^2 = \frac{\sum\left(x - \bar{x}\right)^2}{n-1} = \frac{73,225,929.88}{7} \approx 10,460,847.13$$

$$s = \sqrt{\frac{\sum\left(x - \bar{x}\right)^2}{n-1}} = \sqrt{\frac{73,225,929.88}{7}} \approx \$3234.32$$

29. 99.7% of the distribution lies within 3 standard deviations of the mean.

$$\bar{x} - 3s = 70 - (3)(14.50) = 26.50$$

$$\bar{x} + 3s = 70 + (3)(14.50) = 113.50$$

99.7% of the distribution lies between \$26.50 and \$113.50.

30. $(60.00,\ 85.00) \to (72.50 - 1(12.50),\ 72.50 + 1(12.50)) \to (\bar{x} - s, \bar{x} + s)$

68% of the cable rates lie between \$60.00 and \$85.00.

31. $(\bar{x} - 2s,\ \bar{x} + 2s) \to (20,\ 52)$ are 2 standard deviations from the mean.

$$1 - \frac{1}{k^2} = 1 - \frac{1}{(2)^2} = 1 - \frac{1}{4} = 0.75$$

At least (40)(0.75) = 30 customers have a mean sale between \$20 and \$52.

32. $(\bar{x} - 2s,\ \bar{x} + 2s) \to (3,\ 11)$ are 2 standard deviations from the mean.

$$1 - \frac{1}{k^2} = 1 - \frac{1}{(2)^2} = 1 - \frac{1}{4} = 0.75$$

At least (20)(0.75) = 15 shuttle flights lasted between 3 days and 11 days.

33. $\bar{x} = \dfrac{\sum xf}{n} = \dfrac{99}{40} \approx 2.5$

x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
0	1	0	–2.5	6.25	6.25
1	8	8	–1.5	2.25	18.00
2	13	26	–0.5	0.25	3.25
3	10	30	0.5	0.25	2.50
4	5	20	1.5	2.25	11.25
5	3	15	2.5	6.25	18.75
	$n = 40$	$\sum xf = 99$			$\sum(x - \bar{x})^2 f = 60$

$$s = \sqrt{\frac{\sum(x-\bar{x})^2 f}{n-1}} = \sqrt{\frac{60}{39}} \approx 1.2$$

34. $\bar{x} = \dfrac{\sum xf}{n} = \dfrac{61}{25} \approx 2.4$

x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
0	4	0	–2.4	5.76	23.04
1	5	5	–1.4	1.96	9.80
2	2	4	–0.4	0.16	0.32
3	9	27	0.6	0.36	3.24
4	1	4	1.6	2.56	2.56
5	3	15	2.6	6.76	20.28
6	1	6	3.6	12.96	12.96
	$n = 25$	$\sum xf = 61$			$\sum(x - \bar{x})^2 f = 72.2$

$$s = \sqrt{\frac{\sum(x-\bar{x})^2 f}{n-1}} = \sqrt{\frac{72.2}{24}} \approx 1.7$$

35. Freshmen: $\bar{x} = \dfrac{\sum x}{n} = \dfrac{23.1}{9} \approx 2.567$

x	$x - \bar{x}$	$(x - \bar{x})^2$
2.8	0.233	0.0543
1.8	-0.767	0.5833
4.0	1.433	2.0535
3.8	1.233	1.5203
2.4	-0.167	0.0279
2.0	-0.567	0.3215
0.9	-1.667	2.7789
3.6	1.033	1.0671
1.8	-0.767	0.5883
		$\sum(x - \bar{x})^2 = 9.0000$

$$s=\sqrt{\frac{\sum\left(x-\overline{x}\right)^2}{n-1}}=\sqrt{\frac{9.0000}{8}}\approx 1.061$$

$$CV=\frac{s}{\overline{x}}=\frac{1.061}{2.567}\cdot 100\%\approx 41.3\%$$

Seniors: $\overline{x}=\frac{\sum x}{n}=\frac{26.6}{9}\approx 2.956$

x	$x-\overline{x}$	$\left(x-\overline{x}\right)^2$
2.3	-0.656	0.4303
3.3	0.344	0.1183
1.8	-1.156	1.3363
4.0	1.044	1.0899
3.1	0.144	0.0207
2.7	-0.256	0.0655
3.9	0.944	0.8911
2.6	-0.356	0.1267
2.9	-0.056	0.0031
		$\sum\left(x-\overline{x}\right)^2=4.0822$

$$s=\sqrt{\frac{\sum\left(x-\overline{x}\right)^2}{n-1}}=\sqrt{\frac{4.0822}{8}}\approx 0.714$$

$$CV=\frac{s}{\overline{x}}=\frac{0.714}{2.956}\cdot 100\%\approx 24.2\%$$

Grade point averages are more variable for freshmen than seniors.

36. Ages: $\mu=\frac{\sum x}{N}=\frac{396}{8}=49.5$

x	$x-\mu$	$\left(x-\mu\right)^2$
66	16.5	272.25
54	4.5	20.25
37	-12.5	156.25
61	11.5	132.25
36	-13.5	182.25
59	9.5	90.25
50	0.5	0.25
33	-16.5	272.25
		$\sum\left(x-\mu\right)^2=1126$

$$\sigma=\sqrt{\frac{\sum\left(x-\overline{x}\right)^2}{N}}=\sqrt{\frac{1126}{8}}\approx 11.9$$

$$CV=\frac{\sigma}{\mu}=\frac{11.9}{49.5}\cdot 100\%\approx 24.0\%$$

Years of Experience: $\mu = \frac{\sum x}{N} = \frac{185}{8} \approx 23.1$

x	$x-\mu$	$(x-\mu)^2$
37	13.9	193.21
20	-3.1	9.61
23	-0.1	0.01
32	8.9	79.21
14	-9.1	82.81
29	5.9	34.81
22	-1.1	1.21
8	-15.1	228.01
		$\sum(x-\mu)^2 = 628.88$

$$\sigma = \sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{N}} = \sqrt{\frac{628.88}{8}} \approx 8.866$$

$$CV = \frac{\sigma}{\mu} = \frac{8.866}{23.13} \cdot 100\% \approx 38.3\%$$

Years of experience are more variable than ages for all lawyers at a firm.

37.

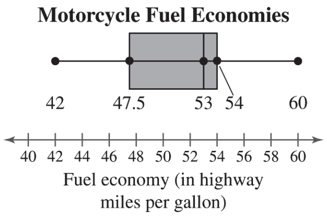
$$\overbrace{42\ \ 42\ \ 42\ \ 47\ \ 47\ \ 48\ \ 48\ \ 50\ \ 53\ \ 53}^{\text{lower half}}\ \ 53\ \ \overbrace{53\ \ 54\ \ 54\ \ 54\ \ 54\ \ 54\ \ 57\ \ 57\ \ 60\ \ 60}^{\text{upper half}}$$

$$\underset{Q_1}{\uparrow} \qquad\qquad \underset{Q_2}{\uparrow} \qquad\qquad \underset{Q_3}{\uparrow}$$

Min = 42, $Q_1 = 47.5$, $Q_2 = 53$, $Q_3 = 54$, Max = 60

38. IQR $= Q_3 - Q_1 = 54 - 47.5 = 6.5$ highway miles per gallon

39.



40. 17

41.

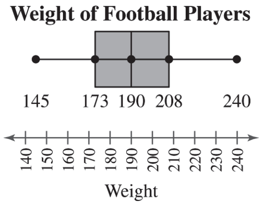
$$24.5\ \ 24.5\ \ \underset{\underset{Q_1 = 25.5}{\uparrow}}{\mathbf{25.5}}\ \ 28.0\ \ \mathbf{28.5}\ \underset{\underset{Q_2 = 29}{\uparrow}}{}\ \mathbf{29.5}\ \ 30.0\ \ \underset{\underset{Q_3 = 31}{\uparrow}}{\mathbf{31.0}}\ \ 32.5\ \ 34.0$$

IQR $= Q_3 - Q_1 = 31 - 25.5 = 5.5$ inches

42.

$$\overbrace{145\ \ 156\ \ 167\ \ 172\ \ \underset{\overset{\uparrow}{Q_1}}{173}\ \ 184\ \ 185\ \ 185\ \ 190}^{\text{lower half}}\ \ \underset{\overset{\uparrow}{Q_2}}{190}\ \ \overbrace{192\ \ 195\ \ 197\ \ 205\ \ \underset{\overset{\uparrow}{Q_3}}{208}\ \ 212\ \ 227\ \ 228\ \ 240}^{\text{upper half}}$$

Min = 145, $Q_1 = 173$, $Q_2 = 190$, $Q_3 = 208$, Max = 240



The distribution of weights is symmetric.

43. The 65th percentile means that 65% had a test grade of 75 or less. So, 35% scored higher than 75.

44. If there are 106 stations with a larger daily audience, then this station has the 665 – 106 = 559$^{\text{th}}$ largest audience. The percentile of $559 = \frac{559}{665} \cdot 100 = 84^{\text{th}}$ percentile

45. (a) $z = \dfrac{16{,}500 - 11{,}830}{2370} \approx 1.97$

(b) A towing capacity of 16,500 pounds is about 1.97 standard deviations above the mean.
(c) Not unusual

46. (a) $z = \dfrac{5500 - 11{,}830}{2370} \approx -2.67$

(b) A towing capacity of 5,500 pounds is about 2.67 standard deviations below the mean.
(c) Unusual

47. (a) $z = \dfrac{18{,}000 - 11{,}830}{2370} = 2.60$

(b) A towing capacity of 18,000 pounds is about 2.60 standard deviations above the mean.
(c) Unusual

48. (a) $z = \dfrac{11{,}300 - 11{,}830}{2370} = -0.22$

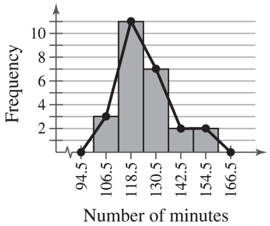
(b) A towing capacity of 11,300 pounds is about 0.22 standard deviations below the mean.
(c) Not unusual

CHAPTER 2 QUIZ SOLUTIONS

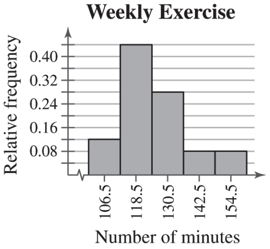
1. (a) Class width $= \dfrac{\text{Max} - \text{Min}}{\text{Number of classes}} = \dfrac{157-101}{5} = 11.2 \Rightarrow 12$

Class	**Midpoint**	**Class boundaries**	**Frequency, f**	**Relative frequency**	**Cumulative frequency**
101-112	106.5	100.5-112.5	3	0.12	3
113-124	118.5	112.5-124.5	11	0.44	14
125-136	130.5	124.5-136.5	7	0.28	21
137-148	142.5	136.5-148.5	2	0.08	23
149-160	154.5	148.5-160.5	2	0.08	25
			$\sum f = 25$	$\sum \frac{f}{n} = 1$	

(b) Frequency histogram and polygon



(c) Relative frequency histogram



(d) Skewed right

(e)

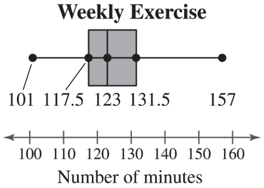
10	1 8
11	1 4 6 7 8 9 9
12	0 0 3 3 4 7 7 8
13	1 1 2 5 9 9
14	
15	0 7

Key: $10|8 = 108$

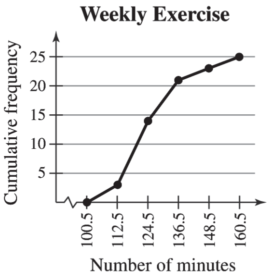
(f)

$$\overbrace{101\ \ 108\ \ 111\ \ 114\ \ 116\ \ 117\ \underset{\underset{Q_1}{\uparrow}}{\ }\ 118\ \ 119\ \ 119\ \ 120\ \ 120\ \ 123}^{\text{lower half}}\ \ \underset{\underset{Q_2}{\uparrow}}{123}\ \ \overbrace{124\ \ 127\ \ 127\ \ 128\ \ 131\ \ 131\ \underset{\underset{Q_3}{\uparrow}}{\ }\ 132\ \ 135\ \ 139\ \ 139\ \ 150\ \ 157}^{\text{upper half}}$$

Min = 101, $Q_1 = 117.5$, $Q_2 = 123$, $Q_3 = 131.5$, Max = 157



(g)



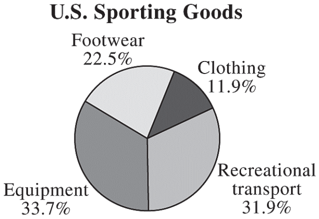
2. $\bar{x} = \dfrac{\sum xf}{n} = \dfrac{3130.5}{25} \approx 125.2$

Midpoint, x	**Frequency, f**	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
106.5	3	319.5	–18.7	349.69	1049.07
118.5	11	1303.5	–6.7	44.89	493.79
130.5	7	913.5	5.3	28.09	196.63
142.5	2	285.0	17.3	299.29	598.58
154.5	2	309.0	29.3	858.49	1716.98
	$n = 25$	$\sum xf = 3130.5$			$\sum (x - \bar{x})^2 f = 4055.05$

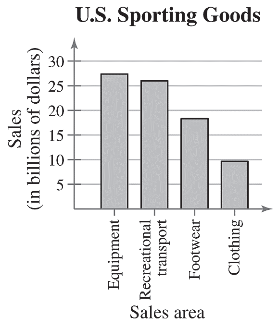
$$s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n - 1}} = \sqrt{\frac{4055.05}{24}} \approx 13.0$$

3. (a)

Category	**Frequency**	**Relative frequency**	**Degrees**
Clothing	9.7	0.1187	43°
Footwear	18.4	0.2252	81°
Equipment	27.5	0.3366	121°
Rec. Transport	26.1	0.3195	115°
	$n = 81.7$	$\sum \frac{f}{n} = 1$	



(b)



4. (a) $\bar{x} = \frac{\sum x}{n} = \frac{7413}{8} \approx 926.6$

619 621 842 **949** **970** 1083 1135 1194

$$\uparrow$$

$$\text{median} = \frac{949 + 970}{2} = 959.5$$

mode = none

The mean best describes a typical salary because there are no outliers.

(b) Range = Max – Min = 1194 – 619 = 575

x	$x - \bar{x}$	$(x - \bar{x})^2$
949	22.4	501.76
621	–305.6	93,391.36
1194	267.4	71,502.76
970	43.4	1883.56
1083	156.4	24,460.96
842	–84.6	7157.16
619	–307.6	94,617.76
1135	208.4	43,430.56
		$\sum(x - \bar{x})^2 = 336,945.88$

$$s^2 = \frac{\sum(x - \bar{x})^2}{n-1} = \frac{336,945.88}{7} \approx 48,135.1$$

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\frac{336,945.88}{7}} \approx 219.4$$

(c) $CV = \frac{s}{\bar{x}} = \frac{219.4}{926.6} \cdot 100\% \approx 23.7\%$

5. $\bar{x} - 2s = 155,000 - 2 \cdot 15,000 = \$125,000$

$\bar{x} + 2s = 155,000 + 2 \cdot 15,000 = \$185,000$

95% of the new home prices fall between \$125,000 and \$185,000.

6. (a) $x = 200,000:\ z = \frac{x - \bar{x}}{s} = \frac{200,000 - 155,000}{15,000} = 3.0 \Rightarrow$ unusual price

(b) $x = 55{,}000: \quad z = \dfrac{x - \overline{x}}{s} = \dfrac{55{,}000 - 155{,}000}{15{,}000} \approx -6.67 \Rightarrow$ very unusual price

(c) $x = 175{,}000: \quad z = \dfrac{x - \overline{x}}{s} = \dfrac{175{,}000 - 155{,}000}{15{,}000} \approx 1.33 \Rightarrow$ not unusual price

(d) $x = 122{,}000: \quad z = \dfrac{x - \overline{x}}{s} = \dfrac{122{,}000 - 155{,}000}{15{,}000} = -2.2 \Rightarrow$ unusual price

7. (a)

55 61 64 66 68 69 69 **72** 73 74 75 76 79 81 **81** **83** 85 86 88 88 89 90 **93** 93 94 94 94 95 97 98

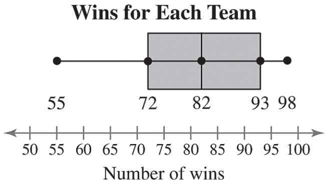
↑ ↑ ↑

$Q_1 = 72$ $Q_2 = 82$ $Q_3 = 93$

Min = 55, $Q_1 = 72$, $Q_2 = 82$, $Q_3 = 93$, Max = 98

(b) $\text{IQR} = Q_3 - Q_1 = 93 - 72 = 21$

(c)

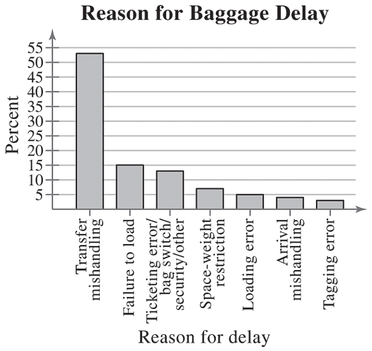


CUMULATIVE REVIEW FOR CHAPTERS 1 AND 2

1. Systematic sampling is used because every fortieth toothbrush from each assembly line is tested. It is possible for bias to enter into the sample if, for some reason, an assembly line makes a consistent error.

2. Simple random sampling is used because each telephone number has an equal chance of being dialed, and all samples of 1200 phone numbers have an equal chance of being selected. The sample may be biased because telephone sampling only samples those individuals who have telephones, who are available, and who are willing to respond.

3.



4. \$3,213,479 is a parameter because it is describing the average salary of all Major League Baseball players.

5. 10% is a statistic because it is describing a proportion within a sample of 1000 likely voters.

6. (a) $\bar{x} = 83{,}500,\ s = \$1500$

$(80{,}500\ 86{,}500) = 83{,}500 \pm 2(1500) \Rightarrow$ 2 standard deviations away form the mean.

Approximately 95% of the electrical engineers will have salaries between \$80,500 and \$86,500.

(b) 40(0.95) = 38

(c) $x = \$90{,}500:\ z = \dfrac{x - \bar{x}}{s} = \dfrac{90{,}500 - 83{,}500}{1500} \approx 4.67$

$x = \$79{,}750:\ z = \dfrac{x - \bar{x}}{s} = \dfrac{79{,}750 - 83{,}500}{1500} = -2.5$

$x = \$82{,}600:\ z = \dfrac{x - \bar{x}}{s} = \dfrac{82{,}600 - 83{,}500}{1500} = -0.6$

The salaries of \$90,500 and \$79,750 are unusual.

7. Population: Collection of opinions of all adults in the United States
Sample: Collection of opinions of the 1009 U.S. adults surveyed

8. Population: Prescription refilling persistency of all prescription drug patients
Sample: Prescription refilling persistency of the 61,522 prescription drug patients studied

9. Experiment. The study applies a treatment (stroke prevention devise) to the subjects.

10. Observational study. The study does not attempt to influence the responses of the subjects.

11. Quantitative: The data are at the ratio level.

12. Qualitative: The data are at the nominal level.

13. (a)

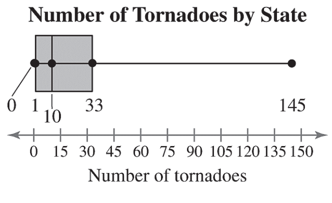
Q_1 ↘ Q_2 ↘

0 0 0 0 0 0 0 0 0 1 1 1 **1** 1 1 2 2 3 3 4 6 7 8 8 **10** **10** 15 16 17 17
18 19 20 25 26 29 32 **33** 37 39 39 40 41 48 53 65 75 87 114 145

Q_3 ↗

Min = 0, $Q_1 = 1$, $Q_2 = 10$, $Q_3 = 33$, Max = 145

(b)



(c) The distribution of the number of tornadoes is skewed right.

14.

Source	Score, x	Weight, w	$x \cdot w$
Test 1	85	0.15	12.75
Test 2	92	0.15	13.80
Test 3	84	0.15	12.60
Test 4	89	0.15	13.35
Test 5	91	0.40	36.40
		$\sum w = 1$	$\sum (x \cdot w) = 88.9$

$$\bar{x} = \frac{\sum (x \cdot w)}{\sum w} = \frac{88.9}{1} = 88.9$$

15. (a) $\bar{x} = \dfrac{49.4}{9} \approx 5.49$

3.4 3.9 4.2 4.6 5.4 6.5 6.8 7.1 7.5

median = 5.4

mode = none

Both the mean and median accurately describe a typical American alligator tail length. (Answers will vary.)

(b) Range – Max – Min – 7.5 – 3.4 = 4.1

x	$x - \bar{x}$	$(x - \bar{x})^2$
3.4	–2.09	4.3681
3.9	–1.59	2.5281
4.2	–1.29	1.6641
4.6	–0.89	0.7921
5.4	–0.09	0.0081
6.5	1.01	1.0201
6.8	1.31	1.7161
7.1	1.61	2.5921
7.5	2.01	4.0401
		$\sum (x - \bar{x})^2 = 18.7289$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1} = \frac{18.7289}{8} \approx 2.34$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} = \sqrt{\frac{18.7289}{8}} \approx 1.53$$

16. (a) An inference drawn from the sample is that the number of deaths due to heart disease for women will continue to decrease.

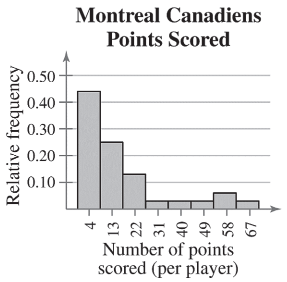
(b) This inference may incorrectly imply that women will have less of a chance of dying of heart disease in the future. The study was only conducted over the past 5 years and deaths may not decrease in the next year.

17. Class width $= \dfrac{\text{Max} - \text{Min}}{\text{Number of classes}} = \dfrac{65-0}{8} = 8.125 \Rightarrow 9$

Class limits	Midpoint	Class boundaries	Frequency	Relative frequency	Cumulative frequency
0-8	4	–0.5-8.5	14	0.47	14
9-17	13	8.5-17.5	8	0.27	22
18-26	22	17.5-26.5	4	0.13	26
27-35	31	26.5-35.5	1	0.03	27
36-44	40	35.5-44.5	1	0.03	28
45-53	49	44.5-53.5	1	0.03	29
54-62	58	53.5-62.5	2	0.07	31
63-71	67	62.5-71.5	1	0.03	32
			$\sum f = 30$	$\sum \frac{f}{n} \approx 1$	

18. The distribution is skewed right.

19.



Class with greatest frequency: 0-8
Classes with least frequency: 27-35, 36-44, 45-53, and 63-71

CHAPTER 2 TEST SOLUTIONS

1. (a) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{234}{12} = 19.5$

$$6\ 8\ 14\ 15\ 18\ \underbrace{19\ 19}\ 22\ 22\ 28\ 29\ 24$$

median = 19
mode = 19, 22
The mean and median both represent the center of the data well.

(b) Range = Max – Min = 34 – 6 = 28

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
29	9.5	90.25
15	-4.5	20.25
14	-5.5	30.25
22	2.5	6.25
22	2.5	6.25
8	-11.5	132.25
19	-0.5	0.25
6	-13.5	182.25
28	8.5	72.25
18	-1.5	2.25
19	-0.5	0.25
34	14.5	210.25
	$\sum\left(x-\bar{x}\right)=0$	$\sum\left(x-\bar{x}\right)^2=753$

$$s^2=\frac{\Sigma\left(x-\bar{x}\right)^2}{n-1}=\frac{753}{11}\approx 68.5$$

$$s=\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n-1}}=\sqrt{\frac{753}{11}}\approx 8.3$$

(c) $CV=\frac{s}{\bar{x}}=\frac{8.27}{19.5}\cdot 100\%\approx 42.4\%$

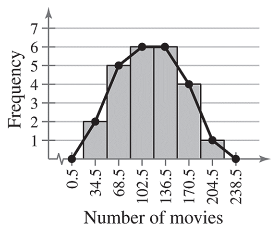
(d) **Points scored** Key: $1|4=14$

0	6
1	4 5 8 9 9
2	2 2 8 9
3	4

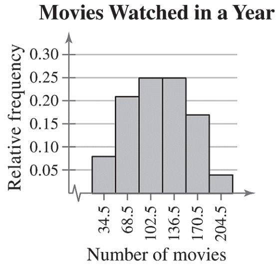
2. (a) Class width = $\frac{\text{Range}}{\text{Number of classes}}=\frac{221-18}{6}\approx 33.8\Rightarrow 34$

Class	**Midpoint**	**Class boundaries**	**Frequency, *f***	**Relative frequency**	**Cumulative frequency**
18-51	34.5	17.5-51.5	2	0.08	2
52-85	68.5	51.5-85.5	5	0.21	7
86-119	102.5	85.5-119.5	6	0.25	13
120-153	136.5	119.5-153.5	6	0.25	19
154-187	170.5	153.5-187.5	4	0.17	23
188-221	204.5	187.5-221.5	1	0.04	24
			$\sum f=24$	$\sum\frac{f}{n}=1$	

(b)

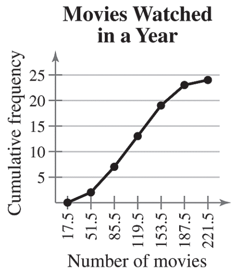


(c)



(d) Symmetric

(e)



3. $\bar{x} = \frac{\sum xf}{n} = \frac{2732}{24} \approx 113.8$

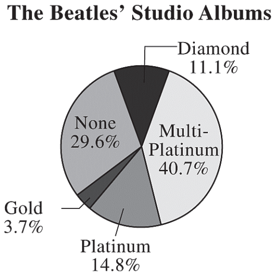
Midpoint, x	**Frequency, f**	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
34.5	2	69.0	-79.33	6293.2489	12,586.4978
68.5	5	342.5	-45.33	2054.8089	10,274.0445
102.5	6	615.0	-11.33	128.3689	770.2134
136.5	6	819.0	22.67	513.9289	3,083.5734
170.5	4	682.0	56.67	3211.4889	12,845.9556
204.5	1	204.5	90.67	8221.0489	8,221.0489
	$n = 24$	$\sum xf = 2732$			$\sum (x - \bar{x})^2 f = 47,781.3336$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n - 1}} = \sqrt{\frac{47,781.3336}{23}} \approx 45.6$$

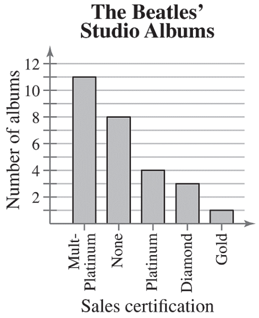
4. 149 is the 19th observation when the data are ordered. The percentile for 149

$$= \frac{\text{number of data entries less than 149}}{\text{total number of data entries}} \cdot 100 = \frac{18}{24} \cdot 100 = 75^{\text{th}} \text{ percentile}$$

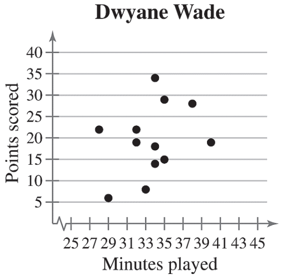
5. (a)



(b)



6.



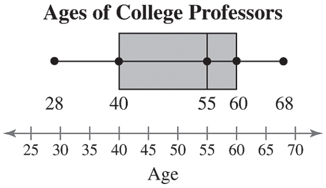
Sample answer: It appears that there is no relation between points scored and minutes played.

7. (a) $28\ 30\ 37\ \underbrace{40}\ 42\ 46\ 51\ \underbrace{55}\ 56\ 58\ 59\ \underbrace{60}\ 62\ 65\ 68$

$Q_1 = 40 \qquad Q_2 = 55 \qquad Q_3 = 60$

Min = 28, $Q_1 = 40$, $Q_2 = 55$, $Q_3 = 60$, Max = 68

(b)



(c) About 75%

8. (a) $(4.1,\ 5.5) \to (4.8 - 1(0.7),\ 4.8 + 1(0.7)) \to (\bar{x} - s,\ \bar{x} + s)$

About 68% of the iguanas are between 4.1 and 5.5 feet long. Thus, about .68(125) = 85 iguanas are between 4.1 and 5.5 feet long.

(b) $z = \dfrac{x - \bar{x}}{s} = \dfrac{3.1 - 4.8}{0.7} = -2.43$; unusual