2 Review and Applications of Algebra

Exercise 2.1

1. $(-p) + (-3p) + 4p = -p - 3p + 4p = \underline{\underline{0}}$

2. $(5s - 2t) - (2s - 4t) = 5s - 2t - 2s + 4t = \underline{\underline{3s + 2t}}$

3. $4x^2y + (-3x^2y) - (-5x^2y) = 4x^2y - 3x^2y + 5x^2y = \underline{\underline{6x^2y}}$

4. $1 - (7e^2 - 5 + 3e - e^3) = 1 - 7e^2 + 5 - 3e + e^3 = \underline{\underline{e^3 - 7e^2 - 3e + 6}}$

5. $(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2) = 6x^2 - 3xy + 4y^2 - 8y^2 + 10xy + x^2$
$= \underline{\underline{7x^2 + 7xy - 4y^2}}$

6. $(7m^3 - m - 6m^2 + 10) - (5m^3 - 9 + 3m - 2m^2)$
$= 7m^3 - m - 6m^2 + 10 - 5m^3 + 9 - 3m + 2m^2$
$= \underline{\underline{2m^3 - 4m^2 - 4m + 19}}$

7. $2(7x - 3y) - 3(2x - 3y) = 14x - 6y - 6x + 9y = \underline{\underline{8x + 3y}}$

8. $4(a^2 - 3a - 4) - 2(5a^2 - a - 6) = 4a^2 - 12a - 16 - 10a^2 + 2a + 12$
$= \underline{\underline{-6a^2 - 10a - 4}}$

9. $15x - [4 - 2(5x - 6)] = 15x - 4 + 10x - 12 = \underline{\underline{25x - 16}}$

10. $6a - [3a - 2(2b - a)] = 6a - 3a + 4b - 2a = \underline{\underline{a + 4b}}$

11. $\frac{2x + 9}{4} - 1.2(x - 1) = 0.5x + 2.25 - 1.2x + 1.2 = \underline{\underline{-0.7x + 3.45}}$

12. $\frac{x}{2} - x^2 + \frac{4}{5} - 0.2x^2 - \frac{4}{5}x + \frac{1}{2} = 0.5x - x^2 + 0.8 - 0.2x^2 - 0.8x + 0.5$
$= \underline{\underline{-1.2x^2 - 0.3x + 1.3}}$

13. $\frac{8x}{0.5} + \frac{5.5x}{11} + 0.5(4.6x - 17) = 16x + 0.5x + 2.3x - 8.5 = \underline{\underline{18.8x - 8.5}}$

14. $\frac{2x}{1.045} - \frac{2.016x}{3} + \frac{x}{2} = 1.9139x - 0.6720x + 0.5x = \underline{\underline{1.7419x}}$

15. $\frac{P}{1 + 0.095 \times \frac{5}{12}} + 2P\left(1 + 0.095 \times \frac{171}{365}\right) = 0.96192P + 2.08901P = \underline{\underline{3.0509P}}$

16. $y\left(1 - 0.125 \times \frac{213}{365}\right) + \frac{2y}{1 + 0.125 \times \frac{88}{365}} = 0.92706y + 1.94149y = \underline{\underline{2.8685y}}$

17. $k(1 + 0.04)^2 + \frac{2k}{(1 + 0.04)^2} = 1.08160k + 1.84911k = \underline{\underline{2.9307k}}$

18. $\frac{h}{(1 + 0.055)^2} - 3h(1 + 0.055)^3 = 0.89845h - 3.52272h = \underline{\underline{-2.6243h}}$

19. $4a(3ab - 5a + 6b) = \underline{\underline{12a^2b - 20a^2 + 24ab}}$

Exercise 2.1 *(continued)*

20. $9k(4 - 8k + 7k^2) = \underline{\underline{36k - 72k^2 + 63k^3}}$

21. $-5xy(2x^2 - xy - 3y^2) = \underline{\underline{-10x^3y + 5x^2y^2 + 15xy^3}}$

22. $-(p^2 - 4pq - 5p)\left(\frac{2q}{p}\right) = \underline{\underline{-2pq + 8q^2 + 10q}}$

23. $(4r - 3t)(2t + 5r) = 8rt + 20r^2 - 6t^2 - 15rt = \underline{\underline{20r^2 - 7rt - 6t^2}}$

24. $(3p^2 - 5p)(-4p + 2) = -12p^3 + 6p^2 + 20p^2 - 10p = \underline{\underline{-12p^3 + 26p^2 - 10p}}$

25 $3(a - 2)(4a + 1) - 5(2a + 3)(a - 7) = 3(4a^2 + a - 8a - 2) - 5(2a^2 - 14a + 3a - 21)$
$= 12a^2 - 21a - 6 - 10a^2 + 55a + 105$
$= \underline{\underline{2a^2 + 34a + 99}}$

26. $5(2x - y)(y + 3x) - 6x(x - 5y) = 5(2xy + 6x^2 - y^2 - 3xy) - 6x^2 + 30xy$
$= -5xy + 30x^2 - 5y^2 - 6x^2 + 30xy$
$= \underline{\underline{24x^2 + 25xy - 5y^2}}$

27. $\frac{18x^2}{3x} = \underline{\underline{6x}}$

28. $\frac{6a^2b}{-2ab^2} = \underline{\underline{-3\frac{a}{b}}}$

29. $\frac{x^2y - xy^2}{xy} = \underline{\underline{x - y}}$

30. $\frac{-4x + 10x^2 - 6x^3}{-0.5x} = \underline{\underline{8 - 20x + 12x^2}}$

31. $\frac{12x^3 - 24x^2 + 36x}{48x} = \underline{\underline{\frac{x^2 - 2x + 3}{4}}}$

32. $\frac{32a^2b - 8ab + 14ab^2}{2ab} = \underline{\underline{16a - 4 + 7b}}$

33. $\frac{4a^2b^3 - 6a^3b^2}{2ab^2} = \underline{\underline{2ab - 3a^2}}$

34. $\frac{120(1+i)^2 + 180(1+i)^3}{360(1+i)} = \underline{\underline{\frac{2(1+i) + 3(1+i)^2}{6}}}$

35. $3d^2 - 4d + 15 = 3(2.5)^2 - 4(2.5) + 15$
$= 18.75 - 10 + 15$
$= \underline{\underline{23.75}}$

36. $15g - 9h + 3 = 15(14) - 9(15) + 3 = \underline{\underline{78}}$

37. $7x(4y - 8) = 7(3.2)(4 \times 1.5 - 8) = 22.4(6 - 8) = \underline{\underline{-44.8}}$

38. $I \div Pr = \frac{\$13.75}{\$500 \times 0.11} = \underline{\underline{0.250}}$

39. $\frac{I}{rt} = \frac{\$23.21}{0.095 \times \frac{283}{365}} = \frac{\$23.21}{0.073658} = \underline{\underline{\$315.11}}$

Exercise 2.1 *(continued)*

40. $\frac{N}{1-d} = \frac{\$89.10}{1-0.10}$ = $\underline{\underline{\$99.00}}$

41. $L(1 - d_1)(1 - d_2)(1 - d_3)$ = \$490(1 – 0.125)(1 – 0.15)(1 – 0.05) = $\underline{\underline{\$346.22}}$

42. $P(1 + rt)$ = $\$770\left(1+0.013\times\frac{223}{365}\right)$ = \$770(1.0079425) = $\underline{\underline{\$776.12}}$

43. $\frac{S}{1+rt} = \frac{\$2500}{1+0.085\times\frac{123}{365}} = \frac{\$2500}{1.028644}$ = $\underline{\underline{\$2430.38}}$

44. $(1 + i)^m - 1 = (1 + 0.0225)^4 - 1 = \underline{\underline{0.093083}}$

45. $P(1 + i)^n = \$1280(1 + 0.025)^3 = \underline{\underline{\$1378.42}}$

46. $\frac{S}{(1+i)^n} = \frac{\$850}{(1+0.0075)^6} = \frac{\$850}{1.045852}$ = $\underline{\underline{\$812.73}}$

47. $R\left[\frac{(1+i)^n - 1}{i}\right] = \$550\left(\frac{1.085^3 - 1}{0.085}\right) = \$550\left(\frac{0.2772891}{0.085}\right)$ = $\underline{\underline{\$1794.22}}$

48. $R\left[\frac{(1+i)^n - 1}{i}\right](1+i) = \$910\left(\frac{1.1038129^4 - 1}{0.1038129}\right)(1.1038129)$

$$= \$910\left(\frac{0.4845057}{0.1038129}\right)(1.1038129)$$

$$= \underline{\underline{\$4687.97}}$$

49. $\frac{R}{i}\left[1-\frac{1}{(1+i)^n}\right] = \frac{\$630}{0.115}\left(1-\frac{1}{1.115^2}\right)$ = $\underline{\underline{\$1071.77}}$

50. $P(1 + rt_1) + \frac{S}{1+rt_2} = \$470\left(1+0.075\times\frac{104}{365}\right) + \frac{\$390}{1+0.075\times\frac{73}{365}}$

$$= \$470(1.021370) + \frac{\$390}{1.01500}$$

$$= \$480.044 + \$384.236$$

$$= \underline{\underline{\$864.28}}$$

Exercise 2.2

1. $I = Prt$

$\$6.25 = P(0.05)0.25$

$\$6.25 = 0.0125P$

$P = \frac{\$6.25}{0.0125} = \underline{\underline{\$500.00}}$

Exercise 2.2 *(continued)*

2. $$PV = \frac{PMT}{i}$$
$$\$150{,}000 = \frac{\$900}{i}$$
$$\$150{,}000i = \$900$$
$$i = \frac{\$900}{\$150{,}000} = \underline{\underline{0.00600}}$$

3. $$S = P(1 + rt)$$
$$\$3626 = P(1 + 0.004 \times 9)$$
$$\$3626 = 1.036P$$
$$P = \frac{\$3626}{1.036} = \underline{\underline{\$3500.00}}$$

4. $$N = L(1 - d)$$
$$\$891 = L(1 - 0.10)$$
$$\$891 = 0.90L$$
$$L = \frac{\$891}{0.90} = \underline{\underline{\$9900.00}}$$

5. $$N = L(1 - d)$$
$$\$410.85 = \$498(1 - \mathrm{d})$$
$$\frac{\$410.85}{\$498} = 1 - d$$
$$0.825 = 1 - d$$
$$d = 1 - 0.825 = \underline{\underline{0.175}}$$

6. $$S = P(1 + rt)$$
$$\$5100 = \$5000(1 + 0.0025t)$$
$$\$5100 = \$5000 + \$12.5t$$
$$\$5100 - \$5000 = \$12.5t$$
$$t = \frac{\$100}{\$12.5} = \underline{\underline{8.00}}$$

7. $$NI = (CM)X - FC$$
$$\$15{,}000 = CM(5000) - \$60{,}000$$
$$\$15{,}000 + \$60{,}000 = 5000CM$$
$$CM = \frac{\$75{,}000}{5000} = \underline{\underline{\$15.00}}$$

8. $$NI = (CM)X - FC$$
$$-\$542.50 = (\$13.50)X - \$18{,}970$$
$$\$18{,}970 - \$542.50 = (\$13.50)X$$
$$X = \frac{\$18{,}427.50}{\$13.50} = \underline{\underline{1365}}$$

9. $$N = L(1 - d_1)(1 - d_2)(1 - d_3)$$
$$\$1468.80 = L(1 - 0.20)(1 - 0.15)(1 - 0.10)$$
$$\$1468.80 = L(0.80)(0.85)(0.90)$$
$$L = \frac{\$1468.80}{0.6120} = \underline{\underline{\$2400.00}}$$

Exercise 2.2 *(continued)*

10. $$N = L(1-d_1)(1-d_2)(1-d_3)$$

$\$70.29 = \$99.99(1 – 0.20)(1 – d_2)(1 – 0.05)$

$\$70.29 = \$75.9924(1 – d_2)$

$$\frac{\$70.29}{\$75.9924} = (1 - d_2)$$

$d_2 = 1 – 0.92496 = \underline{\underline{0.0750}}$

11. $$FV = PV(1+i_1)(1+i_2)(1+i_3)\cdots(1+i_n)$$

$\$1094.83 = \$1000(1 + i_1)(1 + 0.03)(1 + 0.035)$

$\$1094.83 = \$1066.05(1 + i_1)$

$$\frac{\$1094.83}{\$1066.05} = 1 + i_1$$

$i_1 = 1.02700 – 1 = \underline{\underline{0.0270}}$

12. $$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\$1508.54 = PMT\left[\frac{(1+0.05)^4 - 1}{0.05}\right]$$

$$\$1508.54 = PMT\left(\frac{1.21550625 - 1}{0.05}\right)$$

$$PMT = \$1508.54 \times \frac{0.05}{0.21550625} = \underline{\underline{\$350.00}}$$

13. $$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\$6595.20 = PMT\left[\frac{1-(1+0.06)^{-20}}{0.06}\right]$$

$$\$6595.20 = PMT\left[\frac{1-0.31180473}{0.06}\right]$$

$$PMT = \$6595.20 \times \frac{0.06}{0.68819527} = \underline{\underline{\$575.00}}$$

14. $$FV = PV(1+i)^n$$

$\$9321.91 = \$2000(1 + i)^{20}$

$$\left(\frac{\$9321.91}{\$2000}\right)^{1/20} = 1 + i$$

$1.0800 = 1 + i$

$i = 1.08000 – 1 = \underline{\underline{0.0800}}$

Exercise 2.2 *(continued)*

15. $PV = FV(1+i)^{-n}$

$\$5167.20 = \$10,000$

$$\frac{\$5167.20}{\$10,000} = \frac{1}{(1+i)^{15}}$$

$$(1+i)^{15} = \frac{\$10,000}{\$5167.20}$$

$1 + i = (1.935284)^{1/15} = 1.0450$

$i = \underline{\underline{0.0450}}$

16. $I = Prt$

$$\frac{I}{Pr} = \frac{Prt}{Pr}$$

$$t = \frac{I}{Pr}$$

17. $PV = \frac{PMT}{i}$

$i(PV) = PMT$

$$i = \frac{PMT}{PV}$$

18. $N = L(1 - d)$

$$\frac{N}{L} = 1 - d$$

$$d = 1 - \frac{N}{L}$$

19. $NI = (CM)X - FC$

$NI + FC = (CM)X$

$$CM = \frac{NI + FC}{X}$$

20. $NI = (CM)X - FC$

$NI + FC = (CM)X$

$$X = \frac{NI + FC}{CM}$$

21. $S = P(1 + rt)$

$S = P + Prt$

$S - P = Prt$

$r = (S - P)/Pt$

22. $S = P(1 + rt)$

$S = P + Prt$

$S - P = Prt$

$t = (S - P)/Pr$

23. $N = L(1 - d_1)(1 - d_2)(1 - d_3)$

$$\frac{N}{L(1-d_2)(1-d_3)} = (1 - d_1)$$

$$d_1 = 1 - \frac{N}{L(1-d_2)(1-d_3)}$$

24. $N = L(1 - d_1)(1 - d_2)(1 - d_3)$

$$\frac{N}{L(1-d_1)(1-d_2)} = (1 - d_3)$$

$$d_3 = 1 - \frac{N}{L(1-d_1)(1-d_2)}$$

25. $FV = PV(1+i)^n$

$$\frac{FV}{(1+i)^n} = PV$$

$PV = FV(1+i)^{-n}$

26. $FV = PV(1+i)^n$

$$\left(\frac{FV}{PV}\right)^{1/n} = (1+i)$$

$$i = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

Exercise 2.2 *(continued)*

27. $a^2 \times a^3 = \underline{\underline{a^5}}$

28. $(x^6)(x^{-4}) = \underline{\underline{x^2}}$

29. $b^{10} \div b^6 = b^{10-6} = \underline{\underline{b^4}}$

30. $h^7 \div h^{-4} = h^{7-(-4)} = \underline{\underline{h^{11}}}$

31. $(1 + i)^4 \times (1 + i)^9 = \underline{\underline{(1 + i)^{13}}}$

32. $(1 + i) \times (1 + i)^n = \underline{\underline{(1 + i)^{n+1}}}$

33. $(x^4)^7 = x^{4\text{x}7} = \underline{\underline{x^{28}}}$

34. $(y^3)^3 = \underline{\underline{y^9}}$

35. $\left(t^6\right)^{\frac{1}{3}} = \underline{\underline{t^2}}$

36. $(n^{0.5})^8 = \underline{\underline{n^4}}$

37. $\dfrac{\left(x^5\right)\left(x^6\right)}{x^9} = x^{5+6-9} = \underline{\underline{x^2}}$

38. $\dfrac{\left(x^5\right)^6}{x^9} = x^{5\times6-9} = \underline{\underline{x^{21}}}$

39. $\left[2(1+i)\right]^2 = \underline{\underline{4(1+i)^2}}$

40. $\left(\dfrac{1+i}{3i}\right)^3 = \underline{\underline{\dfrac{(1+i)^3}{27i^3}}}$

41. $\dfrac{4r^5t^6}{\left(2r^2t\right)^3} = \dfrac{4r^5t^6}{8r^6t^3} = \dfrac{r^{5-6}t^{6-3}}{2} = \underline{\underline{\dfrac{t^3}{2r}}}$

42. $\dfrac{\left(-r^3\right)(2r)^4}{\left(2r^{-2}\right)^2} = \dfrac{-r^3\left(16r^4\right)}{4r^{-4}} = -4r^{3+4-(-4)} = \underline{\underline{-4r^{11}}}$

43. $8^{4/3} = \left(8^{1/3}\right)^4 = 2^4 = \underline{\underline{16.0000}}$

44. $-27^{2/3} = -\left(27^{1/3}\right)^2 = \underline{\underline{-9.00000}}$

45. $7^{3/2} = 7^{1.5} = \underline{\underline{18.5203}}$

46. $5^{3/4} = 5^{-0.75} = \underline{\underline{0.299070}}$

47. $(0.001)^{-2} = \underline{\underline{1,000,000}}$

48. $0.893^{-1/2} = 0.893^{-0.5} = \underline{\underline{1.05822}}$

Exercise 2.2 *(continued)*

49. $(1.0085)^5(1.0085)^3 = 1.0085^8 = \underline{\underline{1.07006}}$

50. $(1.005)^3(1.005)^{-6} = 1.005^{-3} = \underline{\underline{0.985149}}$

51. $\sqrt[3]{1.03} = 1.03^{0.\overline{3}} = \underline{\underline{1.00990}}$

52. $\sqrt[6]{1.05} = \underline{\underline{1.00816}}$

53. $\left(4^4\right)\left(3^{-3}\right)\left(-\frac{3}{4}\right)^3 = \frac{4^4}{3^3}\left(-\frac{3^3}{4^3}\right) = \underline{\underline{4.00000}}$

54. $\left[\left(-\frac{3}{4}\right)^2\right]^{-2} = \left(-\frac{3}{4}\right)^{-4} = \left(-\frac{4}{3}\right)^4 = \frac{256}{81} = \underline{\underline{3.16049}}$

55. $\left(\frac{2}{3}\right)^3\left(-\frac{3}{2}\right)^2\left(-\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3\left(\frac{3}{2}\right)^2\left(-\frac{2}{3}\right)^3 = \frac{2}{3}\left(-\frac{2}{3}\right)^3 = -\frac{16}{81} = \underline{\underline{-0.197531}}$

56. $\left(-\frac{2}{3}\right)^3 + \left(\frac{3}{2}\right)^{-2} = \frac{\left(-\frac{2}{3}\right)^3}{\left(\frac{2}{3}\right)^2} = -\frac{2}{3} = \underline{\underline{-0.666667}}$

57. $\frac{1.03^{16} - 1}{0.03} = \underline{\underline{20.1569}}$

58. $\frac{\left(1.008\overline{3}\right)^{30} - 1}{0.008\overline{3}} = \frac{0.2826960}{0.008333333} = \underline{\underline{33.9235}}$

59. $\frac{1 - 1.0225^{-20}}{0.0225} = \frac{0.3591835}{0.0225} = \underline{\underline{15.9637}}$

60. $\frac{1 - \left(1.00\overline{6}\right)^{-32}}{0.00\overline{6}} = \frac{0.1915410}{0.00\overline{6}} = \underline{\underline{28.7312}}$

61. $(1 + 0.0275)^{1/3} = \underline{\underline{1.00908}}$

62. $(1 + 0.055)^{1/6} - 1 = \underline{\underline{0.00896339}}$

Exercise 2.3

1. $10a + 10 = 12 + 9a$
   $10a - 9a = 12 - 10$
   $a = \underline{\underline{2}}$

2. $29 - 4y = 2y - 7$
   $36 = 6y$
   $y = \underline{\underline{6}}$

3. $0.5\,(x - 3) = 20$
   $x - 3 = 40$
   $x = \underline{\underline{43}}$

Exercise 2.3 *(continued)*

4. $\frac{1}{3}(x-2)=4$

$$x - 2 = 12$$
$$x = \underline{\underline{14}}$$

5. $y = 192 + 0.04y$

$$y - 0.04y = 192$$
$$y = \frac{192}{0.96} = \underline{\underline{200}}$$

6. $x - 0.025x = 341.25$

$$0.975x = 341.25$$
$$x = \frac{341.25}{0.975} = \underline{\underline{350}}$$

7. $12x - 4(2x - 1) = 6(x + 1) - 3$

$$12x - 8x + 4 = 6x + 6 - 3$$
$$-2x = -1$$
$$x = \underline{\underline{0.5}}$$

8. $3y - 4 = 3(y + 6) - 2(y + 3)$

$$= 3y + 18 - 2y - 6$$
$$2y = 16$$
$$y = \underline{\underline{8}}$$

9. $8 - 0.5(x + 3) = 0.25(x - 1)$

$$8 - 0.5x - 1.5 = 0.25x - 0.25$$
$$-0.75x = -6.75$$
$$x = \underline{\underline{9}}$$

10. $5(2 - c) = 10(2c - 4) - 6(3c + 1)$

$$10 - 5c = 20c - 40 - 18c - 6$$
$$-7c = -56$$
$$c = \underline{\underline{8}}$$

11. $3.1t + 145 = 10 + 7.6t$

$$-4.5t = -135$$
$$t = \underline{\underline{30}}$$

12. $1.25y - 20.5 = 0.5y - 11.5$

$$0.75y = 9$$
$$y = \underline{\underline{12}}$$

13. $\frac{x}{1.1^2} + 2x(1.1)^3 = \$1000$

$$0.8264463x + 2.622x = \$1000$$
$$3.488446x = \$1000$$
$$x = \underline{\underline{\$286.66}}$$

14. $\frac{3x}{1.025^6} + x(1.025)^8 = \$2641.35$

$$2.586891x + 1.218403x = \$2641.35$$
$$x = \underline{\underline{\$694.13}}$$

Exercise 2.3 *(continued)*

15. $$\frac{2x}{1.03^7} + x + x(1.03^{10}) = \$1000 + \frac{\$2000}{1.03^4}$$
$$1.626183x + x + 1.343916x = \$1000 + \$1776.974$$
$$3.970099x = \$2776.974$$
$$x = \underline{\underline{\$699.47}}$$

16. $$x(1.05)^3 + \$1000 + \frac{x}{1.05^7} = \frac{\$5000}{1.05^2}$$
$$1.157625x + 0.7106813x = \$4535.147 - \$1000$$
$$x = \underline{\underline{\$1892.17}}$$

17. $$x\left(1 + 0.095 \times \frac{84}{365}\right) + \frac{2x}{1 + 0.095 \times \frac{108}{365}} = \$1160.20$$
$$1.021863x + 1.945318x = \$1160.20$$
$$2.967181x = \$1160.20$$
$$x = \underline{\underline{\$391.01}}$$

18. $$\frac{x}{1 + 0.115 \times \frac{78}{365}} + 3x\left(1 + 0.115 \times \frac{121}{365}\right) = \$1000\left(1 + 0.115 \times \frac{43}{365}\right)$$
$$0.9760141x + 3.114370x = \$1013.548$$
$$x = \underline{\underline{\$247.79}}$$

19.
$$x - y = 2 \quad ①$$
$$3x + 4y = 20 \quad ②$$
① × 3: $\underline{3x - 3y = 6}$
Subtract: $7y = 14$
$y = 2$

Substitute into equation ①:
$x - 2 = 2$
$x = 4$
$(x, y) = \underline{\underline{(4, 2)}}$

Check: LHS of ② = 3(4) + 4(2) = 20 = RHS of ②

20.
$$y - 3x = 11 \quad ①$$
$$-4y + 5x = -30 \quad ②$$
① × 4: $\underline{4y - 12x = 44}$
Add: $-7x = 14$
$x = -2$

Substitute into equation ①:
$y - 3(-2) = 11$
$y = 11 - 6 = 5$
$(x, y) = \underline{\underline{(-2, 5)}}$

Check: LHS of ② = – 4(5) + 5(–2) = –30= RHS of ②

Exercise 2.3 *(continued)*

21.

$$\begin{aligned} 4a - 3b &= -3 \quad ① \\ 5a - b &= 10 \quad ② \\ ① \times 1: \quad 4a - 3b &= -3 \\ ② \times 3: \quad \underline{15a - 3b} &= \underline{30} \\ \text{Subtract:} \quad -11a &= -33 \\ a &= 3 \end{aligned}$$

Substitute into equation ②:

$$\begin{aligned} 5(3) - b &= 10 \\ b &= 5 \\ (a, b) &= \underline{(3, 5)} \end{aligned}$$

Check: LHS of ① = 4(3) – 3(5) = –3 = RHS of ①

22.

$$\begin{aligned} 7p - 3q &= 23 \quad ① \\ \underline{-2p - 3q} &= \underline{5} \quad ② \\ \text{Subtract:} \quad 9p &= 18 \\ p &= 2 \end{aligned}$$

Substitute into equation ①:

$$\begin{aligned} 7(2) - 3q &= 23 \\ 3q &= -23 + 14 \\ q &= -3 \\ (p, q) &= \underline{(2, -3)} \end{aligned}$$

Check: LHS of ② = –2(2) –3(–3) = 5 = RHS of ②

23.

$$\begin{aligned} y &= 2x \quad ① \\ \underline{7x - y} &= \underline{35} \quad ② \\ \text{Add:} \quad 7x &= 2x + 35 \\ 5x &= 35 \\ x &= 7 \end{aligned}$$

Substitute into ①:

$$\begin{aligned} y &= 2(7) = 14 \\ (x, y) &= \underline{(7, 14)} \end{aligned}$$

Check: LHS of ② = 7(7) – 14 = 49 – 14 = 35 = RHS of ②

24.

$$\begin{aligned} g - h &= 17 \quad ① \\ \tfrac{4}{3}g + \tfrac{3}{2}h &= 0 \quad ② \\ 1.\overline{3}g + 1.5h &= 0 \quad ② \\ ① \times 1.5: \quad \underline{1.5g - 1.5h} &= \underline{25.5} \\ \text{Add:} \quad 2.8\overline{3}g &= 25.5 \\ g &= 9 \end{aligned}$$

Substitute into ②:

$$\begin{aligned} 9 - h &= 17 \\ h &= -8 \\ (h, g) &= \underline{(-8, 9)} \end{aligned}$$

Check: LHS of ② = $\frac{4}{3}(9) + \frac{3}{2}(-8)$ = 12 – 12 = 0 = RHS of ②

Exercise 2.3 *(continued)*

25.

$$d = 3c - 500 \quad ①$$
$$0.7c + 0.2d = 550 \quad ②$$

To eliminate d,

$$
\begin{aligned}
① \times 0.2: &\quad -0.6c + 0.2d = -100 \\
②: &\quad \underline{0.7c + 0.2d} = \underline{\ 550} \\
\text{Subtract:} &\quad -1.3c + 0 = -650 \\
&\quad c = 500
\end{aligned}
$$

Substitute into ①: $d = 3(500) - 500 = 1000$

$$(c, d) = \underline{\underline{(500, 1000)}}$$

Check: LHS of ② = 0.7(500) + 0.2(1000) = 550 = RHS of ②

26.

$$0.03x + 0.05y = 51 \quad ①$$
$$0.8x - 0.7y = 140 \quad ②$$

To eliminate y,

$$
\begin{aligned}
① \times 0.7: &\quad 0.021x + 0.035y = 35.7 \\
② \times 0.05: &\quad \underline{0.04x - 0.035y} = \underline{\ 7\ } \\
\text{Add:} &\quad 0.061x + 0 = 42.7 \\
&\quad x = 700
\end{aligned}
$$

Substitute into ②:

$$
\begin{aligned}
0.8(700) - 0.7y &= 140 \\
-0.7y &= -420 \\
y &= 600 \\
(x, y) &= \underline{\underline{(700, 600)}}
\end{aligned}
$$

Check: LHS of ① = 0.03(700) + 0.05(600) = 51 = RHS of ①

27.

$$2v + 6w = 1 \quad ①$$
$$10v - 9w = 18 \quad ②$$

To eliminate v,

$$
\begin{aligned}
① \times 10: &\quad 20v + 60w = 10 \\
② \times 2: &\quad \underline{20v - 18w} = \underline{36} \\
\text{Subtract:} &\quad 0 + 78w = -26 \\
&\quad w = -\tfrac{1}{3}
\end{aligned}
$$

Substitute into ①:

$$
\begin{aligned}
2v + 6\left(-\tfrac{1}{3}\right) &= 1 \\
2v &= 1 + 2 \\
v &= \tfrac{3}{2} \\
(v, w) &= \underline{\underline{\left(\tfrac{3}{2}, -\tfrac{1}{3}\right)}}
\end{aligned}
$$

Check: LHS of ② = $10\left(\frac{3}{2}\right) - 9\left(-\frac{1}{3}\right)$ = 18 = RHS of ②

Exercise 2.3 *(continued)*

28. $2.5a + 2b = 11$ ①
$8a + 3.5b = 13$ ②

To eliminate b,

① × 3.5: $8.75a + 7b = 38.5$
② × 2: $\underline{16a + 7b} = \underline{26}$
Subtract: $-7.25a + 0 = 12.5$
$a = -1.724$

Substitute into ①:

$2.5(-1.724) + 2b = 11$
$2b = 11 + 4.31$
$b = 7.655$
$(a, b) = \underline{(-1.72, 7.66)}$

Check: LHS of ② = $8(-1.724) + 3.5(7.655) = 13.00$ = RHS of ②

29. $37x - 63y = 235$ ①
$18x + 26y = 468$ ②

To eliminate x,

① × 18: $666x - 1134y = 4230$
② × 37: $\underline{666x + 962y} = \underline{17,316}$
Subtract: $0 - 2096y = -13,086$
$y = 6.243$

Substitute into ①:

$37x - 63(6.243) = 235$
$37x = 628.3$
$x = 16.98$
$(x, y) = \underline{(17.0, 6.24)}$

Check: LHS of ② = $18(16.98) + 26(6.243) = 468.0$ = RHS of ②

30. $68.9n - 38.5m = 57$ ①
$45.1n - 79.4m = -658$ ②

To eliminate n,

① × 45.1: $3107n - 1736.4m = 2571$
② × 68.9: $\underline{3107n - 5470.7m} = \underline{-45,336}$
Subtract: $0 + 3734.3m = 47,907$
$m = 12.83$

Substitute into ①:

$68.9n - 38.5(12.83) = 57$
$68.9n = 551.0$
$n = 7.996$
$(m, n) = \underline{(12.8, 8.00)}$

Check: LHS of ② = $45.1(7.996) - 79.4(12.83) = -658.1$ = RHS of ②

Exercise 2.3 *(continued)*

31.

$$0.33e + 1.67f = 292 \quad ①$$
$$1.2e + 0.61f = 377 \quad ②$$

To eliminate e,

① ÷ 0.33:	$e + 5.061f = 884.8$
② ÷ 1.2:	$\underline{e + 0.508f} = \underline{314.2}$
Subtract:	$0 + 4.552f = 570.6$
	$f = 125.4$

Substitute into ①:

$$0.33e + 1.67(125.4) = 292$$
$$0.33e = 82.58$$
$$e = 250.2$$
$$(e, f) = \underline{(250, 125)}$$

Check: LHS of ② = 1.2(250.2) + 0.61(125.4) = 376.7 = RHS of ②

32.

$$318j - 451k = 7.22 \quad ①$$
$$-249j + 193k = -18.79 \quad ②$$

To eliminate k,

① ÷451:	$0.7051j - k = 0.01601$
② ÷193:	$\underline{-1.2902j + k} = \underline{-0.09736}$
Add:	$-0.5851j + 0 = -0.08135$
	$j = 0.1390$

Substitute into ②:

$$-249(0.1390) + 193k = -18.79$$
$$193k = 15.82$$
$$k = 0.08197$$
$$(j, k) = \underline{(0.139, 0.0820)}$$

Check: LHS of ① =318(0.1390) – 451(0.08197) = 7.23 = RHS of ① (within rounding errors.)

Point of Interest (Section 2.4)

A "Trick" Question

The element of mathematical misdirection in the question is that it presumes (and attempts to get you thinking) that there really is a missing dollar, and that the $3 difference between the $90 originally paid and the net $87 paid consists of the $2 kept by the bellhop and the missing dollar.

But the $3 refund sitting in the workers' pockets explains the difference between the $90 and the $87. The $2 pilfered by the bellhop explains the $2 difference between the net amount ($87) paid by the workers and the amount ($85) in the hotel's till. There is no missing $1!

Exercise 2.4

1. Step 2: Hits last month = 2655 after the $\frac{2}{7}$ increase.

   Let the number of hits 1 year ago be n.

   Step 3: Hits last month = Hits 1 year ago + $\frac{2}{7}$(Hits 1 year ago)

   Step 4: $2655 = n + \frac{2}{7}n$

   Step 5: $2655 = \frac{9}{7}n$

   Multiply both sides by $\frac{7}{9}$.

   $n = 2655 \times \frac{7}{9} = 2065$

   The Web site had 2065 hits in the same month 1 year ago.

2. Step 2: Retail price = \$712; Markup = 60% of wholesale of cost.
   Let the wholesale cost be C.
   Step 3: Retail price = Cost + 0.60(Cost)
   Step 4: \$712 = C + 0.6C
   Step 5: \$712 = 1.6C

   $C = \frac{\$712}{1.6} = \$445.00$. The wholesale cost is \$445.00.

3. Step 2: Tag price = \$39.55 (including 13% HST). Let the plant's pretax price be P.
   Step 3: Tag price = Pre-tax price + HST

   Step 4: \$39.55 = P + 0.13P

   Step 5: \$39.55 = 1.13P

   $P = \frac{\$39.55}{1.13} = \$35.00$

   The amount of HST is \$39.55 – \$35.00 = \$4.55

4. Step 2: Commission rate = 2.5% on the first \$5000 and 1.5% on the remainder
   Commission amount = \$227. Let the transaction amount be x.

   Step 3: Commission amount = 0.025(\$5000) + 0.015(Remainder)

   Step 4: \$227 = \$125.00 + 0.015(x – \$5000)

   Step 5: \$102 = 0.015x – \$75.00
   \$102 + \$75 = 0.015x

   $x = \frac{\$177}{0.015} = \$11,800.00$

   The amount of the transaction was \$11,800.00.

5. Step 2: Let the basic price be P. First 20 meals at P.
   Next 20 meals at P – \$2. Additional meals at P – \$3.

   Step 3: Total price for 73 meals = \$1686

   Step 4: 20P + 20(P – \$2) + (73 – 40)(P – \$3) = \$1686

   Step 5: 20P + 20P – \$40 + 33P – \$99 = \$1686
   73P = \$1686 + \$99 + \$40

   $P = \frac{\$1825}{73} = \$25.00$

   The basic price per meal is \$25.00.

Exercise 2.4 *(continued)*

6. Step 2: Rental Plan 1: \$295 per week + \$0.15 × (Distance in excess of 1000 km)
   Rental Plan 2: \$389 per week
   Let d represent the distance at which the costs of both plans are equal.

   Step 3: Cost of Plan 1 = Cost of Plan 2

   Step 4: \$295 + \$0.15(d – 1000) = \$389

   Step 5: \$295 + \$0.15d – \$150 = \$389
   \$0.15d = \$244
   d = 1627 km

   The unlimited driving plan will be cheaper if you drive more than 1626.7 km in the one-week interval.

7. Step 2: Tax rate = 38%; Overtime hourly rate = 1.5(\$23.50) = \$35.25
   Cost of canoe = \$2750
   Let h represent the hours of overtime Alicia must work.

   Step 3: Gross overtime earnings – Income tax = Cost of the canoe

   Step 4: \$35.25h – 0.38(\$35.25h) = \$2750

   Step 5: \$21.855h = \$2750
   h = 125.83 hours

   Alicia must work 125¾ hours of overtime to earn enough money to buy the canoe.

8. Step 2: Number of two-bedroom homes = 0.4(Number of three-bedroom homes)
   Number of two-bedroom homes = 2(Number of four-bedroom homes)
   Total number of homes = 96
   Let h represent the number of two-bedroom homes

   Step 3: # 2-bedroom homes + # 3-bedroom homes + # 4-bedroom homes = 96

   Step 4: $h + \frac{h}{0.4} + \frac{h}{2} = 96$

   Step 5: h + 2.5h + 0.5h = 96
   4h = 96
   h = 24

   There should be 24 two-bedroom homes, 2.5(24) = 60 three-bedroom homes, and 0.5(24) = 12 four-bedroom homes.

9. Step 2: Cost of radio advertising = 0.5(Cost of newspaper advertising)
   Cost of TV advertising = 0.6(Cost of radio advertising)
   Total advertising budget = \$160,000
   Let r represent the amount allocated to radio advertising

   Step 3: Radio advertising + TV advertising + Newspaper advertising = \$160,000

   Step 4: $r + 0.6r + \frac{r}{0.5} = \$160{,}000$

   Step 5: 3.6r = \$160,000
   r = \$44,444.44

   The advertising budget allocations should be:
   \$44,444 to radio advertising,
   0.6(\$44,444.44) = \$26,667 to TV advertising, and
   2(\$44,444.44) = \$88,889 to newspaper advertising.

Exercise 2.4 *(continued)*

10. Step 2: By-laws require: 5 parking spaces per 100 square meters,
4% of spaces for physically handicapped
In remaining 96%, # regular spaces = 1.4(# small car spaces)
Total area = 27,500 square meters
Let s represent the number of small car spaces.

Step 3: Total # spaces = # spaces for handicapped + # regular spaces + # small spaces

Step 4: $\frac{27{,}500}{100} \times 5 = 0.04 \times \frac{27{,}500}{100} \times 5 + s + 1.4s$

Step 5: $1375 = 55 + 2.4s$
$s = 550$

The shopping centre must have 55 parking spaces for the physically handicapped, 550 small-car spaces, and 770 regular parking spaces.

11. Step 2: Overall portfolio's rate return = 1.1%, equity fund's rate of return = –3.3%,
bond fund's rate of return = 7.7%.
Let e represent the fraction of the portfolio initially invested in the equity fund.

Step 3: Overall rate of return = Weighted average rate of return
= (Equity fraction)(Equity return) + (Bond fraction)(Bond return)

Step 4: $1.1\% = e(-3.3\%) + (1 - e)(7.7\%)$

Step 5: $1.1 = -3.3e + 7.7 - 7.7e$
$-6.6 = -11.0e$
$e = 0.600$

Therefore, 60.0% of Erin's original portfolio was invested in the equity fund.

12. Step 2: Pile A steel is 5.25% nickel; pile B steel is 2.84% nickel.
We want a 32.5-tonne mixture from A and B averaging 4.15% nickel.
Let A represent the tonnes of steel required from pile A.

Step 3: Wt. of nickel in 32.5 tonnes of mixture
= Wt. of nickel in steel from pile A + Wt. of nickel in steel from pile B
= (% nickel in pile A)(Amount from A) + (% nickel in pile B)(Amount from B)

Step 4: $0.0415(32.5) = 0.0525A + 0.0284(32.5 - A)$

Step 5: $1.34875 = 0.0525A + 0.9230 - 0.0284A$
$0.42575 = 0.0241A$
$A = 17.67$ tonnes

The recycling company should mix 17.67 tonnes from pile A with 14.83 tonnes from pile B.

13. Step 2: Total options = 100,000
# of options to an executive = 2000 + # of options to a scientist or engineer
# of options to a scientist or engineer = 1.5(# of options to a technician)
There are 3 executives, 8 scientists and engineers, and 14 technicians.
Let t represent the number of options to each technician.

Step 3: Total options = Total options to scientists and engineers
+ Total options to technicians + Total options to executives

Step 4: $100{,}000 = 8(1.5t) + 14t + 3(2000 + 1.5t)$

Step 5: $= 12t + 14t + 6000 + 4.5t$
$94{,}000 = 30.5t$
$t = 3082$ options

Each technician will receive 3082 options,
each scientist and engineer will receive 1.5(3082) = 4623 options,
and each executive will receive 2000 + 4623 = 6623 options.

Exercise 2.4 *(continued)*

14. Step 2: Plan X: 6.5 cents/minute (in business hours) and 4.5 cents/minute (at other times)
Plan Y: 5.3 cents/minute any time
Let b represent the fraction of business-hour usage at which costs are equal.

Step 3: Cost of Plan X = Cost of plan Y

Step 4: Pick any amount of usage in a month—say 1000 minutes.
$b(1000)\$0.065 + (1 - b)(1000)\$0.045 = 1000(\$0.053)$

Step 5:
$$\$65b + \$45 - \$45b = \$53$$
$$\$20b = \$8$$
$$b = 0.40$$
If business-hour usage exceeds 40% of overall usage, plan Y will be cheaper.

15. Step 2: Raisins cost \$3.75 per kg; peanuts cost \$2.89 per kg.
Cost per kg of ingredients in 50 kg of "trail mix" is to be \$3.20.
Let p represent the weight of peanuts in the mixture.

Step 3: Cost of 50 kg of trail mix = Cost of p kg peanuts + Cost of $(50 - p)$ kg of raisins

Step 4: $50(\$3.20) = p(\$2.89) + (50 - p)(\$3.75)$

Step 5: $\$160.00 = \$2.89p + \$187.50 - \$3.75p$
$-\$27.50 = -\$0.86p$
$p = 31.98$ kg
32.0 kg of peanuts should be mixed with 18.0 kg of raisins.

16. Step 2: Total bill = \$3310. Total hours = 41.
Hourly rate = \$120 for CGA
= \$50 for technician.
Let x represent the CGA's hours.
Step 3: Total bill = (CGA hours x CGA rate) + (Technician hours x Technician rate)

Step 4: $\$3310 = x(\$120) + (41 - x)\$50$

Step 5: $\$3310 = \$120x + \$2050 - \$50x$
$1260 = 70x$
$x = 18$
The CGA worked 18 hours and the technician worked 41 – 18 = 23 hours.

17. Step 2: Total investment = \$32,760
Sue's investment = 1.2(Joan's investment)
Joan's investment = 1.2(Stella's investment)
Let L represent Stella's investment.

Step 3: Sue's investment + Joan's investment + Stella's investment = Total investment

Step 4: Joan's investment = 1.2L
Sue's investment = 1.2L(1.2L) = 1.44L
1.44L + 1.2L + L = \$32,760

Step 5:
$$3.64L = \$32,760$$
$$L = \frac{\$32,760}{3.64} = \$9000$$
Stella will invest \$9000, Joan will invest 1.2(\$9000) = \$10,800, and
Sue will invest 1.2(\$10,800) = \$12,960

18. Step 2: Sven receives 30% less than George (or 70% of George's share).
    Robert receives 25% more than George (or 1.25 times George's share).
    Net income = $88,880
    Let G represent George's share.

    Step 3: George's share + Robert's share + Sven's share = Net income

    Step 4: G + 1.25G + 0.7G = $88,880

    Step 5: 2.95G = $88,880
    G = $30,128.81
    George's share is $30,128.81, Robert's share is 1.25($30,128.81) = $37,661.02, and Sven's share is 0.7($30,128.81) = $21,090.17.

19. Step 2: Time to make X is 20 minutes.
    Time to make Y is 30 minutes.
    Total time is 47 hours. Total units = 120. Let Y represent the number of units of Y.

    Step 3: Total time = (Number of X) × (Time for X) + (Number of Y) × (Time for Y)

    Step 4: 47 × 60 = (120 – Y)20 + Y(30)

    Step 5: 2820 = 2400 – 20Y + 30Y
    420 = 10Y
    Y = 42
    Forty-two units of product Y were manufactured.

20. Step 2: Price of blue ticket = $19.00. Price of red ticket = $25.50.
    Total tickets = 4460. Total revenue = $93,450.
    Let the number of tickets in the red section be R.

    Step 3: Total revenue = (Number of red × Price of red) + (Number of blue × Price of blue)

    Step 4: $93,450 = R($25.50) + (4460 – R)$19.00

    Step 5: 93,450 = 25.5R + 84,740 – 19R
    6.5R = 8710
    R = 1340
    1340 seats were sold in the red section and 4460 – 1340 = 3120 seats were sold in the blue section.

21. Step 2: $3/5$ of a $3/7$ interest was sold for $27,000.
    Let the V represent the implied value of the entire partnership.

    Step 3: $3/5$ of a $3/7$ interest is worth $27,000.

    Step 4: $\frac{3}{5} \times \frac{3}{7} V = \$27,000$

    Step 5: $V = \frac{5 \times 7}{3 \times 3} \times \$27,000 = \$105,000$

    b. The implied value of the entire partnership is $105,000.

    a. The implied value of Shirley's remaining interest is
    $$\frac{2}{5} \times \frac{3}{7} V = \frac{6}{35} \times \$105,000 = \$18,000$$

Exercise 2.4 *(continued)*

22. Step 2: Regal owns a 58% interest in a mineral claim. Yukon owns the remainder (42%).
Regal sells one fifth of its interest for $1.2 million.
Let the V represent the implied value of the entire mineral claim.

Step 3: $\frac{1}{5}$ (or 20%) of a 58% interest is worth $1.2 million

Step 4: 0.20(0.58)V = $1,200,000

Step 5: $V = \frac{\$1,200,000}{0.20 \times 0.58} = \$10,344,828$

The implied value of Yukon's interest is

$0.42V = 0.42 \times \$10,344,828 = \underline{\underline{\$4,344,828}}$

23. Step 2: $\frac{5}{7}$ of entrants complete Level 1. $\frac{2}{9}$ of Level 1 completers fail Level 2.
587 students completed Level 2 last year.
Let the N represent the original number who began Level 1.

Step 3: $\frac{7}{9}$ of $\frac{5}{7}$ of entrants will complete Level 2.

Step 4: $\frac{7}{9} \times \frac{5}{7} N = 587$

Step 5: $N = \frac{9 \times 7}{7 \times 5} \text{ x } 587 = 1056.6$

$\underline{\underline{1057}}$ students began Level 1.

24. Step 2: $\frac{4}{7}$ of inventory was sold at cost.
$\frac{3}{7}$ inventory was sold to liquidators at 45% of cost, yielding $6700.
Let C represent the original cost of the entire inventory.

Step 3: $\frac{3}{7}$ of inventory was sold to liquidators at 45% of cost, yielding $6700.

Step 4: $\frac{3}{7}(0.45C) = \$6700$

Step 5: $C = \frac{7 \times \$6700}{3 \times 0.45} = \$34,740.74$

a. The cost of inventory sold to liquidators was

$\frac{3}{7}(\$34,740.74) = \underline{\underline{\$14,888.89}}$

b. The cost of the remaining inventory sold in the bankruptcy sale was

$\$34,740.74 - \$14,888.89 = \underline{\underline{\$19,851.85}}$

25. Let r represent the number of regular members and s the number of student members.

Then	$r + s = 583$		①
Total revenue:	$\$2140r + \$856s = \$942,028$		②
① × $856:	$\underline{\$856r + \$856s} = \underline{\$499,048}$		
Subtract:	$\$1284r + 0 = \$442,980$		
	$r = 345$		
Substitute into ①:	$345 + s = 583$		
	$s = 238$		

The club had $\underline{\underline{\text{238 student members}}}$ and $\underline{\underline{\text{345 regular members}}}$.

Exercise 2.4 *(continued)*

26. Let c represent the number of children and a represent the number of adults.

Then $c + a = 266$ ①

$\$17.90c + \$25.90a = \$6609.40$ ②

① × \$25.90: $\underline{\$25.90c + \$25.90a = \$6889.40}$

Subtract: $-\$8c + 0 = -\$280$

$c = 35$

That is, 35 of the 266 customers were children.

27. Let s represent the distance travelled at the lower speed (50 km/h).
Let h represent the distance travelled at the higher speed (100 km/h).
Since the total distance = 1000 km,

then $s + h = 1000$ ①

Since travelling time = $\frac{\text{Distance}}{\text{Speed}}$,

then Time at slower speed = $\frac{s}{50}$ and Time at higher speed = $\frac{h}{100}$

Since the total time = 12.3 hours,

then $\frac{s}{50} + \frac{h}{100} = 12.3$ ②

② × 100: $2s + h = 1230$

Repeat ①: $\underline{s + h = 1000}$ ①

Subtract: $s + 0 = 230$

Hence, Tina drive 230 km at 50 km/h and 1000 – 230 = 770 km at 100 km/h.

28. Let a represent the adult airfare and c represent the child airfare.

Mrs. Ramsey's cost: $a + 2c = \$610$ ①

Chudnowskis' cost: $2a + 3c = \$1050$ ②

① × 2: $\underline{2a + 4c = \$1220}$

Subtract: $0 + -c = -\$170$

Substitute c = \$170 into ①: $a + 2(\$170) = \$610$

$a = \$610 - \$340 = \$270$

The airfare is \$270 per adult and \$170 per child.

29. Let h represent the rate per hour and k represent the rate per km.

Vratislav's cost: $2h + 47k = \$54.45$ ①

Bryn's cost: $5h + 93k = \$127.55$ ②

To eliminate x,

① × 5: $10h + 235k = \$272.25$ ①

② × 2: $\underline{10h + 186k = \$255.10}$ ②

Subtract: $0 + 49k = \$\ 17.15$

$k = \$0.35$ per km

Substitute into ①:

$2h + 47(\$0.35) = \$54.45$

$2h = \$54.45 - \$16.45$

$= \$38.00$ per hour

$h = \$19.00$ per hour

Budget Truck Rentals charged \$19.00 per hour plus \$0.35 per km.

Exercise 2.4 *(continued)*

30. Let s represent the weight of 6% nitrogen fertilizer.
Let t represent the weight of 22% nitrogen fertilizer.

Total weight:	$s + t = 300$	①
Total nitrogen:	$0.06s + 0.22t = 0.16(300)$	
Multiply by 100:	$6s + 22t = 4800$	②
① × 6:	$\underline{6s + 6t} = \underline{1800}$	①
Subtract:	$0 + 16t = 3000$ ②	
	$t = 187.5$ kg	
	$s = 300 - 187.5 = 112.5$ kg	

Buckerfield's should mix 112.5 kg of 6% fertilizer with 187.5 kg of 22% fertilizer.

31. Let C represent the interest rate on Canada Savings Bonds.
Let O represent the interest rate on Ontario Savings Bonds.

Year 1 interest:	$4(\$1000)C + 6(\$1000)O = \$438$	①
Year 2 interest:	$3(\$1000)C + 4(\$1000)O = \$306$	②
① × 3:	$\$12{,}000C + \$18{,}000O = \$1314$	①
② × 4:	$\underline{\$12{,}000C + \$16{,}000O} = \underline{\$1224}$	②
Subtract:	$0 + \$2000O = \$\ 90$	

$$O = \frac{\$90}{\$2000} = 0.045 = 4.5\%$$

Substitute into ②: $\$3000C + \$4000(0.045) = \$306$

$$C = \frac{\$306 - \$180}{\$3000} = 0.042 = 4.2\%$$

The Canada Savings Bonds earn 4.2% per annum and
the Ontario Savings Bonds earn 4.5% per annum.

32. Let r represent the tax rate on residences and
let f represent the tax rate on land with farm buildings.

LeClair tax:	$\$400{,}000r + \$300{,}000f = \$3870$	①
Bartoli tax:	$\$350{,}000r + \$380{,}000f = \$3774$	②
① × 7:	$\$2{,}800{,}000r + \$2{,}100{,}000f = \$27{,}090$	①
② × 8:	$\underline{\$2{,}800{,}000r + \$3{,}040{,}000f} = \underline{\$30{,}192}$	②
Subtract:	$0 \quad -\$940{,}000f = -\$3102$	

$$f = \frac{\$3102}{\$940{,}000} = 0.0033 = 0.33\%$$

Substitute into ①: $\$400{,}000r + \$300{,}000(0.0033) = \$3870$

$$r = \frac{\$3870 - \$990}{\$400{,}000} = 0.0072 = 0.72\%$$

The tax rates are 0.72% on residences and 0.33% on land with farm buildings.

33. Let x represent the number of units of product X and
y represent the number of units of product Y. Then

	$x + y = 93$	①
	$0.5x + 0.75y = 60.5$	②
① × 0.5:	$\underline{0.5x + 0.5y} = \underline{46.5}$	
Subtract:	$0 + 0.25y = 14$	
	$y = 56$	
Substitute into ①:	$x + 56 = 93$	
	$x = 37$	

Therefore, 37 units of X and 56 units of Y were produced last week.

Exercise 2.4 *(continued)*

34. Let the price per litre of milk be m and the price per dozen eggs be e. Then

$$5m + 4e = \$19.51 \quad ①$$
$$9m + 3e = \$22.98 \quad ②$$

To eliminate e,

① × 3:	15m + 12e =	\$58.53
② × 4:	36m + 12e =	\$91.92
Subtract:	–21m + 0 =	–\$33.39
	m =	\$1.59

Substitute into ①: $5(\$1.59) + 4e = \$19.51$
$e = \$2.89$

Milk costs <u>\$1.59 per litre</u> and eggs cost <u>\$2.89 per dozen</u>.

35. Let M be the number of litres of milk and J be the number of cans of orange juice per week.

$$\$1.50M + \$1.30J = \$57.00 \quad ①$$
$$\$1.60M + \$1.37J = \$60.55 \quad ②$$

To eliminate M,

① × 1.6:	\$2.40M + \$2.080J =	\$91.200
② × 1.5:	\$2.40M + \$2.055J =	\$90.825
Subtract:	0 + \$0.025J =	\$0.375
	J =	15

Substitution of J = 15 into either equation will give M = 25. Hence, <u>25 litres of milk</u> and <u>15 cans of orange</u> juice are purchased each week.

36. Let S represent the selling price of a case of beer and R represent the refund per case of empties. Then

$$871S – 637R = \$12,632.10 \quad ①$$
$$932S – 805R = \$13,331.70 \quad ②$$

To eliminate S,

① × 932:	811,772S – 593,684R =	\$11,773,117.20
② × 871:	811,772S – 701,155R =	\$11,611,910.70
Subtract:	0 + 107,471R =	\$161,206.50
	R =	\$1.50

The store paid a refund of <u>\$1.50</u> per case.

37. Let S represent the number of people who bought single tickets and T represent the number of people who bought at three-for-\$5. Then

$$S + 3T = 3884 \quad ①$$
$$\$2S + \$5T = \$6925 \quad ②$$

To eliminate S,

① × \$2:	\$2S + \$6T =	\$7768
②:	\$2S + \$5T =	\$6925
Subtract:	0 + \$1T =	\$843
	T =	843

Hence, <u>843</u> people bought tickets at the three-for-\$5 discount.

Exercise 2.4 *(continued)*

38. Let P represent the number of six-packs and C represent the number of single cans sold.

Then $\$4.35P + \$0.90C = \$178.35$ ①

$6P + C = 225$ ②

To eliminate C,

①: $\$4.35P + \$0.90C = \$178.35$

② × \$0.90: $\underline{\$5.40P + \$0.90C = \$202.50}$

Subtract: $-\$1.05P + 0 = -\$24.15$

$P = 23$

Substitute into ②: $6(23) + C = 225$

$C = 87$

The store sold <u>23 six-packs</u> and <u>87 single cans</u>.

39. Let P represent the annual salary of a partner and T represent the annual salary of a technician. Then

$7P + 12T = \$1{,}629{,}000$ ①

$1.05(7P) + 1.08(12T) = \$1{,}734{,}750$ ②

① × 1.05: $\underline{1.05(7P) + 1.05(12T) = \$1{,}710{,}450}$

Subtract: $0 + 0.03(12T) = \$24{,}300$

$T = \$67{,}500$

Substitute into ①: $7P + 12(\$67{,}500) = \$1{,}629{,}000$

$P = \$117{,}000$

The current annual salary of a partner is <u>\$117,000</u> and of a technician is <u>\$67,500</u>.

40. Let P represent the current number of production workers and A the current number of assembly workers. Then

$\$5100P + \$4200A = \$380{,}700$ ①

$\$5100(0.8P) + \$4200(0.75A) = \$297{,}000$ ②

To eliminate P,

① × 0.8: $\$5100(0.8P) + \$4200(0.8A) = \$304{,}560$

②: $\underline{\$5100(0.8P) + \$4200(0.75A) = \$297{,}000}$

Subtract: $\$4200(0.05A) = \$7560$

$A = 36$

Substitute into ①: $\$5100P + \$4200(36) = \$380{,}700$

$P = 45$

Therefore, 0.2P = <u>9 production</u> workers and 0.25A = <u>9 assembly workers</u> will be laid off.

41. Step 2: Each of 4 children receive 0.5(Wife's share).

Each of 13 grandchildren receive $0.\overline{3}$ (Child's share).

Total distribution = \$759,000. Let w represent the wife's share.

Step 3: Total amount = Wife's share + 4(Child's share) + 13(Grandchild's share)

Step 4: $\$759{,}000 = w + 4(0.5w) + 13\left(0.\overline{3}\right)(0.5w)$

Step 5: $\$759{,}000 = w + 2w + 2.1\overline{6}w$

$= 5.1\overline{6}w$

$w = \$146{,}903.23$

Each child will receive 0.5(\$146,903.23) = <u>\$73,451.62</u>

and each grandchild will receive $0.\overline{3}$ (\$73,451.62) = <u>\$24,483.87</u>.

Exercise 2.4 *(continued)*

42. Step 2: Stage B workers = 1.6(Stage A workers)
Stage C workers = 0.75(Stage B workers)
Total workers = 114. Let A represent the number of Stage A workers.

Step 3: Total workers = A workers + B workers + C workers

Step 4: 114 = A + 1.6A + 0.75(1.6A)

Step 5: 114 = 3.8A
A = 30
30 workers should be allocated to Stage A, 1.6(30) = 48 workers to Stage B, and 114 – 30 – 48 = 36 workers to Stage C.

43. Step 2: Hillside charge = 2(Barnett charge) – $1000
Westside charge = Hillside charge + $2000
Total charges = $27,600. Let B represent the Barnett charge.

Step 3: Total charges = Barnett charge + Hillside charge + Westside charge

Step 4: $27,600 = B + 2B – $1000 + 2B – $1000 + $2000

Step 5: $27,600 = 5B
B = $5520
Hence, the Westside charge is 2($5520) – $1000 + $2000 = $12,040

44. Step 2: There are 3 managers and 26 production workers. Total distribution = $100,000.
Manager's share = 1.2 (Production worker's share).
Let p represent a production worker's share.

Step 3: 3(Manager's share) + 26(Production worker's share) = $100,000

Step 4: 3(1.2p) + 26p = $100,000
Step 5: 29.6p = $100,000
p = $3378.38
Each production worker will receive $3378.38 and each manager will receive 1.2($3378.38) = $4054.05.

45. Step 2: Assembly time = 0.5(Cutting time) + 2 minutes
Painting time = 0.5(Assembly time) + 0.5 minutes
Total units = 72. Total time = 42 hours. Let C represent the cutting time.

Step 3: Time to produce one toy = Cutting time + Assembly time + Painting time

Step 4: $\frac{42 \times 60}{72}$ = C + 0.5C + 2 + 0.5(0.5C + 2) + 0.5

Step 5: 35 = 1.75C + 3.5
C = 18 minutes
Cutting requires 18 minutes (per unit), assembly requires 0.5(18)+2 = 11 minutes, and painting requires 0.5(11) + 0.5 = 6 minutes.

Exercise 2.5

1. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{\$100 - \$95}{\$95} \times 100\% = \underline{\underline{5.26\%}}$

2. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{\$95 - \$100}{\$100} \times 100\% = \underline{\underline{-5.00\%}}$

3. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{135kg - 35kg}{35kg} \times 100\% = \underline{\underline{285.71\%}}$

4 $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{35kg - 135kg}{135kg} \times 100\% = \underline{\underline{-74.07\%}}$

5. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{0.13 - 0.11}{0.11} \times 100\% = \underline{\underline{18.18\%}}$

6. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{0.085 - 0.095}{0.095} \times 100\% = \underline{\underline{-10.53\%}}$

7. $V_f = V_i(1+c)$ = \$134.39[1 + (–0.12)] = \$134.39(0.88) = $\underline{\underline{\$118.26}}$

8. $V_f = V_i(1+c)$ = 112g(1 + 1.12) = $\underline{\underline{237.44\text{g}}}$

9. $V_f = V_i(1+c)$ = (26.3 cm)(1 + 3.00) = $\underline{\underline{105.2 \text{ cm}}}$

10. $V_f = V_i(1+c)$ = 0.043[1 + (–0.30)] = $\underline{\underline{0.0301}}$

11. $V_i = \dfrac{V_f}{1+c} = \dfrac{\$75}{1+2.00} = \underline{\underline{\$25.00}}$

12. $V_i = \dfrac{V_f}{1+c} = \dfrac{\$75}{1+(-0.50)} = \underline{\underline{\$150.00}}$

13. Given: V_i = \$90, V_f = \$100

$$c = \frac{\$100 - \$90}{\$90} \times 100\% = \underline{\underline{11.11\%}}$$

\$100 is 11.11% more than \$90.

14. Given: V_i = \$110, V_f = \$100

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$100 - \$110}{\$110} \times 100\% = \underline{\underline{-9.09\%}}$$

\$100 is 9.09% less than \$110.

15. Given: c = 25%, V_f = \$100

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+0.25} = \underline{\underline{\$80.00}}$$

\$80.00 increased by 25% equals \$100.00.

16. Given: c = 7%, V_f = \$52.43

$$V_i = \frac{V_f}{1+c} = \frac{\$52.43}{1+0.07} = \underline{\underline{\$49.00}}$$

\$49.00 increased by 7% equals \$52.43.

Exercise 2.5 *(continued)*

17. Given: V_f = \$75, c = 75%

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+0.75} = \underline{\underline{\$42.86}}$$

\$75 is 75% more than \$42.86.

18. Given: V_i = \$56, c = 65%

$$V_f = V_i(1+c) = \$56(1.65) = \underline{\underline{\$92.40}}$$

\$56 after an increase of 65% is \$92.40.

19. Given: V_i = \$759.00, V_f = \$754.30

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$754.30 - \$759.00}{\$759.00} \times 100\% = \underline{\underline{-0.62\%}}$$

\$754.30 is 0.62% less than \$759.00.

20. Given: V_i = 77,400, V_f = 77,787

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{77{,}787 - 77{,}400}{77{,}400} \times 100\% = \underline{\underline{0.50\%}}$$

77,787 is 0.50% more than 77,400.

21 Given: V_i = \$75, c = 75%

$$V_f = V_i(1 + c) = \$75(1 + 0.75) = \underline{\underline{\$131.25}}$$

\$75.00 becomes \$131.25 after an increase of 75%.

22. Given: V_f = \$100, c = – 10%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.10)} = \underline{\underline{\$111.11}}$$

\$100.00 is 10% less than \$111.11.

23. Given: V_f = \$100, c = – 20%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.20)} = \underline{\underline{\$125.00}}$$

\$125 after a reduction of 20% equals \$100.

24. Given: V_f = \$50, c = – 25%

$$V_i = \frac{V_f}{1+c} = \frac{\$50}{1+(-0.25)} = \underline{\underline{\$66.67}}$$

\$66.67 after a reduction of 25% equals \$50.

25. Given: V_f = \$549, c = $-16.\overline{6}\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$549}{1+(-0.1\overline{6})} = \underline{\underline{\$658.80}}$$

\$658.80 after a reduction of $16.\overline{6}\%$ equals \$549.

26. Given: V_i = \$900, c = –90%

$$V_f = V_i(1 + c) = \$900[1 + (-0.9)] = \underline{\underline{\$90.00}}$$

\$900 after a decrease of 90% is \$90.00.

Exercise 2.5 *(continued)*

27. Given: V_i = \$102, c = –2%

    $V_f = V_j(1 + c)$ = \$102(1 – 0.02) = $\underline{\underline{\$99.96}}$

    \$102 after a decrease of 2% is \$99.96.

28. Given: V_i = \$102, c = –100%

    $V_f = V_i(1 + c)$ = \$102[1 + (–1.00)] = \$102(0) = $\underline{\underline{\$0.00}}$

    Any positive amount after a decrease of 100% is zero.

29. Given: V_i = \$250, V_f = \$750

    $$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$750 - \$250}{\$250} \times 100\% = \underline{\underline{200.00\%}}$$

    \$750 is 200.00% more than \$250.

30. Given: V_i = \$750, V_f = \$250

    $$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$250 - \$750}{\$750} \times 100\% = \underline{\underline{-66.67\%}}$$

    \$250 is <u>66.67% less</u> than \$750.

31. Given: c = 0.75%, V_i = \$10,000

    $V_f = V_i (1 + c)$ = \$10,000(1 + 0.0075) = $\underline{\underline{\$10,075.00}}$

    \$10,000 after an increase of $\frac{3}{4}$% is \$10,075.00.

32. Given: V_i = \$1045, c = – 0.5%

    $V_f = V_i (1 + c) = \$1045\left[1+\left(-0.005\right)\right] = \underline{\underline{\$1039.78}}$

    \$1045 after an decrease of 0.5% is \$1039.78.

33. Given: c = 150%, V_f = \$575

    $$V_i = \frac{V_f}{1+c} = \frac{\$575}{1+1.5} = \underline{\underline{\$230.00}}$$

    \$230.00 when increased by 150% equals \$575.

34. Given: c = 210%, V_f = \$465

    $$V_i = \frac{V_f}{1+c} = \frac{\$465}{1+2.1} = \underline{\underline{\$150.00}}$$

    \$150.00 after being increased by 210% equals \$465.

35. Given: V_i = \$150, c = 150%

    $V_f = V_i (1 + c)$ = \$150(1 + 1.5) = $\underline{\underline{\$375.00}}$

    \$150 after an increase of 150% is \$375.00.

36. Let the retail price be p. Then

    $p + 0.13\,p$ = \$281.37

    $$p = \frac{\$281.37}{1.13} = \underline{\underline{\$249.00}}$$

    The coat's sticker price was \$249.00.

Exercise 2.5 *(continued)*

37. Let the TV's pre-tax price be p. Then

$$p + 0.05p + 0.07p = \$2797.76$$

$$p = \frac{\$2797.76}{1.12} = \$2498.00$$

Then, GST = $0.05p$ = 0.05($2498) = $\underline{\underline{\$124.90}}$
and PST = $0.07p$ = 0.07($2498) = $\underline{\underline{\$174.86}}$

38. Let the population figure for 1999 be p. Then

$$p + 0.1056p = 33{,}710{,}000$$

$$p = \frac{\$33{,}710{,}000}{1.1056} = 30{,}490{,}232$$

Rounded to the nearest 10,000, the population in 1999 was $\underline{\underline{30{,}490{,}000}}$.

39. *a.* . Given: V_i = 32,400, V_f = 27,450

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{27{,}450 - 32{,}400}{32{,}400} \times 100\% = \underline{\underline{-15.28\%}}$$

The number of hammers sold declined by 15.28%.

b. Given: V_i = $15.10, V_f = $15.50

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15.50 - \$15.10}{\$15.10} \times 100\% = \underline{\underline{2.65\%}}$$

The average selling price increased by 2.65%.

c. Year 1 revenue = 32,400($15.10) = $489,240
Year 2 revenue = 27,450($15.50) = $425,475

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$425{,}475 - \$489{,}240}{\$489{,}240} \times 100\% = \underline{\underline{-13.03\%}}$$

The revenue decreased by 13.03%.

40. *a.* Given: V_i = $0.55, V_f = $1.55

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1.55 - \$0.55}{\$0.55} \times 100\% = \underline{\underline{181.82\%}}$$

The share price rose by 181.82% in the first year.

b. Given: V_i = $1.55, V_f = $0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$1.55}{\$1.55} \times 100\% = \underline{\underline{-51.61\%}}$$

The share price declined by 51.61% in the second year.

c. Given: V_i = $0.55, V_f = $0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$0.55}{\$0.55} \times 100\% = \underline{\underline{36.36\%}}$$

The share price rose by 36.36% over 2 years.

Exercise 2.5 *(continued)*

41. Pick an arbitrary price, say $1.00, for a bar of the soap.

The former unit price was $V_i = \dfrac{\$1.00}{100\text{ g}} = \$0.01$ per gram.

The new unit price is $V_f = \dfrac{\$1.00}{90\text{ g}} = \$0.011111$ per gram.

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.011111 - \$0.01}{\$0.01} \times 100\% = \underline{\underline{11.11\%}}$$

42. Initial unit price = $\dfrac{\$5.49}{1.65\,l}$ = $3.327 per litre

Final unit price = $\dfrac{\$7.98}{2.2\,l}$ = $3.627 per litre

The percent increase in the unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$3.627 - \$3.327}{\$3.327} \times 100\% = \underline{\underline{9.02\%}}$$

43. Initial unit price = $\dfrac{\$7.98}{3.6\text{ kg}}$ = $2.2167 per kg

Final unit price = $\dfrac{\$6.98}{3\text{ kg}}$ = $2.3267 per kg

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$2.3267 - \$2.2167}{\$2.2167} \times 100\% = \underline{\underline{4.96\%}}$$

44. Initial unit price = $\dfrac{1098\text{ cents}}{700\text{ g}}$ = 1.5686 cents per g

Final unit price = $\dfrac{998\text{ cents}}{600\text{ g}}$ = 1.6633 cents per g

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{1.6633 - 1.5686}{1.5686} \times 100\% = \underline{\underline{6.04\%}}$$

45. Current unit price = $\dfrac{449\text{ cents}}{500\text{ ml}}$ = 0.8980 cents per ml

New unit price = 1.10(0.8980 cents per ml) = 0.9878 cents per ml

Price of a 425-ml container = (425 ml) × (0.9878 cents per ml) = 419.8 cents = $\underline{\underline{\$4.20}}$

46. Current unit price = $\dfrac{115\text{ cents}}{100\text{ g}}$ = 1.15 cents per g

New unit price = 1.075(1.15 cents per g) = 1.23625 cents per g

Price of an 80-g bar = (80 g) × (1.23625 cents per g) = 98.9 cents = $\underline{\underline{\$0.99}}$

Exercise 2.5 *(continued)*

47. Given: V_f = \$338,500, c = 8.7%

$$V_i = \frac{V_f}{1+c} = \frac{\$338{,}500}{1.087} = \underline{\underline{\$311{,}400}}$$

The average price one year ago was \$311,400.

48. Given: V_f = \$348.60, c = –0.30

$$V_i = \frac{V_f}{1+c} = \frac{\$348.60}{1+(-0.30)} = \frac{\$348.60}{0.70} = \underline{\underline{\$498.00}}$$

The regular price of the boots is \$498.00.

49. For Year 1, V_f = \$6 and $V_f - V_i$ = – \$4

Therefore, $V_i = V_f$ + \$4 = \$6 + \$4 = \$10

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{-\$4}{\$10} \times 100\% = \underline{\underline{-40.00\%}}$$

For Year 2, V_i = \$6 and $V_f - V_i$ = \$4

Therefore, $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$4}{\$6} \times 100\% = \underline{\underline{66.67\%}}$

The percent change was -40.00% in Year 1 and 66.67% in Year 2.

50. Given: For Q2 of 2009, V_f = 5.21 million, c = 626%

$$V_i = \frac{V_f}{1+c} = \frac{5.21\text{ million}}{1+6.26} = 0.7176\text{ million} = 717{,}600$$

Rounded to the nearest 10,000, Apple sold $\underline{\underline{720{,}000}}$ iPhones in Q2 of 2008.

51. Given: In February of 2008, V_i = 475,000 visitors and c = 1382%

In February of 2009, the number of visitors was

$V_f = V_i(1 + c)$ = 475,000(1+13.82) = 7,039,500

Rounded to the nearest 1000, Twitter.com had $\underline{\underline{7{,}040{,}000}}$ visitors in February of 2009.

52. The fees to Fund A will be

$$\frac{(\text{Fees to Fund A}) - (\text{Fees to Fund B})}{(\text{Fees to Fund B})} \times 100\% = \frac{2.38\% - 1.65\%}{1.65\%} \times 100\% = \underline{\underline{44.24\%}}$$

more than the fees to Fund B.

53. Percent change in the GST rate

$$= \frac{(\text{Final GST rate}) - (\text{Initial GST rate})}{(\text{Initial GST rate})} \times 100\% = \frac{5\% - 6\%}{6\%} \times 100\% = -16.67\%$$

The GST paid by consumers was reduced by $\underline{\underline{16.67\%}}$.

54. Given: For February of 2009, V_f = 65,704,000 visitors, c = 228.2%

Then, $V_i = \frac{V_f}{1+c} = \frac{65{,}704{,}000}{1+2.282} = 20{,}019{,}500$

That is, Facebook had 20,019,500 unique visitors in February of 2008

Therefore, the absolute increase from February of 2008 to February of 2009 was

65,704,000 – 20,019,500 = $\underline{\underline{45{,}680{,}000}}$ (rounded to the nearest 10,000)

Exercise 2.5 *(continued)*

55. Given: V_f = \$0.45, c = 76%

$$V_i = \frac{V_f}{1+c} = \frac{\$0.45}{1+(-0.76)} = \$1.88$$

Price decline = $V_i - V_f$ = \$1.88 – \$0.45 = \$1.43

The share price dropped by \$1.43.

56. Given: V_f = \$24,300, c = – 55%

$$V_i = \frac{V_f}{1+c} = \frac{\$24,300}{1+(-0.55)} = \$54,000$$

The amount of depreciation is \$54,000 – \$24,300 = \$29,700.

57. Given: For the appreciation, V_i = Purchase price, c = 140%, V_f = List price

For the price reduction, V_i = List price, c = –10%, V_f = \$172,800

List price = $\frac{V_f}{1+c} = \frac{\$172,800}{1+(-0.1)} = \$192,000$

Original purchase price = $\frac{V_f}{1+c} = \frac{\$192,000}{1+1.4} = \$80,000$

The owner originally paid \$80,000 for the property.

58. Given: For the markup, V_i = Cost, c = 22%, V_f = List price

For the markdown, V_i = List price, c = –10%, V_f = \$17,568

List price = $\frac{V_f}{1+c} = \frac{\$17,568}{1+(-0.10)} = \$19,520$

Cost (to dealer) = $\frac{V_f}{1+c} = \frac{\$19,520}{1+0.22} = \$16,000$

The dealer paid \$16,000 for the car.

59. If General Paint's prices are marked down by 30%, then

General Paint's prices = 0.70(Cloverdale Paint's prices)

Hence, Cloverdale's prices = $\frac{\text{General Paint's prices}}{0.70}$ = 1.4286(General Paint's prices)

Therefore, you will pay 42.86% more at Cloverdale Paint.

60. If the Canadian dollar is worth 6.5% less than the US dollar,

Canadian dollar = (1 – 0.065)(US dollar) = 0.935(US dollar)

Hence, US dollar = $\frac{\text{Canadian dollar}}{0.935}$ = 1.0695(Canadian dollar)

Therefore, the US dollar is worth 6.95% more than the Canadian dollar.

61. Canada's exports to US exceeded imports from the US by 23%.

That is, Exports = 1.23(Imports)

Therefore, Imports = $\frac{\text{Exports}}{1.23}$ = 0.8130(Exports)

That is, Canada's imports from US (= US exports to Canada) were

1 – 0.8130 = 0.1870 = 18.70%

less than Canada's exports to US (= US imports from Canada.)

Exercise 2.5 *(continued)*

62. Given: January sales were 17.4% less than December sales
Hence, January sales = (1 – 0.174)(December sales) = 0.826(December sales)

$$\text{Therefore, December sales} = \frac{\text{January sales}}{0.826} = 1.2107(\text{January sales})$$

That is, December sales were 121.07% of January sales.

63. Suppose the initial ratio is $\frac{x}{y}$.

If the denominator is reduced by 20%, then

$$\text{Final ratio} = \frac{x}{y - 0.20y} = \frac{x}{0.8y} = 1.25\frac{x}{y}$$

That is, the value of the ratio increases by 25%.

64. Next year there must be 15% fewer students per teacher.
With the same number of students,

$$\frac{\text{Students}}{\text{Teachers next year}} = 0.85\left(\frac{\text{Students}}{\text{Teachers now}}\right)$$

$$\text{Therefore,} \quad \text{Teachers next year} = \frac{\text{Teachers now}}{0.85} = 1.1765(\text{Teachers now})$$

That is, if the number of students does not change, the number of teachers must be increased by 17.65%.

65. Given: Operating expenses = 0.40(Revenue)

$$\text{Then} \quad \text{Revenue} = \frac{\text{Operating expenses}}{0.40} = 2.5(\text{Operating expenses})$$

That is, Revenue is 250% of Operating expenses, or
Revenue exceeds Operating expenses by 250% – 100% = 150%.

66. Given: Equity = (100% – 50%) of Debt = 50% of Debt = 0.50(Debt)

$$\text{Therefore,} \ \frac{\text{Debt}}{\text{Equity}} = \frac{\text{Debt}}{0.5(\text{Debt})} = \frac{1}{0.5} = 2$$

Since Debt is twice (or 200% of) Equity, then debt financing is 100% more than equity financing.

67. Use ppm as the abbreviation for “pages per minute”.
Given: Lightning printer prints 30% more ppm than the Reliable printer.
That is, the Lightning’s printing speed is 1.30 times the Reliable’s printing speed.
Therefore, the Reliable’s printing speed is

$$\frac{1}{1.3} = 0.7692 = 76.92\% \text{ of the Lightning's printing speed}$$

Therefore, the Reliable’s printing speed is
100% – 76.92% = 23.08% less than the Lighting’s speed.
The Lightning printer will require 23.08% less time than the Reliable for a long printing job.

68. Given: Euro is worth 39% more than the Canadian dollar.
That is, Euro = 1.39(Canadian dollar)

$$\text{Therefore,} \quad \text{Canadian dollar} = \frac{\text{Euro}}{1.39} = 0.7914(\text{Euro}) = 79.14\% \text{ of a Euro.}$$

That is, the Canadian dollar is worth 100% – 79.14% = 28.06% less than the Euro.

Exercise 2.5 *(continued)*

69. Let us use OT as an abbreviation for "overtime".
The number of OT hours permitted by this year's budget is

$$\text{OT hours (this year)} = \frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

The number of overtime hours permitted by next year's budget is

$$\text{OT hours (next year)} = \frac{\text{OT budget (next year)}}{\text{OT hourly rate (next year)}} = \frac{1.03\,[\text{OT budget (this year)}]}{1.05\,[\text{OT hourly rate (this year)}]}$$

$$= 0.980952\,\frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

= 98.0952% of this year's OT hours

The number of OT hours must be reduced by 100% – 98.0952% = 1.90%.

Review Problems

1. 4(3a + 2b)(2b – a) – 5a(2a – b) = 4(6ab – 3a^2 + 4b^2 – 2ab) – 10a^2 + 5ab
= – 22a^2 + 21ab + 16b^2

2. *a.* Given: c = 17.5%, V_i = \$29.43
$V_f = V_i(1 + c)$ = \$29.43(1.175) = \$34.58
\$34.58 is 17.5% more than \$29.43.

b. Given: V_f = \$100, c = – 80%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.80} = \$500.00$$

80% off \$500 leaves \$100.

c. Given: V_f = \$100, c = – 15%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.15} = \$117.65$$

\$117.65 reduced by 15% equals \$100.

d. Given: V_i = \$47.50, c = 320%
$V_f = V_i(1 + c)$ = \$47.50(1 + 3.2) = \$199.50
\$47.50 after an increase of 320% is \$199.50.

e. Given: c = – 62%, V_f = \$213.56

$$V_i = \frac{V_f}{1+c} = \frac{\$213.56}{1-0.62} = \$562.00$$

\$562 decreased by 62% equals \$213.56.

f. Given: c = 125%, V_f = \$787.50

$$V_i = \frac{V_f}{1+c} = \frac{\$787.50}{1+1.25} = \$350.00$$

\$350 increased by 125% equals \$787.50.

g. Given: c = – 30%, V_i = \$300

$$V_f = V_i(1+c) = \$300(1-0.30) = \$210.00$$

\$210 is 30% less than \$300.

Review Problems ***(continued)***

3. *a.* $\dfrac{9y-7}{3}-2.3(y-2)=3y-2.\overline{3}-2.3y+4.6=\underline{\underline{0.7y+2.2\overline{6}}}$

   b. $P\left(1+0.095\times\dfrac{135}{365}\right)+\dfrac{2P}{1+0.095\times\frac{75}{365}}=1.035137P+1.961706P=\underline{\underline{2.996843P}}$

4. *a.* $6(4y-3)(2-3y)-3(5-y)(1+4y)=6(8y-12y^2-6+9y)-3(5+20y-y-4y^2)$
   $=\underline{\underline{-60y^2+45y-51}}$

   b. $\dfrac{5b-4}{4}-\dfrac{25-b}{1.25}+\dfrac{7}{8}b=1.25b-1-20+0.8b+0.875b=\underline{\underline{2.925b-21}}$

   c. $\dfrac{x}{1+0.085\times\frac{63}{365}}+2x\left(1+0.085\times\dfrac{151}{365}\right)=0.985541x+2.070329x=\underline{\underline{3.05587x}}$

   d. $\dfrac{96nm^2-72n^2m^2}{48n^2m}=\dfrac{4m-3nm}{2n}=\dfrac{4m}{2n}-\dfrac{3nm}{2n}=\underline{\underline{2\dfrac{m}{n}-1.5m}}$

5. $P(1+i)^n+\dfrac{S}{1+rt}=\$2500(1.1025)^2+\dfrac{\$1500}{1+0.09\times\frac{93}{365}}=\$3038.766+\$1466.374=\underline{\underline{\$4505.14}}$

6. *a.* $L(1-d_1)(1-d_2)(1-d_3)=\$340(1-0.15)(1-0.08)(1-0.05)=\underline{\underline{\$252.59}}$

   b. $\dfrac{R}{i}\left[1-\dfrac{1}{(1+i)^n}\right]=\dfrac{\$575}{0.085}\left[1-\dfrac{1}{(1+0.085)^3}\right]=\$6764.706(1-0.7829081)=\underline{\underline{\$1468.56}}$

7. *a.* $\dfrac{(-3x^2)^3(2x^{-2})}{6x^5}=\dfrac{(-27x^6)(2x^{-2})}{6x^5}=\underline{\underline{-\dfrac{9}{x}}}$

   b. $\dfrac{(-2a^3)^{-2}(4b^4)^{3/2}}{(-2b^3)(0.5a)^3}=\dfrac{\left(\dfrac{1}{4a^6}\right)(8b^6)}{(-2b^3)(0.125a^3)}=\underline{\underline{-\dfrac{8b^3}{a^9}}}$

8. $\left(-\dfrac{2x^2}{3}\right)^{-2}\left(\dfrac{5^2}{6x^3}\right)\left(-\dfrac{15}{x^5}\right)^{-1}=\left(\dfrac{3}{2x^2}\right)^2\left(\dfrac{25}{6x^3}\right)\left(-\dfrac{x^5}{15}\right)=\underline{\underline{-\dfrac{5}{8x^2}}}$

9. *a.* $1.0075^{24}=\underline{\underline{1.19641}}$

   b. $(1.05)^{1/6}-1=\underline{\underline{0.00816485}}$

   c. $\dfrac{(1+0.0075)^{36}-1}{0.0075}=\underline{\underline{41.1527}}$

   d. $\dfrac{1-(1+0.045)^{-12}}{0.045}=\underline{\underline{9.11858}}$

10. *a.* $\dfrac{(1.00\overline{6})^{240}-1}{0.00\overline{6}}=\dfrac{4.926802-1}{0.00\overline{6}}=\underline{\underline{589.020}}$

    b. $(1+0.025)^{1/3}-1=\underline{\underline{0.00826484}}$

11. *a.* $\frac{2x}{1+0.13\times\frac{92}{365}}+x\left(1+0.13\times\frac{59}{365}\right)=\$831$

$1.936545x + 1.021014x = \$831$

$2.957559x = \$831$

$x = \underline{\underline{\$280.97}}$

b. $3x(1.03^5) + \frac{x}{1.03^3}+x=\frac{\$2500}{1.03^2}$

$3.47782x + 0.91514x + x = \$2356.49$

$x = \underline{\underline{\$436.96}}$

12. *a.* $\frac{x}{1.08^3}+\frac{x}{2}(1.08)^4=\$850$

$0.793832x + 0.680245x = \$850$

$x = \underline{\underline{\$576.63}}$

Check: $\frac{\$576.63}{1.08^3}+\frac{\$576.63}{2}(1.08)^4=\$457.749+\$392.250=\$850.00$

b. $2x\left(1+0.085\times\frac{77}{365}\right)+\frac{x}{1+0.085\times\frac{132}{365}}=\$1565.70$

$2.03586x + 0.97018x = \$1565.70$

$x = \underline{\underline{\$520.85}}$

Check:

$2(\$520.85)\left(1+0.085\times\frac{77}{365}\right)+\frac{\$520.85}{1+0.085\times\frac{132}{365}}=\$1060.38+\$505.32=\$1565.70$

13. $N = L(1-d_1)(1-d_2)(1-d_3)$

$\$324.30 = \$498(1 - 0.20)(1 - d_2)(1 - 0.075)$

$\$324.30 = \$368.52(1 - d_2)$

$\frac{\$324.30}{\$368.52} = (1 - d_2)$

$d_2 = 1 - 0.8800 = \underline{\underline{0.120}} = \underline{\underline{12.0\%}}$

14. $V_f = V_i(1 + c_1)(1 + c_2)(1 + c_3)$

$\$586.64 = \$500(1 + 0.17)(1 + c_2)(1 + 0.09)$

$\$586.64 = \$637.65(1 + c_2)$

$1 + c_2 = \frac{\$586.64}{\$637.65}$

$c_2 = 0.9200 - 1 = \underline{\underline{-0.0800 = -8.00\%}}$

15. 3x + 5y = 11 ①

2x – y = 16 ②

To eliminate y,

①: 3x + 5y = 11

② × 5: 10x – 5y = 80

Add: 13x + 0 = 91

x = 7

Substitute into equation ②: 2(7) – y = 16

y = –2

Hence, $\underline{\underline{(x, y)}} = \underline{\underline{(7, -2)}}$

Review Problems *(continued)*

16\. *a.*

$$4a - 5b = 30 \quad ①$$
$$2a - 6b = 22 \quad ②$$

To eliminate a,

① × 1: $a - 5b = 30$

② × 2: $\underline{4a - 12b} = \underline{44}$

Subtract: $7b = -14$

$b = -2$

Substitute into ①: $4a - 5(-2) = 30$

$4a = 30 - 10$

$a = 5$

Hence, $\underline{(a, b)} = \underline{(5, -2)}$

b.

$$76x - 29y = 1050 \quad ①$$
$$-13x - 63y = 250 \quad ②$$

To eliminate ①,

① × 13: $988x - 377y = 13,650$

② × 76: $-\underline{988x - 4788y} = \underline{19,000}$

Add: $-5165y = 32,650$

$y = -6.321$

Substitute into ①: $76x - 29(-6.321) = 1050$

$76x = 1050 - 183.31$

$x = 11.40$

Hence, $\underline{(x, y) = (11.40, -6.32)}$

17\. $FV = PV(1 + i_1)(1 + i_2)$

$$\frac{FV}{PV(1+i_2)} = (1 + i_1)$$

$$i_1 = \frac{FV}{PV(1+i_2)} - 1$$

18\. Given:

	Year 1 value (V_i)	Year 2 *value* (V_f)
Gold produced:	34,300 oz.	23,750 oz.
Average price:	$1160	$1280

a. Percent change in gold production = $\frac{23,750 - 34,300}{34,300} \times 100\% = \underline{\underline{-30.76\%}}$

b. Percent change in price = $\frac{\$1280 - \$1160}{\$1160} \times 100\% = \underline{\underline{10.34\%}}$

c. Year 1 revenue, V_i = 34,300($1160) = $39.788 million
Year 2 revenue, V_f, = 23,750($1280) = $30.400 million

Percent change in revenue = $\frac{\$30.400 - \$39.788}{\$39.788} \times 100\% = \underline{\underline{-23.60\%}}$

19\. Given: For the first year, V_i = $3.40, V_f = $11.50.
For the second year, V_i = $11.50, c = – 35%.

a. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11.50 - \$3.40}{\$3.40} \times 100\% = \underline{\underline{238.24\%}}$

The share price increased by 238.24% in the first year.

b. Current share price, $V_f = V_i(1 + c)$ = $11.50(1 – 0.35) = $\underline{\underline{\$7.48}}$.

Review Problems *(continued)*

20. Given: For the first year, c = 150%

For the second year, c = – 40%, V_f = \$24

The price at the beginning of the second year was

$$V_i = \frac{V_f}{1+c} = \frac{\$24}{1-0.40} = \$40.00 = V_f \text{ for the first year.}$$

The price at the beginning of the first year was

$$V_i = \frac{V_f}{1+c} = \frac{\$40.00}{1+1.50} = \underline{\underline{\$16.00}}$$

Barry bought the stock for \$16.00 per share.

21. Given: Last year's revenue = \$2,347,000

Last year's expenses = \$2,189,000

a. Given: Percent change in revenue = 10%; Percent change in expenses = 5%

Anticipated revenues, $V_f = V_i(1 + c)$ = \$2,347,000(1.1) = \$2,581,700

Anticipated expenses = \$2,189,000(1.05) = \$2,298,450

Anticipated profit = \$283,250

Last year's profit = \$2,347,000 – \$2,189,000 = \$158,000

Percent increase in profit = $\frac{\$283,250-\$158,000}{\$158,000} \times 100\% = \underline{\underline{79.27\%}}$

b. Given: c(revenue) = –10%; c(expenses) = – 5%

Anticipated revenues = \$2,347,000(1 – 0.10) = \$2,112,300

Anticipated expenses = \$2,189,000(1 – 0.05) = \$2,079,550

Anticipated profit \$32,750

Percent change in profit = $\frac{\$32,750-\$158,000}{\$158,000} \times 100\% = \underline{\underline{-79.27\%}}$

The operating profit will decline by 79.27%.

22. Given: Ken's share = 0.80(Hugh's share) + \$15,000; Total distribution = \$98,430

Let H represent Hugh's share. Then

Hugh's share + Ken's share = Total distribution

H + 0.8H + \$15,000 = \$98,430

1.8H = \$83,430

H = \$46,350

Hugh should receive \$46,350 and Ken should receive \$98,430 – \$46,350 = $\underline{\underline{\$52,080}}$.

23. Given: Grace's share = 1.2(Kajsa's share); Mary Anne's share = $\frac{5}{8}$ (Grace's share)

Total allocated = \$36,000

Let K represent Kajsa's share.

(Kajsa's share) + (Grace's share) + (Mary Anne's share) = \$36,000

K + 1.2K + $\frac{5}{8}$(1.2K) = \$36,000

2.95 K = \$36,000

K $\underline{\underline{= \$12,203.39}}$

Kajsa's should receive \$12,203.39. Grace should receive 1.2K = $\underline{\underline{\$14,644.07}}$.

Mary Anne should receive $\frac{5}{8}$ (\$14,644.07) = $\underline{\underline{\$9152.54}}$.

Review Problems *(continued)*

24. Let R represent the price per kg for red snapper and let L represent the price per kg for ling cod. Then

$$370R + 264L = \$2454.20 \quad ①$$
$$255R + 304L = \$2124.70 \quad ②$$

To eliminate R,

① ÷ 370: $R + 0.71351L = \$6.6330$

② ÷ 255: $\underline{R + 1.19216L} = \underline{\$8.3322}$

Subtract: $-0.47865L = -\$1.6992$

$L = \$3.55$

Substitute into ①: $370R + 264(\$3.55) = \$2454.20$

$370R = \$1517.00$

$R = \$4.10$

Nguyen was paid $\underline{\underline{\text{\$3.55 per kg for ling cod}}}$ and $\underline{\underline{\text{\$4.10 per kg for red snapper}}}$.

25. Let b represent the base salary and r represent the commission rate. Then

$$r(\$27{,}000) + b = \$2815.00 \quad ①$$
$$\underline{r(\$35{,}500) + b} = \underline{\$3197.50} \quad ②$$

Subtract: $-\$8500r = \$382.50$

$r = 0.045$

Substitute into ①: $0.045(\$27{,}000) + b = \$2815$

$b = \$1600$

Deanna's base salary is $\underline{\underline{\text{\$1600 per month}}}$ and her commission rate is $\underline{\underline{4.5\%}}$.

26. Given: Total initial investment = \$7800; Value 1 year later = \$9310
Percent change in ABC portion = 15%
Percent change in XYZ portion = 25%

Let X represent the amount invested in XYZ Inc.

The solution "idea" is:

(Amount invested in ABC)1.15 + (Amount invested in XYZ)1.25 = \$9310

Hence,

$$(\$7800 - X)1.15 + (X)1.25 = \$9310$$
$$\$8970 - 1.15X + 1.25X = \$9310$$
$$0.10X = \$9310 - \$8970$$
$$X = \$3400$$

Rory invested $\underline{\underline{\text{\$3400 in XYZ}}}$ Inc. and \$7800 – \$3400 = $\underline{\underline{\text{\$4400 in ABC}}}$ Ltd.

27. Let the regular season ticket prices be R for the red section and B for the blue section. Then

$$2500R + 4500B = \$50{,}250 \quad ①$$
$$2500(1.3R) + 4500(1.2B) = \$62{,}400 \quad ②$$

① × 1.2: $\underline{2500(1.2R) + 4500(1.2B)} = \underline{\$60{,}300}$

Subtract: $2500(0.1R) + 0 = \$2100$

$R = \$8.40$

Substitute into ①: $2500(\$8.40) + 4500B = \$50{,}250$

$B = \$6.50$

The ticket prices for the playoffs cost

$1.3 \times \$8.40 = \underline{\underline{\$10.92}}$ in the "reds"

and $1.2 \times \$6.50 = \underline{\underline{\$7.80}}$ in the "blues".

Review Problems *(continued)*

28. 60% of a $\frac{3}{8}$ interest was purchased for $25,000.

Let the V represent the implied value of the entire partnership.

Then $0.60 \times \frac{3}{8} V = \$25{,}000$

$$V = \frac{8 \times \$25{,}000}{0.60 \times 3} = \underline{\underline{\$111{,}111}}$$

The implied value of the chalet was $111,111.

29. Let S represent the number of cucumbers sold individually and let F represent the number of four-cucumber packages sold in the promotion. Then

$$S + 4F = 541 \quad ①$$
$$\$0.98S + \$2.94F = \$418.46 \quad ②$$

To eliminate S,

$$① \times \$0.98: \quad \$0.98S + \$3.92F = \$530.18$$
$$②: \quad \underline{\$0.98S + \$2.94F} = \underline{\$418.46}$$
$$\text{Subtract:} \quad 0 + \$0.98F = \$111.72$$
$$F = 114$$

Hence, a total of $4 \times 114 = \underline{\underline{456 \text{ cucumbers}}}$ were sold on the four-for-the-price-of-three promotion.