1. You are in a nicely heated cabin in the winter. Deciding that it's too warm, you open a small window. Let T be the temperature in the room, t minutes after the window was opened, x feet from the window. Is T an increasing or decreasing function of x?
   A) Increasing B) Decreasing C) Neither
   Ans: A difficulty: easy section: 12.1

2. The following table gives the number $f(x, y)$ of grape vines, in thousands, of age x in year y.

Year \ Age of Vine	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1980	3	3	3	3	3	2	2	0	0	2	2	2	1	0	0	0	0
1981	4	3	3	3	3	3	2	2	0	0	2	2	2	1	0	0	0
1982	4	4	3	3	3	1	1	1	1	0	0	1	1	1	0	0	0
1983	0	0	0	1	1	3	1	1	1	1	0	0	1	1	1	0	0
1984	7	0	0	0	1	1	3	1	1	1	1	0	0	1	1	1	0
1985	4	7	0	0	0	1	1	3	1	1	1	1	0	0	1	1	1
1986	4	4	7	0	0	0	1	1	3	1	1	1	1	0	0	1	1
1987	7	4	4	7	0	0	0	1	1	3	1	1	1	1	0	0	1
1988	12	7	4	4	7	0	0	0	1	1	3	1	1	1	1	0	0
1989	12	12	7	4	4	7	0	0	0	1	1	3	1	1	1	1	0
1990	9	12	12	7	4	4	7	0	0	0	1	1	3	1	1	1	1
1991	8	9	12	12	7	4	4	7	0	0	0	1	1	3	1	1	1
1992	8	8	9	12	12	7	4	4	7	0	0	0	1	1	3	1	1

   In one year a fungal disease killed most of the older grapevines, and in the following year a long freeze killed most of the young vines. Which are these years?
   Ans: 1982 and 1983
   difficulty: easy section: 12.1

3. The following table gives the number $f(x, y)$ of grape vines, in thousands, of age x in year y.

Age of Vine

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1980	3	3	3	3	3	2	2	0	0	2	2	2	1	0	0	0	0
1981	4	3	3	3	3	3	2	2	0	0	2	2	2	1	0	0	0
1982	4	4	3	3	3	1	1	1	1	0	0	1	1	1	0	0	0
1983	0	0	0	1	1	3	1	1	1	1	0	0	1	1	1	0	0
1984	7	0	0	0	1	1	3	1	1	1	1	0	0	1	1	1	0
1985	4	7	0	0	0	1	1	3	1	1	1	1	0	0	1	1	1
1986	4	4	7	0	0	0	1	1	3	1	1	1	1	0	0	1	1
1987	7	4	4	7	0	0	0	1	1	3	1	1	1	1	0	0	1
1988	12	7	4	4	7	0	0	0	1	1	3	1	1	1	1	0	0
1989	12	12	7	4	4	7	0	0	0	1	1	3	1	1	1	1	0
1990	9	12	12	7	4	4	7	0	0	0	1	1	3	1	1	1	1
1991	8	9	12	12	7	4	4	7	0	0	0	1	1	3	1	1	1
1992	8	8	9	12	12	7	4	4	7	0	0	0	1	1	3	1	1

In 1986 a successful advertising campaign led to a dramatic increase in demand for premium wines. The growers followed by adding many more plants. Suppose a vine (the plant) produces the first harvestable grapes at age five, and is removed after sixteen years. How many (thousand) grape vines that bear fruit were there in the year 1986 and how many will be there in the year 1992 (assuming that no current vines die before 1992)?

Enter your answers separated by a semi-colon.
Ans: 11,000; 29,000
difficulty: medium section: 12.1

4. You are at (4, 2, 4) facing the *yz*-plane. You walk 3 units, turn right and walk for another 2 units. What are your coordinates now? Are you above or below the *xy*-plane?
Ans: My coordinates are (1, 4, 4) and I am above the *xy*-plane.
difficulty: easy section: 12.1

5. (a) Find an equation of the largest sphere that can fit inside the cubical space enclosed by the planes $x = 1$, $x = 5$, $y = 2$, $y = 6$, $z = 2$ and $z = 6$.
(b) If we replace the plane $z = 6$ in part (a) with $z = 7$, what will be the new equation of the largest sphere?
Ans: (a) $(x-3)^2+(y-4)^2+(z-4)^2=4$
(b) $(x-3)^2+(y-4)^2+(z-c)^2=4$, $4 \le c \le 5$
difficulty: medium section: 12.1

6. Consider the sphere
$$(x+1)^2+(y-0)^2+(z-1)^2=4$$
(a) What are the center and radius of this sphere?
(b) Find an equation of the circle (if any) where the sphere intersects the plane $x = -2$.
Ans: (a) Center (–1, 0, 1), Radius 2.

(b) $(y-0)^2+(z-1)^2=3$.
difficulty: medium section: 12.1

7. The points $A = (4, 1, 2)$, $B = (3, -2, 3)$, and $C = (-2, 3, -4)$ are the vertices of a triangle in space.
Which of the vertices is closest to the *yz*-plane?
A) *C* B) *A* C) *B*
Ans: A difficulty: easy section: 12.1

8. The points $A = (1, 1, 1)$, $B = (2, 4, 2)$, and $C = (3, 2, 2)$ are the vertices of a triangle in space.
Which of the vertices is closest to the origin?
A) *A* B) *B* C) *C*
Ans: A difficulty: easy section: 12.1

9. The points $A = (-4, 5, -3)$, $B = (-1, -3, -4)$, and $C = (-2, 4, -4)$ are the vertices of a triangle in space.
What is the length of the longest side of the triangle?
Ans: $\sqrt{74}$
difficulty: easy section: 12.1

10. A certain piece of electronic surveying equipment is designed to operate in temperatures ranging from 0° C to 30° C. Its performance index, $p(t, h)$, measured on a scale from 0 to 1, depends on both the temperature t and the humidity h of its surrounding environment. Values of the function $p = f(t, h)$ are given in the following table. (The higher the value of p, the better the performance.)

		Humidity, h				
		0	25	50	75	100
temperature, t°C	0	0.38	0.46	0.43	0.28	0
	10	0.65	0.79	0.73	0.47	0.01
	20	0.81	0.99	0.91	0.59	0.02
	30	0.71	0.87	0.81	0.52	0.01

What is the value of $p(0, 25)$?
Ans: 0.46
difficulty: easy   section: 12.1

11. A certain piece of electronic surveying equipment is designed to operate in temperatures ranging from 0° C to 30° C. Its performance index, $p(t, h)$, measured on a scale from 0 to 1, depends on both the temperature t and the humidity h of its surrounding environment. Values of the function $p = f(t, h)$ are given in the following table. (The higher the value of p, the better the performance.)

		Humidity, h				
		0	25	50	75	100
temperature, t°C	0	0.38	0.46	0.43	0.28	0
	10	0.65	0.79	0.73	0.47	0.01
	20	0.81	0.99	0.91	0.59	0.02
	30	0.71	0.87	0.81	0.52	0.01

Describe the function $p(10, h)$ and explain its meaning.
Ans: The value of $p(10, h)$ will first increase (as h increases from 0 to 25) then decrease (as h increases from 25 to 100). This means that when the temperature is fixed at 10° C, the equipment works best in low humidity, with optimal performance around 25% humidity. The performance will degrade severely as the humidity rises.
difficulty: easy   section: 12.1

12. Yummy Potato Chip Company has manufacturing plants in N.Y. and N.J. The cost of manufacturing depends on the quantities (in thousand of bags), q_1 and q_2, produced in the N.Y. and N.J. factories respectively. Suppose the cost function is given by

$$C(q_1, q_2) = 2q_1^2 + q_1 q_2 + q_2^2 + 420$$

(a) Find $C(10, 25)$

(b) By comparing the terms $2q_1^2$ and q_2^2 in the above expression, the manager concluded that it is more expensive to produce in the N.Y. factory. Will shifting all the production to the N.J. factory minimize the production cost?

Ans: (a) 1495

(b) No, the move will not minimize the production cost. To produce 100,000 bags, it is cheaper to have N.Y. produce 25,000 bags and N.J. produce 75,000 bags, rather than to have N.J. produce all 100,000 bags. The manager failed to notice from the formula that as the production in a factory increases, the cost will rise quadratically.

difficulty: easy section: 12.1

13. Your monthly payment, $C(s, t)$, on a car loan depends on the amount, s, of the loan (in thousands of dollars), and the time, t, required to pay it back (in months). What is the meaning of $C(7, 48) = 250$?

A) If you borrow $7,000 from the bank for 48 months (4 year loan), your monthly car loan payment is $250.
B) If you borrow $4,000 from the bank for 48 months (7 year loan), your monthly car loan payment is $250.
C) If you borrow $250 from the bank for 48 months (4 year loan), your monthly car loan payment is $7.
D) If you borrow $7 from the bank for 48 months (4 year loan), your monthly car loan payment is $250.

Ans: A difficulty: easy section: 12.1

14. Your monthly payment, $C(s, t)$, on a car loan depends on the amount, s, of the loan (in thousands of dollars), and the time, t, required to pay it back (in months). Is C an increasing or decreasing function of t?

A) Decreasing
B) Increasing

Ans: A difficulty: easy section: 12.1

15. Find a possible formula for a function $f(x, y)$ with the given values.

			y	
		1	2	3
	1	1	4	7
x	2	–1	2	5
	3	–3	0	3

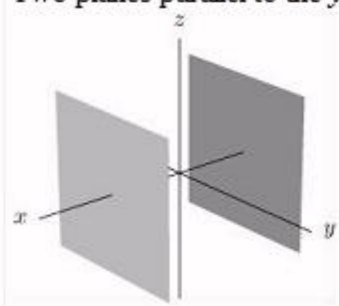
Ans: $-2x + 3y$
difficulty: hard section: 12.1

16. Describe in words, write equations, and give a sketch for the following set of points.

Consider the set of points (x, y, z) whose distance from the yz-plane is 3.

Ans:

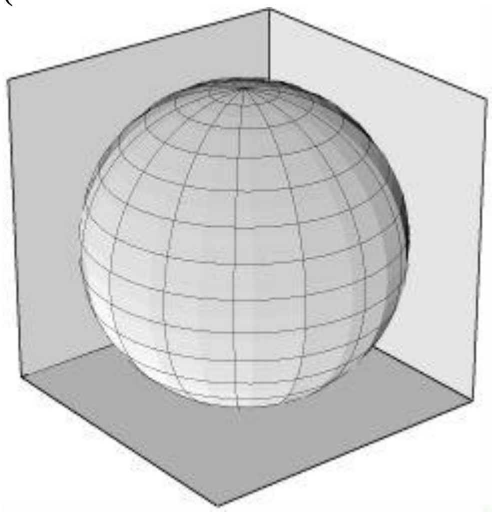
Two planes parallel to the yz-plane at $x = 3$ and $x = -3$. Its equation is $x^2 = 9$.



difficulty: easy section: 12.2

17. Describe in words the intersection of the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 7 - 6(x^2 + y^2)$.
Ans: A circle (of radius 1) in the plane $z = 1$.
difficulty: medium section: 12.2

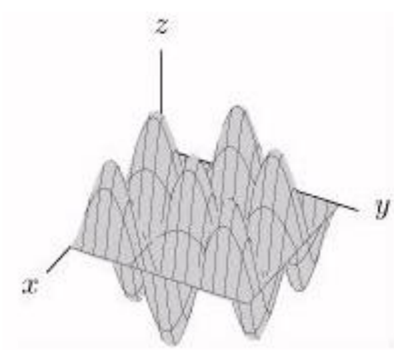
18. A spherical ball of radius four units is in a corner touching both walls and floor. What is the radius of the largest spherical ball that can be fit into the corner behind the given ball? (Hint: The smaller ball will not touch the corner point where the walls meet the floor.)



Ans: $r = \dfrac{4\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)}$

difficulty: medium section: 12.2

19. Match the graph with the function.



A)

$z = -\sin x \sin y$

B)

$z = 3e^{-\frac{x}{3}} \sin y$

C)

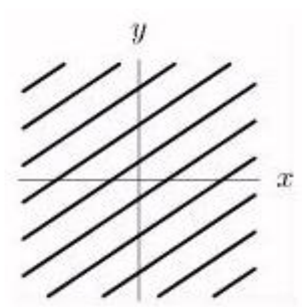
$z = \frac{1}{2}x^2 + \sin^2 y$

D)

$z = \sin y$

Ans: A difficulty: easy section: 12.2

20. Match the graph with the function.



A)

$f(x, y) = 6 - 2x + 3y$

B)

$f(x, y) = 6 - 2x - 3y$

C)

$f(x, y) = \sqrt{1 - x^2 - y^2}$

D)

$f(x, y) = \sqrt{x^2 + y^2}$

Ans: A difficulty: medium section: 12.2

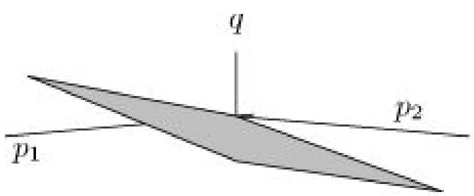
21. What is the slope of the contour lines of the function $f(x, y) = -3 + 9x + 10y$?

Ans: $-\dfrac{9}{10}$

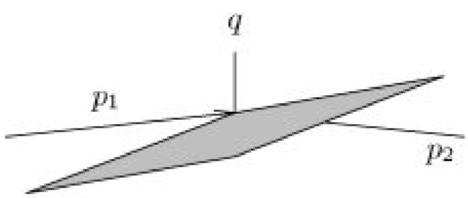
difficulty: easy section: 12.2

22. A soft drink company is interested in seeing how the demand for its products is affected by price. The company believes that the quantity, q, of soft drinks sold depends on p_1, the average price of the company's soft drinks, and p_2, the average price of competing soft drinks. Which of the graphs below is most likely to represent q as a function of p_1 and p_2?

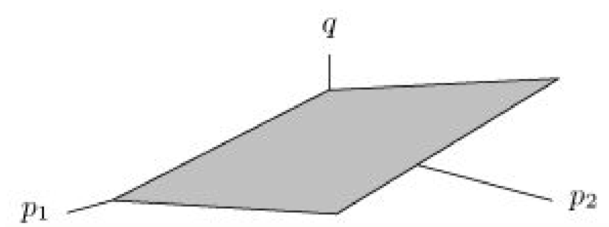
A)



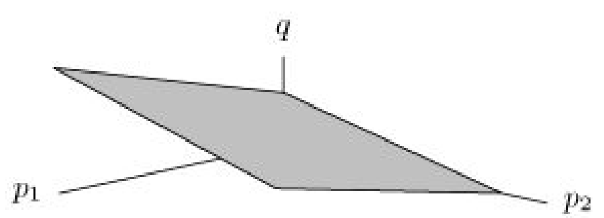
B)



C)



D)



Ans: C difficulty: medium section: 12.2

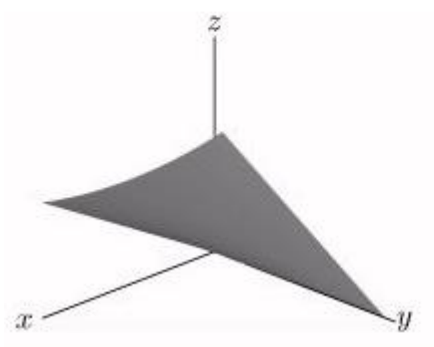
23. For what values of the constant *k* is the intersection between the set of points $y = x$ and the graph of $f(x, y) = 4x^2 - ky^2$ a straight line?
Ans: 4
difficulty: easy section: 12.2

24. Match the following function with the graphs below.

The function $z = f(x, y)$ giving happiness as a function of health *y* and money *x* according to the statement of a fortune cookie: 'Whoever said money cannot buy happiness does not know where to shop.'

A)

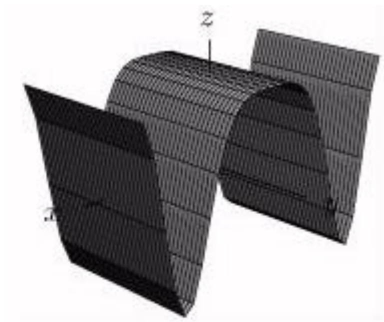
B)



C)

Ans: C difficulty: easy section: 12.2

25. Match the function with the graph below.



A)

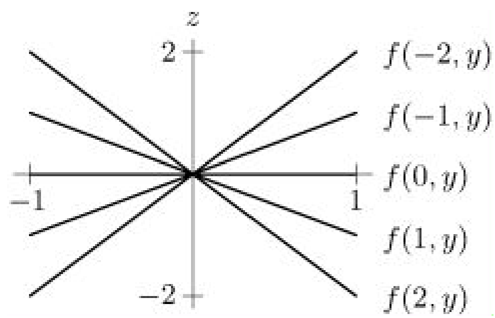
$z = \sin x^2$

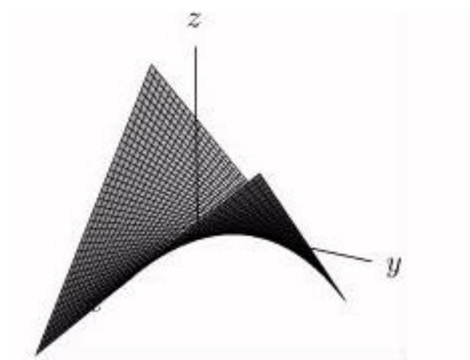
B)

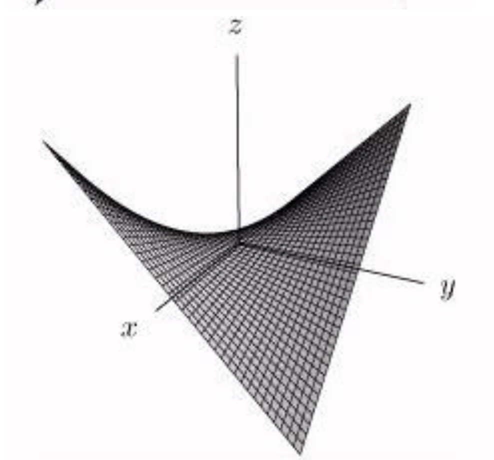
$z = \cos x^2$

Ans: B difficulty: easy section: 12.2

26. The following figure contains the graphs of the cross sections $z = f(a, y)$ for a = -2, -1, 0, 1, 2. Which of the graphs of $z = f(x, y)$ in A and B best fits this information?

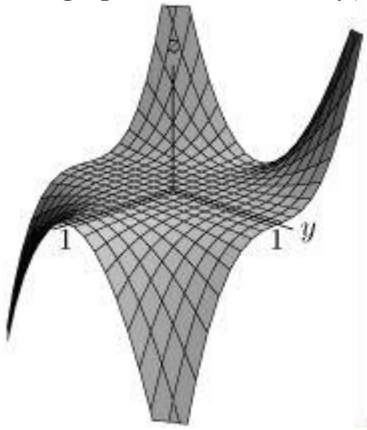


A) 

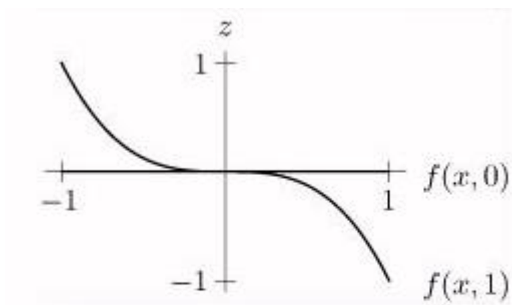
B) 

Ans: B difficulty: easy section: 12.2

27. The graph of the function $f(x, y)$ is shown below.



Draw graph of cross-sections with y fixed at $y = 0$, and $y = 1$.
Ans:

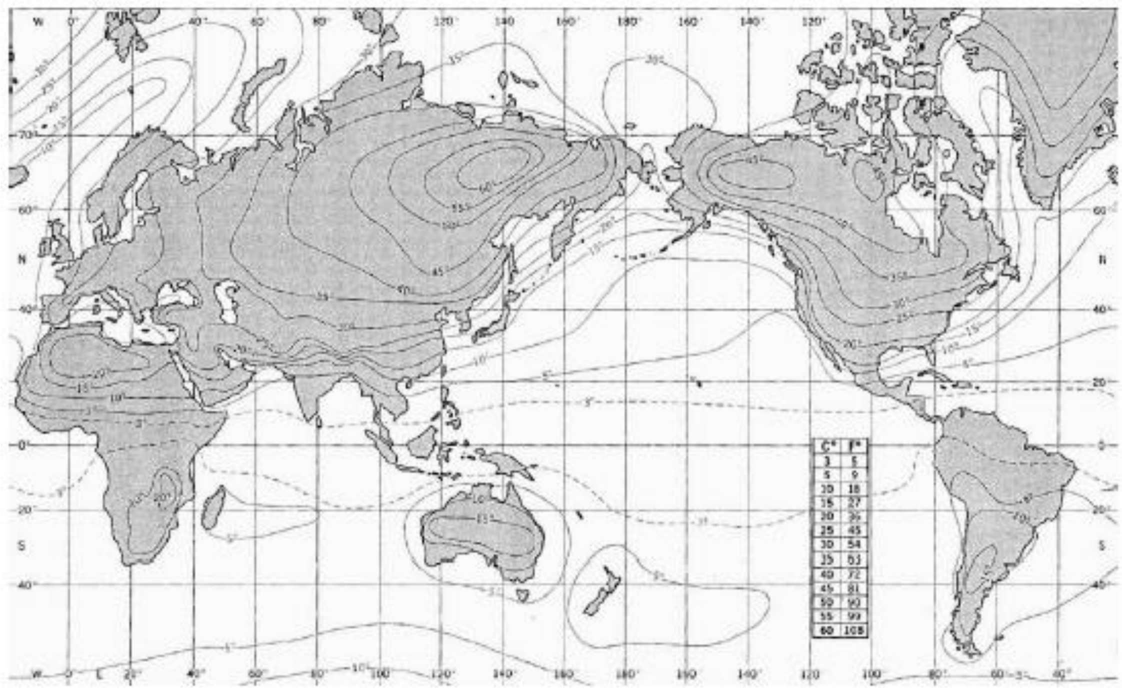


difficulty: easy section: 12.2

28. Two contours of the function $f(x, y)$ corresponding to different values of f cannot ever cross.
Ans: True difficulty: easy section: 12.3

29. The contours of the function $f(x, y) = 8x + 4y$ are all parallel lines with slope 2.
A) False B) True
Ans: A difficulty: easy section: 12.3

30. The contour diagram below shows the level curves of the difference between July and January mean temperatures in ° *F*.

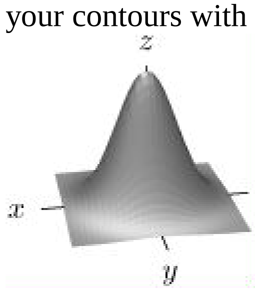


Does this graph support or contradict the claim that the largest annual temperature variations are found on the coasts of continents?

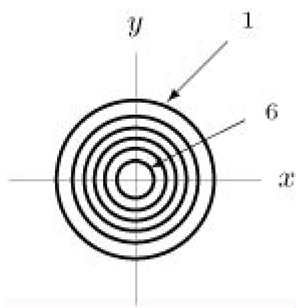
Ans: This graph supports the claim that the largest annual temperature variations are found on the coasts of continents, as level curves are very close together near the coasts of continents.

difficulty: easy section: 12.3

31. Draw a possible contour diagram for the function whose graph is shown below. Label your contours with reasonable *z*-values.



Ans:

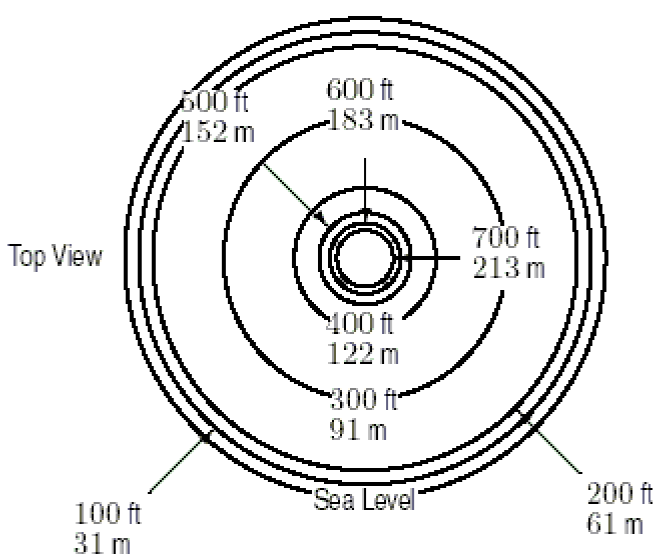


difficulty: easy section: 12.3

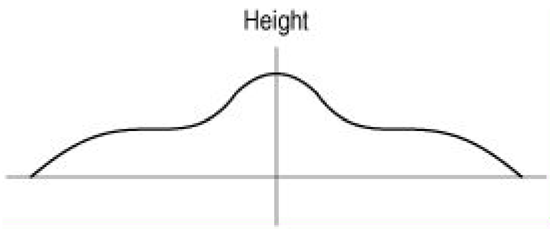
32. Consider the function $z = f(x, y) = -3y - 2x^2$. Suppose you are standing on the surface at the point where $x = 2$, $y = -1$. What is your altitude?
Ans: –5
difficulty: easy section: 12.3

33. Consider the function $z = f(x, y) = 3y - 4x^3$. Suppose you are standing on the surface at the point where $x = 3$, $y = 1$. If you start to move on the surface parallel to the y-axis in the direction of increasing y, does your height increase or decrease?
Ans: Increase
difficulty: easy section: 12.3

34. The diagram below shows the contour map for a circular island. Sketch the vertical cross-section of the island through the center. Your sketch should show concavity clearly.



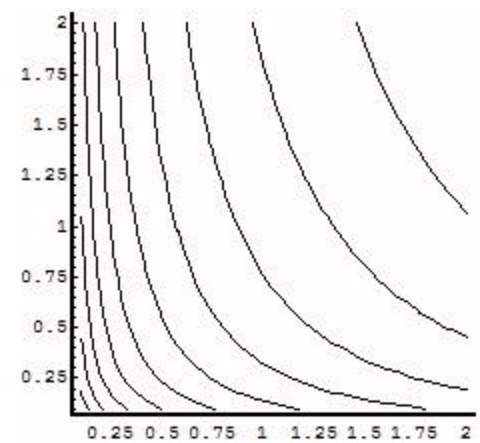
Ans:



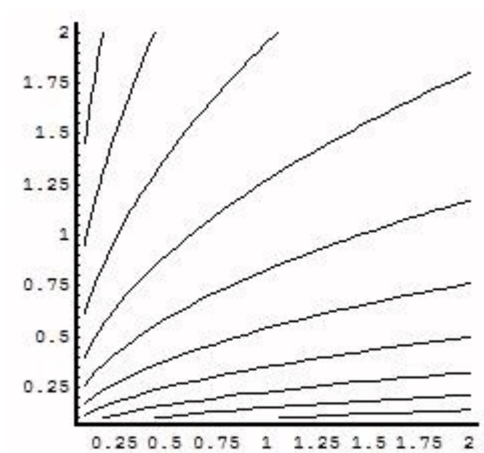
difficulty: easy section: 12.3

35. Draw the level curves for $z = 2\ln(x) - \ln(y)$.

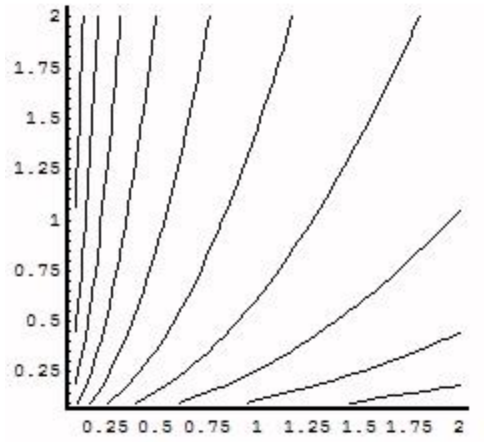
A)



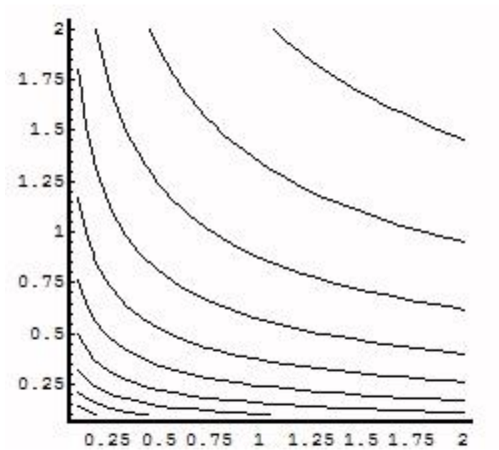
B)



C)



D)

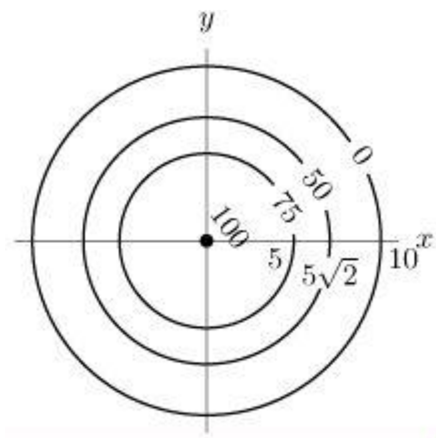


Ans: C difficulty: easy section: 12.3

36. Suppose that the temperature T of any point (x, y) is given by $T(x, y) = 100 - x^2 - y^2$.

Sketch isothermal curves (i.e. contours) for $T = 100$, $T = 75$, $T = 50$ and $T = 0$. Be sure to label each contour.

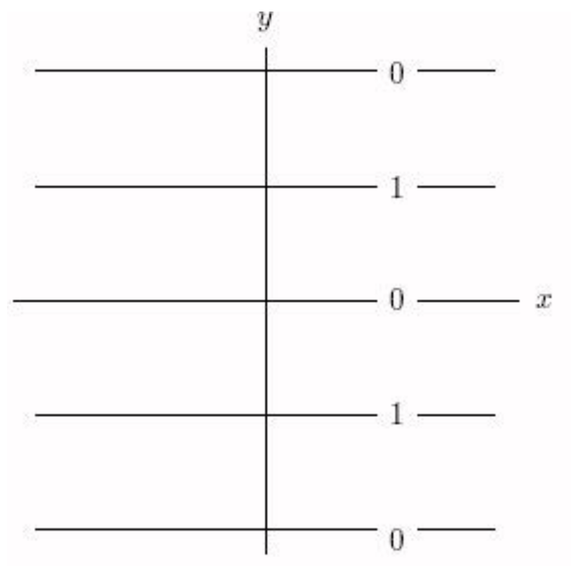
What does the graph of $T(x, y)$ look like if it is sliced by the plane $x = 4$?
Ans:



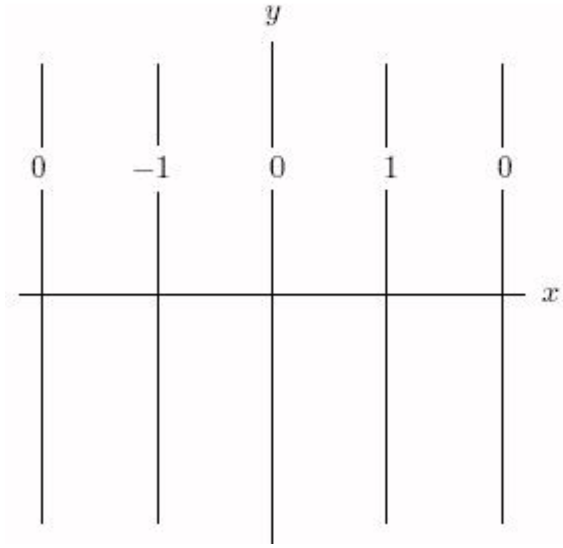
The parabola $z = 84 - y^2$

difficulty: easy section: 12.3

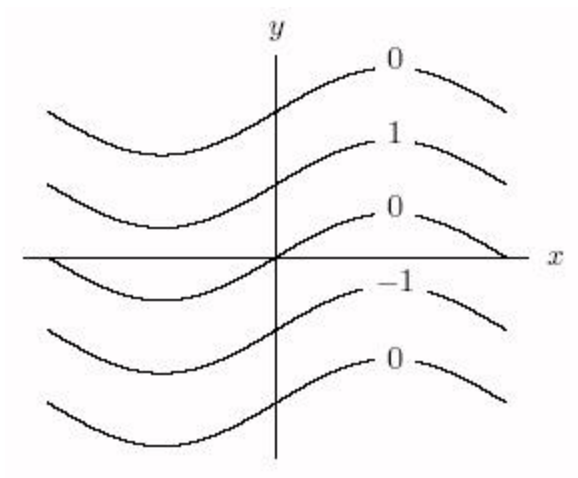
37. Which of the following is a contour diagram for $f(x, y) = \sin x$?
A)



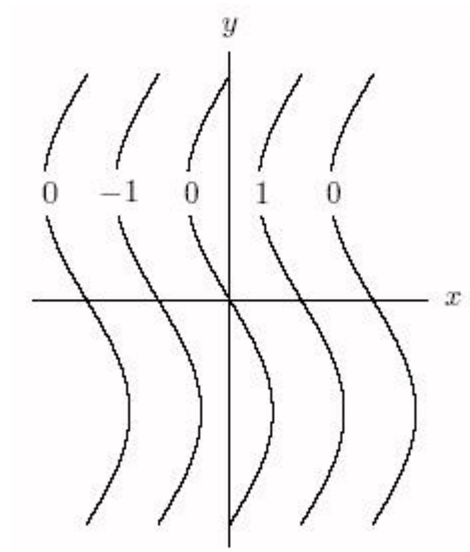
B)



C)

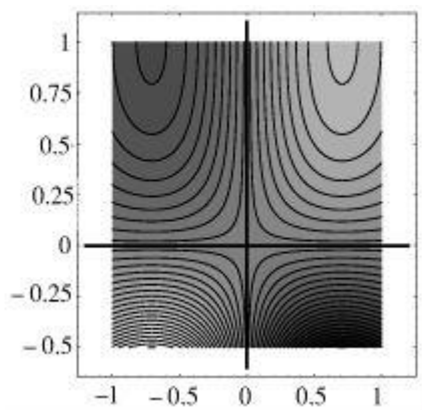


D)



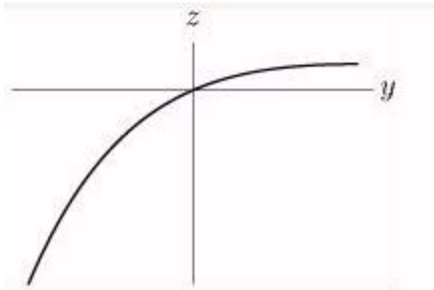
Ans: B difficulty: easy section: 12.3

38. The picture below is the contour diagram of $f(x, y)$. The areas between contours have been shaded. Lighter shades represent higher levels, while darker shades represent lower levels.



Sketch the cross section of $f(x, y)$ with x fixed at $x = 0.5$.

Ans:

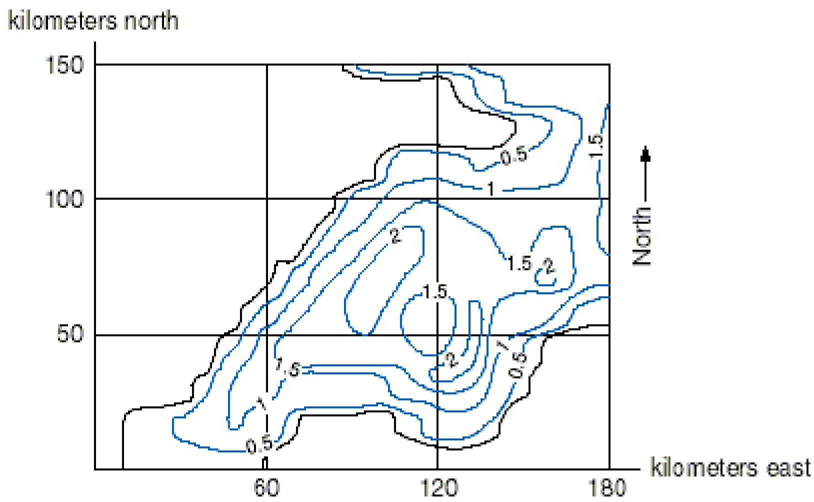


difficulty: easy section: 12.3

39. Let $f(x,y) = y^2 - 8yx + 16x^2$. Find the contour of f that passes through the point $(0,2)$.
Ans: $y = 4x + 2$ and $y = 4x - 2$
difficulty: medium section: 12.3

40. Find a formula for a function $f(x, y)$, given that its contour at level 9 has equation $x^2 - 10xy = -1$.
Ans: $f(x,y) = x^2 - 10xy + 10$
difficulty: medium section: 12.3

41. Below is a contour diagram depicting *D*, the average fox population density as a function of x_E, kilometers east of the western end of England, and x_N, kilometers north of the same point.



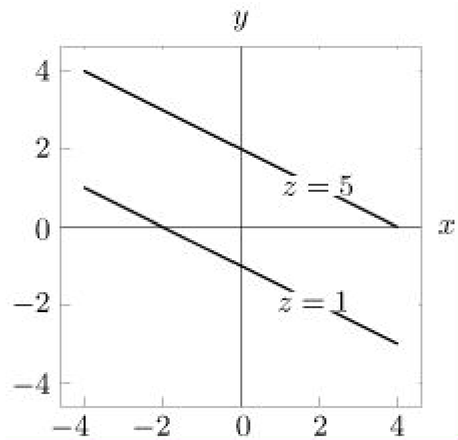
Is *D* increasing or decreasing at the point (120, 25) in the northern direction?
Ans: The function is increasing in the northern direction since as we go north the number of foxes increases.
difficulty: easy section: 12.3

42. True or False: If all of the contours of a function $f(x, y)$ are parallel lines, then the function must be linear.
Ans: False difficulty: easy section: 12.4

43. True or False: If *f* is a linear function, then $f(4,2) - f(4,1) = f(0,2) - f(0,1)$.
Ans: True difficulty: easy section: 12.4

44. True or False: If *f* is any linear function of two variables, then $f(x, y + 1) = f(x + 1, y)$.
Ans: False difficulty: easy section: 12.4

45. Consider the (partial) contour diagram below for a linear function



Find an equation $z = f(x, y)$ for the function.

Ans: $z = \frac{7}{3} + \frac{2}{3}x + \frac{4}{3}y$

difficulty: medium section: 12.4

46. A plane passes through the points (1, 3, 7), (-1, 0, 6), and (2, 1, –3). Determine the equation of the plane.

Ans: $z = 2 - 4x + 3y$

difficulty: medium section: 12.4

47. Consider the plane that passes through the points (1, 3, 10), (-1, –1, 0), and (2, 1, –1). If you were walking on this plane with no change in altitude, what would be the slope of your path in the *xy*-plane?

Ans: $\frac{3}{4}$

difficulty: medium section: 12.4

48. Given the table of some values of a linear function complete the table:

$y \backslash x$	2.5	3	3.5
-1	13		15
1		8	9
3	1		

Ans:

$y \backslash x$	2.5	3	3.5
-1	13	14	15
1	7	8	9
3	1	2	3

difficulty: easy section: 12.4

49. Given the table of some values of a linear function determine a formula for the function.

$y \backslash x$	2.5	3	3.5
-1	–7		–11
1		–13	–15
3	–15		

Ans: $z = -4x - 2y + 1$

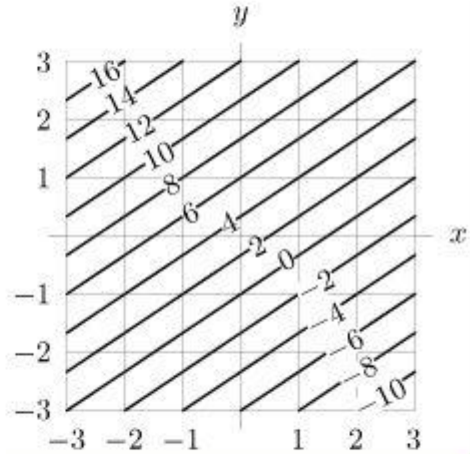
difficulty: medium section: 12.4

50. Find an equation for the plane passing through (1, –5, –2) and containing the *x*-axis.

Ans: $z = \frac{2}{5}y$

difficulty: medium section: 12.4

51. Find a formula for the linear function whose contours are shown below.



Ans: $f(x, y) = 3y - 2x + 3$

difficulty: easy section: 12.4

52. A linear function $f(x, y)$ has the values $f(4, 2) = 10$, $f(1, 2) = 4$ and $f(4, 1) = 7$. Find an equation for f.

Ans: $f(x, y) = 2x + 3y - 4$

difficulty: medium section: 12.4

53. Determine a formula for the linear function $f(x, y)$, such that its cross-section with y fixed at $y = 1$ has equation $z = 4x - 1$, and its contour at level 0 is the line $y = (4 + 4x)/5$.

Ans: $f(x, y) = 4x - 5y + 4$

difficulty: medium section: 12.4

54. A linear function $f(x,y)$ has cross-sections $f(x,4) = 2x - 14$ and $f(2,y) = -4y + 6$. Find an equation for f.

Ans: $f(x, y) = 2x - 4y + 2$

difficulty: medium section: 12.4

55. Determine the equation of the plane which passes through the point (1, 3, –1), has slope 5 in the x-direction and slope –3 in the y-direction.

Ans: $z = 5x - 3y + 3$

difficulty: easy section: 12.4

56. Find a formula for a linear function $z = f(x, y)$ whose $f = 0$ contour is the line $y = 2x + 2$.

Ans: $z = 2x - y + 2$

difficulty: easy section: 12.4

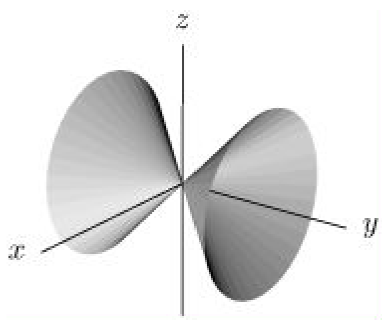
57. Describe the set of points whose distance from the *y*-axis equals the distance from the *xz*-plane.

Write an equation for this set in the form $G(x, y, z) = 0$.

Can the set be described as the graph of a function of two variables?
Ans: This is two cones centered around the *y*-axis with vertex at the origin as shown below.

The equation is $x^2 - y^2 + z^2 = 0$.

It cannot be expressed as the graph of a function of two variables, because there are two values of *z* for each value of *x* and *y* (except when $x = \pm y$): $z^2 = \pm\sqrt{y^2 - x^2}$.



difficulty: easy section: 12.5

58. Does the point (–5, –1, –5) lie on any of the level surfaces of $f(x, y, z) = x^2 + 4y^2 + z^2$? If so, what is the equation of that level surface?
Ans: Yes: $x^2 + 4y^2 + z^2 = 54$
difficulty: easy section: 12.5

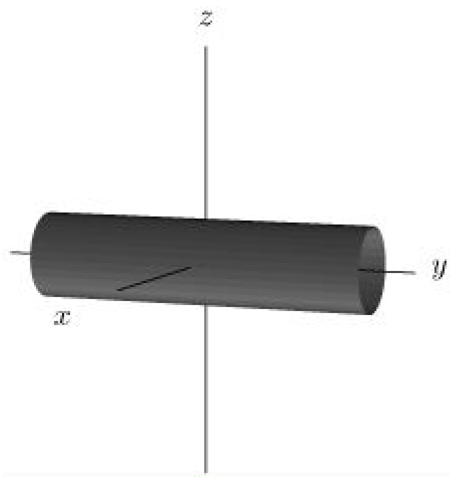
59. What is the domain of $g(x, y, z) = \dfrac{x^2 + y^2 + z^2}{x^2 + y^2 + z^2 - 3}$?

Describe the level surface $g = 5$.
Ans: The domain of *g* consists of all points in 3-space except the points that satisfy $x^2 + y^2 + z^2 = 3$, which lie on a sphere of radius $\sqrt{3}$ centered at the origin.

The level surface is a sphere $x^2 + y^2 + z^2 = \dfrac{15}{4}$.

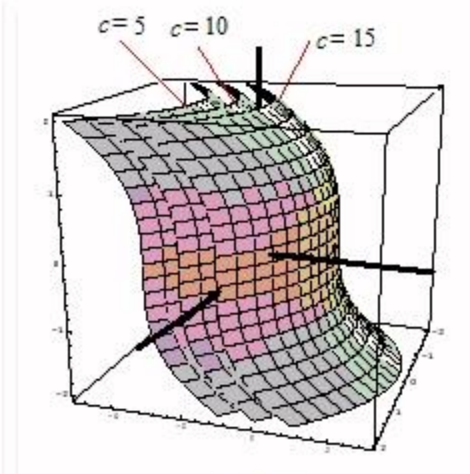
difficulty: easy section: 12.5

60. Sketch the level set of $f(x, y) = x^2 + z^2$ corresponding to $f = 16$ and describe it in words.
Ans: The level set is a cylinder of radius 4, with its axis along the y-axis.

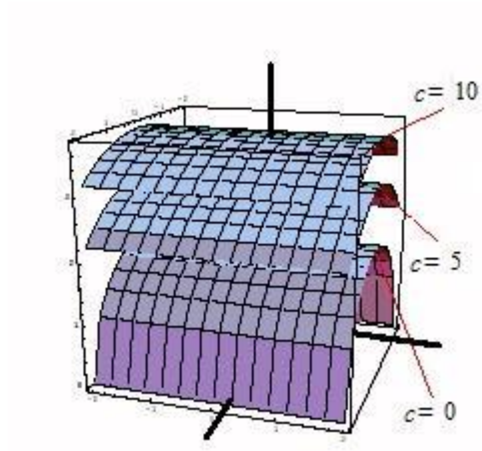


difficulty: easy section: 12.5

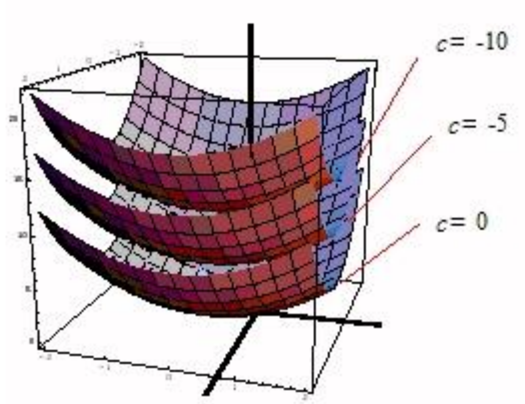
61. Some level surfaces of the functions $g_1(x,y,z)$, $g_2(x,y,z)$ and $g_3(x,y,z)$ are shown below, respectively.



$g_1(x,y,z)$



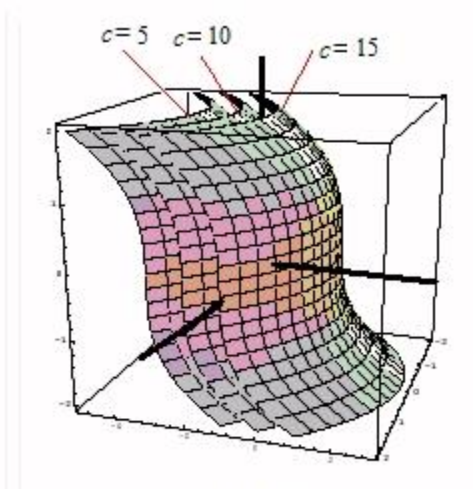
$g_2(x,y,z)$



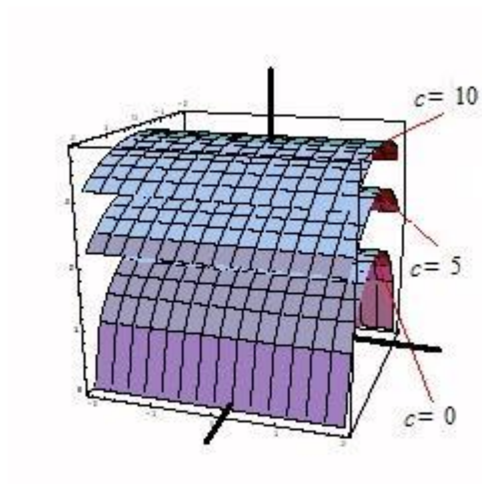
$g_2(x,y,z)$

Which function takes the value 0 at (0, 0, 0)?
Ans: g_3
difficulty: easy section: 12.5

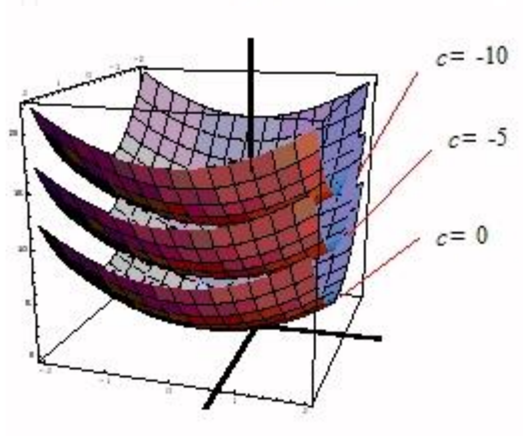
62. The level surfaces of the functions $g_1(x, y, z)$, $g_2(x, y, z)$ and $g_3(x, y, z)$ are shown in below respectively.



$g_1(x, y, z)$



$g_2(x, y, z)$



$g_2(x, y, z)$

Which function is is decreasing in the positive z-direction?
Ans: g_3
difficulty: easy section: 12.5

63. Classify the following surface as ellipsoid, elliptical paraboloid, hyperbolic paraboloid, hyperboloid of one sheet, hyperboloid of two sheets, or cone.

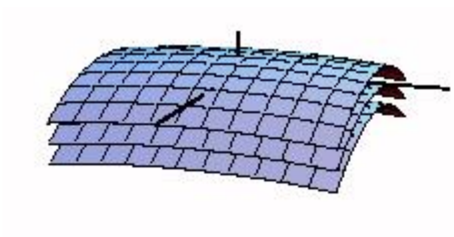
$-9x^2 - 4y^2 + z^2 = 9$
Ans: hyperboloid of two sheets
difficulty: medium section: 12.5

64. Find a function $f(x, y, z)$ whose level surface at level 4 is the graph of the function $h(x,y)=5x^2y+5y-1$.

Ans: $f(x,y,z)=z-5x^2y-5y+5$. (There are other possible answers.)
difficulty: medium section: 12.5

65. Given that the equation of the level surface of $f(x, y, z)$ at level –3 is $3x^2+xy+z=15$, determine $f(2, 3, -3)$ if possible.
Ans: –3
difficulty: medium section: 12.5

66. Consider the figure shown below.



Match the level curve with the function.

A) $f_1(x,y,z)=5x+y+z$

B) $f_2(x,y,z)=5x^2+y^2+z$

C) $f_2(x,y,z)=5x-y+z$

Ans: B difficulty: easy section: 12.5

67. What value of c (if any) makes the following function continuous at (0, 0)?

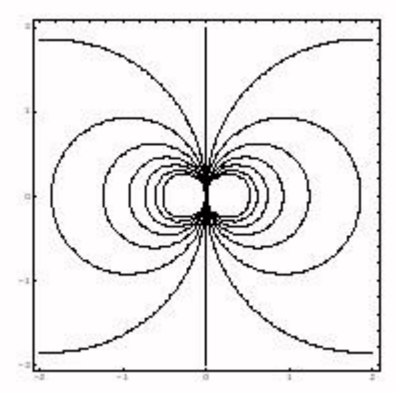
$$f(x,y)=\begin{cases}\dfrac{x^4-y^4}{x-y} & \text{if } (x,y)\neq(0,0)\\ c & \text{otherwise}\end{cases}$$

Ans: c=0
difficulty: easy section: 12.6

68. Let $f(x,y)=\dfrac{x-5y}{x+5y}$. Describe the level curve of f at level 2.

Ans: $y=-\dfrac{1}{15}x$, provided that $x\neq 0$.
difficulty: easy section: 12.6

69. A function $f(x, y)$ is defined for $(x, y) \neq (0, 0)$. Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist if f has the contour diagram below?



Assume that different contours represent different values of the function. Explain your answer.

Ans: The limit does not exist. The different contours meet at the origin. If it existed the limit would have to be equal to the value on each of the contours. This is impossible since the values are different. Hence the function is not continuous at the origin.

difficulty: medium section: 12.6

70. Determine the values of a and b such that the following function is continuous everywhere.

$$f(x, y) = \begin{cases} 5 + ax + by, & y \geq 2 \\ 4 + 5x + 2y, & y < 2 \end{cases}$$

Ans: $a = 5,\ b = \frac{3}{2}$

difficulty: medium section: 12.6

71. Suppose that for all x and y the function f satisfies

$$|f(x,y)-3| > \sqrt{x^2+y^2} \text{ and } |f(x,y)-4| \le 5\left((x-4)^2+(y-2)^2\right).$$

Determine, if possible, the values of the following limits. Explain your answer.
Note that the limit may not exist nor be determined from the given information.
(a) $\lim_{(x,y)\to(0,0)} f(x,y)$
(b) $\lim_{(x,y)\to(4,2)} f(x,y)$
Ans: (a) It cannot be determined from the given information. Since $|f(x,y)-3| > \sqrt{x^2+y^2}$, we do not know whether $|f(x, y) - 3|$ will approach 0 as $\sqrt{x^2+y^2}$ approaches 0.
(b) As (x, y) approaches (4, 2), we have $5\left((x-4)^2+(y-2)^2\right)$ approaches 0, and hence by the inequality, $|f(x,y)-4|$ will also approach zero. This implies that $\lim_{(x,y)\to(4,2)} f(x,y)=4$.
difficulty: medium section: 12.6

72. Let $f(x,y)=\cos\sqrt{9-x^2-y^2}$. What is the domain of f? What are the possible values of f?
Ans: Domain: $x^2+y^2 \le 9$.
The value of f can be any real number between -1 and 1.
difficulty: easy section: 12R

73. Consider the function $f(x,y)=\dfrac{ax^2-bx}{-7+y}$, where a, b are positive numbers. For what values of (x, y) will the function f not be defined?
Ans: $y = 7$
difficulty: easy section: 12R

74. Find a linear function with $f(-5, 5) = 1$, whose graph is parallel to the level surfaces of $g(x, y, z) = 10x + 15y + 5z$.
Ans: $f(x,y)=-2x-3y+6$
difficulty: medium section: 12R

75. Represent (if possible) the following surfaces as graphs of functions *f*(*x*, *y*), and as level surfaces of the form *g*(*x*, *y*, *z*) = *c*. (There are many possible answers.)
(a) The upper half of the sphere of radius 3, centered at (5, 5, 0).
(b) The lower half of the cylinder of radius 4 around the *x*-axis.
(c) The cone $z^2 = 4x^2 + 4y^2$.
Ans: (a) The equation of the sphere of radius 3, centered at (5, 5,0) is:

$$(x-5)^2 + (y-5)^2 + z^2 = 9, \cdot$$

The upper part of this sphere is given by

$$z = +\sqrt{9-(x-5)^2-(y-5)^2}.$$

Therefore, we can choose

$$f(x,y) = +\sqrt{9-(x-5)^2-(y-5)^2} \quad \text{and} \quad g(x,y,z) = z-\sqrt{9-(x-5)^2-(y-5)^2}\ .$$

(b) The equation of the cylinder is $y^2 + z^2 = 16$. The lower part of the cylinder is given by $z = -\sqrt{16-y^2}$.
Therefore, we can choose

$$f(x,y) = -\sqrt{16-y^2} \quad \text{and} \quad g(x,y,z) = z+\sqrt{16-y^2}\ .$$

(c) We cannot represent the cone as the graph of a function. We can represent it as a level surface of $g(x,y,z) = z^2 - 4x^2 - 4y^2$.
difficulty: medium section: 12R

76. Let *f*(*x*, *y*, *z*) = *ax* + *by* + *cz* + *d* be a linear function of three variables, for constants *a*, *b*, *c* and *d*.
Given that some of the cross sections of *f* are *f*(*x*, 1, 1) = 0 + 4*x*, *f*(0, *y*, –3) = 4 + 4*y*, and *f*(1, 1, *z*) = 6 - 2*z*, find a formula for *f*.
Ans: *f*(*x*, *y*, *z*) = 4*x* + 4*y* - 2*z* - 2.
difficulty: medium section: 12R

77. Consider the function $f(x,y) = a^x b^y$, for certain constants *a* and *b*. Simplify the following expression.

$$\frac{f(x+4,y+5)}{f(x,y)}$$

Ans: $a^4 b^5$
difficulty: easy section: 12.1

78. Which of the following gives the domain and range of $f(x, y) = \ln\left(\frac{x}{y}\right)$?
   A) domain: $\{(x, y) : x, y \neq 0\}$; range: $\{z : z \neq 0\}$
   B) domain: $\{(x, y) : x > 0, y > 0\}$; range: $\{z : z > 0\}$
   C) domain: $\{(x, y) : xy > 0\}$; range: all real numbers
   Ans: C difficulty: easy section: 12.1

79. Which of the following best describes the graph of $g(x, y) = \sqrt{x^2 - y^2}$?
   A) cone centered on *x*-axis
   B) hyperbola centered at the origin
   C) upper half of a cone centered on the *x*-axis
   D) one branch of a hyperbola centered at the origin
   Ans: C difficulty: easy section: 12.2

80. Describe the set of points in 3-space for which the distance from the x-axis equals the distance from the y-axis.
   A) two planes that intersect in the z-axis
   C) two planes that intersect in the y-axis
   B) two planes that intersect in the x-axis
   Ans: A difficulty: easy section: 12.2

81. Suppose that $g(x, y) = \cos(Ax + By)$. Find *A* and *B* if you know that the period of $g(x, 0)$ is $\frac{2\pi}{11}$ and the period of $g(0, y)$ is 3.
   Ans: $A = 11;\ B = \frac{2\pi}{3}$.
   difficulty: easy section: 12.2

82. Suppose *f* is a function of two variables. True or False: Given a point (*x*,*y*) in the domain of *f*, there is a level curve of *f* passing through (*x*,*y*).
   Ans: True difficulty: easy section: 12.3

83. Suppose *f* is a function of two variables. True or False: Given a point (*x*,*y*) in the plane, there is a level curve of *f* passing through (*x*,*y*).
   Ans: False difficulty: easy section: 12.3

84. Suppose *f* is a function of two variables. True or False: The contours of *f* must also be functions.
   Ans: False difficulty: easy section: 12.3

85. Suppose *f* is a function of two variables. True or False: The cross-sections of *f* with one variable fixed must also be functions.
   Ans: True difficulty: easy section: 12.2

86. Suppose that the graph of a linear function *f* passes through the point (3,1,6) and that the $z=9$ level curve of *f* is the line $y=\frac{1}{2}x-2$. Find a formula for *f*.

Ans: $f(x,y)=x-2y+5$

difficulty: medium   section: 12.4

87. Lawn King sells a push mower at price p_1 and a ride-on mower at price p_2. The demand function for each mower is a linear function of p_1 and p_2. Market research shows that if the price of the push mower is increased by \$40, then the demand for it drops by 6400 units, while the demand for the ride-on mower increases by 1200 units. On the other hand, if the price of the ride-on mower is decreased by \$60, then demand for it increases by 3000 units, while demand for the push mower drops by 9000 units. Currently, the company is selling 142,000 push mowers at a price of \$300 and 19,000 ride-on mowers at a price of \$1200. Find the two demand functions.

Ans: $d_1(p_1,p_2)=-160p_1+150p_2+10{,}000$, $d_2(p_1,p_2)=30p_1-50p_2+70{,}000$

difficulty: hard   section: 12.4

88. True or False: Level surfaces of *g(x,y,z)* corresponding to different levels cannot intersect.

Ans: True   difficulty: easy   section: 12.5

89. Suppose *f* and *g* are different functions of three variables. Is it possible for the level surface *f* = 1 and the level surface *g* = 1 to be the same surface?

Ans: Yes. For example, $f(x,y,z)=x^2+y^2+z^2$ and $g(x,y,z)=\frac{1}{x^2+y^2+z^2}$ both have the unit sphere as their level surface at level 1, but the functions are different. (There are other such examples.)

difficulty: easy   section: 12.5

90. Match the appropriate function $f(x,y,z)$ with the geometric description of its level surface *f* = 0.

(i) $f(x,y,z)=\sin x$	(a) Sinusoidal curves in the *xy*-plane expanded in the *z* direction
(ii) $f(x,y,z)=y+\sin x$	(b) A sphere of radius 1 centered at the origin.
(iii) $f(x,y,z)=x^2+y^2+z^2-1$	(c) A elliptical paraboloid.
(iv) $f(x,y,z)=x^2+y^2-z^2$	(d) An infinite number of planes parallel to the *yz*-plane.
(v) $f(x,y,z)=6x^2+3y^2-z$	(e) Two cones with vertices at (0,0,0) centered about the *z*-axis.

Ans: (i)=(d), (ii)=(a), (iii)=(b), (iv)=(e), (v)=(c)

difficulty: medium   section: 12.5

91. Consider $f(x,y)=\dfrac{(x-2)^2}{(x-2)^2+y-5}$. Use the cross-sections $f(2,y)$ and $f(x,5)$ to explain why $\lim_{(x,y)\to(2,5)} f(x,y)$ does not exist.

Ans: Notice that $f(2,y)=\frac{0}{y-5}=0$ if $y\neq 5$, whereas $f(x,5)=\frac{(x-2)^2}{(x-2)^2+0}=1$ if $x\neq a$.
So, within any distance of $(2,5)$, one can always find points of the form (2,*y*) with $y\neq 5$ and points of the form (*x*,5) with $x\neq 2$. In other words, no matter how close we are to $(2,5)$, there are points where *f* = 0 and points where *f* = 1.
Therefore $\lim_{(x,y)\to(2,5)} f(x,y)$ does not exist.

difficulty: hard section: 12.6

92. Using the fact that

$$\left|\frac{y^2(x-2)}{(x-2)^2+y^2}\right|\leq\sqrt{(x-2)^2+y^2} \text{ for all } x \text{ and } y \text{ except } (x,y)=(2,0),$$

show that $\lim_{(x,y)\to(2,0)}\dfrac{y^2(x-2)}{(x-2)^2+y^2}=0$.

Ans: We note that $\sqrt{(x-2)^2+y^2}$ is equal to the distance from (x,y) to $(2,0)$. Using the given inequality, we see that, if *u* is any small positive number, then

$$\left|\frac{y^2(x-2)}{(x-2)^2+y^2}-0\right|=\left|\frac{y^2(x-2)}{(x-2)^2+y^2}\right|\leq\sqrt{(x-2)^2+y^2}<u.$$

This means that the difference between $\frac{y^2(x-2)}{(x-2)^2+y^2}$ and 0 can be made as small as we wish by choosing the distance from (x,y) to $(2,0)$ to be sufficiently small. By the definition of limit, we can conclude that $\lim_{(x,y)\to(2,0)}\dfrac{y^2(x-2)}{(x-2)^2+y^2}=0$.

difficulty: hard section: 12.6

93. True or False: If all the level surfaces of a function $f(x,y,z)$ are parallel planes, then *f* must be a linear function.

Ans: False difficulty: easy section: 12R

94. Let $f(x,y)=3$. Describe the cross-sections, the contours, and the graph of *f*.

Ans: All the cross sections of *f* are the same line *z* = 3, which is horizontal in a cross-section diagram. There is only one contour of *f*, which is of level 3, and contains all the points of 2-space. The graph of *f* is the plane *z* = 3, which is parallel to the *xy*-plane.

difficulty: easy section: 12R

95. Let $g(x, y, z) = 5$. Describe the level surfaces of *g*.
Ans: There is only one level surface of *g*. It is of level 5 and it contains all the points of 3-space.
difficulty: easy section: 12R

96. In an exam, a student wrote the following answer.
"The level curve of $f(x, y)$ at the point (3,–4) is $x^2 - 2xy = 34$."
How can you tell that the answer is wrong without even knowing the formula for *f*?
Ans: The answer is wrong because the point (3,–4) does not lie on the curve $x^2 - 2xy = 34$.
difficulty: easy section: 12R