Chapter 2

Consumption, investment and the capital market

Solutions to questions

1. The role of the company is to invest the funds supplied by its owners in activities that will increase the wealth of its owners.

   The role of shareholders is to supply capital to the company. Each shareholder can allocate his or her wealth between consumption and investment, and can also borrow or lend via the capital market.

   The role of the capital market is to facilitate borrowing or lending by individuals. By so doing, individuals can allocate their wealth between consumption and investment so as to achieve a pattern of consumption over time that maximises their utility. The role of the company then is reduced to that of maximising the wealth (present value) of the shareholders.

2. Modern finance theory is built on the behaviour of individual shareholders, companies and the capital market. Since Fisher's Separation Theorem combines these three elements, it is not surprising that many basic notions of modern finance theory can be found in the theorem. Rational individuals are interested in maximising their utility and an investor's utility is directly related to his wealth level. Companies can maximise investors' wealth by investing in all projects with a net present value greater than zero.

3. It follows from Fisher's Separation Theorem that a company's investment decisions can be made independently of the consumption preferences of the company's shareholders. As a result, there is a unique preferred level of investment that will make all the company's shareholders better off. It also follows that the company's financing and dividend decisions are irrelevant.

4. With certainty, and perfect and complete markets, Fisher's analysis shows that in order to maximise shareholders' wealth, a company should accept all investments that have a positive net present value. The financing and dividend decisions are irrelevant. In this case, financial decision-making is, indeed, trivial.

5. The most important implication of different borrowing and lending rates is that (except under fairly restrictive assumptions) Fisher's Separation Theorem breaks down—that is, there is no unique set of financial decisions which a company can make that will be favoured by all shareholders.

Solutions to problems

1. (a) The net present values of the projects are:

$$NPV(Small) = \frac{\$12\,100}{1.05} - \$11\,000$$
$$= \$523.81$$
$$NPV(Upgrade) = \frac{\$6\,500}{1.05} - \$6\,000$$
$$= \$190.48$$

Because both NPVs are positive, both projects are acceptable investments.

(b) First calculate the total dividend payouts, and then the dividends per shareholder.
Total dividend payouts are:
Today: \$20 000 − \$11 000 − \$6 000 = \$3 000
Later: \$12 100 + \$6 500 = \$18 600.

The shareholders hold equal numbers of shares so dividends per shareholder are:
Today: \$3 000 / 2 = \$1 500
Later: \$18 600 / 2 = \$9 300.

(c) Shareholder A will need to borrow \$4 000 − \$1 500 = \$2 500. The repayment required, with 5% interest, is \$2 500 × 1.05 = \$2 625.

Her consumption per period is shown in the table below:

	Now	**Later**
Dividend	\$1 500	\$9 300
Borrow / Repay	\$2 500	−\$2 625
Consumption	\$4 000	\$6 675

If the company had instead undertaken only Project Small, then Shareholder A's consumption outcomes would have been:

	Now	**Later**
Dividend	\$4 500[(a)]	\$6 050[(b)]
Lend / Receive	–\$500	\$525[(c)]
Consumption	\$4 000	\$6 575

(a) (\$20 000 − \$11 000) / 2 = \$4 500
(b) \$12 100 / 2 = \$6 050
(c) \$500 × 1.05 = \$525

Under this alternative, Shareholder A has the same consumption now (\$4 000), but lower consumption later (\$6 575 < \$6 675).

(d) Let *P* be the amount that Shareholder A must borrow now, which, with interest added, will require a repayment of 1.05*P* later. The consumption outcomes are shown in the table below:

	Now	Later
Dividend	\$1 500	\$9 300
Borrow / Repay	P	$-1.05P$
Consumption	\$1 500 + P	\$9 300 − $1.05P$

Equate consumption now and consumption later and solve for P:

$$\$1\,500 + P = \$9\,300 - 1.05P$$

$$2.05P = \$7\,800$$

$$P = \$3\,804.88$$

Shareholder A should borrow \$3 804.88 today. The repayment required is, thus, \$3 804.88 × 1.05 = \$3 995.12.

The consumption outcomes are shown in the table below.

	Now	Later
Dividend	\$1 500.00	\$9 300.00
Borrow / Repay	\$3 804.88	−\$3 995.12
Consumption	\$5 304.88	\$5 304.88

As required, Shareholder A is able to consume the same amount in both periods.

2. (a) The net present values of the projects are:

$$NPV(Small) = \frac{\$12\,100}{1.09} - \$11\,000$$

$$= \$100.92$$

$$NPV(Upgrade) = \frac{\$6500}{1.09} - \$6000$$

$$= -\$36.70$$

Project Small is accepted because it has a positive NPV. Project Upgrade is rejected because it has a negative NPV.

(b) First calculate the total dividend payouts, and then the dividends per shareholder.
Total dividend payouts are:
Today: \$20 000 − \$11 000 = \$9 000
Later: \$12 100

The shareholders hold equal shares. Therefore, dividends per shareholder are:
Today: \$9 000 / 2 = \$4 500
Later: \$12 100 / 2 = \$6 050.

(c) Shareholder A will lend \$4 500 − \$4 000 = \$500. The future repayment received, with 9% interest, is \$500 × 1.09 = \$545.

Her consumption per period is shown in the table below:

	Now	Later
Dividend	$4 500	$6 050
Lend / Receive	–$500	$545
Consumption	$4 000	$6 575

(d) In Problem 1(c), consumption is $4 000 today and $6 675 later. In Problem 2(c), consumption is $4 000 today and $6 595 later. Therefore, Shareholder A has greater consumption, and hence greater utility, in Problem 1 than in Problem 2. Note that in Problem 1, Shareholder A borrows at the interest rate of 5%, while in Problem 2, Shareholder A lends at the interest rate of 9%. Thus, counter-intuitively, it is possible that a lender (like Shareholder A in Problem 2) can *benefit* if interest rates fall. A more general case of this phenomenon is presented in Problem 4.

3. (a) The net present value of the investment by the company is $2 million. Therefore, the present value of the investment is $8 million. As the return occurs in 1 year's time, the amount to be received at that time is $9.2 million (= $8 million × 1.15).

(b) See Figure 2.1.

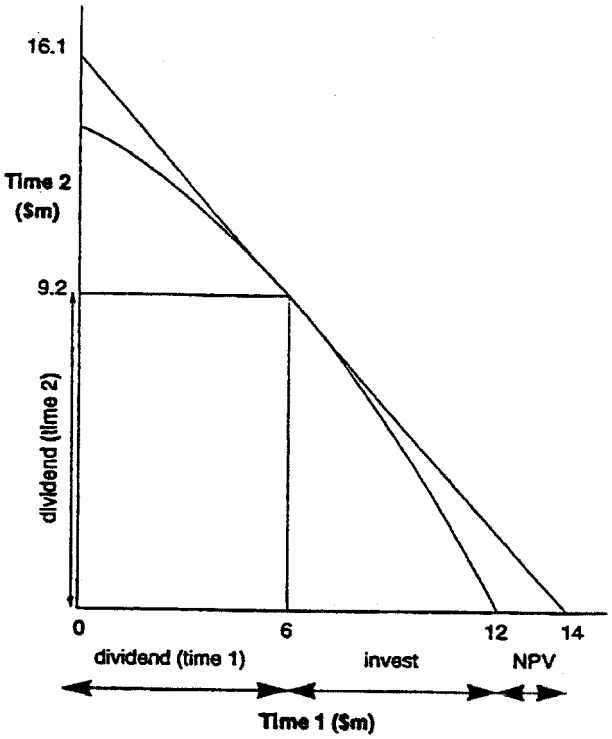

Figure 2.1

(c) The marginal rate of return on the investment—that is, the return on the incremental unit of investment—is 15 per cent. The average rate of return on the investment is 53.33 per cent (= $3.2 million/$6 million).

(d) The total wealth of the company's shareholders, immediately after the investment plan is announced, is $14 million (= $12 million + $2 million).

4. See Figure 2.2 which shows that the net present value of the revised investment plan is approximately $3 million. This is $1 million greater than previously—that is, before the

fall in the interest rate—and occurs because some investments that had a negative net present value at the higher interest rate now have a positive net present value (as is evidenced by the increased level of investment).

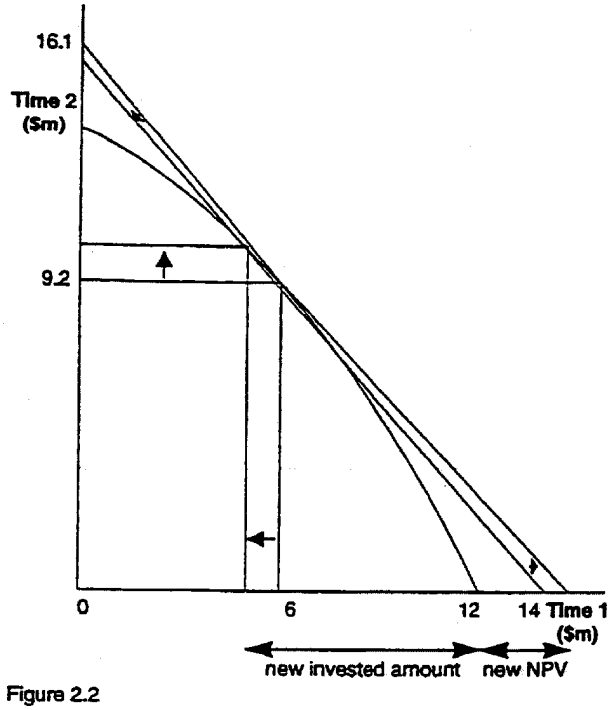

Figure 2.2

Investors are affected in two ways by the fall in the interest rate:

First, all investors benefit from the increase in the net present value (which was discussed above).

Second, all investors now face a different interest rate. Investors who were previously borrowers are better off after the decrease in the interest rate, since they gain from both the increase in the company's net present value and the fall in the interest rate they must pay on their borrowings. However, it is not obvious that investors who were also lenders have been made better off. While they gain from the increase in the company's net present value, they lose because they receive a lower rate of interest on the funds that they lend. Figure 2.3 shows the potential for some investors to be made worse off by the fall in the interest rate, as the new market opportunity line intersects the old market opportunity line. However, it should be noted that the point of intersection is to the left of the point of tangency of the old market opportunity line with the production possibility curve. This indicates that some investors, who were previously small lenders, are now small borrowers and have been made better off by the fall in interest rate.

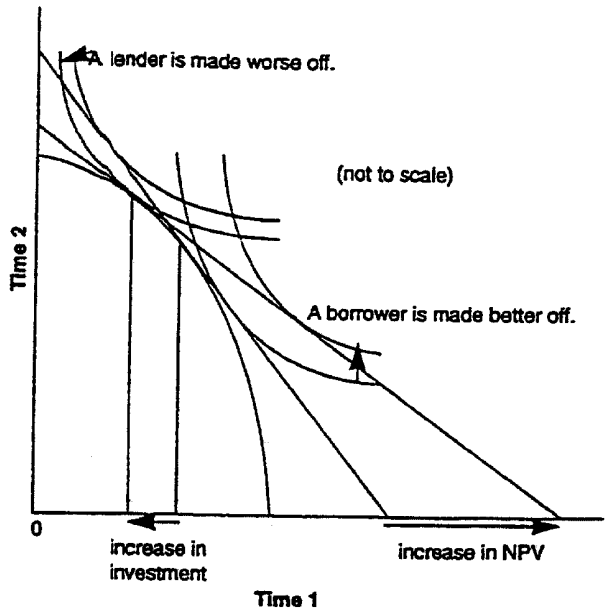

Figure 2.3

5. Figure 2.4 shows that all shareholders have been made worse off by the company's increased investment. This is because the company has invested in projects that have a negative net present value.

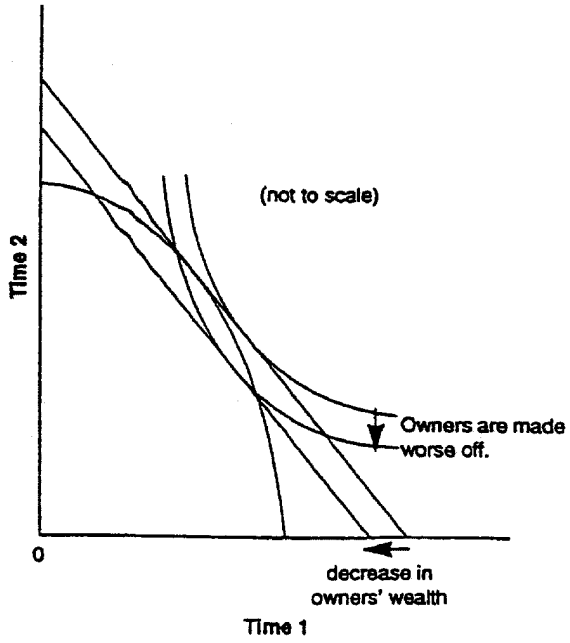

Figure 2.4

6. (a) In the absence of a capital market, the person cannot easily engage in any borrowing or lending activities. At time zero, she will consume \$40, which means she has \$160 available for her consumption in Year 1. Together with her endowment in Year 1, the person has a total of \$260 available for consumption. In Year 2, the person wants to consume \$120, which is \$70 more than her endowment in that period (\$50). This means that \$70 has to be put aside in Year 1 for consumption in Year 2. Hence, the

person's consumption in Year 1 is $190 (= $260 – $70). This is shown in the table below:

	Year 0	**Year 1**	**Year 2**
Desired consumption	$40	?	$120
Endowment	$200	$100	$50
Distribution of Year 0 surplus	–$160	+$160	
Distribution of Year 1 surplus	___	–$70	+$70
Actual consumption	$40	$190	$120

(b) With a capital market, the person can lend $160 now at i = 5 per cent. This would give her $168 in Year 1 (= $160 × 1.05). Since she needs an extra $70 to achieve her consumption target in Year 2, the person could set aside $66.67 in Year 1 and lend this amount at i = 5 per cent, (i.e. $70 = $66.67 × 1.05). Therefore, her consumption in Year 1 is $100 + $168 – $66.67, which is equal to $201.33. Compared with (a) above, the existence of a capital market has enabled the person to increase her consumption in Year 1 without causing any reduction in consumption in the other two years. This is shown in the table below.

	Year 0	**Year 1**	**Year 2**
Desired consumption	$40	?	$120
Endowment	$200	$100.00	$50
Distribution of Year 0 surplus[(a)]	–$160	+$168.00	
Distribution of Year 1 surplus[(a)]		–$66.67	+$70
Actual consumption	$40	$201.33	$120

[(a)] with interest

Note: $160 × 1.05 = $168
$66.67 × 1.05 = $70