Chapter 2
Linear Equations and Inequalities in One Variable

2.1 The Addition Property of Equality

Classroom Examples, Now Try Exercises

1. Note: When solving equations we will write "Add 5" as a shorthand notation for "Add 5 to each side" and "Subtract 5" as a notation for "Subtract 5 from each side."

$x-12=-3$ Given

$x-12+12=-3+12$ Add 12.

$x=9$ Combine like terms.

We check by substituting 9 for x in the *original* equation.

Check $x-12=-3$ Original equation

$9-12\stackrel{?}{=}-3$ Let $x=9$.

$-3=-3$ True

Since a true statement results, $\{9\}$ is the solution set.

N1. $x-13=4$ Given

$x-13+13=4+13$ Add 13.

$x=17$ Combine like terms.

We check by substituting 17 for x in the *original* equation.

Check $x-13=4$ Original equation

$17-13\stackrel{?}{=}4$ Let $x=17$.

$4=4$ True

Since a true statement results, $\{17\}$ is the solution set.

2. $m-4.1=-6.3$

$m-4.1+4.1=6.3+4.1$ Add 4.1.

$m=-2.2$

Check $m=-2.2$: $-6.3=-6.3$ True

This is a shorthand notation for showing that if we substitute -2.2 for m, both sides are equal to -6.3, and hence a true statement results. In practice, this is what you will do, especially if you're using a calculator.

The solution set is $\{-2.2\}$.

N2. $t-5.7=-7.2$

$t-5.7+5.7=-7.2+5.7$ Add 5.7.

$t=-1.5$

Check $t=-1.5$: $-7.2=-7.2$ True

The solution set is $\{-1.5\}$.

3. $-22=x+16$

$-22-16=x+16-16$ Subtract 16.

$-38=x$

Check $x=-38$: $-22=-22$ True

The solution set is $\{-38\}$.

N3. $-15=x+12$

$-15-12=x+12-12$ Subtract 12.

$-27=x$

Check $x=-27$: $-15=-15$ True

The solution set is $\{-27\}$.

4. $11z-9=12z$

$11z-9-11z=12z-11z$ Subtract $11z$.

$-9=z$

Check $z=-9$: $-108=-108$ True

The solution set is $\{-9\}$.

N4. $x-5=2x$

$x-5-x=2x-x$ Subtract x.

$-5=x$ Combine terms.

Check $x=-5$: $-10=-10$ True

The solution set is $\{-5\}$.

5. $\frac{7}{2}p+1=\frac{9}{2}p$

$\frac{7}{2}p+1-\frac{7}{2}p=\frac{9}{2}p-\frac{7}{2}p$ Subtract $\frac{7}{2}p$.

$1=p$

Check $p=1$: $\frac{9}{2}=\frac{9}{2}$ True

The solution set is $\{1\}$.

N5. $\frac{2}{3}x+4=\frac{5}{3}x$

$\frac{2}{3}x+4-\frac{2}{3}x=\frac{5}{3}x-\frac{2}{3}x$ Subtract $\frac{2}{3}x$.

$4=x$ Combine terms.

Check $x=4$: $\frac{20}{3}=\frac{20}{3}$ True

The solution set is $\{4\}$.

6. $10 - x = -2x + 9$ Given
$10 - x + 2x = -2x + 9 + 2x$ Add $2x$.
$10 + x = 9$ Combine terms.
$10 + x - 10 = 9 - 10$ Subtract 10.
$x = -1$ Combine terms.
Check $x = -1$: $11 = 11$ True
The solution set is $\{-1\}$.

N6. $6x - 8 = 12 + 5x$
$6x - 8 - 5x = 12 + 5x - 5x$ Subtract $5x$.
$x - 8 = 12$ Combine terms.
$x - 8 + 8 = 12 + 8$ Add 8.
$x = 20$ Combine terms.
Check $x = 20$: $112 = 112$ True
The solution set is $\{20\}$.

7. $9r + 4r + 6 - 2 = 9r + 4 + 3r$
$13r + 4 = 12r + 4$ Combine terms.
$13r + 4 - 12r = 12r + 4 - 12r$ Subtract $12r$.
$r + 4 = 4$ Combine terms.
$r + 4 - 4 = 4 - 4$ Subtract 4.
$r = 0$ Combine terms.
Check $r = 0$: $4 = 4$ True
The solution set is $\{0\}$.

N7. $5x - 10 - 12x = 4 - 8x - 9$
$-7x - 10 = -8x - 5$ Combine terms.
$-7x - 10 + 8x = -8x - 5 + 8x$ Add $8x$.
$x - 10 = -5$ Combine terms.
$x - 10 + 10 = -5 + 10$ Add 10.
$x = 5$ Combine terms.
Check $x = 5$: $-45 = -45$ True
The solution set is $\{5\}$.

8. $4(x + 1) - (3x + 5) = 1$
$4(x + 1) - 1(3x + 5) = 1$ $-a = -1a$
$4x + 4 - 3x - 5 = 1$ Distributive prop.
$x - 1 = 1$ Combine terms.
$x - 1 + 1 = 1 + 1$ Add 1.
$x = 2$
Check $x = 2$: $1 = 1$ True
The solution set is $\{2\}$.

N8. $4(3x - 2) - (11x - 4) = 3$
$4(3x - 2) - 1(11x - 4) = 3$ $-a = -1a$
$12x - 8 - 11x + 4 = 3$ Distributive prop.
$x - 4 = 3$ Combine terms.
$x - 4 + 4 = 3 + 4$ Add 4.
$x = 7$
Check $x = 7$: $3 = 3$ True
The solution set is $\{7\}$.

Exercises

1. An equation includes an equality symbol, while an expression does not.

2. A linear equation in one variable (here x) can be written in the form $Ax + B = C$.

3. Equations that have exactly the same solution set are equivalent equations.

4. The addition property of equality states that the same expression may be added to or subtracted from each side of an equation without changing the solution set.

5. **(a)** $5x + 8 - 4x + 7$
This is an expression, not an equation, since there is no equals symbol. It can be simplified by rearranging terms and then combining like terms.
$5x + 8 - 4x + 7 = 5x - 4x + 8 + 7$
$= x + 15$

(b) $-6y + 12 + 7y - 5$
This is an expression, not an equation, since there is no equals symbol. It can be simplified by rearranging terms and then combining like terms.
$-6y + 12 + 7y - 5 = -6y + 7y + 12 - 5$
$= y + 7$

(c) $5x + 8 - 4x = 7$
This is an equation because of the equals symbol.
$5x + 8 - 4x = 7$
$x + 8 = 7$
$x = -1$
The solution set is $\{-1\}$.

(d) This is an equation because of the equals symbol.

$$-6y+12+7y=-5$$
$$y+12=-5$$
$$y=-17$$

The solution set is $\{-17\}$.

6. Equations that have exactly the same solution sets are equivalent equations.

$$x+2=6$$
$$x+2-2=6-2 \quad \text{Subtract 2.}$$
$$x=4$$

So $x+2=6$ and $x=4$ *are* equivalent equations.

$$10-x=5$$
$$10-x-10=5-10 \quad \text{Subtract 10.}$$
$$-x=-5$$
$$-1(-x)=-1(-5) \quad \text{Multiply by } -1.$$
$$x=5$$

So $10-x=5$ and $x=-5$ *are not* equivalent equations.

Subtract 3 from both sides to get $x=6$, so $x+3=9$ and $x=6$ *are* equivalent equations.

Subtract 4 from both sides to get $x=4$. The second equation is $x=-4$, so $4+x=8$ and $x=-4$ *are not* equivalent equations.

The pairs of equations in A and C are equivalent.

7. Equations A $\left(x^2-5x+6=0\right)$ and B $\left(x^3=x\right)$ are *not* linear equations in one variable because they cannot be written in the form $Ax+B=C$. Note that in a linear equation the exponent on the variable must be 1.

8. Check by replacing the variable(s) in the *original* equation with the proposed solution. A true statement will result if the proposed solution is correct.

9.
$$x-3=9$$
$$x-3+3=9+3$$
$$x=12$$

Check this solution by replacing x with 12 in the original equation.

$$x-3=9$$
$$12-3\stackrel{?}{=}9 \quad \text{Let } x=12.$$
$$9=9 \quad \text{True}$$

Because the final statement is true, $\{12\}$ is the solution set.

10.
$$x-9=8$$
$$x-9+9=8+9$$
$$x=17$$

Check $x=17$

$$17-9\stackrel{?}{=}8 \quad \text{Let } x=17.$$
$$8=8 \quad \text{True}$$

Thus, $\{17\}$ is the solution set.

11.
$$x-12=19$$
$$x-12+12=19+12$$
$$x=31$$

Check $x=31$

$$31-12\stackrel{?}{=}19 \quad \text{Let } x=31.$$
$$19=19 \quad \text{True}$$

Thus, $\{31\}$ is the solution set.

12.
$$x-18=22$$
$$x-18+18=22+18$$
$$x=40$$

Checking yields a true statement, so $\{40\}$ is the solution set.

13.
$$x-6=-9$$
$$x-6+6=-9+6$$
$$x=-3$$

Checking yields a true statement, so $\{-3\}$ is the solution set.

14.
$$x-5=-7$$
$$x-5+5=-7+5$$
$$x=-2$$

Checking yields a true statement, so $\{-2\}$ is the solution set.

15.
$$r+8=12$$
$$r+8-8=12-8$$
$$r=4$$

Checking yields a true statement, so $\{4\}$ is the solution set.

16.
$$x+7=11$$
$$x+7-7=11-7$$
$$x=4$$

Checking yields a true statement, so $\{4\}$ is the solution set.

17. $x+28=19$

$x+28-28=19-28$

$x=-9$

Checking yields a true statement, so $\{-9\}$ is the solution set.

18. $x+47=26$

$x+47-47=26-47$

$x=-21$

Checking yields a true statement, so $\{-21\}$ is the solution set.

19. $x+\frac{1}{4}=-\frac{1}{2}$

$x+\frac{1}{4}-\frac{1}{4}=-\frac{1}{2}-\frac{1}{4}$

$x=-\frac{2}{4}-\frac{1}{4}$

$x=-\frac{3}{4}$

Check $x=-\frac{3}{4}:-\frac{1}{2}=-\frac{1}{2}$ True

The solution set is $\left\{-\frac{3}{4}\right\}$.

20. $x+\frac{2}{3}=-\frac{1}{6}$

$x+\frac{2}{3}-\frac{2}{3}=-\frac{1}{6}-\frac{2}{3}$

$x=-\frac{1}{6}-\frac{4}{6}$

$x=-\frac{5}{6}$

Check $x=-\frac{5}{6}:-\frac{1}{6}=-\frac{1}{6}$ True

The solution set is $\left\{-\frac{5}{6}\right\}$.

21. $7+r=-3$

$r+7=-3$

$r+7-7=-3-7$

$r=-10$

The solution set is $\{-10\}$.

22. $8+k=-4$

$k+8=-4$

$k+8-8=-4-8$

$k=-12$

The solution set is $\{-12\}$.

23. $2=p+15$

$2-15=p+15-15$

$-13=p$

The solution set is $\{-13\}$.

24. $5=z+19$

$5-19=z+19-19$

$-14=z$

The solution set is $\{-14\}$.

25. $-4=x-14$

$-4+14=x-14+14$

$10=x$

The solution set is $\{10\}$.

26. $-7=x-22$

$-7+22=x-22+22$

$15=x$

The solution set is $\{15\}$.

27. $-\frac{1}{3}=x-\frac{3}{5}$

$-\frac{1}{3}+\frac{3}{5}=x-\frac{3}{5}+\frac{3}{5}$

$-\frac{5}{15}+\frac{9}{15}=x$

$\frac{4}{15}=x$

Check $x=\frac{4}{15}:-\frac{5}{15}=\frac{4}{15}-\frac{9}{15}$ True

The solution set is $\left\{\frac{4}{15}\right\}$.

28. $-\frac{1}{4}=x-\frac{2}{3}$

$-\frac{1}{4}+\frac{2}{3}=x-\frac{2}{3}+\frac{2}{3}$

$-\frac{3}{12}+\frac{8}{12}=x$

$\frac{5}{12}=x$

Check $x=\frac{5}{12}:-\frac{3}{12}=\frac{5}{12}-\frac{8}{12}$ True

The solution set is $\left\{\frac{5}{12}\right\}$.

29. $x - 8.4 = -2.1$

$x - 8.4 + 8.4 = -2.1 + 8.4$

$x = 6.3$

The solution set is $\{6.3\}$.

30. $x - 15.5 = -5.1$

$x - 15.5 + 15.5 = -5.1 + 15.5$

$x = 10.4$

The solution set is $\{10.4\}$.

31. $t + 12.3 = -4.6$

$t + 12.3 - 12.3 = -4.6 - 12.3$

$t = -16.9$

The solution set is $\{-16.9\}$.

32. $x + 21.5 = -13.4$

$x + 21.5 - 21.5 = -13.4 - 21.5$

$x = -34.9$

The solution set is $\{-34.9\}$.

33. $3x = 2x + 7$

$3x - 2x = 2x + 7 - 2x$ Subtract $2x$.

$1x = 7$ or $x = 7$

Check $x = 7$: $21 = 21$ True

The solution set is $\{7\}$.

34. $5x = 4x + 9$

$5x - 4x = 4x + 9 - 4x$ Subtract $4x$.

$1x = 9$ or $x = 9$

Check $x = 9$: $45 = 45$ True

The solution set is $\{9\}$.

35. $10x + 4 = 9x$

$10x + 4 - 9x = 9x - 9x$ Subtract $9x$.

$1x + 4 = 0$

$x + 4 - 4 = 0 - 4$ Subtract 4.

$x = -4$

Check $x = -4$: $-36 = -36$ True

The solution set is $\{-4\}$.

36. $8t + 5 = 7t$

$8t + 5 - 7t = 7t - 7t$ Subtract $7t$.

$1t + 5 = 0$

$t + 5 - 5 = 0 - 5$ Subtract 5.

$t = -5$

Check $t = -5$: $-35 = -35$ True

The solution set is $\{-5\}$.

37. $8x - 3 = 9x$

$8x - 3 - 8x = 9x - 8x$ Subtract $8x$.

$-3 = x$

Check $x = -3$: $8(-3) - 3 = 9(-3)$ True

The solution set is $\{-3\}$.

38. $6x - 4 = 7x$

$6x - 4 - 6x = 7x - 6x$ Subtract $7x$.

$-4 = x$

Check $x = -4$: $6(-4) - 4 = 7(-4)$ True

The solution set is $\{-4\}$.

39. $6t - 2 = 5t$

$6t - 2 - 5t = 5t - 5t$ Subtract $5t$.

$t - 2 = 0$

$t - 2 + 2 = 2$ Add 2.

$t = 2$

Check $t = 2$: $6(2) - 2 = 5(2)$ True

The solution set is $\{2\}$.

40. $4z - 6 = 3z$

$4z - 6 - 3z = 3z - 3z$ Subtract $3z$.

$z - 6 = 0$

$z - 6 + 6 = 6$ Add 6.

$z = 6$

Check $z = 6$: $4(6) - 6 = 3(6)$ True

The solution set is $\{6\}$.

41. $\frac{2}{5}w - 6 = \frac{7}{5}w$

$\frac{2}{5}w - 6 - \frac{2}{5}w = \frac{7}{5}w - \frac{2}{5}w$ Subtract $\frac{2}{5}w$.

$-6 = \frac{5}{5}w$

$-6 = w$

Check $w = -6$: $\frac{2}{5}(-6) - 6 = \frac{7}{5}(-6)$ True

The solution set is $\{-6\}$.

42. $\frac{2}{7}z-2=\frac{9}{7}z$

$\frac{2}{7}z-2-\frac{2}{7}z=\frac{9}{7}z-\frac{2}{7}z$ Subtract $\frac{2}{7}z$.

$-2=\frac{7}{7}z$

$-2=z$

Check $z=-2$: $\frac{2}{7}(-2)-2=\frac{9}{7}(-2)$ True

The solution set is $\{-2\}$.

43. $\frac{1}{2}x+5=-\frac{1}{2}x$

$\frac{1}{2}x+\frac{1}{2}x+5=-\frac{1}{2}x+\frac{1}{2}x$

$x+5=0$

$x+5-5=0-5$

$x=-5$

The solution set is $\{-5\}$.

44. $\frac{1}{5}x+7=-\frac{4}{5}x$

$\frac{1}{5}x+7+\frac{4}{5}x=-\frac{4}{5}x+\frac{4}{5}x$

$\frac{5}{5}x+7=0$

$x+7-7=0-7$

$x=-7$

The solution set is $\{-7\}$.

45. $5.6x+2=4.6x$

$5.6x+2-4.6x=4.6x-4.6x$

$1.0x+2=0$

$x+2-2=0-2$

$x=-2$

The solution set is $\{-2\}$.

46. $9.1x+5=8.1x$

$9.1x+5-8.1x=8.1x-8.1x$

$1.0x+5=0$

$x+5-5=0-5$

$x=-5$

The solution set is $\{-5\}$.

47. $1.4x-3=0.4x$

$1.4x-3-0.4x=0.4x-0.4x$

$1.0x-3=0$

$1.0x-3+3=0+3$

$x=3$

The solution set is $\{3\}$.

48. $1.9t-6=0.9t$

$1.9t-6-0.9t=0.9t-0.9t$

$1.0t-6=0$

$1.0t-6+6=0+6$

$t=6$

The solution set is $\{6\}$.

49. $5p=4p$

$5p-4p=4p-4p$

$p=0$

The solution set is $\{0\}$.

50. $8z=7z$

$8z-7z=7z-7z$

$z=0$

The solution set is $\{0\}$.

51. $3x+7-2x=0$

$x+7=0$

$x+7-7=0-7$

$x=-7$

The solution set is $\{-7\}$.

52. $5x+4-4x=0$

$x+4=0$

$x+4-4=0-4$

$x=-4$

The solution set is $\{-4\}$.

53. $3x+7=2x+4$

$3x+7-2x=2x+4-2x$

$x+7=4$

$x+7-7=4-7$

$x=-3$

The solution set is $\{-3\}$.

54. $9x+5=8x+4$

$9x+5-8x=8x+4-8x$

$x+5=4$

$x+5-5=4-5$

$x=-1$

Check $x=-1$: $-4=-4$ True

The solution set is $\{-1\}$.

55. $8t+6=7t+6$

$8t+6-7t=7t+6-7t$

$t+6=6$

$t+6-6=6-6$

$t=0$

The solution set is $\{0\}$.

56. $13t+9=12t+9$

$13t+9-12t=12t+9-12t$

$t+9=9$

$t+9-9=9-9$

$t=0$

The solution set is $\{0\}$.

57. $-4x+7=-5x+9$

$-4x+7+5x=-5x+9+5x$

$x+7=9$

$x+7-7=9-7$

$x=2$

The solution set is $\{2\}$.

58. $-6x+3=-7x+10$

$-6x+3+7x=-7x+10+7x$

$x+3=10$

$x+3-3=10-3$

$x=7$

The solution set is $\{7\}$.

59. $5-x=-2x-11$

$5-x+2x=-2x-11+2x$ Add $2x$.

$5+x-5=-11-5$ Subtract 5.

$x=-16$

The solution set is $\{-16\}$.

60. $3-8x=-9x-1$

$3-8x+9x=-9x-1+9x$ Add $9x$.

$3+x=-1$

$3+x-3=-1-3$ Subtract 3.

$x=-4$

The solution set is $\{-4\}$.

61. $1.2y-4=0.2y-4$

$1.2y-4-0.2y=0.2y-4-0.2y$

$1.0y-4=-4$

$y-4+4=-4+4$

$y=0$

The solution set is $\{0\}$.

62. $7.7r-6=6.7r-6$

$7.7r-6-6.7r=6.7r-6-6.7r$

$1.0r-6=-6$

$r-6+6=-6+6$

$r=0$

The solution set is $\{0\}$.

63. $3x+6-10=2x-2$

$3x-4=2x-2$ Combine terms.

$3x-4-2x=2x-2-2x$ Subtract $2x$.

$x-4=-2$

$x-4+4=-2+4$

$x=2$

The solution set is $\{2\}$.

64. $8x+4-8=7x-1$

$8x-4=7x-1$ Combine terms.

$8x-4-7x=7x-1-7x$ Subtract $7x$.

$x-4=-1$

$x-4+4=-1+4$

$x=3$

The solution set is $\{3\}$.

65. $5t+3+2t-6t=4+12$

$(5+2-6)t+3=16$

$t+3-3=16-3$

$t=13$

Check $t=13$: $16=16$ True

The solution set is $\{13\}$.

66. $4x-6+3x-6x=3+10$

$x-6=13$

$x-6+6=13+6$ Add 6.

$x=19$

Check $x=19$: $13=13$ True

The solution set is $\{19\}$.

67. $6x+5+7x+3=12x+4$

$13x+8=12x+4$

$13x+8-12x=12x+4-12x$

$x+8=4$

$x+8-8=4-8$

$x=-4$

Check $x=-4$: $-44=-44$ True

The solution set is $\{-4\}$.

68. $4x+3+8x+1=11x+2$

$12x+4=11x+2$

$12x+4-11x=11x+2-11x$

$x+4=2$

$x+4-4=2-4$

$x=-2$

Check $x=-2$: $-20=-20$ True

The solution set is $\{-2\}$.

69. $5.2q-4.6-7.1q=-0.9q-4.6$

$-1.9q-4.6=-0.9q-4.6$

$-1.9q-4.6+0.9q=-0.9q-4.6+0.9q$

$-1.0q-4.6=-4.6$

$-1.0q-4.6+4.6=-4.6+4.6$

$-q=0$

$q=0$

Check $q=0$: $-4.6=-4.6$ True

The solution set is $\{0\}$.

70. $4.0x+2.7-9.6x=-4.6x+2.7$

$2.7-5.6x=-4.6x+2.7$

$2.7-5.6x+5.6x=-4.6x+2.7+5.6x$

$2.7=2.7+x$

$2.7-2.7=x+2.7-2.7$

$0=x$

Check $x=0$: $2.7=2.7$ True

The solution set is $\{0\}$.

71. $\frac{5}{7}x+\frac{1}{3}=\frac{2}{5}-\frac{2}{7}x+\frac{2}{5}$

$\frac{5}{7}x+\frac{1}{3}=\frac{4}{5}-\frac{2}{7}x$

$\frac{5}{7}x+\frac{2}{7}x+\frac{1}{3}=\frac{4}{5}-\frac{2}{7}x+\frac{2}{7}x$ Add $\frac{2}{7}x$.

$\frac{7}{7}x+\frac{1}{3}=\frac{4}{5}$ Combine terms.

$1x+\frac{1}{3}-\frac{1}{3}=\frac{4}{5}-\frac{1}{3}$ Subtract $\frac{1}{3}$.

$x=\frac{12}{15}-\frac{5}{15}$ LCD = 15

$x=\frac{7}{15}$

Check $x=\frac{7}{15}$: $\frac{2}{3}=\frac{2}{3}$ True

The solution set is $\left\{\frac{7}{15}\right\}$.

72. To solve the equation, follow the simplification steps below.

$\frac{6}{7}s-\frac{3}{4}=\frac{4}{5}-\frac{1}{7}s+\frac{1}{6}$

$\frac{6}{7}s-\frac{3}{4}=\frac{24}{30}-\frac{1}{7}s+\frac{5}{30}$ LCD = 30

$\frac{6}{7}s-\frac{3}{4}=\frac{29}{30}-\frac{1}{7}s$ Add.

$\frac{6}{7}s-\frac{3}{4}+\frac{1}{7}s=\frac{29}{30}-\frac{1}{7}s+\frac{1}{7}s$ Add $\frac{1}{7}s$.

$\frac{7}{7}s-\frac{3}{4}=\frac{29}{30}$ Combine terms.

$1s-\frac{3}{4}+\frac{3}{4}=\frac{29}{30}+\frac{3}{4}$ Add $\frac{3}{4}$.

$s=\frac{58}{60}+\frac{45}{60}$ LCD = 60

$s=\frac{103}{60}$

Check $s=\frac{103}{60}$: $\frac{101}{140}=\frac{101}{140}$ True

The solution set is $\left\{\frac{103}{60}\right\}$.

73. $(5y+6)-(3+4y)=10$

$5y+6-3-4y=10$ Distributive prop.

$y+3=10$ Combine terms.

$y+3-3=10-3$ Subtract 3.

$y=7$

Check $y=7$: $10=10$ True

The solution set is $\{7\}$.

74. $(8r+3)-(1+7r)=6$

$8r+3-1-7r=6$

$r+2=6$

$r+2-2=6-2$

$r=4$

Check $r=4$: $6=6$ True

The solution set is $\{4\}$.

75. $2(p+5)-(9+p)=-3$

$2p+10-9-p=-3$

$p+1=-3$

$p+1-1=-3-1$

$p=-4$

Check $p=-4$: $-3=-3$ True

The solution set is $\{-4\}$.

76. $4(k+6)-(8+3k)=-5$

$4k+24-8-3k=-5$

$k+16=-5$

$k+16-16=-5-16$

$k=-21$

Check $k=-21$: $-5=-5$ True

The solution set is $\{-21\}$.

77. $-6(2b+1)+(13b-7)=0$

$-12b-6+13b-7=0$

$b-13=0$

$b-13+13=0+13$

$b=13$

Check $b=13$: $0=0$ True

The solution set is $\{13\}$.

78. $-5(3w-3)+(16w+1)=0$

$-15w+15+16w+1=0$

$w+16=0$

$w+16-16=0-16$

$w=-16$

Check $w=-16$: $0=0$ True

The solution set is $\{-16\}$.

79. $10(-2x+1)=-19(x+1)$

$-20x+10=-19x-19$

$-20x+10+19x=-19x-19+19x$

$-x+10=-19$

$-x+10-10=-19-10$

$-x=-29$

$x=29$

Check $x=29$: $-570=-570$ True

The solution set is $\{29\}$.

80. $2(-3r+2)=-5(r-3)$

$-6r+4=-5r+15$

$-6r+4+5r=-5r+15+5r$

$4-r=15$

$4-r-4=15-4$

$-r=11$

$r=-11$

Check $r=-11$: $70=70$ True

The solution set is $\{-11\}$.

81. $-2(8p+2)-3(2-7p)-2(4+2p)=0$

$-16p-4-6+21p-8-4p=0$

$p-18=0$

$p-18+18=0+18$

$p=18$

Check $p=18$: $0=0$ True

The solution set is $\{18\}$.

82. $-5(1-2z)+4(3-z)-7(3+z)=0$

$-5+10z+12-4z-21-7z=0$

$-z-14=0$

$-z-14+z=0+z$

$-14=z$

Check $z=-14$: $0=0$ True

The solution set is $\{-14\}$.

83. $4(7x-1)+3(2-5x)-4(3x+5)=-6$

$28x-4+6-15x-12x-20=-6$

$x-18=-6$

$x-18+18=-6+18$

$x=12$

Check $x=12: -6=-6$ True

The solution set is $\{12\}$.

84. $9(2m-3)-4(5+3m)-5(4+m)=-3$

$18m-27-20-12m-20-5m=-3$

$m-67=-3$

$m-67+67=-3+67$

$m=64$

Check $m=64: -3=-3$ True

The solution set is $\{64\}$.

85. Answers will vary. One example is $x-6=-8$.

86. Answers will vary. One example is $x+\frac{1}{2}=1$.

87. "Three times a number is 17 more than twice the number."

$3x=2x+17$

$3x-2x=2x+17-2x$

$x=17$

The number is 17 and $\{17\}$ is the solution set.

88. "One added to three times a number is three less than four times the number."

$1+3x=4x-3$

$1+3x-3x=4x-3-3x$

$1=x-3$

$1+3=x-3+3$

$4=x$

The number is 4 and $\{4\}$ is the solution set.

89. "If six times a number is subtracted from seven times the number, the result is -9."

$7x-6x=-9$

$x=-9$

The number is -9 and $\{-9\}$ is the solution set.

90. "If five times a number is added to three times the number, the result is the sum of seven times the number and 9."

$5x+3x=7x+9$

$8x=7x+9$

$8x-7x=7x+9-7x$

$x=9$

The number is 9 and $\{9\}$ is the solution set.

2.2 The Multiplication Property of Equality

Classroom Examples, Now Try Exercises

1. $15x=75$

$\frac{15x}{15}=\frac{75}{15}$ Divide by 15.

$x=5$

Check $x=5: 75=75$ True

The solution set is $\{5\}$.

N1. $8x=80$

$\frac{8x}{8}=\frac{80}{8}$ Divide by 8.

$x=10$

Check $x=10: 80=80$ True

The solution set is $\{10\}$.

2. $8x=-20$

$\frac{8x}{8}=-\frac{20}{8}$ Divide by 8.

$x=-\frac{20}{8}=-\frac{5}{2}$ Write in lowest terms.

Check $x=-\frac{5}{2}: -20=-20$ True

The solution set is $\left\{-\frac{5}{2}\right\}$.

N2. $10x=-24$

$\frac{10x}{10}=-\frac{24}{10}$ Divide by 10.

$x=-\frac{24}{10}=-\frac{12}{5}$ Write in lowest terms.

Check $x=-\frac{12}{5}: -24=-24$ True

The solution set is $\left\{-\frac{12}{5}\right\}$.

3. $5.04 = -0.7x$

$\frac{5.04}{-0.7} = \frac{-0.7x}{-0.7}$ Divide by -0.7.

$x = -7.2$

Check $x = -7.2$: $5.04 = 5.04$ True

The solution set is $\{-7.2\}$.

N3. $7.02 = -1.3x$

$\frac{7.02}{-1.3} = \frac{-1.3x}{-1.3}$ Divide by -1.3.

$x = -5.4$

Check $x = -5.4$: $7.02 = 7.02$ True

The solution set is $\{-5.4\}$.

4. $\frac{x}{4} = -6$

$\frac{1}{4}x = -6$

$4 \cdot \frac{1}{4}x = 4(-6)$ Multiply by 4.

$x = -24$

Check $x = -24$: $-6 = -6$ True

The solution set is $\{-24\}$.

N4. $\frac{x}{5} = -7$

$\frac{1}{5}x = -7$

$5 \cdot \frac{1}{5}x = 5(-7)$ Multiply by 5.

$p = -35$

Check $p = -35$: $-7 = -7$ True

The solution set is $\{-35\}$.

5. $-\frac{2}{3}t = -12$

$-\frac{3}{2}\left(-\frac{2}{3}t\right) = -\frac{3}{2}(-12)$ Multiply by $-\frac{3}{2}$.

$1 \cdot t = -\frac{3}{2} \cdot \frac{-12}{1}$

$t = 18$

Check $t = 18$: $-12 = -12$ True

The solution set is $\{18\}$.

N5. $\frac{4}{7}z = -16$

$\frac{7}{4}\left(\frac{4}{7}z\right) = \frac{7}{4}(-16)$ Multiply by $\frac{7}{4}$.

$1 \cdot t = \frac{7}{4} \cdot \frac{-16}{1}$

$t = -28$

Check $t = -28$: $-16 = -16$ True

The solution set is $\{-28\}$.

6. $-p = -7$

$-1 \cdot p = -7$ $\quad -p = -1 \cdot p$

$(-1)(-1) \cdot p = (-1)(-7)$

$1 \cdot p = 7$

$p = 7$

Check $p = 7$: $-7 = -7$ True

The solution set is $\{7\}$.

N6. $-x = -9$

$-1 \cdot x = -9$ $\quad -x = -1 \cdot x$

$(-1)(-1) \cdot x = (-1)(-9)$ Multiply by -1.

$1 \cdot x = 9$

$x = 9$

Check $x = 9$: $-9 = -9$ True

The solution set is $\{9\}$.

7. $4r - 9r = 20$

$-5r = 20$ Combine terms.

$\frac{-5r}{-5} = \frac{20}{-5}$ Divide by -5.

$r = -4$

Check $r = -4$: $20 = 20$ True

The solution set is $\{-4\}$.

N7. $9n - 6n = 21$

$3n = 21$ Combine terms.

$\frac{3n}{3} = \frac{21}{3}$ Divide by 3.

$n = 7$

Check $n = 7$: $21 = 21$ True

The solution set is $\{7\}$.

Exercises

1. **(a)** multiplication property of equality; to get x alone on the left side of the equation, multiply each side by $\frac{1}{3}$ (or divide each side by 3).

(b) addition property of equality; to get x alone on the left side of the equation, add -3 (or subtract 3) on each side.

(c) multiplication property of equality; to get x alone on the left side of the equation, multiply each side by -1 (or divide each side by -1).

(d) addition property of equality; to get x alone on the right side of the equation, add -6 (or subtract 6) on each side.

2. Choice C doesn't require the use of the multiplicative property of equality. After the equation is simplified, the variable x is alone on the left side.

$$5x - 4x = 7$$
$$x = 7$$

3. Choice B; to find the solution of $-x = -\frac{3}{4}$, multiply (or divide) each side by -1, or use the rule "If $-x = a$, then $x = -a$."

4. Choice A; to find the solution of $-x = -24$, multiply (or divide) each side of the equation by -1, or use the rule "If $-x = a$, then $x = -a$."

5. To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{4}{5}$, which is $\frac{5}{4}$.

6. To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

7. This equation is equivalent to $\frac{1}{10}x = 5$. To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{1}{10}$, which is 10.

8. This equation is equivalent to $\frac{1}{100}x = 10$. To get just x on the left side, multiply both sides of the equation by the reciprocal of $\frac{1}{100}$, which is 100.

9. To get just x on the left side, multiply both sides of the equation by the reciprocal of $-\frac{9}{2}$, which is $-\frac{2}{9}$.

10. To get just x on the left side, multiply both sides of the equation by the reciprocal of $-\frac{8}{3}$, which is $-\frac{3}{8}$.

11. This equation is equivalent to $-1x = 0.75$. To get just x on the left side, multiply both sides of the equation by the reciprocal of -1, which is -1.

12. This equation is equivalent $-1x = 0.48$. To get just x on the left side, multiply both sides of the equation by the reciprocal of -1, which is -1.

13. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 6.

14. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 7.

15. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -4.

16. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -13.

17. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 0.12.

18. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is 0.21.

19. This equation is equivalent to $-1x = 25$. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -1.

20. This equation is equivalent to $-1x = 50$. To get just x on the left side, divide both sides of the equation by the coefficient of x, which is -1.

21. $6x = 36$

$\frac{6x}{6} = \frac{36}{6}$ Divide by 6.

$1x = 6$

$x = 6$

Check $x = 6: 36 = 36$ True

The solution set is $\{6\}$.

22. $8x = 64$

$\frac{8x}{8} = \frac{64}{8}$ Divide by 8.

$x = 8$

Check $x = 8: 64 = 64$ True

The solution set is $\{8\}$.

23. $2m = 15$

$\frac{2m}{2} = \frac{15}{2}$ Divide by 2.

$m = \frac{15}{2}$

Check $m = \frac{15}{2}: 15 = 15$ True

The solution set is $\left\{\frac{15}{2}\right\}$.

24. $3m = 10$

$\frac{3m}{3} = \frac{10}{3}$ Divide by 3.

$m = \frac{10}{3}$

Check $m = \frac{10}{3}: 10 = 10$ True

The solution set is $\left\{\frac{10}{3}\right\}$.

25. $4x = -20$

$\frac{4x}{4} = \frac{-20}{4}$ Divide by 4.

$x = -5$

Check $x = -5: -20 = -20$ True

The solution set is $\{-5\}$.

26. $5x = -60$

$\frac{5x}{5} = \frac{-60}{5}$ Divide by 5.

$x = -12$

Check $x = -12: -60 = -60$ True

The solution set is $\{-12\}$.

27. $-7x = 28$

$\frac{-7x}{-7} = \frac{28}{-7}$ Divide by -7.

$x = -4$

Check $x = -4: 28 = 28$ True

The solution set is $\{-4\}$.

28. $-9x = 36$

$\frac{-9x}{-9} = \frac{36}{-9}$ Divide by -9.

$x = -4$

Check $x = -4: 36 = 36$ True

The solution set is $\{-4\}$.

29. $10t = -36$

$\frac{10t}{10} = \frac{-36}{10}$ Divide by 10.

$t = -\frac{36}{10} = -\frac{18}{5}$ Lowest terms

Check $t = -\frac{18}{5}: -36 = -36$ True

The solution set is $\left\{-\frac{18}{5}\right\}$, or $\{-3.6\}$.

30. $10s = -54$

$\frac{10s}{10} = \frac{-54}{10}$ Divide by 10.

$s = -\frac{54}{10} = -\frac{27}{5}$ Lowest terms

Check $s = -\frac{27}{5}: -54 = -54$ True

The solution set is $\left\{-\frac{27}{5}\right\}$, or $\{-5.4\}$.

31. $-6x = -72$

$\frac{-6x}{-6} = \frac{-72}{-6}$ Divide by -6.

$x = 12$

Check $x = 12: -72 = -72$ True

The solution set is $\{12\}$.

32. $-4x = -64$

$\frac{-4x}{-4} = \frac{-64}{-4}$ Divide by -4.

$x = 16$

Check $x = 16: -64 = -64$ True

The solution set is $\{16\}$.

33. $4r = 0$

$\frac{4r}{4} = \frac{0}{4}$ Divide by 4.

$r = 0$

Check $r = 0$: $0 = 0$ True

The solution set is $\{0\}$.

34. $7x = 0$

$\frac{7x}{7} = \frac{0}{7}$ Divide by 7.

$x = 0$

Check $x = 0$: $0 = 0$ True

The solution set is $\{0\}$.

35. $-x = 12$

$-1 \cdot (-x) = -1 \cdot 12$ Multiply by -1.

$x = -12$

Check $x = -12$: $12 = 12$ True

The solution set is $\{-12\}$.

36. $-t = 14$

$-1 \cdot (-t) = -1 \cdot 14$ Multiply by -1.

$t = -14$

Check $t = -14$: $14 = 14$ True

The solution set is $\{-14\}$.

37. $-x = -\frac{3}{4}$

$-1 \cdot (-x) = -1 \cdot \left(-\frac{3}{4}\right)$

$x = \frac{3}{4}$

Check $x = \frac{3}{4}$: $-\frac{3}{4} = -\frac{3}{4}$ True

The solution set is $\left\{\frac{3}{4}\right\}$.

38. $-x = -\frac{1}{2}$

$-1 \cdot (-x) = -1 \cdot \left(-\frac{1}{2}\right)$

$x = \frac{1}{2}$

Check $x = \frac{1}{2}$: $-\frac{1}{2} = -\frac{1}{2}$ True

The solution set is $\left\{\frac{1}{2}\right\}$.

39. $0.2t = 8$

$\frac{0.2t}{0.2} = \frac{8}{0.2}$

$t = 40$

Check $t = 40$: $8 = 8$ True

The solution set is $\{40\}$.

40. $0.9x = 18$

$\frac{0.9x}{0.9} = \frac{18}{0.9}$

$x = 20$

Check $x = 20$: $18 = 18$ True

The solution set is $\{20\}$.

41. $-0.3x = 9$

$\frac{-0.3x}{-0.3} = \frac{9}{-0.3}$

$x = -30$

Check $x = -30$: $9 = 9$ True

The solution set is $\{-30\}$.

42. $-0.5x = 20$

$\frac{-0.5x}{-0.5} = \frac{20}{-0.5}$ Divide by -0.5.

$x = -40$

Check $x = -40$: $20 = 20$ True

The solution set is $\{-40\}$.

43. $0.6x = -1.44$

$\frac{0.6x}{0.6} = \frac{-1.44}{0.6}$ Divide by 0.6.

$x = -2.4$

Check $x = -2.4$: $-1.44 = -1.44$ True

The solution set is $\{-2.4\}$.

44. $0.8x = -2.96$

$\frac{0.8x}{0.8} = \frac{-2.96}{0.8}$ Divide by 0.8.

$x = -3.7$

Check $x = -3.7$: $-2.96 = -2.96$ True

The solution set is $\{-3.7\}$.

45. $-9.1 = -2.6x$

$\frac{-9.1}{-2.6} = \frac{-2.6x}{-2.6}$ Divide by -2.6.

$x = 3.5$

Check $x = 3.5$: $-9.1 = -9.1$ True

The solution set is $\{3.5\}$.

46. $-7.2 = -4.5x$

$\frac{-7.2}{-4.5} = \frac{-4.5x}{-4.5}$ Divide by -4.5.

$x = 1.6$

Check $x = 1.6$: $-7.2 = -7.2$ True

The solution set is $\{1.6\}$.

47. $-2.1m = 25.62$

$\frac{-2.1m}{-2.1} = \frac{25.62}{-2.1}$ Divide by -2.1.

$m = -12.2$

Check $m = -12.2$: $25.62 = 25.62$ True

The solution set is $\{-12.2\}$.

48. $-3.9x = 32.76$

$\frac{-3.9x}{-3.9} = \frac{32.76}{-3.9}$

$x = -8.4$

Check $x = -8.4$: $32.76 = 32.76$ True

The solution set is $\{-8.4\}$.

49. $\frac{1}{4}x = -12$

$4 \cdot \frac{1}{4}x = 4(-12)$ Multiply by 4.

$1x = -48$

$x = -48$

Check $x = -48$: $-12 = -12$ True

The solution set is $\{-48\}$.

50. $\frac{1}{5}p = -3$

$5 \cdot \frac{1}{5}p = 5(-3)$ Multiply by 5.

$p = -15$

Check $p = -15$: $-3 = -3$ True

The solution set is $\{-15\}$.

51. $\frac{z}{6} = 12$

$\frac{1}{6}z = 12$

$6 \cdot \frac{1}{6}z = 6 \cdot 12$

$z = 72$

Check $z = 72$: $12 = 12$ True

The solution set is $\{72\}$.

52. $\frac{x}{5} = 15$

$\frac{1}{5}x = 15$

$5 \cdot \frac{1}{5}x = 5 \cdot 15$

$x = 75$

Check $x = 75$: $15 = 15$ True

The solution set is $\{75\}$.

53. $\frac{x}{7} = -5$

$\frac{1}{7}x = -5$

$7\left(\frac{1}{7}x\right) = 7(-5)$

$x = -35$

Check $x = -35$: $-5 = -5$ True

The solution set is $\{-35\}$.

54. $\frac{r}{8} = -3$

$\frac{1}{8}r = -3$

$8\left(\frac{1}{8}r\right) = 8(-3)$

$r = -24$

Check $r = -24$: $-3 = -3$ True

The solution set is $\{-24\}$.

55. $\frac{2}{7}p = 4$

$\frac{7}{2}\left(\frac{2}{7}p\right) = \frac{7}{2}(4)$ Multiply by $\frac{7}{2}$.

$p = 14$

Check $p = 14$: $4 = 4$ True

The solution set is $\{14\}$.

56. $\frac{3}{8}x = 9$

$\left(\frac{8}{3}\right)\left(\frac{3}{8}x\right) = \left(\frac{8}{3}\right)(9)$ Multiply by $\frac{8}{3}$.

$x = 24$

Check $x = 24$: $9 = 9$ True

The solution set is $\{24\}$.

57. $-\frac{5}{6}t = -15$

$-\frac{6}{5}\left(-\frac{5}{6}t\right) = -\frac{6}{5}(-15)$ Multiply by $-\frac{6}{5}$.

$t = 18$

Check $t = 18: -15 = -15$ True

The solution set is $\{18\}$.

58. $-\frac{3}{4}z = -21$

$-\frac{4}{3}\left(-\frac{3}{4}z\right) = -\frac{4}{3}(-21)$ Multiply by $-\frac{4}{3}$.

$z = 28$

Check $z = 28: -21 = -21$ True

The solution set is $\{28\}$.

59. $-\frac{7}{9}x = \frac{3}{5}$

$-\frac{9}{7}\left(-\frac{7}{9}x\right) = -\frac{9}{7}\cdot\frac{3}{5}$ Multiply by $-\frac{9}{7}$.

$x = -\frac{27}{35}$

Check $x = -\frac{27}{35}: \frac{3}{5} = \frac{3}{5}$ True

The solution set is $\left\{-\frac{27}{35}\right\}$.

60. $-\frac{5}{6}x = \frac{4}{9}$

$\left(-\frac{6}{5}\right)\left(-\frac{5}{6}x\right) = \left(-\frac{6}{5}\right)\left(\frac{4}{9}\right)$ Multiply by $-\frac{6}{5}$.

$x = -\frac{2\cdot 3\cdot 4}{5\cdot 3\cdot 3} = -\frac{8}{15}$

Check $x = -\frac{8}{15}: \frac{4}{9} = \frac{4}{9}$ True

The solution set is $\left\{-\frac{8}{15}\right\}$.

61. $4x + 3x = 21$

$7x = 21$

$\frac{7x}{7} = \frac{21}{7}$

$x = 3$

Check $x = 3: 21 = 21$ True

The solution set is $\{3\}$.

62. $8x + 3x = 121$

$11x = 121$

$\frac{11x}{11} = \frac{121}{11}$

$x = 11$

Check $x = 11: 121 = 121$ True

The solution set is $\{11\}$.

63. $6r - 8r = 10$

$-2r = 10$

$\frac{-2r}{-2} = \frac{10}{-2}$

$r = -5$

Check $r = -5: 10 = 10$ True

The solution set is $\{-5\}$.

64. $3p - 7p = 24$

$-4p = 24$

$\frac{-4p}{-4} = \frac{24}{-4}$

$p = -6$

Check $p = -6: 24 = 24$ True

The solution set is $\{-6\}$.

65. $\frac{2}{5}x - \frac{3}{10}x = 2$

$\frac{4}{10}x - \frac{3}{10}x = 2$

$\frac{1}{10}x = 2$

$10\cdot\frac{1}{10}x = 10\cdot 2$

$x = 20$

Check $x = 20: \ 8 - 6 = 2$ True

The solution set is $\{20\}$.

66. $\frac{2}{3}x - \frac{5}{9}x = 4$

$\frac{6}{9}x - \frac{5}{9}x = 4$

$\frac{1}{9}x = 4$

$9\cdot\frac{1}{9}x = 9\cdot 4$

$x = 36$

Check $x = 36: 24 - 20 = 4$ True

The solution set is $\{36\}$.

67. $7m+6m-4m=63$

$$9m=63$$
$$\frac{9m}{9}=\frac{63}{9}$$
$$m=7$$

Check $m=7: 63=63$ True

The solution set is $\{7\}$.

68. $9r+2r-7r=68$

$$4r=68$$
$$\frac{4r}{4}=\frac{68}{4}$$
$$r=17$$

Check $r=17:\ 68=68$ True

The solution set is $\{17\}$.

69. $-6x+4x-7x=0$

$$-9x=0$$
$$\frac{-9x}{-9}=\frac{0}{-9}$$
$$x=0$$

Check $x=0: 0=0$ True

The solution set is $\{0\}$.

70. $-5x+4x-8x=0$

$$-9x=0$$
$$\frac{-9x}{-9}=\frac{0}{-9}$$
$$x=0$$

Check $x=0: 0=0$ True

The solution set is $\{0\}$.

71. $8w-4w+w=-3$

$$5w=-3$$
$$\frac{5w}{5}=\frac{-3}{5}$$
$$w=-\frac{3}{5}$$

Check $w=-\frac{3}{5}: -3=-3$ True

The solution set is $\left\{-\frac{3}{5}\right\}$.

72. $9x-3x+x=-4$

$$7x=-4$$
$$\frac{7x}{7}=-\frac{4}{7}$$
$$x=-\frac{4}{7}$$

Check $x=-\frac{4}{7}: -4=-4$ True

The solution set is $\left\{-\frac{4}{7}\right\}$.

73. $\frac{1}{3}x-\frac{1}{4}x+\frac{1}{12}x=3$

$\left(\frac{1}{3}-\frac{1}{4}+\frac{1}{12}\right)x=3$ Distributive property

$\left(\frac{4}{12}-\frac{3}{12}+\frac{1}{12}\right)x=3$ LCD = 12

$\frac{1}{6}x=3$ Lowest terms

$6\left(\frac{1}{6}x\right)=6(3)$ Multiply by 6.

$x=18$

Check $x=18: 6-4.5+1.5=3$ True

The solution set is $\{18\}$.

74. $\frac{2}{5}x+\frac{1}{10}x-\frac{1}{20}x=18$

$\left(\frac{2}{5}+\frac{1}{10}-\frac{1}{20}\right)x=18$ Distributive prop.

$\left(\frac{8}{20}+\frac{2}{20}-\frac{1}{20}\right)x=18$ LCD = 20

$\frac{9}{20}x=18$ Add fractions.

$\frac{20}{9}\left(\frac{9}{20}x\right)=\frac{20}{9}(18)$ Multiply by $\frac{20}{9}$.

$x=40$

Check $x=40: 16+4-2=18$ True

The solution set is $\{40\}$.

75. $0.9w-0.5w+0.1w=-3$

$0.5w=-3$ Combine terms.

$\frac{0.5w}{0.5}=\frac{-3}{0.5}$ Divide by 0.5.

$w=-6$

Check $w=-6: -3=-3$ True

The solution set is $\{-6\}$.

76. $0.5x - 0.6x + 0.3x = -1$

$0.2x = -1$ Simplify.

$\frac{0.2x}{0.2} = \frac{-1}{0.2}$ Divide by 0.2.

$x = -5$

Check $x = -5$: $-1 = -1$ True

The solution set is $\{-5\}$.

77. Answers will vary. One example is $\frac{3}{2}x = -6$.

78. Answers will vary. One example is $100x = 17$.

79. "When a number is multiplied by 4, the result is 6."

$4x = 6$

$\frac{4x}{4} = \frac{6}{4}$

$x = \frac{3}{2}$

The number is $\frac{3}{2}$ and $\left\{\frac{3}{2}\right\}$ is the solution set.

80. "When a number is multiplied by -4, the result is 10."

$-4x = 10$

$\frac{-4x}{-4} = \frac{10}{-4}$

$x = -\frac{10}{4} = -\frac{5}{2}$

The number is $-\frac{5}{2}$ and $\left\{-\frac{5}{2}\right\}$ is the solution set.

81. "When a number is divided by -5, the result is 2."

$\frac{x}{-5} = 2$

$(-5)\left(-\frac{1}{5}x\right) = (-5)(2)$

$x = -10$

The number is -10 and $\{-10\}$ is the solution set.

82. "If twice a number is divided by 5, the result is 4."

$\frac{2x}{5} = 4$

$\left(\frac{5}{2}\right)\left(\frac{2}{5}x\right) = \left(\frac{5}{2}\right)\left(\frac{4}{1}\right)$

$x = 10$

The number is 10 and $\{10\}$ is the solution set.

2.3 More on Solving Linear Equations

Classroom Examples, Now Try Exercises

1. *Step 1* (not necessary)

Step 2

$-5p + 4 = 19$

$-5p + 4 - 4 = 19 - 4$ Subtract 4.

$-5p = 15$ Combine terms.

Step 3

$\frac{-5p}{-5} = \frac{15}{-5}$ Divide by -5.

$p = -3$

Step 4

Check $p = -3$: $15 + 4 = 19$ True

The solution set is $\{-3\}$.

N1. *Step 1* (not necessary)

Step 2

$7 + 2m = -3$

$7 + 2m - 7 = -3 - 7$ Subtract 7.

$2m = -10$ Combine terms.

Step 3

$\frac{2m}{2} = \frac{-10}{2}$ Divide by 2.

$m = -5$

Step 4

Check $m = -5$: $7 - 10 = -3$ True

The solution set is $\{-5\}$.

2. *Step 1* (not necessary)
Step 2

$$5-8x=2x-5$$
$$5-8x+8x=2x-5+8x \quad \text{Add } 8x.$$
$$5=10x-5 \quad \text{Combine terms.}$$
$$5+5=10x-5+5 \quad \text{Add 5.}$$
$$10=10x \quad \text{Combine terms.}$$

Step 3

$$\frac{10}{10}=\frac{10x}{10} \quad \text{Divide by 10.}$$
$$1=x$$

Step 4
Check $x=1$: $5-8=2-5$ True
The solution set is $\{1\}$.

N2. *Step 1* (not necessary)
Step 2

$$2q+3=4q-9$$
$$2q+3-2q=4q-9-2q \quad \text{Subtract } 2q.$$
$$3=2q-9 \quad \text{Combine terms.}$$
$$3+9=2q-9+9 \quad \text{Add 9.}$$
$$12=2q \quad \text{Combine terms.}$$

Step 3

$$\frac{12}{2}=\frac{2q}{2} \quad \text{Divide by 2.}$$
$$6=q$$

Step 4
Check $q=6$: $12+3=24-9$ True
The solution set is $\{6\}$.

3. *Step 1*

$$11+3(x+1)=5x+16$$
$$11+3x+3=5x+16 \quad \text{Distributive property}$$
$$3x+14=5x+16 \quad \text{Combine terms.}$$

Step 2

$$3x+14-14=5x+16-14 \quad \text{Subtract 14.}$$
$$3x=5x+2$$
$$3x-5x=5x+2-5x \quad \text{Subtract } 5x.$$
$$-2x=2$$

Step 3

$$\frac{-2x}{-2}=\frac{2}{-2} \quad \text{Divide by } -2.$$
$$x=-1$$

Step 4
Check $x=-1$: $11=11$ True
The solution set is $\{-1\}$.

N3. *Step 1*

$$3(z-6)-5z=-7z+7$$
$$3z-18-5z=-7z+7 \quad \text{Distributive property}$$
$$-2z-18=-7z+7 \quad \text{Combine terms.}$$

Step 2

$$-2z-18+18=-7z+7+18 \quad \text{Add 18.}$$
$$-2z=-7z+25$$
$$-2z+7z=-7z+25+7z \quad \text{Add } 7z.$$
$$5z=25$$

Step 3

$$\frac{5z}{5}=\frac{25}{5} \quad \text{Divide by 5.}$$
$$z=5$$

Step 4
Check $z=5$: $-28=-28$ True
The solution set is $\{5\}$.

4. *Step 1*

$$4x-(x+7)=9$$
$$4x-x-7=9 \quad \text{Distributive property}$$
$$3x-7=9$$

Step 2

$$3x-7+7=9+7 \quad \text{Add 7.}$$
$$3x=16$$

Step 3

$$\frac{3x}{3}=\frac{16}{3} \quad \text{Divide by 3.}$$
$$x=\frac{16}{3}$$

Step 4
Check $x=\frac{16}{3}$: $9=9$ True
The solution set is $\left\{\frac{16}{3}\right\}$.

N4. *Step 1*

$$5x-(x+9)=x-4$$
$$5x-x-9=x-4 \quad \text{Distributive property}$$
$$4x-9=x-4$$

Step 2

$$4x-9+9=x-4+9 \quad \text{Add 9.}$$
$$4x=x+5$$
$$4x-x=x+5-x \quad \text{Subtract } x.$$
$$3x=5$$

Step 3

$$\frac{3x}{3}=\frac{5}{3} \quad \text{Divide by 3.}$$
$$x=\frac{5}{3}$$

Step 4

Check $x=\frac{5}{3}$: $-\frac{7}{3}=-\frac{7}{3}$ True

The solution set is $\left\{\frac{5}{3}\right\}$.

5. *Step 1*

$$2-3(2+6z)=4(z+1)-8$$
$$2-6-18z=4z+4-8 \quad \text{Distributive property}$$
$$-4-18z=4z-4$$

Step 2

$$-4-18z+4=4z-4+4 \quad \text{Add 4.}$$
$$-18z=4z$$
$$-18z-4z=4z-4z \quad \text{Subtract } 4z.$$
$$-22z=0$$

Step 3

$$\frac{-22z}{-22}=\frac{0}{-22} \quad \text{Divide by } -22.$$
$$z=0$$

Step 4

Check $z=0$: $2-3(2)\stackrel{?}{=}4(1)-8$

$-4=-4$ True

The solution set is $\{0\}$.

N5. *Step 1*

$$24-4(7-2t)=4(t-1)$$
$$24-28+8t=4t-4 \quad \text{Distributive property}$$
$$-4+8t=4t-4$$

Step 2

$$-4+8t+4=4t-4+4 \quad \text{Add 4.}$$
$$8t=4t$$
$$8t-4t=4t-4t \quad \text{Subtract } 4t.$$
$$4t=0$$

Step 3

$$\frac{4t}{4}=\frac{0}{4} \quad \text{Divide by 4.}$$
$$t=0$$

Step 4

Check $t=0$: $24-4(7)\stackrel{?}{=}4(-1)$

$-4=-4$ True

The solution set is $\{0\}$.

6.
$$3x-x+10=2x-4+14$$
$$2x+10=2x+10$$
$$2x+10-10=2x+10-10 \quad \text{Subtract 10.}$$
$$2x=2x$$
$$2x-2x=2x-2x \quad \text{Subtract } 2x.$$
$$0=0 \quad \text{True}$$

The variable x has "disappeared," and a true statement has resulted. The original equation is an identity. This means that for every real number value of x, the equation is true. Thus, the solution set is {all real numbers}.

N6.
$$-3(x-7)=2x-5x+21$$
$$-3x+21=-3x+21$$
$$-3x+21-21=-3x+21-21 \quad \text{Subtract 21.}$$
$$-3x=-3x$$
$$-3x+3x=-3x+3x \quad \text{Add } 3x.$$
$$0=0 \quad \text{True}$$

The variable x has "disappeared," and a true statement has resulted. The original equation is an identity. This means that for every real number value of x, the equation is true. Thus, the solution set is {all real numbers}.

7.
$$3x+8=6(x-1)-3x$$
$$3x+8=6x-6-3x \quad \text{Distributive prop.}$$
$$3x+8=3x-6 \quad \text{Combine terms.}$$
$$3x-8-3x=3x-6-3x \quad \text{Subtract } 3x.$$
$$8=-6 \quad \text{False}$$

The variable x has "disappeared," and a *false* statement has resulted. This means that for every real number value of x, the equation is false. Thus, the equation has no solution and its solution set is the empty set, or null set, symbolized $\varnothing$.

N7.
$$-4x+12=3-4(x-3)$$
$$-4x+12=3-4x+12 \quad \text{Distr. prop.}$$
$$-4x+12=-4x+15 \quad \text{Combine.}$$
$$-4x+12+4x=-4x+15+4x \quad \text{Add } 4x.$$
$$12=15 \quad \text{False}$$

The variable x has "disappeared," and a *false* statement has resulted. This means that for every real number value of x, the equation is false. Thus, the equation has no solution and its solution set is the empty set, or null set, symbolized $\varnothing$.

8. *Step 1*

$$\frac{1}{3}x-\frac{5}{12}=\frac{3}{4}+\frac{1}{2}x$$

The LCD of all the fractions in the equation is 12, so multiply each side by 12 to clear the fractions.

$$12\left(\frac{1}{3}x-\frac{5}{12}\right)=12\left(\frac{3}{4}+\frac{1}{2}x\right)$$

$$12\left(\frac{1}{3}x\right)-12\left(\frac{5}{12}\right)=12\left(\frac{3}{4}\right)+12\left(\frac{1}{2}x\right)$$

$$4x-5=9+6x$$

Step 2

$4x-6x-5=9+6x-6x$ Subtract $6x$.

$$-2x-5=9$$

$-2x-5+5=9+5$ Add 5.

$$-2x=14$$

Step 3

$\frac{-2x}{-2}=\frac{14}{-2}$ Divide by -2.

$$x=-7$$

Step 4

Check $x=-7$: $-\frac{7}{3}-\frac{5}{12}\stackrel{?}{=}\frac{3}{4}-\frac{7}{2}$

$$\frac{-28-5}{12}\stackrel{?}{=}\frac{3-14}{4}$$

$-\frac{33}{12}=-\frac{11}{4}$ True

The solution set is $\{-7\}$.

N8. *Step 1*

$$\frac{1}{2}x+\frac{5}{8}x=\frac{3}{4}x-6$$

The LCD of all the fractions in the equation is 8, so multiply each side by 8 to clear the fractions.

$$8\left(\frac{1}{2}x+\frac{5}{8}x\right)=8\left(\frac{3}{4}x-6\right)$$

$$8\left(\frac{1}{2}x\right)+8\left(\frac{5}{8}x\right)=8\left(\frac{3}{4}x\right)-8(6)$$

$$4x+5x=6x-48$$

$$9x=6x-48$$

Step 2

$9x-6x=6x-48-6x$ Subtract $6x$.

$$3x=-48$$

Step 3

$\frac{3x}{3}=\frac{-48}{3}$ Divide by 3.

$$x=-16$$

Step 4

Check $x=-16$: $-8-10\stackrel{?}{=}-12-6$

$-18=-18$ True

The solution set is $\{-16\}$.

9. *Step 1*

$$\frac{1}{4}(x+3)-\frac{2}{3}(x+1)=-2$$

$$12\left[\frac{1}{4}(x+3)-\frac{2}{3}(x+1)\right]=12(-2)$$

$$12\left[\frac{1}{4}(x+3)\right]-12\left[\tfrac{2}{3}(x+1)\right]=-24$$

$$3(x+3)-8(x+1)=-24$$

$$3x+9-8x-8=-24$$

$$-5x+1=-24$$

Step 2

$-5x+1-1=-24-1$ Subtract 1.

$-5x=-25$ Combine like terms.

Step 3

$\frac{-5x}{-5}=\frac{-25}{-5}$ Divide by -5.

$$x=5$$

Step 4

Check $x=5$: $2-4=-2$ True

The solution set is $\{5\}$.

N9. *Step 1*

$$\frac{2}{3}(x+2)-\frac{1}{2}(3x+4)=-4$$

$$6\left[\frac{2}{3}(x+2)-\frac{1}{2}(3x+4)\right]=6(-4)$$

$$6\left[\frac{2}{3}(x+2)\right]-6\left[\frac{1}{2}(3x+4)\right]=-24$$

$$4(x+2)-3(3x+4)=-24$$

$$4x+8-9x-12=-24$$

$$-5x-4=-24$$

Step 2

$-5x-4+4=-24+4$ Add 4.

$-5x=-20$ Combine like terms.

Step 3

$\frac{-5x}{-5}=\frac{-20}{-5}$ Divide by -5.

$$x=4$$

Step 4

Check $x=4$: $4-8=-4$ True

The solution set is $\{4\}$.

10. *Step 1*

$0.5(2-3x) = 4.5 - 0.1(x+7)$

To clear decimals, multiply both sides by 10.

$10[0.5(2-3x)] = 10[4.5 - 0.1(x+7)]$

$5(2-3x) = 45 - 1(x+7)$

$10 - 15x = 45 - x - 7$

$10 - 15x = 38 - x$

Step 2

$10 - 15x + x = 38 - x + x$

$10 - 14x = 38$

$10 - 10 - 14x = 38 - 10$

$-14x = 28$

Step 3

$\frac{-14x}{-14} = \frac{28}{-14}$

$x = -2$

Step 4

Check $x = -2$: $0.5(8) \stackrel{?}{=} 4.5 - 0.1(5)$

$4 = 4$ True

The solution set is $\{-2\}$.

N10. *Step 1*

$0.05(13-t) - 0.2t = 0.08(30)$

To clear decimals, multiply both sides by 100.

$100[0.05(13-t) - 0.2t] = 100[0.08(30)]$

$5(13-t) - 20t = 8(30)$

$65 - 5t - 20t = 240$

$65 - 25t = 240$

Step 2

$65 - 25t - 65 = 240 - 65$

$-25t = 175$

Step 3

$\frac{-25t}{-25} = \frac{175}{-25}$

$t = -7$

Step 4

Check $t = -7$: $1 + 1.4 \stackrel{?}{=} -2.4$

$2.4 = 2.4$ True

The solution set is $\{-7\}$.

11. To find the other number, you would divide 36 by x, so an expression for the other number is $\frac{36}{x}$.

N11. First, suppose that the sum of two numbers is 18, and one of the numbers is 10. How would you find the other number? You would subtract 10 from 18. Instead of using 10 as one of the numbers, use m. This gives us the expression $18 - m$ for the other number.

Exercises

1. Use the addition property of equality to subtract 8 from each side.

2. Combine like terms on the left.

3. Clear the parentheses by using the distributive property.

4. Use the multiplication property of equality to multiply each side by $\frac{4}{3}$.

5. Clear fractions by multiplying by the LCD, 6.

6. Clear the decimals by multiplying each side by 10.

7. **(a)** $6 = 6$ (The original equation is a(n) identity.) This goes with choice B, {all real numbers}.

(b) $x = 0$ (The original equation is a(n) conditional.) This goes with choice A, $\{0\}$.

(c) $-5 = 0$ (The original equation is a(n) contradiction.) This goes with choice C, $\varnothing$.

8. $100[0.03(x-10)] = 3(x-10) = 3x - 30$, so choice D is correct.

9. $3x + 2 = 14$

$3x + 2 - 2 = 14 - 2$ Subtract 2.

$3x = 12$ Combine like terms.

$\frac{3x}{3} = \frac{12}{3}$ Divide by 3.

$x = 4$

Check $x = 4$: $12 + 2 = 14$ True

The solution set is $\{4\}$.

10. $4x + 3 = 27$

$4x + 3 - 3 = 27 - 3$ Subtract 3.

$4x = 24$ Combine like terms.

$\frac{4x}{4} = \frac{24}{4}$ Divide by 4.

$x = 6$

Check $x = 6$: $24 + 3 = 27$ True

The solution set is $\{6\}$.

11. $-5z-4=21$

$-5z-4+4=21+4$ Add 4.

$-5z=25$ Combine like terms.

$\frac{-5z}{-5}=\frac{25}{-5}$ Divide by -5.

$z=-5$

Check $z=-5$: $25-4=21$ True

The solution set is $\{-5\}$.

12. $-7w-4=10$

$-7w-4+4=10+4$ Add 4.

$-7w=14$ Combine like terms.

$\frac{-7w}{-7}=\frac{14}{-7}$ Divide by -7.

$w=-2$

Check $w=-2$: $14-4=10$ True

The solution set is $\{-2\}$.

13. $4p-5=2p$

$4p-5-4p=2p-4p$ Subtract $4p$.

$-5=-2p$ Combine like terms.

$\frac{-5}{-2}=\frac{-2p}{-2}$ Divide by -2.

$\frac{5}{2}=p$

Check $p=\frac{5}{2}$: $10-5=5$ True

The solution set is $\left\{\frac{5}{2}\right\}$.

14. $6q-2=3q$

$6q-2-6q=3q-6q$ Subtract $6q$.

$-2=-3q$ Combine like terms.

$\frac{-2}{-3}=\frac{-3q}{-3}$ Divide by -3.

$\frac{2}{3}=q$

Check $q=\frac{2}{3}$: $4-2=2$ True

The solution set is $\left\{\frac{2}{3}\right\}$.

15. $2x+9=4x+11$

$-2x+9=11$ Subtract $4x$.

$-2x=2$ Subtract 9.

$x=-1$ Divide by -2.

Check $x=-1$: $7=7$ True

The solution set is $\{-1\}$.

16. $7p+8=9p-2$

$-2p+8=-2$ Subtract $9p$.

$-2p=-10$ Subtract 8.

$p=5$ Divide by -2.

Check $p=5$: $43=43$ True

The solution set is $\{5\}$.

17. For this equation, Step 1 is not needed.

Step 2

$5m+8-8=7+3m-8$ Subtract 8.

$5m=3m-1$

$5m-3m=3m-1-3m$ Subtract $3m$.

$2m=-1$

Step 3

$\frac{2m}{2}=\frac{-1}{2}$

$m=-\frac{1}{2}$

Step 4

Substitute $-\frac{1}{2}$ for m in the original equation.

$5m+8=7+3m$

$5\left(-\frac{1}{2}\right)+8\stackrel{?}{=}7+3\left(-\frac{1}{2}\right)$ Let $m=-\frac{1}{2}$.

$-\frac{5}{2}+8\stackrel{?}{=}7+\left(-\frac{3}{2}\right)$

$\frac{11}{2}=\frac{1}{2}$ True

The solution set is $\left\{-\frac{1}{2}\right\}$.

18. $4r+2=r-6$

$4r+2-r=r-6-r$ Subtract r.

$3r+2=-6$

$3r+2-2=-6-2$ Subtract 2.

$3r=-8$

$\frac{3r}{3}=-\frac{-8}{3}$ Divide by 3.

$r=-\frac{8}{3}$

Check $r=-\frac{8}{3}$: $-\frac{32}{3}+\frac{6}{3}=-\frac{8}{3}-\frac{18}{3}$ True

The solution set is $\left\{-\frac{8}{3}\right\}$.

19. $-12x-5=10-7x$

$-12x-5+7x=10-7x+7x$ Add $7x$.

$-5x-5=10$

$-5x-5+5=10+5$ Add 5.

$-5x=15$

$\frac{-5x}{-5}=\frac{15}{-5}$ Divide by -5.

$x=-3$

Check $x=-3$: $36-5=10+21$ True

The solution set is $\{-3\}$.

20. $-16w-3=13-8w$

$-16w-3+8w=13-8w+8w$ Add $8w$.

$-8w-3=13$

$-8w-3+3=13+3$ Add 3.

$-8w=16$

$\frac{-8w}{-8}=\frac{16}{-8}$ Divide by -8.

$w=-2$

Check $w=-2$: $32-3=13+16$ True

The solution set is $\{-2\}$.

21. $12h-5=11h+5-h$

$12h-5=10h+5$ Combine like terms.

$2h-5=5$ Subtract $10h$.

$2h=10$ Add 5.

$h=5$ Divide by 2.

Check $h=5$: $55=55$ True

The solution set is $\{5\}$.

22. $-4x-1=-5x+1+3x$

$-4x-1=-2x+1$ Combine like terms.

$-2x-1=1$ Add $2x$.

$-2x=2$ Add 1.

$x=-1$ Divide by -2.

Check $x=-1$: $3=3$ True

The solution set is $\{-1\}$.

23. $7r-5r+2=5r+2-r$

$2r+2=4r+2$ Combine like terms.

$2=2r+2$ Subtract $2r$.

$0=2r$ Subtract 2.

$0=r$ Divide by 2.

Check $r=0$: $2=2$ True

The solution set is $\{0\}$.

24. $9p-4p+6=7p+6-3p$

$5p+6=4p+6$ Combine terms.

$p+6=6$ Subtract $4p$.

$p=0$ Subtract 6.

Check $p=0$: $6=6$ True

The solution set is $\{0\}$.

25. $3(4x+2)+5x=30-x$

$12x+6+5x=30-x$ Distributive prop.

$17x+6=30-x$ Combine like terms.

$18x+6=30$ Add $1x$.

$18x=24$ Subtract 6.

$x=\frac{24}{18}=\frac{4}{3}$ Divide by 18.

Check $x=\frac{4}{3}$: $\frac{86}{3}=\frac{86}{3}$ True

The solution set is $\left\{\frac{4}{3}\right\}$.

26. $5(2m+3)-4m=2m+25$

$10m+15-4m=2m+25$ Distributive prop.

$6m+15=2m+25$ Combine terms.

$6m=2m+10$ Subtract 15.

$4m=10$ Subtract $2m$.

$m=\frac{10}{4}=\frac{5}{2}$ Divide by 4.

Check $m=\frac{5}{2}$: $30=30$ True

The solution set is $\left\{\frac{5}{2}\right\}$.

27. $-2p+7=3-(5p+1)$

$-2p+7=3-5p-1$ Distributive property

$-2p+7=-5p+2$ Combine like terms.

$3p+7=2$ Add $5p$.

$3p=-5$ Subtract 7.

$p=-\frac{5}{3}$

Check $p=-\frac{5}{3}$: $\frac{31}{3}=\frac{31}{3}$ True

The solution set is $\left\{-\frac{5}{3}\right\}$.

28. $4x+9=3-(x-2)$

$4x+9=3-x+2$ Distributive property

$4x+9=-x+5$ Combine like terms.

$5x+9=5$ Add $1x$.

$5x=-4$ Subtract 9.

$x=-\frac{4}{5}$ Divide by 5.

Check $x=-\frac{4}{5}$: $\frac{29}{5}=\frac{29}{5}$ True

The solution set is $\left\{-\frac{4}{5}\right\}$.

29. $11x-5(x+2)=6x+5$

$11x-5x-10=6x+5$

$6x-10=6x+5$

$6x-10-6x=6x+5-6x$

$-10=5$

Since $-10=5$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

30. $6x-4(x+1)=2x+4$

$6x-4x-4=2x+4$

$2x-4=2x+4$

$-4=4$ Subtract $2x$.

Since $-4=4$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

31. $6(3w+5)=2(10w+10)$

$18w+30=20w+20$

$18w=20w-10$ Subtract 30.

$-2w=-10$ Subtract $20w$.

$w=5$ Divide by -2.

Check $w=5$: $120=120$ True

The solution set is $\{5\}$.

32. $4(2x-1)=-6(x+3)$

$8x-4=-6x-18$ Distributive property

$14x-4=-18$ Add $6x$.

$14x=-14$ Add 4.

$x=-1$ Divide by 14.

Check $x=-1$: $-12=-12$ True

The solution set is $\{-1\}$.

33. $-(4x+2)-(-3x-5)=3$

$-1(4x+2)-1(-3x-5)=3$

$-4x-2+3x+5=3$

$-x+3=3$

$-x=0$

$x=0$

Check $x=0$: $3=3$ True

The solution set is $\{0\}$.

34. $-(6k-5)-(-5k+8)=-3$

$-1(6k-5)-1(-5k+8)=-3$

$-6k+5+5k-8=-3$

$-k-3=-3$

$-k=0$

$k=0$

Check $k=0$: $-3=-3$ True

The solution set is $\{0\}$.

35. $3(2x-4)=6(x-2)$

$6x-12=6x-12$

$-12=-12$ Subtract $6x$.

$0=0$ Add 12.

The variable has "disappeared." Since the resulting statement is a true one, any real number is a solution. We indicate the solution set as {all real numbers}.

36. $3(6-4x)=2(-6x+9)$

$18-12x=-12x+18$

$18=18$ Add $12x$.

Since $18=18$ is a true statement, the solution set is {all real numbers}.

37. $6(4x-1)=12(2x+3)$

$24x-6=24x+36$

$-6=36$ Subtract $24x$.

The variable has "disappeared," and the resulting equation is false. Therefore, the equation has no solution set, symbolized by $\varnothing$.

38. $6(2x+8) = 4(3x-6)$

$12x + 48 = 12x - 24$

$48 = -24$ Subtract $12x$.

Since $48 = -24$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

39. The least common denominator of all the fractions in the equation is 10.

$$10\left(\frac{3}{5}t - \frac{1}{10}t\right) = 10\left(t - \frac{5}{2}\right)$$

$$10\left(\frac{3}{5}t\right) + 10\left(-\frac{1}{10}t\right) = 10t + 10\left(-\frac{5}{2}\right)$$

$$6t - t = 10t - 25$$

$$5t = 10t - 25$$

$$-5t = -25$$

$$\frac{-5t}{-5} = \frac{-25}{-5}$$

$$t = 5$$

Check $t = 5$: $\frac{5}{2} = \frac{5}{2}$ True

The solution set is $\{5\}$.

40. The least common denominator of all the fractions in the equation is 14, so multiply both sides by 14 and solve for r.

$$14\left(-\frac{2}{7}r + 2r\right) = 14\left(\frac{1}{2}r + \frac{17}{2}\right)$$

$$-4r + 28r = 7r + 119$$

$$24r = 7r + 119$$

$$17r = 119$$

$$r = \frac{119}{17} = 7$$

Check $r = 7$: $12 = 12$ True

The solution set is $\{7\}$.

41. The least common denominator of all the fractions in the equation is 12, so multiply both sides by 12 and solve for x.

$$12\left(\frac{3}{4}x - \frac{1}{3}x + 5\right) = 12\left(\frac{5}{6}x\right)$$

$$9x - 4x + 60 = 10x$$

$$5x + 60 = 10x$$

$$60 = 5x$$

$$\frac{60}{5} = \frac{5x}{5}$$

$$12 = x$$

Check $x = 12$: $9 - 4 + 5 = 10$ True

The solution set is $\{12\}$.

42. The least common denominator of all the fractions in the equation is 15, so multiply both sides by 15 and solve for x.

$$15\left(\frac{1}{5}x - \frac{2}{3}x - 2\right) = 15\left(-\frac{2}{5}x\right)$$

$$3x - 10x - 30 = -6x$$

$$-7x - 30 = -6x$$

$$-30 = x$$

Check $x = -30$: $-6 + 20 - 2 = 12$ True

The solution set is $\{-30\}$.

43. The least common denominator of all the fractions in the equation is 35, so multiply both sides by 35 and solve for x.

$$35\left[\frac{1}{7}(3x+2) - \frac{1}{5}(x+4)\right] = 35(2)$$

$$5(3x+2) - 7(x+4) = 70$$

$$15x + 10 - 7x - 28 = 70$$

$$8x - 18 = 70$$

$$8x = 88$$

$$\frac{8x}{8} = \frac{88}{8}$$

$$x = 11$$

Check $x = 11$: $5 - 3 = 2$ True

The solution set is $\{11\}$.

44. The least common denominator of all the fractions in the equation is 12, so multiply both sides by 12 and solve for x.

$$12\left[\frac{1}{4}(3x-1) + \frac{1}{6}(x+3)\right] = 12(3)$$

$$3(3x-1) + 2(x+3) = 36$$

$$9x - 3 + 2x + 6 = 36$$

$$11x + 3 = 36$$

$$11x = 33$$

$$\frac{11x}{11} = \frac{33}{11}$$

$$x = 3$$

Check $x = 3$: $2 + 1 = 3$ True

The solution set is $\{3\}$.

45. The LCD of all the fractions is 4.

$$4\left[-\frac{1}{4}(x-12)+\frac{1}{2}(x+2)\right]=4(x+4)$$
$$4\left(-\frac{1}{4}\right)(x-12)+4\left(\frac{1}{2}\right)(x+2)=4x+16$$
$$(-1)(x-12)+2(x+2)=4x+16$$
$$-x+12+2x+4=4x+16$$
$$x+16=4x+16$$
$$-3x+16=16$$
$$-3x=0$$
$$\frac{-3x}{-3}=\frac{0}{-3}$$
$$x=0$$

Check $x=0$: $4=4$ True

The solution set is $\{0\}$.

46. The least common denominator of all the fractions in the equation is 9, so multiply both sides by 9 and solve for p.

$$9\left[\tfrac{1}{9}(p+18)+\tfrac{1}{3}(2p+3)\right]=9(p+3)$$
$$p+18+3(2p+3)=9p+27$$
$$p+18+6p+9=9p+27$$
$$7p+27=9p+27$$
$$-2p+27=27$$
$$-2p=0$$
$$\frac{-2p}{-2}=\frac{0}{-2}$$
$$p=0$$

Check $p=0$: $3=3$ True

The solution set is $\{0\}$.

47. The least common denominator of all the fractions in the equation is 6, so multiply both sides by 6 and solve for k.

$$6\left[\frac{2}{3}k-\left(k-\frac{1}{2}\right)\right]=6\left[\frac{1}{6}(k-51)\right]$$
$$6\left(\frac{2}{3}k\right)-6\left(k-\frac{1}{2}\right)=6\left[\frac{1}{6}(k-51)\right]$$
$$4k-6k+3=1(k-51)$$
$$-2k+3=k-51$$
$$-3k+3=-51$$
$$-3k=-54$$
$$k=18$$

Check $k=18$: $-\frac{11}{2}=-\frac{11}{2}$ True

The solution set is $\{18\}$.

48. The least common denominator is 12.

$$12\left[-\frac{5}{6}q-(q-1)\right]=12\left[\frac{1}{4}(-q+80)\right]$$
$$12\left(-\frac{5}{6}q\right)-12(q-1)=12\left[\frac{1}{4}(-q+80)\right]$$
$$-10q-12q+12=3(-q+80)$$
$$-22q+12=-3q+240$$
$$-19q+12=240$$
$$-19q=228$$
$$q=\frac{228}{-19}=-12$$

Check $q=-12$: $23=23$ True

The solution set is $\{-12\}$.

49. To clear the equation of decimals, we multiply by 100.

$$100(0.75x-3.2)=100(0.55-0.5x)$$
$$75x-320=55-50x$$
$$75x-320+50x=55-50x+50x$$
$$125x-320=55$$
$$125x-320+320=55+320$$
$$125x=375$$
$$\frac{125x}{125}=\frac{375}{125}$$
$$x=3$$

Check $x=3$: $-0.95=-0.95$ True

The solution set is $\{3\}$.

50. $100(1.35x-0.6)=100(1.65+2.1x)$

$$135x-60=165+210x$$
$$135x-60-135x=165+210x-135x$$
$$-60=165+75x$$
$$-60-165=165+75x-165$$
$$-225=75x$$
$$\frac{-225}{75}=\frac{75x}{75}$$
$$x=-3$$

Check $x=-3$: $-4.65=-4.65$ True

The solution set is $\{-3\}$.

51. Solve the equation for t.

$$0.8t + 0.15 = 2t - 1.35$$
$$100(0.8t + 0.15) = 100(2t - 1.35)$$
$$80t + 15 = 200t - 135$$
$$80t + 15 - 80t = 200t - 135 - 80t$$
$$15 = 120t - 135$$
$$15 + 135 = 120t - 135 + 135$$
$$150 = 120t$$
$$\frac{150}{120} = \frac{120t}{120}$$
$$t = \frac{5}{4}$$

Check $t = \frac{5}{4}$: $1.15 = 1.15$ True

The solution set is $\left\{\frac{5}{4}\right\}$.

52. To eliminate the decimal in 3.4, we need to multiply both sides by 10. But to eliminate the decimal in –0.12 and 0.84, we need to multiply by 100, so we choose 100.

$$-0.12p + 3.4 = 0.84 + 5p$$
$$100[-0.12p + 3.4] = 100[0.84 + 5p]$$
$$-12p + 340 = 84 + 500p$$
$$-12p + 340 + 12p = 84 + 500p + 12p$$
$$340 = 84 + 512p$$
$$340 - 84 = 84 + 512p - 84$$
$$256 = 512p$$
$$\frac{256}{512} = \frac{512p}{512}$$
$$p = \frac{1}{2}$$

Check $p = \frac{1}{2}$: $3.34 = 3.34$ True

The solution set is $\left\{\frac{1}{2}\right\}$.

53. To eliminate the decimal in 0.2 and 0.1, we need to multiply both sides by 10. But to eliminate the decimal in 0.05, we need to multiply by 100, so we choose 100.

$$100[0.2(60) + 0.05x] = 100[0.1(60 + x)]$$
$$20(60) + 5x = 10(60 + x)$$
$$1200 + 5x = 600 + 10x$$
$$1200 - 5x = 600$$
$$-5x = -600$$
$$x = \frac{-600}{-5} = 120$$

Check $x = 120$: $18 = 18$ True

The solution set is $\{120\}$.

54.

$$0.3(30) + 0.15x = 0.2(30 + x)$$
$$100[0.3(30) + 0.15x] = 100[0.2(30 + x)]$$
$$30(30) + 15x = 20(30 + x)$$
$$900 + 15x = 600 + 20x$$
$$900 - 5x = 600$$
$$-5x = -300$$
$$x = 60$$

Check $x = 60$: $18 = 18$ True

The solution set is $\{60\}$.

55. $1.00x + 0.05(12 - x) = 0.10(63)$

To clear the equation of decimals, we multiply both sides by 100.

$$100[1.00x + 0.05(12 - x)] = 100[0.10(63)]$$
$$100x + 5(12 - x) = 10(63)$$
$$100x + 60 - 5x = 630$$
$$95x + 60 = 630$$
$$95x = 570$$
$$x = \frac{570}{95} = 6$$

Check $x = 6$: $6.3 = 6.3$ True

The solution set is $\{6\}$.

56. $0.92x + 0.98(12 - x) = 0.96(12)$

$100[0.92x + 0.98(12 - x)] = 100[0.96(12)]$

$92x + 98(12 - x) = 96(12)$

$92x + 1176 - 98x = 1152$

$-6x + 1176 = 1152$

$-6x = -24$

$x = \frac{-24}{-6} = 4$

Check $x = 4$: $11.52 = 11.52$ True

The solution set is $\{4\}$.

57. $0.6(10,000) + 0.8x = 0.72(10,000 + x)$

$60(10,000) + 80x = 72(10,000 + x)$

$600,000 + 80x = 720,000 + 72x$

$600,000 + 8x = 720,000$

$8x = 120,000$

$x = \frac{120,000}{8}$

$x = 15,000$

Check $x = 15,000$: $18,000 = 18,000$ True

The solution set is $\{15,000\}$.

58. $0.2(5000) + 0.3x = 0.25(5000 + x)$

$20(5000) + 30x = 25(5000 + x)$

$100,000 + 30x = 125,000 + 25x$

$5x + 100,000 = 125,000$

$5x = 25,000$

$x = \frac{25,000}{5} = 5000$

Check $x = 5000$: $2500 = 2500$ True

The solution set is $\{5000\}$.

59. $10(2x - 1) = 8(2x + 1) + 14$

$20x - 10 = 16x + 8 + 14$

$20x - 10 = 16x + 22$

$4x - 10 = 22$

$4x = 32$

$x = 8$

Check $x = 8$: $150 = 150$ True

The solution set is $\{8\}$.

60. $9(3k - 5) = 12(3k - 1) - 51$

$27k - 45 = 36k - 12 - 51$

$27k - 45 = 36k - 63$

$-45 = 9k - 63$

$18 = 9k$

$2 = k$

Check $k = 2$: $9 = 9$ True

The solution set is $\{2\}$.

61. $24 - 4(7 - 2t) = 4(t - 1)$

$24 - 28 + 8t = 4t - 4$

$-4 + 8t = 4t - 4$

$-4 + 8t - 4t = 4t - 4 - 4t$

$-4 + 4t = -4$

$4t = 0$

$t = 0$

Check $t = 0$: $-4 = -4$ True

The solution set is $\{0\}$.

62. $8 - 2(2 - x) = 4(x + 1)$

$8 - 4 + 2x = 4x + 4$

$4 + 2x = 4x + 4$

$4 + 2x - 2x = 4x + 4 - 2x$

$4 = 2x + 4$

$2x = 0$

$x = 0$

Check $x = 0$: $4 = 4$ True

The solution set is $\{0\}$.

63. $4(x + 8) = 2(2x + 6) + 20$

$4x + 32 = 4x + 12 + 20$

$4x + 32 = 4x + 32$

$4x = 4x$

$0 = 0$

Since $0 = 0$ is a true statement, the solution set is {all real numbers}.

64. $4(x + 3) = 2(2x + 8) - 4$

$4x + 12 = 4x + 16 - 4$

$4x + 12 = 4x + 12$

$12 = 12$

Since $12 = 12$ is a true statement, the solution set is {all real numbers}.

65. To clear fractions, multiply both sides by the LCD, which is 4.

$$4\left[\frac{1}{2}(x+2)+\frac{3}{4}(x+4)\right]=4(x+5)$$
$$4\left(\frac{1}{2}\right)(x+2)+4\left(\frac{3}{4}\right)(x+4)=4x+20$$
$$2(x+2)+3(x+4)=4x+20$$
$$2x+4+3x+12=4x+20$$
$$5x+16=4x+20$$
$$x+16=20$$
$$x=4$$

Check $x=4$: $9=9$ True

The solution set is $\{4\}$.

66. To clear fractions, multiply both sides by the LCD, which is 6.

$$6\left[\frac{1}{3}(x+3)+\frac{1}{6}(x-6)\right]=6(x+3)$$
$$6\left(\frac{1}{3}\right)(x+3)+6\left(\frac{1}{6}\right)(x-6)=6(x+3)$$
$$2(x+3)+1(x-6)=6(x+3)$$
$$2x+6+x-6=6x+18$$
$$3x=6x+18$$
$$-3x=18$$
$$x=\frac{18}{-3}=-6$$

Check $x=-6$: $-3=-3$ True

The solution set is $\{-6\}$.

67. To eliminate the decimals, multiply both sides by 10.

$$10[0.1(x+80)+0.2x]=10(14)$$
$$1(x+80)+2x=140$$
$$x+80+2x=140$$
$$3x+80=140$$
$$3x=60$$
$$x=20$$

Check $x=20$: $14=14$ True

The solution set is $\{20\}$.

68. To clear the decimals, multiply both sides by 10.

$$3(x+15)+4(x+25)=250$$
$$3x+45+4x+100=250$$
$$7x+145=250$$
$$7x=105$$
$$x=15$$

Check $x=15$: $25=25$ True

The solution set is $\{15\}$.

69.
$$9(v+1)-3v=2(3v+1)-8$$
$$9v+9-3v=6v+2-8$$
$$6v+9=6v-6$$
$$9=-6$$

Because $9=-6$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

70.
$$8(t-3)+4t=6(2t+1)-10$$
$$8t-24+4t=12t+6-10$$
$$12t-24=12t-4$$
$$-24=-4$$

Because $-24=-4$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

71. The sum of q and the other number is 11. To find the other number, you would subtract q from 11, so an expression for the other number is $11-q$.

72. The sum of r and the other number is 34. To find the other number, you would subtract r from 34, so an expression for the other number is $34-r$.

73. The product of x and the other number is 9. To find the other number, you would divide 9 by x, so an expression for the other number is $\frac{9}{x}$.

74. The product of m and the other number is -6. To find the other number, you would divide -6 by m, so an expression for the other number is $\frac{-6}{m}$.

75. If a baseball player gets 65 hits in one season, and h of the hits are in one game, then $65-h$ of the hits came in the rest of the games.

76. If a hockey player scores 42 goals in one season, and n of the goals are in one game, then $42-n$ of the goals came in the rest of the games.

77. If Monica is x years old now, then 15 years from now she will be $x+15$ years old. Five years ago, she was $x-5$ years old.

78. If Chandler is y years old now, then four years ago he was $y-4$ years old. Eleven years from now, he will be $y+11$ years old.

79. Since the value of each quarter is 25 cents, the value of r quarters is $25r$ cents.

80. Since the value of each dime is 10 cents, the value of y dimes is $10y$ cents.

81. Since each bill is worth 5 dollars, the number of bills is $\frac{t}{5}$.

82. Since each bill is worth 10 dollars, the number of bills is $\frac{v}{10}$.

83. Since each adult ticket costs x dollars, the cost of 3 adult tickets is $3x$. Since each child's ticket costs y dollars, the cost of 2 children's tickets is $2y$. Therefore, the total cost is $3x + 2y$ (dollars).

84. Since each adult ticket costs p dollars, the cost of 4 adult tickets is $4p$. Since each child's ticket costs q dollars, the cost of 6 children's tickets is $6q$. Therefore, the total cost is $4p + 6q$ (dollars).

Summary Exercises Applying Methods for Solving Linear Equations

1. This is an equation since it has an equals sign.

$x + 2 = -3$

$x = -5$ Subtract 2.

Check $x = -5$: $-3 = -3$ True

The solution set is $\{-5\}$.

2. This is an expression since it does not have an equals sign.

$4p - 6 + 3p - 8 = 7p - 14$

3. This is an expression since it does not have an equals sign.

$$-(m-1) - (3+2m) = -m + 1 - 3 - 2m$$
$$= -3m - 2$$

4. This is an equation since it has an equals sign.

$6q - 9 = 12 + 3q$

$3q - 9 = 12$

$3q = 21$

$q = 7$

Check $q = 7$: $33 = 33$ True

The solution set is $\{7\}$.

5. This is an equation since it has an equals sign.

$5x - 9 = 3(x - 3)$

$5x - 9 = 3x - 9$ Distributive property

$2x - 9 = -9$ Subtract $3x$.

$2x = 0$ Add 9.

$x = 0$ Divide by 2.

Check $x = 0$: $-9 = -9$ True

The solution set is $\{0\}$.

6. This is an equation since it has an equals sign. To clear fractions, multiply both sides by the LCD, which is 12.

$$12\left(\frac{2}{3}x + 8\right) = 12\left(\frac{1}{4}x\right)$$

$8x + 96 = 3x$

$5x + 96 = 0$

$5x = -96$

$x = -\frac{96}{5}$

Check $x = -\frac{96}{5}$: $-\frac{24}{5} = -\frac{24}{5}$ True

The solution set is $\left\{-\frac{96}{5}\right\}$.

7. This is an expression since it does not have an equals sign.

$$2 - 6(z+1) - 4(z-2) - 10$$
$$= 2 - 6z - 6 - 4z + 8 - 10$$
$$= -10z - 6$$

8. This is an equation since it has an equals sign.

$7(p-2) + p = 2(p+2)$

$7p - 14 + p = 2p + 4$

$8p - 14 = 2p + 4$

$6p - 14 = 4$

$6p = 18$

$p = 3$

Check $p = 3$: $10 = 10$ True

The solution set is $\{3\}$.

9. This is an expression since it does not have an equals sign.

$$\frac{1}{2}(x+10) - \frac{2}{3}x = \frac{1}{2}x + 5 - \frac{2}{3}x$$
$$= \frac{3}{6}x + 5 - \frac{4}{6}x$$
$$= -\frac{1}{6}x + 5$$

10. This is an expression since it does not have an equals sign.

$$-4(k+2)+3(2k-1) = -4k-8+6k-3$$
$$= 2k-11$$

11. $-6z = -14$

$$z = \frac{-14}{-6} \quad \text{Divide by } -6.$$
$$= \frac{7}{3}$$

Check $z = \frac{7}{3}$: $-14 = -14$ True

The solution set is $\left\{\frac{7}{3}\right\}$.

12. $2m+8 = 16$

$2m = 8$ Subtract 8.

$m = 4$ Divide by 2.

Check $m = 4$: $16 = 16$ True

The solution set is $\{4\}$.

13. $12.5x = -63.75$

$$x = \frac{-63.75}{12.5} \quad \text{Divide by } 12.5.$$
$$= -5.1$$

Check $x = -5.1$: $-63.75 = -63.75$ True

The solution set is $\{-5.1\}$.

14. $-x = -12$

$x = 12$ Multiply by -1.

Check $x = 12$: $-12 = -12$ True

The solution set is $\{12\}$.

15. $\frac{4}{5}x = -20$

$$x = \left(\frac{5}{4}\right)(-20) \quad \text{Multiply by } \frac{5}{4}.$$
$$= -25$$

Check $x = -25$: $-20 = -20$ True

The solution set is $\{-25\}$.

16. $7m-5m = -12$

$2m = -12$ Combine like terms.

$m = -6$ Divide by 2.

Check $m = -6$: $-12 = -12$ True

The solution set is $\{-6\}$.

17. $-x = 6$

$x = -6$ Multiply by -1.

Check $x = -6$: $6 = 6$ True

The solution set is $\{-6\}$.

18. $\frac{x}{-2} = 8$

$$-\frac{1}{2}x = 8$$
$$x = -2(8) \quad \text{Multiply by } -2.$$
$$= -16$$

Check $x = -16$: $8 = 8$ True

The solution set is $\{-16\}$.

19. $4x+2(3-2x) = 6$

$$4x+6-4x = 6$$
$$6 = 6$$

Since $6 = 6$ is a true statement, the solution set is $\{\text{all real numbers}\}$.

20. $x-16.2 = 7.5$

$x = 23.7$ Add 16.2.

Check $x = 23.7$: $7.5 = 7.5$ True

The solution set is $\{23.7\}$.

21. $7m-(2m-9) = 39$

$$7m-2m+9 = 39$$
$$5m+9 = 39$$
$$5m = 30$$
$$m = 6$$

Check $m = 6$: $39 = 39$ True

The solution set is $\{6\}$.

22. $2-(m+4) = 3m-2$

$$2-m-4 = 3m-2$$
$$-m-2 = 3m-2$$
$$-4m-2 = -2$$
$$-4m = 0$$
$$m = \frac{0}{-4} = 0$$

Check $m = 0$: $-2 = -2$ True

The solution set is $\{0\}$.

23. $-3(m-4)+2(5+2m)=29$

$-3m+12+10+4m=29$

$m+22=29$

$m=7$

Check $m=7$: $29=29$ True

The solution set is $\{7\}$.

24. To eliminate the decimals, multiply both sides by 10.

$10[-0.3x+2.1(x-4)]=10(-6.6)$

$-3x+21(x-4)=-66$

$-3x+21x-84=-66$

$18x-84=-66$

$18x=18$

$x=1$

Check $x=1$: $-0.3-6.3=-6.6$ True

The solution set is $\{1\}$.

25. To eliminate the decimals, multiply both sides by 100.

$100[0.08x+0.06(x+9)]=100(1.24)$

$8x+6(x+9)=124$

$8x+6x+54=124$

$14x+54=124$

$14x=70$

$x=5$

Check $x=5$: $0.4+0.84=1.24$ True

The solution set is $\{5\}$.

26. $3(m+5)-1+2m=5(m+2)$

$3m+15-1+2m=5m+10$

$5m+14=5m+10$

$14=10$

Because $14=10$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

27. $-2t+5t-9=3(t-4)-5$

$-2t+5t-9=3t-12-5$

$3t-9=3t-17$

$-9=-17$

Because $-9=-17$ is a false statement, the equation has no solution set, symbolized by $\varnothing$.

28. To eliminate the decimals, multiply both sides by 10.

$10[2.3x+13.7]=10[1.3x+2.9]$

$23x+137=13x+29$

$10x+137=29$

$10x=-108$

$x=-10.8$

Check $x=-10.8$: $-11.14=-11.14$ True

The solution set is $\{-10.8\}$.

29. To eliminate the decimals, multiply both sides by 10.

$10[0.2(50)+0.8r]=10[0.4(50+r)]$

$2(50)+8r=4(50+r)$

$100+8r=200+4r$

$100+4r=200$

$4r=100$

$r=25$

Check $r=25$: $10+20=30$ True

The solution set is $\{25\}$.

30. $r+9+7r=4(3+2r)-3$

$8r+9=12+8r-3$

$8r+9=8r+9$

$9=9$

Since $9=9$ is a true statement, the solution set is {all real numbers}.

31. $2(3+7x)-(1+15x)=2$

$6+14x-1-15x=2$

$-x+5=2$

$-x=-3$

$x=3$

Check $x=3$: $48-46=2$ True

The solution set is $\{3\}$.

32. To eliminate the decimals, multiply both sides by 10.

$10[0.6(100-x)+0.4x]=10[0.5(92)]$

$6(100-x)+4x=5(92)$

$600-6x+4x=460$

$600-2x=460$

$-2x=-140$

$x=70$

Check $x=70$: $18+28=46$ True

The solution set is $\{70\}$.

33. To clear fractions, multiply both sides by the LCD, which is 4.

$$4\left(\frac{1}{4}x-4\right)=4\left(\frac{3}{2}x+\frac{3}{4}x\right)$$
$$x-16=6x+3x$$
$$x-16=9x$$
$$-16=8x$$
$$x=-2$$

Check $x=-2$: $-4.5=-3-1.5$ True

The solution set is $\{-2\}$.

34. To clear fractions, multiply both sides by the LCD, which is 12.

$$12\left[\frac{3}{4}(z-2)-\frac{1}{3}(5-2z)\right]=12(-2)$$
$$9(z-2)-4(5-2z)=-24$$
$$9z-18-20+8z=-24$$
$$17z-38=-24$$
$$17z=14$$
$$z=\frac{14}{17}$$

Check $z=\frac{14}{17}$: $-\frac{15}{17}-\frac{19}{17}=-2$ True

The solution set is $\left\{\frac{14}{17}\right\}$.

2.4 Applications of Linear Equations

Classroom Examples, Now Try Exercises

1. *Step 2*

Let $x=$ the number.

Step 3

The product of 5, and	a number increased by 8,	is	−60.
↓	↓	↓	↓
$5\cdot$	$(x+8)$	$=$	-60

Step 4

Solve the equation.

$$5(x+8)=-60$$
$5x+40=-60$ Distributive property

$5x=-100$ Subtract 40.

$x=-20$ Divide by 5.

Step 5

The number is -20.

Step 6

-20 plus 8 is -12 times 5 is -60, so -20 is the number.

N1. *Step 2*

Let $x=$ the number.

Step 3

The product of 7, and	a number increased by 3,	is	−63.
↓	↓	↓	↓
$7\cdot$	$(x+3)$	$=$	-63

Step 4

Solve the equation.

$$7(x+3)=-63$$
$7x+21=-63$ Distributive property

$7x=-84$ Subtract 21.

$x=-12$ Divide by 7.

Step 5

The number is -12.

Step 6

-12 plus 3 is -9, times 7 is -63, so -12 is the number.

2. *Step 2*

Let $x=$ the number.

Step 3

If 5 is	added to	the product of 9 and a number,	the result is	19 less than the number.
↓	↓	↓	↓	↓
5	$+$	$9x$	$=$	$x-19$

Step 4

Solve the equation.

$$5+9x=x-19$$
$5+9x-5=x-19-5$ Subtract 5.

$9x=x-24$

$9x-x=x-24-x$ Subtract x.

$8x=-24$

$\frac{8x}{8}=\frac{-24}{8}$ Divide by 8.

$x=-3$

Step 5

The number is -3.

Step 6

9 times -3 is -27. 5 added to -27 is -22, which is 19 less than -3, so -3 is the number.

N2. *Step 2*

Let x = the number.

Step 3

If 5	added	a	the result	7 less than 3 times
is	to	number,	is	the number.
↓	↓	↓	↓	↓
5	+	x	=	$3x-7$

Step 4

Solve the equation.

$$5+x=3x-7$$
$$5+x-5=3x-7-5 \quad \text{Subtract 5.}$$
$$x=3x-12$$
$$x-3x=3x-12-3x \quad \text{Subtract } 3x.$$
$$-2x=-12$$
$$\frac{-2x}{-2}=\frac{-12}{-2} \quad \text{Divide by } -2.$$
$$x=6$$

Step 5

The number is 6.

Step 6

5 added to 6 is 11. 3 times 6 is 18, and 7 less than 18 is 11, so 6 is the number.

3. *Step 2*

Let x = the number of medals France won.

Then $x+4$ number of medals Japan won.

Step 3

The		the number of medals		the number of medals
total	is	France won	plus	Japan won.
↓	↓	↓	↓	↓
72	=	x	+	$(x+4)$

Step 4

Solve the equation.

$$x+(x+4)=72$$
$$2x+4=72 \quad \text{Combine like terms.}$$
$$2x=68 \quad \text{Subtract 4.}$$
$$x=34 \quad \text{Divide by 2.}$$

Step 5

France won 34 medals and Japan won $34+4=38$ medals.

Step 6

38 is 4 more than 34, and the sum of 38 and 34 is 72.

N3. *Step 2*

Let x = the number of medals Great Britain won.

Let $x-21$ = the number of medals Germany won.

Step 3

The		the number of medals Great		the number of medals
total	is	Britain won	plus	Germany won.
↓	↓	↓	↓	↓
109	=	x	+	$(x-21)$

Step 4

Solve the equation.

$$x+(x-21)=109$$
$$2x-21=109 \quad \text{Combine like terms.}$$
$$2x=130 \quad \text{Add 21.}$$
$$x=65 \quad \text{Divide by 2.}$$

Step 5

Great Britain won 65 medals and Germany won $65-21=44$ medals.

Step 6

44 is 21 less than 65, and the sum of 65 and 44 is 109.

4. *Step 2*

Let x = the number of orders for muffins.

Then $\frac{1}{6}x$ = the number of orders for croissants.

Step 3

The		orders for		orders for
total	is	muffins	plus	croissants.
↓	↓	↓	↓	↓
56	=	x	+	$\frac{1}{6}x$

Step 4

Solve the equation.

$$56=1x+\frac{1}{6}x \quad x=1x$$
$$56=\frac{6}{6}x+\frac{1}{6}x \quad \text{LCD}=6$$
$$56=\frac{7}{6}x \quad \text{Combine like terms.}$$
$$\frac{6}{7}(56)=\frac{6}{7}\left(\frac{7}{6}x\right) \quad \text{Multiply by } \frac{6}{7}.$$
$$48=x$$

Step 5
The number of orders for muffins was 48, so the number of orders for croissants was
$\frac{1}{6}(48) = 8.$
Step 6
One-sixth of 48 is 8, and the sum of 48 and 8 is 56.

N4. *Step 2*
Let x = the number of orders for chocolate scones.
Then $\frac{2}{3}x$ = the number of orders for bagels.
Step 3

The total	is	orders for chocolate scones	plus	orders for bagels.
↓	↓	↓	↓	↓
525	=	x	+	$\frac{2}{3}x$

Step 4
Solve the equation.

$525 = 1x + \frac{2}{3}x \quad x = 1x$

$525 = \frac{3}{3}x + \frac{2}{3}x \quad \text{LCD} = 3$

$525 = \frac{5}{3}x \quad \text{Combine like terms.}$

$\frac{3}{5}(525) = \frac{3}{5}\left(\frac{5}{3}x\right) \quad \text{Multiply by } \frac{3}{5}.$

$315 = x$

Step 5
The number of orders for chocolate scones was 315, so the number of orders for bagels was
$\frac{2}{3}(315) = 210.$
Step 6
Two-thirds of 315 is 210, and the sum of 315 and 210 is 525.

5. *Step 2*
Let x = the number of members.
Then $2x$ = the number of nonmembers.
(If each member brought two nonmembers, there would be twice as many nonmembers as members.)
Step 3

Number of members	plus	number of nonmembers	is	the total in attendance.
↓	↓	↓	↓	↓
x	+	$2x$	=	27

Step 4
Solve the equation.
$x + 2x = 27$
$3x = 27$
$\frac{3x}{3} = \frac{27}{3}$
$x = 9$
Step 5
There were 9 members and $2 \cdot 9 = 18$ nonmembers.
Step 6
18 is twice as much as 9, and the sum of 9 and 18 is 27.

N5. *Step 2*
Let x = the number of residents.
Then $4x$ = the number of guests.
(If each resident brought four guests, there would be four times as many guests as residents.)
Step 3

Number of residents	plus	number of guests	is	the total in attendance.
↓	↓	↓	↓	↓
x	+	$4x$	=	175

Step 4
Solve the equation.
$x + 4x = 175$
$5x = 175$
$\frac{5x}{5} = \frac{175}{5}$
$x = 35$
Step 5
There were 35 residents and $4 \cdot 35 = 140$ guests.
Step 6
140 is four times as much as 35, and the sum of 35 and 140 is 175.

6. *Step 2*
Let x = the length of the middle-sized piece.
Then $x+10$ = the length of the longest piece
and $x-5$ = the length of the shortest piece.
Step 3

Shortest	plus	middle-sized	plus	longest	is	total length.
↓	↓	↓	↓	↓	↓	↓
$x-5$	+	x	+	$x+10$	=	50

Step 4
Solve the equation.

$$(x-5)+x+(x+10)=50$$
$$3x+5=50$$
$$3x=45$$
$$\frac{3x}{3}=\frac{45}{3}$$
$$x=15$$

Step 5
The middle-sized piece is 15 inches long, the longest piece is $15+10=25$ inches long, and the shortest piece is $15-5=10$ inches long.
Step 6
Since 25 inches is 10 inches longer than 15 inches, 15 inches is 5 inches longer than 10 inches, and $15+25+10=50$ inches (the length of the pipe), the answers are correct.

N6. *Step 2*
Let x = the time spent practicing free throws.
Then $2x$ = the time spent lifting weights and
$x+2$ = the time spent watching game films.
Step 3

Free throws	plus	lifting weights	plus	watching films	is	total time.
↓	↓	↓	↓	↓	↓	↓
x	+	$2x$	+	$x+2$	=	6

Step 4
Solve the equation.

$$x+2x+(x+2)=6$$
$$4x+2=6$$
$$4x=4$$
$$\frac{4x}{4}=\frac{4}{4}$$
$$x=1$$

Step 5
The time spent practicing free throws is 1 hour, the time spent lifting weights is $2(1)=2$ hours, and the time spent watching game films is $1+2=3$ hours.
Step 6
Since 2 hours is twice as much time as 1 hour, 3 hours is 2 more hours than 1 hour, and the sum of the times is $1+2+3=6$ hours (the total time spent), the answers are correct.

7. *Step 2*
Let x = the lesser page number.
Then $x+1$ = the greater page number.
Step 3
Because the sum of the page numbers is 569, an equation is $x+(x+1)=569$.
Step 4
Solve the equation.

$2x+1=569$ Combine like terms.
$2x=568$ Subtract 1.
$x=284$ Divide by 2.

Step 5
The lesser page number is 284, and the greater page number is $284+1=285$.
Step 6
285 is one more than 284, and the sum of 284 and 285 is 569.

N7. *Step 2*
Let x = the lesser page number.
Then $x+1$ = the greater page number.
Step 3
Because the sum of the page numbers is 593, an equation is $x+(x+1)=593$.
Step 4
Solve the equation.

$2x+1=593$ Combine like terms.
$2x=592$ Subtract 1.
$x=296$ Divide by 2.

Step 5
The lesser page number is 296, and the greater page number is $296+1=297$.
Step 6
297 is one more than 296, and the sum of 296 and 297 is 593.

8. Let x = the lesser even integer.
Then $x+2$ = the greater even integer.
From the given information, we have
$6\cdot x+(x+2)=86$.
Solve the equation.

$$7x+2=86$$
$$7x=84$$
$$x=12$$

The lesser even integer is 12 and the greater consecutive even integer is $12+2=14$. Six times 12 is 72 and 72 plus 14 is 86.

N8. Let x = the lesser odd integer.
Then $x+2$ = the greater odd integer.
From the given information, we have
$2\cdot x+3(x+2)=191$.
Solve the equation.
$$2x+3x+6=191$$
$$5x+6=191$$
$$5x=185$$
$$x=37$$
The lesser odd integer is 37 and the greater consecutive odd integer is $37+2=39$. Two times 37 is 74, three times 39 is 117, and 74 plus 117 is 191.

9. Let x = the degree measure of the angle.
Then $90-x$ = the degree measure of its complement.

The		eight times
complement	is	the angle.
↓	↓	↓
$90-x$	=	$8x$

Solve the equation.
$$90-x=8x$$
$$90=9x$$
$$10=x$$
The measure of the angle is 10°.

N9. Let x = the degree measure of the angle.
Then $90-x$ = the degree measure of its complement.

The		twice
complement	is	the angle.
↓	↓	↓
$90-x$	=	$2x$

Solve the equation.
$$90-x=2x$$
$$90=3x$$
$$30=x$$
The measure of the angle is 30°.

10. *Step 2*
Let x = the degree measure of the angle.
Then $90-x$ = the degree measure of its complement, and $180-x=$ the degree measure of its supplement.

Step 3

complement	plus	supplement	equals	174
↓	↓	↓	↓	↓
$90-x$	+	$180-x$	=	174

Step 4
Solve the equation.
$$270-2x=174$$
$$-2x=-96 \quad \text{Subtract 270.}$$
$$x=48 \quad \text{Divide by } -2.$$
Step 5
The measure of the angle is 48°.
Step 6
The complement of 48° is $90°-48°=42°$ and the supplement is $180°-48°=132°$. The sum of 42° and 132° is 174°.

N10. *Step 2*
Let x = the degree measure of the angle.
Then $90-x$ = the degree measure of its complement, and $180-x=$ the degree measure of its supplement.
Step 3

The		46° less than 3 times
supplement	equals	its complement.
↓	↓	↓
$180-x$	=	$3\cdot(90-x)-46$

Step 4
Solve the equation.
$$180-x=3(90-x)-46$$
$$180-x=270-3x-46 \quad \text{Distributive prop.}$$
$$180-x=224-3x \quad \text{Combine terms.}$$
$$180+2x=224 \quad \text{Add } 3x.$$
$$2x=44 \quad \text{Subtract 180.}$$
$$x=22 \quad \text{Divide by 2.}$$
Step 5
The measure of the angle is 22°.
Step 6
The complement of 22° is $90°-22°=68°$, and 46° less than 3 times 68° is $3(68°)-46°=204°-46°=158°$.
The supplement is $180°-22°=158°$.

Exercises

1. Choice D, $6\frac{1}{2}$, is *not* a reasonable answer in an applied problem that requires finding the number of cars on a dealer's lot, since you cannot have $\frac{1}{2}$ of a car. The number of cars must be a whole number.

2. Choice D, 25, is *not* a reasonable answer since the number of hours a light bulb is on during a day cannot be more than 24.

3. Choice A, -10, is *not* a reasonable answer since distance cannot be negative.

4. Choice C, -5, is *not* a reasonable answer since time cannot be negative.

5. *Step 2*
Let x = the number.
Step 3

The sum of			
a number and	9	is	-26.
↓	↓	↓	↓
$x+$	9	=	-26

Step 4
Solve the equation.
$x+9=-26$
$x=-35$
Step 5
The number is -35.
Step 6
The sum of -35 and 9 is -26.

6. *Step 2*
Let x = the number.
Step 3

The difference			
of a number and 11		is	-31.
↓		↓	↓
$x-11$		=	-31

Step 4
Solve the equation.
$x-11=-31$
$x=-20$
Step 5
The number is -20.
Step 6
Check that -20 is the correct answer by substituting this result into the words of the original problem. The difference of -20 and 11 is -31.

7. *Step 2*
Let x = the number.
Step 3

The product		and a number		
of 8,		increased by 6,	is	104.
↓		↓	↓	↓
$8\cdot$		$(x+6)$	=	104

Step 4
Solve the equation.
$8(x+6)=104$
$8x+48=104$
$8x=56$
$x=7$
Step 5
The number is 7.
Step 6
Check that 7 is the correct answer by substituting this result into the words of the original problem. 7 increased by 6 is 13. The product of 8 and 13 is 104, so 7 is the number.

8. *Step 2*
Let x = the number.
Step 3

The product		and 3 more than		
of 5,		twice a number,	is	85.
↓		↓	↓	↓
5	·	$(2x+3)$	=	85

Step 4
Solve the equation.
$10x+15=85$
$10x=70$
$x=7$
Step 5
The number is 7.
Step 6
Check that 7 is the correct answer by substituting this result into the words of the original problem. If the number is 7, then twice the number is 14, and 3 more than twice the number is 17. The product of 5 and 17 is 85, so 7 is the number.

9. *Step 2*
Let x = the unknown number. Then $5x+2$ represents "2 is added to five times a number," and $4x+5$ represents "5 more than four times the number."
Step 3
$5x+2=4x+5$

Step 4
Solve the equation.
$5x+2=4x+5$
$x+2=5$
$x=3$
Step 5
The number is 3.
Step 6
Check that 3 is the correct answer by substituting this result into the words of the original problem. Two added to five times a number is $2+5(3)=17$ and 5 more than four times the number is $5+4(3)=17$. The values are equal, so the number 3 is the correct answer.

10. *Step 2*
Let $x=$ the unknown number. Then $8+4x$ represents "four times a number added to 8," and $5+3x$ represents "three times the number, added to 5."
Step 3
$8+4x=5+3x$
Step 4
Solve the equation.
$8+4x=5+3x$
$8+x=5$
$x=-3$
Step 5
The number is -3.
Step 6
Check that -3 is the correct answer by substituting this result into the words of the original problem. Four times a number is added to 8 is $8+4(-3)=-4$ and three times the number added to 5 is $5+3(-3)=-4$. The values are equal, so the number -3 is the correct answer.

11. *Step 2*
Let $x=$ the unknown number. Then $x-2$ is two subtracted from the number, $3(x-2)$ is triple the difference, and $x+6$ is six more than the number.
Step 3
$3(x-2)=x+6$
Step 4
Solve the equation.
$3x-6=x+6$
$2x-6=6$
$2x=12$
$x=6$
Step 5
The number is 6.
Step 6
Check that 6 is the correct answer by substituting this result into the words of the original problem. Two subtracted from the number is $6-2=4$. Triple this difference is $3(4)=12$, which is equal to 6 more than the number, since $6+6=12$.

12. *Step 2*
Let $x=$ the unknown number. Then $x+3$ is 3 is added to a number, $2(x+3)$ is this sum is doubled, and $x+2$ is 2 more than the number.
Step 3
$2(x+3)=x+2$
Step 4
Solve the equation.
$2x+6=x+2$
$x+6=2$
$x=-4$
Step 5
The number is -4.
Step 6
Check that -4 is the correct answer by substituting this result into the words of the original problem. Three added to the number is -1, double this value is -2. Two more than -4 is also -2, so the number is -4.

13. *Step 2*
Let $x=$ the unknown number. Then $\frac{3}{4}x$ is $\frac{3}{4}$ of the number, and $\frac{3}{4}x+6$ is 6 added to $\frac{3}{4}$ of the number. $x-4$ is 4 less than the number.
Step 3
$\frac{3}{4}x+6=x-4$

Step 4
Solve the equation.

$$\frac{3}{4}x+6=x-4$$
$$-\frac{1}{4}x+6=-4$$
$$-\frac{1}{4}x=-10$$
$$-4\left(-\frac{1}{4}x\right)=-4(-10)$$
$$x=40$$

Step 5
The number is 40.
Step 6
Check that 40 is the correct answer by substituting this result into the words of the original problem. $\frac{3}{4}$ of 40 is 30. When 6 is added to 30, the sum is 36, which is 4 less than 40 because 6 is added to 30.

14. *Step 2*
Let $x =$ the unknown number. Then $\frac{2}{3}x$ is $\frac{2}{3}$ of the number, and $10+\frac{2}{3}x$ is $\frac{2}{3}$ of the number added to 10. $x+5$ is 5 more than the number.
Step 3

$$10+\frac{2}{3}x=x+5$$

Step 4
Solve the equation.

$$10+\frac{2}{3}x=x+5$$
$$10-\frac{1}{3}x=5$$
$$-\frac{1}{3}x=-5$$
$$-3\left(-\frac{1}{3}x\right)=-3(-5)$$
$$x=15$$

Step 5
The number is 15.
Step 6
Check that 15 is the correct answer by substituting this result into the words of the original problem. $\frac{2}{3}$ of 15 is 10. When 10 is added to 10, the sum is 20, which is 5 more than 15 because 5 is added to 10.

15. *Step 2*
Let $x =$ the unknown number. Then $3x$ is three times the number, $x+7$ is 7 more than the number, $2x$ is twice the number, and $-11-2x$ is the difference between -11 and twice the number.
Step 3

$$3x+(x+7)=-11-2x$$

Step 4
Solve the equation.

$$4x+7=-11-2x$$
$$6x+7=-11$$
$$6x=-18$$
$$x=-3$$

Step 5
The number is -3.
Step 6
Check that -3 is the correct answer by substituting this result into the words of the original problem. The sum of three times a number and 7 more than the number is $3(-3)+(-3+7)=-5$ and the difference between –11 and twice the number is $-11-2(-3)=-5$. The values are equal, so the number -3 is the correct answer.

16. *Step 2*
Let $x =$ the unknown number. Then $2x+4$ is 4 is added to twice the number, $2(2x+4)$ is the sum multiplied by 2, and $3x+4$ is the number is multiplied by 3 and 4 is added to the product.
Step 3

$$2(2x+4)=3x+4$$

Step 4
Solve the equation.

$$4x+8=3x+4$$
$$x+8=4$$
$$x=-4$$

Step 5
The number is -4.
Step 6
Check that -4 is the correct answer by substituting this result into the words of the original problem.
Twice the number is $2(-4)=-8$. Four added to twice the number is $-8+4=-4$. This sum multiplied by 2 is $2(-4)=-8$. The number multiplied by 3 is $3(-4)=-12$. Four added to this product is $-12+4=-8$. Because both results are -8, the answer, -4, checks.

17. *Step 1*
We must find the number of Democrats and the number of Republicans.
Step 2
Let x = the number of Republicans.
Then $x-4$ = the number of Democrats.
Step 3

Number of Democrats	+	number of Republicans	equals	total members.
↓	↓	↓	↓	↓
$(x-4)$	+	x	=	150

Step 4
Solve the equation.
$$(x-4)+x=150$$
$$2x-4=150$$
$$2x=154$$
$$x=77$$
Step 5
There were 77 Republicans and $77-4=73$ Democrats.
Step 6
Check that the numbers found are the correct answers by substituting the result into the words of the original problem. 73 is 4 fewer than 77, and $77+73=150$.

18. *Step 1*
We must find the two consecutive integers.
Step 2
Let x = the lesser of the two consecutive even integers.
Then $x+2$ = the greater of the two consecutive even integers.
Step 3

The lesser consecutive even integer	+	the greater consecutive even integer	is	the total.
↓	↓	↓	↓	
x	+	$x+2$	=	254

Step 4
Solve the equation.
$$x+(x+2)=254$$
$$2x+2=254$$
$$2x=252$$
$$x=126$$
Step 5
The lesser even integer is 126, and the greater even integer is $126+2=128$.
Step 6
Check that the numbers found are the correct answers by substituting the result into the words of the original problem. 126 and 128 are consecutive even integers, and $126+128=254$.

19. Let x = the number of drive-in movie screens in Ohio.
Then $x+2$ = the number of drive-in movie screens in New York.
Since the total number of screens was 56, we can write the equation $x+(x+2)=56$.
Solve the equation.
$$2x+2=56$$
$$2x=54$$
$$x=27$$
Since $x=27$, $x+2=29$.
There were 27 drive-in movie screens in Ohio and 29 in New York. Since 29 is 2 more than 27 and $27+29=56$, this answer checks.

20. Let x = the number of *Cheers* viewers.
Then $x+8$ = the number of *M*A*S*H* viewers.
Since the total number of viewers is 92 (all numbers in millions), we can write the equation $x+(x+8)=92$.
Solve the equation.
$$2x+8=92$$
$$2x=84$$
$$x=42$$
Since $x=42$, $x+8=50$.
There were 42 million *Cheers* viewers and 50 million *M*A*S*H* viewers. Since $42+50=92$, this answer checks.

21. Let x = the number of Republicans.
Then $x+6$ = the number of Democrats.
Since the total number of Democrats and Republicans was 98, we can write the equation $x+(x+6)=98$.
Solve the equation.
$$2x+6=98$$
$$2x=92$$
$$x=46$$
Since $x=46$, $x+6=52$.
There were 46 Republicans and 52 Democrats. Since $46+52=98$, this answer checks.

22. Let x = the number of Democrats.
Then $x+32$ = the number of Republicans.
Since the total number of Democrats and Republicans was 432, we can write the equation $x+(x+32)=432$.
Solve the equation.
$$2x+32=432$$
$$2x=400$$
$$x=200$$
Since $x=200$, $x+32=232$.
There were 200 Democrats and 232 Republicans. Since $200+232=432$, this answer checks.

23. Let x = revenue from ticket sales for Madonna.
Then $x-29$ = the revenue from the ticket sales for Bruce Springsteen.
Since the total revenue from ticket sales was \$427 (all numbers in millions), we can write the equation $x+(x-29)=427$.
Solve the equation.
$$2x-29=427$$
$$2x=456$$
$$x=228$$
Since $x=228$, $x-29=199$.
Madonna took in \$228 million and Bruce Springsteen took in \$199 million. Since 199 is 29 less than 228 and $228+199=427$, this answer checks.

24. Let x = the number of Toyota Camry sales.
Then $x-27$ = the number of Honda Accord sales.
Since the total number of sales was 691 (all numbers in thousands), we can write the equation $x+(x-27)=691$.
Solve the equation.
$$2x-27=691$$
$$2x=718$$
$$x=359$$
Since $x=359$, $x-27=332$.
There were 359 thousand Toyota Camry sales and 332 thousand Honda Accord sales. Since 332 is 27 less than 359 and $359+332=691$, this answer checks.

25. Let x = the number of games the Heat lost.
Then $4x+2$ = the number of games the Heat won.
Since the total number of games played was 82, we can write the equation $x+(4x+2)=82$.
Solve the equation.
$$5x+2=82$$
$$5x=80$$
$$x=16$$
Since $x=16$, $4x+2=66$.
The Heat won 66 games and lost 16 games. Since $66+16=82$, this answer checks.

26. Let x = the number of games the Indians lost.
Then $2x-48$ = the number of games the Indians won.
Since the total number of games played was 162, we can write the equation $x+(2x-48)=162$.
Solve the equation.
$$3x-48=162$$
$$3x=210$$
$$x=70$$
Since $x=70$, $2x-48=92$.
The Indians won 92 games and lost 70 games. Since $92+70=162$, this answer checks.

27. Let x = the number of mg of vitamin C in a one-cup serving of pineapple juice.
Then $4x-3$ = the number of mg of vitamin C in a one-cup serving of orange juice.
Since the total amount of vitamin C in a serving of the two juices is 122 mg, we can write
$x+(4x-3)=122.$
Solve the equation.
$$5x-3=122$$
$$5x=125$$
$$x=25$$
Since $x=25, 4x-3=97.$
A one-cup serving of pineapple juice has 25 mg of vitamin C and a one-cup serving of orange juice has 97 mg of vitamin C. Since 97 is 3 less than four times 25 and $25+97=122,$ this answer checks.

28. Let x = the number of calories in a one-cup serving of tomato juice.
Then $3x+9$ = the number of calories in a one-cup serving of pineapple juice.
Since the total number of calories in a serving of the two juices is 173, we can write
$x+(3x+9)=173.$
Solve the equation.
$$4x+9=173$$
$$4x=164$$
$$x=41$$
Since $x=41, 3x+9=132.$
A one-cup serving of tomato juice has 41 calories and a one-cup serving of pineapple juice has 132 calories. Since 132 is 9 more than three times 41 and $41+132=173,$ this answer checks.

29. Let x = the number of Blu-Ray discs sold.
Then $\frac{2}{3}x$ = the number of DVDs sold.
The total number of Blu-Rays and DVDs sold was 280, so $x+\frac{2}{3}x=280.$
Solve the equation.
$$1x+\frac{2}{3}x=280$$
$$\frac{5}{3}x=280$$
$$\frac{3}{5}\left(\frac{5}{3}x\right)=\frac{3}{5}(280)$$
$$x=\frac{\cancel{3}}{\cancel{5}_{1}}\cdot\frac{\cancel{280}^{56}}{1}=168$$
Since $x=168, \frac{2}{3}x=\frac{2}{3}(168)=112.$
There were 112 DVDs sold.

30. Let x = the number of calories burned doing aerobics. Then $\frac{2}{5}x$ = the number of calories burned doing weight training.
The total number of calories burned was 371, so $x+\frac{2}{5}x=371.$
Solve the equation.
$$1x+\frac{2}{5}x=371$$
$$\frac{7}{5}x=371$$
$$\frac{5}{7}\left(\frac{7}{5}x\right)=\frac{5}{7}(371)$$
$$x=\frac{\cancel{5}}{\cancel{7}_{1}}\cdot\frac{\cancel{371}^{53}}{1}=265$$
Since $x=265, \frac{2}{5}x=\frac{2}{5}(265)=106.$
Weight training burns 106 calories.

31. Let x = the number of kg of onions.
Then $6.6x$ = the number of kg of grilled steak.
The total weight of these two ingredients was 617.6 kg, so
$x+6.6x=617.6.$
Solve the equation.
$$1x+6.6x=617.6$$
$$7.6x=617.6$$
$$x=\frac{617.6}{7.6}\approx 81.3$$

Since $x = \frac{617.6}{7.6}$, $6.6x = 6.6\left(\frac{617.6}{7.6}\right) \approx 536.3$.

To the nearest tenth of a kilogram, 81.3 kg of onions and 536.3 kg of grilled steak were used to make the taco.

32. Let x = the population of China.
Then $0.9x$ = the population of India. The combined population of the two countries was 2.5 billion, so
$x + 0.9x = 2.5$.
Solve the equation.
$1x + 0.9x = 2.5$

$1.9x = 2.5$

$x = \frac{2.5}{1.9} = \frac{25}{19}$

Since $x = \frac{25}{19} \approx 1.3$, $0.9x = 0.9\left(\frac{25}{19}\right) = \frac{45}{38} \approx 1.2$.

To the nearest tenth of a billion, the population of China was 1.3 billion and the population of India was 1.2 billion.

33. Let x = the value of the 1945 nickel.
Then $2x$ = the value of the 1950 nickel.
The total value of the two coins is \$24.00, so
$x + 2x = 24$.
Solve the equation.
$3x = 24$

$x = 8$ Divide by 8.

Since $x = 8$, $2x = 2(8) = 16$.

The value of the 1945 Philadelphia nickel is \$8.00 and the value of the 1950 Denver nickel is \$16.00.

34. Let x = the number of pounds of nickel.
Then $3x$ = the number of pounds of copper.
The total number of pounds would be 560, so
$x + 3x = 560$.
Solve the equation.
$4x = 560$

$x = \frac{560}{4} = 140$

Since $x = 140$, $3x = 3(140) = 420$. To make 560 pounds of five-cent coins, use 420 pounds of copper.

35. Let x = the number of ounces of rye flour.
Then $4x$ = the number of ounces of whole-wheat flour.
The total number of ounces would be 32, so
$x + 4x = 32$.
Solve the equation.
$5x = 32$

$x = \frac{32}{5} = 6.4$

Since $x = 6.4$, $4x = 4(6.4) = 25.6$. To make a loaf of bread weighing 32 oz, use 6.4 oz of rye flour and 25.6 oz of whole-wheat flour.

36. Let x = the number of milligrams of inert ingredients.
Then $9x$ = the number of milligrams of active ingredients.
The total number of milligrams would be 250, so $x + 9x = 250$.
Solve the equation.
$10x = 250$

$x = \frac{250}{10} = 25$

Since $x = 25$, $9x = 9(25) = 225$. In a single 250-mg caplet, there would be 25 mg of inert ingredients and 225 mg of active ingredients.

37. Let x = the number of tickets booked on United Airlines.
Then $x + 7$ = the number of tickets booked on American Airlines, and $2x + 4$ = the number of tickets booked on Southwest Airlines.
The total number of tickets booked was 55, so
$x + (x + 7) + (2x + 4) = 55$.
Solve the equation.
$4x + 11 = 55$

$4x = 44$

$x = \frac{44}{4} = 11$

Since $x = 11$, $x + 7 = 11 + 7 = 18$ and $2x + 4 = 2(11) + 4 = 26$. He booked 18 tickets on American, 11 tickets on United, and 26 tickets on Southwest.

38. Let x = the number of hours making telephone calls.
Then $x+0.5$ = the number of hours writing emails, and $2x$ = the number of hours attending meetings.
The total number of hours is 7.5, so
$x+(x+0.5)+2x=7.5.$
Solve the equation.
$$4x+0.5=7.5$$
$$4x=7$$
$$x=\frac{7}{4}=1.75$$
Since $x=1.75$, $x+0.5=1.75+0.5=2.25$ and $2x=2(1.75)=3.5$. In her 7.5 hour-day, she spent 1.75 hours making telephone calls, 2.25 hours writing e-mails, and 3.5 hours attending meetings.

39. Let x = the length of the shortest piece.
Then $x+5$ = the length of the middle piece, and $x+9$ = the length of the longest piece.
The total length is 59 inches, so
$x+(x+5)+(x+9)=59.$
Solve the equation.
$$3x+14=59$$
$$3x=45$$
$$x=15$$
Since $x=15$, $x+5=20$ and $x+9=24$.
The shortest piece should be 15 inches, the middle piece should be 20 inches, and the longest piece should be 24 inches. The answer checks since $15+20+24=59$.

40. Let x = the length of the shortest piece.
Then $2x$ = the length of the middle piece, and $x+8$ = the length of the longest piece.
The total length is 3 feet, or 36 inches, so
$x+(2x)+(x+8)=36.$
Solve the equation.
$$4x+8=36$$
$$4x=28$$
$$x=7$$
Since $x=7$, $2x=14$ and $x+8=15$.
The shortest piece should be 7 inches, the middle piece should be 14 inches, and the longest piece should be 15 inches. The answer checks since $7+14+15=36$.

41. Let x = the number of silver medals.
Then x = the number of bronze medals, and $x+17$ = the number of gold medals.
The total number of medals earned by the United States was 104, so
$x+x+(x+17)=104.$
Solve the equation.
$$3x+17=104$$
$$3x=87$$
$$x=29$$
Since $x=29$, $x+17=46$.
The United States earned 46 gold medals, 29 silver medals, and 29 bronze medals. The answer checks since $46+29+29=104$.

42. Let x = the number of bronze medals.
Then $x+4$ = the number of silver medals, and $x+15$ = the number of gold medals.
The total number of medals earned by China was 88, so $x+(x+4)+(x+15)=88$.
Solve the equation.
$$3x+19=88$$
$$3x=69$$
$$x=23$$
Since $x=23$, $x+4=27$ and $x+15=38$.
China earned 38 gold medals, 27 silver medals, and 23 bronze medals. The answer checks since $23+27+38=88$.

43. Let x = the distance of Mercury from the sun.
Then $x+31.2$ = the distance of Venus from the sun, and $x+57$ = the distance of Earth from the sun.
Since the total of the distances from these three planets is 196.2 (all distances in millions of miles), we can write the equation
$x+(x+31.2)+(x+57)=196.2.$
Solve the equation.
$$3x+88.2=196.2$$
$$3x=108$$
$$x=36$$
Mercury is 36 million miles from the sun, Venus is $36+31.2=67.2$ million miles from the sun, and Earth is $36+57=93$ million miles from the sun. The answer checks since $36+67.2+93=196.2$.

44. Let x = the number of satellites of Saturn.
Then $x+5$ = the number of satellites of Jupiter, and $x-35$ = the number of satellites of Uranus.
Since the total number of satellites is 156, $x+(x+5)+(x-35)=156$.
Solve the equation.
$$3x-30=156$$
$$3x=186$$
$$x=\frac{186}{3}=62$$
Since $x=62$, $x+5=67$ and $x-35=27$.
Uranus has 27 known satellites.

45. Let x = the measure of angles *A* and *B*.
Then $x+60$ = the measure of angle *C*.
The sum of the measures of the angles of any triangle is 180°, so $x+x+(x+60)=180$.
Solve the equation.
$$3x+60=180$$
$$3x=120$$
$$x=40$$
Angles *A* and *B* have measures of 40 degrees, and angle *C* has a measure of $40+60=100$ degrees. The answer checks since $40+40+100=180$.

46. Let x = the measure of angles *B* and *C*.
Then $x+141$ = the measure of angle *A*.
The sum of the measures of the angles of any triangle is 180°, so $(x+141)+x+x=180$.
Solve the equation.
$$3x+141=180$$
$$3x=39$$
$$x=13$$
Since $x=13$, $x+141=154$.
The measure of angle *A* is 154°; the measures of angles *B* and *C* are both 13°. The answer checks since $154+13+13=180$.

47. Let x = the number on the first locker.
Then $x+1$ = the number on the next locker.
Since the numbers have a sum of 137, we can write the equation $x+(x+1)=137$.
Solve the equation.
$$2x+1=137$$
$$2x=136$$
$$x=\frac{136}{2}=68$$
Since $x=68$, $x+1=69$.
The lockers have numbers 68 and 69. Since $68+69=137$, this answer checks.

48. Let x = the number on the first check.
Then $x+1$ = the number on the second check.
Since the sum of the numbers is 357, we can write the equation $x+(x+1)=357$.
Solve the equation.
$$2x+1=357$$
$$2x=356$$
$$x=\frac{356}{2}=178$$
Since $x=178$, $x+1=179$.
The checkbook check numbers are 178 and 179. Since $178+179=357$, this answer checks.

49. Because the two pages are back-to-back, they must have page numbers that are consecutive integers.
Let x = the lesser page number.
Then $x+1$ = the greater page number.
$$x+(x+1)=203$$
$$2x+1=203$$
$$2x=202$$
$$x=101$$
Since $x=101$, $x+1=102$.
The page numbers are 101 and 102. This answer checks since the sum is 203.

50. Let x = the first apartment number.
Then $x+1$ = the second apartment number.
Since the sum of the numbers is 59, we have $x+(x+1)=59$.
Solve the equation.
$$2x+1=59$$
$$2x=58$$
$$x=29$$
Since $x=29$, $x+1=30$.
The apartment numbers are 29 and 30. Since $29+30=59$, this answer checks.

51. Let x = the lesser even integer.
Then $x+2$ = the greater even integer.
"The lesser added to three times the greater gives a sum of 46" can be written as $x+3(x+2)=46$.

$$\begin{aligned} x+3x+6&=46\\ 4x+6&=46\\ 4x&=40\\ x&=10 \end{aligned}$$

Since $x=10$, $x+2=12$.
The integers are 10 and 12. This answer checks since $10+3(12)=46$.

52. Let x = the lesser even integer.
Then $x+2$ = the greater even integer.
"Six times the lesser added to the greater gives a sum of 86" can be written as $6x+x+2=86$.

$$\begin{aligned} 7x+2&=86\\ 7x&=84\\ x&=12 \end{aligned}$$

Since $x=12$, $x+2=14$.
The integers are 12 and 14. This answer checks since 6 times 12 is 72, plus 14 is 86.

53. Let x = the lesser odd integer.
Then $x+2$ = the greater odd integer.
"59 more than the lesser is 4 times the greater" can be written as $x+59=4(x+2)$.

$$\begin{aligned} x+59&=4x+8\\ 59&=3x+8\\ 51&=3x\\ 17&=x \end{aligned}$$

Since $x=17$, $x+2=19$.
The integers are 17 and 19. This answer checks since $17+59=76$ and $4\cdot 19=76$.

54. Let x = the lesser odd integer.
Then $x+2$ = the greater odd integer.
"Twice the greater is 17 more than the lesser" can be written as $2(x+2)=x+17$.

$$\begin{aligned} 2x+4&=x+17\\ x+4&=17\\ x&=13. \end{aligned}$$

Since $x=13$, $x+2=15$.
The integers are 13 and 15. This answer checks since $2(15)=13+17$.

55. Let x = the lesser integer.
Then $x+1$ = the greater integer.

$$\begin{aligned} x+3(x+1)&=43\\ x+3x+3&=43\\ 4x+3&=43\\ 4x&=40\\ x&=10 \end{aligned}$$

Since $x=10$, $x+1=11$.
The integers are 10 and 11. This answer checks since $10+3(11)=43$.

56. Let x = the lesser integer.
Then $x+1$ = the greater integer.

$$\begin{aligned} 5x+3(x+1)&=59\\ 5x+3x+3&=59\\ 8x+3&=59\\ 8x&=56\\ x&=7 \end{aligned}$$

Since $x=7$, $x+1=8$.
The integers are 7 and 8. This answer checks since $5(7)+3(8)=35+24=59$.

57. Let x = the first even integer.
Then $x+2$ = the second even integer, and $x+4$ = the third even integer.

$$\begin{aligned} x+(x+2)+(x+4)&=60\\ 3x+6&=60\\ 3x&=54\\ x&=18 \end{aligned}$$

Since $x=18$, $x+2=20$ and $x+4=22$.
The first even integer is 18. This answer checks since $18+20+22=60$.

58. Let x = the first odd integer.
Then $x+2$ = the second odd integer, and $x+4$ = the third odd integer.

$$\begin{aligned} x+(x+2)+(x+4)&=69\\ 3x+6&=69\\ 3x&=63\\ x&=21 \end{aligned}$$

Since $x=21$, $x+2=23$ and $x+4=25$.
The third odd integer is 25. This answer checks since $21+23+25=69$.

59. Let x = the first odd integer.
Then $x+2$ = the second odd integer, and
$x+4$ = the third odd integer.

$$2[(x+4)-6] = [x+2(x+2)]-23$$
$$2(x-2) = x+2x+4-23$$
$$2x-4 = 3x-19$$
$$-4 = x-19$$
$$15 = x$$

Since $x=15$, $x+2=17$ and $x+4=19$.
The integers are 15, 17, and 19.

60. Let x = the first even integer.
Then $x+2$ = the second even integer, and
$x+4$ = the third even integer.

$$x+(x+4) = 3(x+2)-22$$
$$x+x+4 = 3x+6-22$$
$$2x+4 = 3x-16$$
$$-x+4 = -16$$
$$-x = -20$$
$$x = 20$$

Since $x=20$, $x+2=22$ and $x+4=24$.
The integers are 20, 22, and 24.

61. Let x = the measure of the angle.
Then $90-x$ = the measure of its complement.
The phrase "complement is four times its measure" can be written as $90-x=4x$.
Solve the equation.

$$90 = 5x$$
$$x = \frac{90}{5} = 18$$

The measure of the angle is 18°. The complement is $90°-18° = 72°$, which is four times 18°.

62. Let x = the measure of the angle.
Then $90-x$ = the measure of its complement.
The phrase "complement is five times its measure" can be written as $90-x=5x$.
Solve the equation.

$$90 = 6x$$
$$x = \frac{90}{6} = 15$$

The measure of the angle is 15°. The complement is $90°-15° = 75°$, which is five times 15°.

63. Let x = the measure of the angle.
Then $180-x$ = the measure of its supplement.
The phrase "supplement is eight times its measure" can be written as $180-x=8x$.
Solve the equation.

$$180 = 9x$$
$$x = \frac{180}{9} = 20$$

The measure of the angle is 20°. The supplement is $180°-20° = 160°$, which is eight times 20°.

64. Let x = the measure of the angle.
Then $180-x$ = the measure of its supplement.
The phrase "supplement is three times its measure" can be written as $180-x=3x$.
Solve the equation.

$$180 = 4x$$
$$x = \frac{180}{4} = 45$$

The measure of the angle is 45°. The supplement is $180°-45° = 135°$, which is three times 45°.

65. Let x = the measure of the angle. Then
$90-x$ = the measure of its complement, and
$180-x$ = the measure of its supplement.

Its			more	twice its
supplement	measures	39°	than	complement.
↓	↓	↓	↓	↓
$180-x$	=	39	+	$2(90-x)$

Solve the equation.

$$180-x = 39+2(90-x)$$
$$180-x = 39+180-2x$$
$$180-x = 219-2x$$
$$x+180 = 219$$
$$x = 39$$

The measure of the angle is 39°. The complement is $90°-39° = 51°$. Now 39° more than twice its complement is $39°+2(51°) = 141°$, which is the supplement of 39° since $180°-39° = 141°$.

66. Let x = the measure of the angle.
Then $180 - x$ = the measure of its supplement, and $90 - x$ = the measure of its complement.

The supplement	is	38° less than 3 times its complement.
↓	↓	↓
$180 - x$	=	$3 \cdot (90 - x) - 38$

Solve the equation.

$$\begin{aligned} 180 - x &= 270 - 3x - 38 \\ 180 - x &= 232 - 3x \\ 180 + 2x &= 232 \\ 2x &= 52 \\ x &= 26 \end{aligned}$$

The measure of the angle is $26°$.

67. Let x = the measure of the angle.
Then $180 - x$ = the measure of its supplement, and $90 - x$ = the measure of its complement.
Remember that difference means subtraction.

The supplement	minus	3 times its complement	is	10°.
↓	↓	↓	↓	↓
$(180 - x)$	−	$3(90 - x)$	=	10

Solve the equation.

$$\begin{aligned} (180 - x) - 3(90 - x) &= 10 \\ 180 - x - 270 + 3x &= 10 \\ 2x - 90 &= 10 \\ 2x &= 100 \\ x &= 50 \end{aligned}$$

The measure of the angle is $50°$. The supplement is $180° - 50° = 130°$ and the complement is $90° - 50° = 40°$. The answer checks since $130° - 3(40°) = 10°$.

68. Let x = the measure of the angle.
Then $90 - x$ = the measure of its complement, and $180 - x$ = the measure of its supplement.
"The sum of the measures of its complement and supplement is 160°" can be written as $(90 - x) + (180 - x) = 160$.
Solve the equation.

$$\begin{aligned} -2x + 270 &= 160 \\ -2x &= -110 \\ x &= 55 \end{aligned}$$

The measure of the angle is $55°$. The sum of the measures of its complement $(90° - 55° = 35°)$ and its supplement $(180° - 55° = 125°)$ is 160°.

2.5 Formulas and Additional Applications from Geometry

Classroom Examples, Now Try Exercises

1. $P = 2L + 2W$

$126 = 2L + 2(25)$ Let $P = 126$ and $W = 25$.

$126 = 2L + 50$

$76 = 2L$ Subtract 50.

$38 = L$ Divide by 2.

N1. $P = 2a + 2b$

$78 = 2(12) + 2b$ Let $P = 78$ and $a = 12$.

$78 = 24 + 2b$

$54 = 2b$ Subtract 24.

$27 = b$ Divide by 2.

2. The fence will enclose the perimeter of the rectangular field, so use the formula for the perimeter of a rectangle. Find the length of the field by substituting $P = 800$ and $W = L - 50$ into the formula and solving for L.

$$\begin{aligned} P &= 2L + 2W \\ 800 &= 2L + 2(L - 50) \\ 800 &= 2L + 2L - 100 \\ 900 &= 4L \\ 225 &= L \end{aligned}$$

Since $L = 225, W = 225 - 50 = 175$. The dimensions of the field are 225 m by 175 m.

N2. The fence will enclose the perimeter of the rectangular garden, so use the formula for the perimeter of a rectangle. Find the width of the garden by substituting $P = 160$ and $L = 2W - 10$ into the formula and solving for W.

$$\begin{aligned} P &= 2L + 2W \\ 160 &= 2(2W - 10) + 2W \\ 160 &= 4W - 20 + 2W \\ 180 &= 6W \\ 30 &= W \end{aligned}$$

Since $W = 30, L = 2(30) - 10 = 50$. The dimensions of the garden are 50 ft by 30 ft.

3. Let s = the length of the shortest side, in inches; $s+5$ = the length of the medium side, and $(s+5)+1 = s+6$ = the length of the longest side.
The perimeter is 32 inches, so
$s+(s+5)+(s+6)=32.$
$3s+11=32$
$3s=21$
$s=7$
Since $s=7, s+5=12$ and $s+6=13$. The lengths of the sides are 7, 12, and 13 inches. The perimeter is $7+12+13=32,$ as required.

N3. Let s = the length of the medium side, in feet; $s+1$ = the length of the longest side, and $s-7$ = the length of the shortest side.
The perimeter is 30 feet, so
$s+(s+1)+(s-7)=30.$
$3s-6=30$
$3s=36$
$s=12$
Since $s=12, s+1=13$ and $s-7=5$. The lengths of the sides are 5, 12, and 13 feet. The perimeter is $5+12+13=30,$ as required.

4. Use the formula for the area of a triangle.
$A=\frac{1}{2}bh$
$120=\frac{1}{2}b(24)$ Let $A=120, h=24.$
$120=12b$
$10=b$
The length of the base is 10 meters.

N4. Use the formula for the area of a triangle.
$A=\frac{1}{2}bh$
$77=\frac{1}{2}(14)h$ Let $A=77, h=14.$
$77=7h$
$11=h$
The height is 11 centimeters.

5. The sum of the measures of the two angles is $180°$ because together they form a straight angle.
$(6x+29)+(x+11)=180$
$7x+40=180$
$7x=140$
$x=20$
If $x=20, 6x+29=6(20)+29=149$ and $x+11=20+11=31.$
The measures of the angles are $149°$ and $31°$.

N5. Since the marked angles are vertical angles, they have equal measures.
$6x+2=8x-8$
$2=2x-8$
$10=2x$
$5=x$
If $x=5, 6x+2=6(5)+2=32$ and $8x-8=8(5)-8=32.$
The measure of the angles is $32°$.

6. Solve $I=prt$ for t.
$\frac{I}{pr}=\frac{prt}{pr}$ Divide by pr.
$\frac{I}{pr}=t,$ or $t=\frac{I}{pr}$

N6. Solve $W=Fd$ for F.
$\frac{W}{d}=\frac{Fd}{d}$ Divide by d.
$\frac{W}{d}=F,$ or $F=\frac{W}{d}$

7. Solve $S=2\pi rh+2\pi r^2$ for h.
$S-2\pi r^2=2\pi rh$ Subtract $2\pi r^2$.
$\frac{S-2\pi r^2}{2\pi r}=h,$ Divide by $2\pi r$.
or $h=\frac{S-2\pi r^2}{2\pi r}$

N7. Solve $Ax+By=C$ for A.
$Ax=C-By$ Subtract By.
$\frac{Ax}{x}=\frac{C-By}{x}$ Divide by x.
$A=\frac{C-By}{x}$

8. Solve $F = \frac{9}{5}C + 32$ for C.

$F - 32 = \frac{9}{5}C + 32 - 32$ Subtract 32.

$F - 32 = \frac{9}{5}C$

$\frac{5}{9}(F - 32) = \frac{5}{9}\left(\frac{9}{5}C\right)$ Multiply by $\frac{5}{9}$.

$\frac{5(F - 32)}{9} = C$

$C = \frac{5(F - 32)}{9}$, or $C = \frac{5F - 160}{9}$

N8. Solve $S = \frac{1}{2}(a + b + c)$ for a.

$2S = a + b + c$ Multiply by 2.

$2S - b - c = a$ Subtract b and c.

9. **(a)** Solve $-2x + y = 4$ for y.

$-2x + y + 2x = 4 + 2x$ Add $2x$.

$y = 2x + 4$

(b) Solve $x + 3y = 9$ for y.

$x + 3y - x = 9 - x$ Subtract x.

$y = \frac{9 - x}{3}$ Divide by 3.

$y = 3 - \frac{1}{3}x$, or $y = -\frac{1}{3}x + 3$

N9. **(a)** Solve $5x + y = 3$ for y.

$5x + y - 5x = 3 - 5x$ Subtract $5x$.

$y = 3 - 5x$

(b) Solve $x - 2y = 8$ for y.

$x - 2y - x = 8 - x$ Subtract x.

$y = \frac{8 - x}{-2}$ Divide by -2.

$y = -4 + \frac{1}{2}x$, or $y = \frac{1}{2}x - 4$

Exercises

1. Carpeting for a bedroom covers the surface of the bedroom floor, so area would be used.

2. Sod for a lawn covers the surface of the lawn, so area would be used.

3. To measure fencing for a yard, use perimeter since you would need to measure the lengths of the sides of the yard.

4. The baseboards for a living room go around the edges of the room. The amount of baseboard needed will be the sum of the lengths of the sides of the room, so perimeter would be used.

5. Tile for a bathroom covers the surface of the bathroom floor, so area would be used.

6. Fertilizer for a garden covers the surface of the garden, so area would be used.

7. To determine the cost of replacing a linoleum floor with a wood floor, use area since you need to know the measure of the surface covered by the wood.

8. Determining the cost of planting rye grass in a lawn for the winter requires finding the amount of surface to be covered, so area would be used.

9. $P = 2L + 2W$; $L = 8, W = 5$

$P = 2L + 2W$

$= 2(8) + 2(5)$

$= 16 + 10$

$P = 26$

10. $P = 2L + 2W$; $L = 6$, $W = 4$

$P = 2L + 2W$

$= 2(6) + 2(4)$

$= 12 + 8$

$P = 20$

11. $A = \frac{1}{2}bh$; $b = 8$, $h = 16$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(8)(16)$

$A = 64$

12. $A = \frac{1}{2}bh$; $b = 10$, $h = 14$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(10)(14)$

$= (5)(14)$

$A = 70$

13. $P = a + b + c$; $P = 12$, $a = 3$, $c = 5$

$P = a + b + c$

$12 = 3 + b + 5$

$12 = b + 8$

$4 = b$

14. $P = a + b + c;\ P = 15,\ a = 3,\ b = 7$

$$P = a + b + c$$
$$15 = 3 + 7 + c$$
$$15 = 10 + c$$
$$5 = c$$

15. $d = rt;\ d = 252,\ r = 45$

$$d = rt$$
$$252 = 45t$$
$$\frac{252}{45} = \frac{45t}{45}$$
$$5.6 = t$$

16. $d = rt;\ d = 100,\ t = 2.5$

$$d = rt$$
$$100 = r(2.5)$$
$$100 = 2.5r$$
$$\frac{100}{2.5} = \frac{2.5r}{2.5}$$
$$40 = r$$

17. $A = \frac{1}{2}h(b + B);\ A = 91,\ h = 7,\ b = 12$

$$A = \frac{1}{2}h(b + B)$$
$$91 = \frac{1}{2}(7)(12 + B)$$
$$182 = (7)(12 + B)$$
$$12 + B = \frac{1}{7}(182)$$
$$B = 26 - 12 = 14$$

18. $A = \frac{1}{2}h(b + B);\ A = 75,\ b = 19,\ B = 31$

$$A = \frac{1}{2}h(b + B)$$
$$75 = \frac{1}{2}h(19 + 31)$$
$$150 = h(50)$$
$$h = \frac{150}{50} = 3$$

19. $C = 2\pi r;\ C = 16.328,\ \pi = 3.14$

$$C = 2\pi r$$
$$16.328 = 2(3.14)r$$
$$16.328 = 6.28r$$
$$2.6 = r$$

20. $C = 2\pi r;\ C = 8.164,\ \pi = 3.14$

$$C = 2\pi r$$
$$8.164 = 2(3.14)r$$
$$8.164 = 6.28r$$
$$1.3 = r$$

21. $C = 2\pi r;\ C = 20\pi$

$$C = 2\pi r$$
$$20\pi = 2\pi r$$
$$10 = r \qquad \text{Divide by } 2\pi.$$

22. $C = 2\pi r;\ C = 100\pi$

$$C = 2\pi r$$
$$100\pi = 2\pi r$$
$$50 = r \qquad \text{Divide by } 2\pi.$$

23. $A = \pi r^2;\ r = 4,\ \pi = 3.14$

$$A = \pi r^2$$
$$= 3.14(4)^2$$
$$= 3.14(16)$$
$$A = 50.24$$

24. $A = \pi r^2;\ r = 12,\ \pi = 3.14$

$$A = \pi r^2$$
$$= 3.14(12)^2$$
$$= 3.14(144)$$
$$A = 452.16$$

25. $S = 2\pi rh;\ S = 120\pi,\ h = 10$

$$S = 2\pi rh$$
$$120\pi = 2\pi r(10)$$
$$120\pi = 20\pi r$$
$$6 = r \qquad \text{Divide by } 20\pi.$$

26. $S = 2\pi rh;\ S = 720\pi,\ h = 30$

$$S = 2\pi rh$$
$$720\pi = 2\pi r(30)$$
$$720\pi = 60\pi r$$
$$12 = r \qquad \text{Divide by } 60\pi.$$

27. $V = LWH;\ L = 10,\ W = 5,\ H = 3$

$$V = LWH$$
$$= (10)(5)(3)$$
$$V = 150$$

28. $V = LWH;\ L = 12,\ W = 8,\ H = 4$

$V = LWH$

$= (12)(8)(4)$

$V = 384$

29. $V = \frac{1}{3}Bh;\ B = 12,\ h = 13$

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(12)(13)$

$V = 52$

30. $V = \frac{1}{3}Bh;\ B = 36,\ h = 4$

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(36)(4)$

$V = 48$

31. $V = \frac{4}{3}\pi r^3;\ r = 12,\ \pi = 3.14$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}(3.14)(12)^3$

$= \frac{4}{3}(3.14)(1728)$

$V = 7234.56$

32. $V = \frac{4}{3}\pi r^3;\ r = 6,\ \pi = 3.14$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}(3.14)(6)^3$

$= \frac{4}{3}(3.14)(216)$

$V = 904.32$

33. $I = prt;\ p = \$7500,\ r = 4\%,\ t = 2$ yr

$I = prt$

$= (\$7500)(0.04)(2)$

$= \$600$

34. $I = prt;\ p = \$3600,\ r = 3\%,\ t = 4$ yr

$I = prt$

$= (\$3600)(0.03)(4)$

$= \$432$

35. $I = prt;\ I = \$33,\ r = 2\%,\ t = 3$ yr

$I = prt$

$\$33 = p(0.02)(3)$

$\$33 = (p)(0.06)$

$\$550 = p$

36. $I = prt;\ I = \$270,\ r = 5\%,\ t = 6$ yr

$I = prt$

$\$270 = (p)(0.05)(6)$

$\$270 = (p)(0.3)$

$\$900 = p$

37. $I = prt;\ I = \$180,\ p = \$4800,\ r = 2.5\%$

$I = prt$

$\$180 = (\$4800)(0.025)(t)$

$\$180 = (\$120)(t)$

$1.5 \text{ yr} = t$

38. $I = prt;\ I = \$162,\ p = \$2400,\ r = 1.5\%$

$I = prt$

$\$162 = (\$2400)(0.015)(t)$

$\$162 = (\$36)(t)$

$4.5 \text{ yr} = t$

39. $P = 2L + 2W$

$54 = 2(W + 9) + 2W$ Let $L = W + 9$.

$54 = 2W + 18 + 2W$

$54 = 4W + 18$

$36 = 4W$

$9 = W$

The width is 9 inches and the length is $9 + 9 = 18$ inches.

40. $P = 2L + 2W$

$62 = 2L + 2(L - 3)$ Let $W = L - 3$.

$62 = 2L + 2L - 6$

$62 = 4L - 6$

$68 = 4L$

$17 = L$

The length is 17 feet and the width is $17 - 3 = 14$ feet.

41. $P = 2L + 2W$

$36 = 2(3W + 2) + 2W$ Let $L = 3W + 2$.

$36 = 6W + 4 + 2W$

$36 = 8W + 4$

$32 = 8W$

$4 = W$

The width is 4 meters and the length is $3(4) + 2 = 14$ meters.

42. $P = 2L + 2W$

$36 = 2L + 2(2L - 18)$ Let $W = 2L - 18$.

$36 = 2L + 4L - 36$

$36 = 6L - 36$

$72 = 6L$

$12 = L$

The length is 12 yards and the width is $2(12) - 18 = 6$ yards.

43. Let s = the length of the shortest side, in inches; $s + 2$ = the length of the medium side, and $s + 3$ = the length of the longest side.
The perimeter is 20 inches, so

$s + (s + 2) + (s + 3) = 20.$

$3s + 5 = 20$

$3s = 15$

$s = 5$

Since $s = 5$, $s + 2 = 7$ and $s + 3 = 8$. The lengths of the sides are 5, 7, and 8 inches. The perimeter is $5 + 7 + 8 = 20$, as required.

44. Let s = the length of the shortest side, in feet.
$s + 4$ = the length of the medium side, and
$2s$ = the length of the longest side.
The perimeter is 28 feet, so

$s + (s + 4) + (2s) = 28.$

$4s + 4 = 28$

$4s = 24$

$s = 6$

Since $s = 6$, $s + 4 = 10$ and $2s = 12$. The lengths of the sides are 6, 10, and 12 feet. The perimeter is $6 + 10 + 12 = 28$, as required.

45. Let s = the length of the two sides that have equal length, in meters, and $2s - 4$ = the length of the third side.
The perimeter is 24 meters, so

$s + s + (2s - 4) = 24.$

$4s - 4 = 24$

$4s = 28$

$s = 7$

Since $s = 7$, $2s - 4 = 10$. The lengths of the sides are 7, 7, and 10 meters. The perimeter is $7 + 7 + 10 = 24$, as required.

46. Let s = the length of the shortest side, in yards;
$2s$ = the length of the medium side, and
$3s - 7$ = the length of the longest side.
The perimeter is 47 yards, so

$s + 2s + (3s - 7) = 47.$

$6s - 7 = 47$

$6s = 54$

$s = 9$

Since $s = 9$, $2s = 18$ and $3s - 7 = 20$. The lengths of the sides are 9, 18, and 20 yards. The perimeter is $9 + 18 + 20 = 47$, as required.

47. The page is a rectangle with length 1.5 m and width 1.2 m, so use the formulas for the perimeter and area of a rectangle.

$P = 2L + 2W$

$= 2(1.5) + 2(1.2)$

$= 3 + 2.4$

$P = 5.4$ meters

$A = LW$

$= (1.5)(1.2)$

$A = 1.8$ square meters

48. Since the sand painting is a square with side 12.24 m, use the formulas for the perimeter and area of a square.

$P = 4s$

$= 4(12.24)$

$P = 48.96$ meters

$A = s^2$

$= (12.24)^2$

$A = 149.8176$ square meters

To the nearest hundredth of a square meter, the area is 149.82 square meters.

49. Use the formula for the area of a triangle with $A = 70$ and $b = 14$.

$$A = \frac{1}{2}bh$$
$$70 = \frac{1}{2}(14)h$$
$$70 = 7h$$
$$10 = h$$

The height of the sign is 10 feet.

50. Use the formula for the area of a triangle with $A = 96$ and $h = 12$.

$$A = \frac{1}{2}bh$$
$$96 = \frac{1}{2}b(12)$$
$$96 = 6b$$
$$16 = b$$

The base of the banner is 16 feet.

51. The diameter of the circle is 443 feet, so its radius is $\frac{443}{2} = 221.5$ ft. Use the area of a circle formula to find the enclosed area.

$$A = \pi r^2$$
$$= \pi(221.5)^2$$
$$\approx 154{,}133.6 \text{ ft}^2,$$

or about $154{,}000 \text{ ft}^2$. (If 3.14 is used for π, the value is 154,055.465.)

52. The diameter of the circular dome is 630 feet, so its radius is $\frac{630}{2} = 315$ ft. Use the circumference of a circle formula.

$$C = 2\pi r$$
$$= 2\pi(315)$$
$$\approx 1979.2 \text{ ft}$$

If we use 3.14 for π, the answer is 1978.2 ft, or about 1978 ft.

53. To find the area of the drum face, use the formula for the area of a circle, $A = \pi r^2$.

$$A = \pi r^2$$
$$\approx (3.14)(7.87)^2$$
$$= (3.14)(61.9369)$$
$$A \approx 194.48$$

The area of the drum face is about 194.48 square feet.

Use the circumference of a circle formula.

$$C \approx 2(3.14)(7.87)$$
$$= 49.4236$$

The circumference is about 49.42 feet.

54. To find the area of the drum face, use the formula for the area of a circle, $A = \pi r^2$.

$$A = \pi r^2$$
$$\approx (3.14)(6.5)^2$$
$$= (3.14)(42.25)$$
$$A = 132.665$$

The area of the drum face is about 132.665 square feet.

Use the circumference of a circle formula.

$$C = 2\pi r$$
$$= 2\pi(6.5)$$
$$\approx 40.84 \text{ ft}$$

If we use 3.14 for π, the answer is about 40.82 feet.

55. Use the formula for the area of a trapezoid with $B = 115.80$, $b = 171.00$, and $h = 165.97$.

$$A = \frac{1}{2}(B + b)h$$
$$= \frac{1}{2}(115.80 + 171.00)(165.97)$$
$$= \frac{1}{2}(286.80)(165.97)$$
$$= 23{,}800.098$$

To the nearest hundredth of a square foot, the combined area of the two lots is 23,800.10 square feet.

56. Let A = the area of Lot A. Use the formula for the area of a trapezoid with $B = 82.05$, $b = 26.84$, and $h = 165.97$.

$$A = \frac{1}{2}(B + b)h$$
$$= \frac{1}{2}(82.05 + 26.84)(165.97)$$
$$= 9036.23665$$

To the nearest hundredth of a square foot, the area is 9036.24 square feet.

57. The girth is $4 \cdot 18 = 72$ inches. Since the length plus the girth is 108, we have

$$L + G = 108$$
$$L + 72 = 108$$
$$L = 36 \text{ in.}$$

The volume of the box is
$V = LWH$
$= (36)(18)(18)$
$= 11,664 \text{ in.}^3$

58. To find the volume of the sandwich, use the formula for the volume of a rectangular solid.
$V = LWH$
$= (12)(12)\left(1\frac{11}{24}\right)$
$= \frac{\overset{6}{\cancel{12}}}{1} \cdot \frac{\overset{1}{\cancel{12}}}{1} \cdot \frac{35}{\cancel{24}_{1}}$
$= 210$
The volume of the sandwich was 210 cubic feet.

59. The two angles are supplementary, so the sum of their measures is $180°$.
$(x+1)+(4x-56)=180$
$5x-55=180$
$5x=235$
$x=47$
Since $x=47,\ x+1=47+1=48$ and
$4x-56=4(47)-56=132.$
The measures of the angles are $48°$ and $132°$.

60. In the figure, the two angles are supplementary, so their sum is $180°$.
$(10x+7)+(7x+3)=180$
$17x+10=180$
$17x=170$
$x=10$
Since $x=10, 10x+7=10(10)+7=107$ and
$7x+3=7(10)+3=73.$
The two angle measures are $107°$ and $73°$.

61. In the figure, the two angles are complementary, so their sum is $90°$.
$(8x-1)+5x=90$
$13x-1=90$
$13x=91$
$x=7$
Since $x=7, 8x-1=8(7)-1=55$ and
$5x=5(7)=35.$
The two angle measures are $55°$ and $35°$.

62. In the figure, the two angles are complementary, so their sum is $90°$.
$4x+(3x+13)=90$
$7x+13=90$
$7x=77$
$x=11$
Since $x=11, 4x=4(11)=44$ and
$3x+13=3(11)+13=46.$
The two angle measures are $44°$ and $46°$.

63. The two angles are vertical angles, which have equal measures. Set their measures equal to each other and solve for x.
$5x-129=2x-21$
$3x-129=-21$
$3x=108$
$x=36$
Since $x=36,\ 5x-129=5(36)-129=51$ and
$2x-21=2(36)-21=51.$
The measure of each angle is $51°$.

64. The two angles are vertical angles, which have equal measures. Set their measures equal to each other and solve for x.
$7x+5=3x+45$
$4x+5=45$
$4x=40$
$x=10$
The measure of the first angle is
$7(10)+5=75°$; the measure of the second angle is $3(10)+45,$ which is also $75°$.

65. The angles are vertical angles, so their measures are equal.
$12x-3=10x+15$
$2x-3=15$
$2x=18$
$x=9$
Since $x=9, 12x-3=12(9)-3=105$ and
$10x+15=10(9)+15=105.$
The measure of each angle is $105°$.

66. The angles are vertical angles, which have equal measures. Set $11x-37$ equal to $7x+27$ and solve.

$$11x-37=7x+27$$
$$4x-37=27$$
$$4x=64$$
$$x=16$$

Since $x=16, 11x-37=11(16)-37=139$ and $7x+27=7(16)+27=139$.

The angles both measure 139°.

67. Solve $d=rt$ for t.

$\frac{d}{r}=\frac{rt}{r}$ Divide by r.

$\frac{d}{r}=t$, or $t=\frac{d}{r}$

68. Solve $d=rt$ for r.

$\frac{d}{t}=\frac{rt}{t}$ Divide by t.

$\frac{d}{t}=r$, or $r=\frac{d}{t}$

69. Solve $A=bh$ for b.

$\frac{A}{h}=\frac{bh}{h}$ Divide by h.

$\frac{A}{h}=b$, or $b=\frac{A}{h}$

70. Solve $A=LW$ for L.

$\frac{A}{W}=\frac{LW}{W}$ Divide by W.

$\frac{A}{W}=L$, or $L=\frac{A}{W}$

71. Solve $C=\pi d$ for d.

$\frac{C}{\pi}=\frac{\pi d}{\pi}$ Divide by π.

$\frac{C}{\pi}=d$, or $d=\frac{C}{\pi}$

72. Solve $P=4s$ for s.

$\frac{P}{4}=\frac{4s}{4}$ Divide by 4.

$\frac{P}{4}=s$, or $s=\frac{P}{4}$

73. Solve $V=LWH$ for H.

$\frac{V}{LW}=\frac{LWH}{LW}$ Divide by LW.

$\frac{V}{LW}=H$, or $H=\frac{V}{LW}$

74. Solve $V=LWH$ for W.

$\frac{V}{LH}=\frac{LWH}{LH}$ Divide by LH.

$\frac{V}{LH}=W$, or $W=\frac{V}{LH}$

75. Solve $I=prt$ for r.

$\frac{I}{pt}=\frac{prt}{pt}$ Divide by pt.

$\frac{I}{pt}=r$, or $r=\frac{I}{pt}$

76. $I=prt$ for p.

$\frac{I}{rt}=\frac{prt}{rt}$ Divide by rt.

$\frac{I}{rt}=p$, or $p=\frac{I}{rt}$

77. Solve $A=\frac{1}{2}bh$ for h.

$2A=2\left(\frac{1}{2}bh\right)$ Multiply by 2.

$2A=bh$

$\frac{2A}{b}=\frac{bh}{b}$ Divide by b.

$\frac{2A}{b}=h$, or $h=\frac{2A}{b}$

78. Solve $A=\frac{1}{2}bh$ for b.

$2A=2\left(\frac{1}{2}bh\right)$ Multiply by 2.

$2A=bh$

$\frac{2A}{h}=\frac{bh}{h}$ Divide by h.

$\frac{2A}{h}=b$, or $b=\frac{2A}{h}$

79. Solve $V=\frac{1}{3}\pi r^2h$ for h.

$3V=3\left(\frac{1}{3}\right)\pi r^2h$ Multiply by 3.

$3V=\pi r^2h$

$\frac{3V}{\pi r^2}=\frac{\pi r^2h}{\pi r^2}$ Divide by πr^2.

$\frac{3V}{\pi r^2}=h$, or $h=\frac{3V}{\pi r^2}$

80. Solve $V = \pi r^2 h$ for h.

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \quad \text{Divide by } \pi r^2.$$

$$\frac{V}{\pi r^2} = h, \quad \text{or} \quad h = \frac{V}{\pi r^2}$$

81. Solve $P = a + b + c$ for b.

$$P - a - c = a + b + c - a - c \quad \text{Subtract } a \text{ and } c.$$

$$P - a - c = b, \quad \text{or} \quad b = P - a - c$$

82. Solve $P = a + b + c$ for a.

$$P - b - c = a + b + c - b - c \quad \text{Subtract } b \text{ and } c.$$

$$P - b - c = a, \quad \text{or} \quad a = P - b - c$$

83. Solve $P = 2L + 2W$ for W.

$$P - 2L = 2L + 2W - 2L \quad \text{Subtract } 2L.$$

$$P - 2L = 2W$$

$$\frac{P - 2L}{2} = \frac{2W}{2} \quad \text{Divide by 2.}$$

$$\frac{P - 2L}{2} = W, \quad \text{or} \quad W = \frac{P - 2L}{2}$$

84. Solve $A = p + prt$ for r.

$$A - p = p + prt - p \quad \text{Subtract } p.$$

$$A - p = prt$$

$$\frac{A - p}{pt} = \frac{prt}{pt} \quad \text{Divide by } pt.$$

$$\frac{A - p}{pt} = r, \quad \text{or} \quad r = \frac{A - p}{pt}$$

85. Solve $y = mx + b$ for m.

$$y - b = mx + b - b \quad \text{Subtract } b.$$

$$y - b = mx$$

$$\frac{y - b}{x} = \frac{mx}{x} \quad \text{Divide by } x.$$

$$\frac{y - b}{x} = m, \quad \text{or} \quad m = \frac{y - b}{x}$$

86. Solve $y = mx + b$ for x.

$$y - b = mx + b - b \quad \text{Subtract } b.$$

$$y - b = mx$$

$$\frac{y - b}{m} = \frac{mx}{m} \quad \text{Divide by } m.$$

$$\frac{y - b}{m} = x, \quad \text{or} \quad x = \frac{y - b}{m}$$

87. Solve $Ax + By = C$ for y.

$$By = C - Ax \quad \text{Subtract } Ax.$$

$$\frac{By}{B} = \frac{C - Ax}{B} \quad \text{Divide by } B.$$

$$y = \frac{C - Ax}{B}$$

88. Solve $Ax + By = C$ for x.

$$Ax = C - By \quad \text{Subtract } By.$$

$$\frac{Ax}{A} = \frac{C - By}{A} \quad \text{Divide by } A.$$

$$x = \frac{C - By}{A}$$

89. Solve $M = C(1 + r)$ for r.

$$M = C + Cr \quad \text{Distributive property}$$

$$M - C = Cr \quad \text{Subtract } C.$$

$$\frac{M - C}{C} = \frac{Cr}{C} \quad \text{Divide by } C.$$

$$\frac{M - C}{C} = r, \quad \text{or} \quad r = \frac{M - C}{C}$$

Alternative solution:

$$M = C(1 + r)$$

$$\frac{M}{C} = 1 + r \quad \text{Divide by } C.$$

$$\frac{M}{C} - 1 = r \quad \text{Subtract 1.}$$

90. Solve $A = p(1 + rt)$ for t.

$$A = p + prt \quad \text{Distributive property}$$

$$A - p = p + prt - p \quad \text{Subtract } p.$$

$$A - p = prt$$

$$\frac{A - p}{pr} = \frac{prt}{pr} \quad \text{Divide by } pr.$$

$$\frac{A - p}{pr} = t, \quad \text{or} \quad t = \frac{A - p}{pr}$$

91. Solve $P = 2(a + b)$ for a.

$$P = 2a + 2b \quad \text{Distributive property}$$

$$P - 2b = 2a \quad \text{Subtract } 2b.$$

$$\frac{P - 2b}{2} = \frac{2a}{2} \quad \text{Divide by 2.}$$

$$\frac{P - 2b}{2} = a$$

92. Solve $P = 2(a+b)$ for b.

$$\frac{P}{2} = \frac{2(a+b)}{2} \quad \text{Divide by 2.}$$

$$\frac{P}{2} = a+b$$

$$\frac{P}{2} - a = b \quad \text{Subtract } a.$$

$$\frac{P}{2} - a = b, \quad \text{or} \quad b = \frac{P-2a}{2}$$

The second form of the answer results from first using the distributive property, then subtracting $2a$, and then dividing by 2—similar to the method used in Exercise 91.

93. Solve $S = \frac{1}{2}(a+b+c)$ for b.

$$2S = a+b+c \quad \text{Multiply by 2.}$$

$$2S - a - c = b \quad \text{Subtract } a \text{ and } c.$$

94. Solve $S = \frac{1}{2}(a+b+c)$ for c.

$$2S = a+b+c \quad \text{Multiply by 2.}$$

$$2S - a - b = c \quad \text{Subtract } a \text{ and } b.$$

95. Solve $C = \frac{5}{9}(F-32)$ for F.

$$C = \frac{5}{9}(F-32)$$

$$\frac{9}{5}C = \frac{9}{5}\cdot\frac{5}{9}(F-32)$$

$$\frac{9}{5}C = F-32$$

$$\frac{9}{5}C + 32 = F - 32 + 32$$

$$\frac{9}{5}C + 32 = F$$

96. Solve $A = \frac{1}{2}h(b+B)$ for b.

$$A = \frac{1}{2}h(b+B)$$

$$2A = 2\left(\frac{1}{2}h(b+B)\right)$$

$$2A = hb + hB \quad \text{Distributive Prop.}$$

$$2A - hB = hb + hB - hB \quad \text{Subtract } hB.$$

$$\frac{2A-hB}{h} = \frac{hb}{h} \quad \text{Divide by } h.$$

$$\frac{2A-hB}{h} = b$$

97. Solve $6x + y = 4$ for y.

$$6x + y - 6x = 4 - 6x \quad \text{Subtract } 6x.$$

$$y = 4 - 6x$$

98. Solve $3x + y = 6$ for y.

$$3x + y - 3x = 6 - 3x \quad \text{Subtract } 3x.$$

$$y = 6 - 3x$$

99. Solve $5x - y = 2$ for y.

$$5x - y - 5x = 2 - 5x \quad \text{Subtract } 5x.$$

$$\frac{-y}{-1} = \frac{2-5x}{-1} \quad \text{Divide by } -1.$$

$$y = -2 + 5x, \quad \text{or} \quad y = 5x - 2$$

100. Solve $4x - y = 1$ for y.

$$4x - y - 4x = 1 - 4x \quad \text{Subtract } 4x.$$

$$\frac{-y}{-1} = \frac{1-4x}{-1} \quad \text{Divide by } -1.$$

$$y = -1 + 4x, \quad \text{or} \quad y = 4x - 1$$

101. Solve $-3x + 5y = -15$ for y.

$$-3x + 5y + 3x = -15 + 3x \quad \text{Add } 3x.$$

$$\frac{5y}{5} = \frac{-15+3x}{5} \quad \text{Divide by 5.}$$

$$y = -3 + \frac{3}{5}x, \quad \text{or} \quad y = \frac{3}{5}x - 3$$

102. Solve $-2x + 3y = -9$ for y.

$$-2x + 3y + 2x = -9 + 2x \quad \text{Add } 2x.$$

$$\frac{3y}{3} = \frac{-9+2x}{3} \quad \text{Divide by 3.}$$

$$y = -3 + \frac{2}{3}x, \quad \text{or} \quad y = \frac{2}{3}x - 3$$

103. Solve $x - 3y = 12$ for y.

$$x - 3y - x = 12 - x \quad \text{Subtract } x.$$

$$\frac{-3y}{-3} = \frac{12-x}{-3} \quad \text{Divide by } -3.$$

$$y = -4 + \frac{1}{3}x, \quad \text{or} \quad y = \frac{1}{3}x - 4$$

104. Solve $x - 5y = 10$ for y.

$$x - 5y - x = 10 - x \quad \text{Subtract } x.$$

$$\frac{-5y}{-5} = \frac{10-x}{-5} \quad \text{Divide by } -5.$$

$$y = -2 + \frac{1}{5}x, \quad \text{or} \quad y = \frac{1}{5}x - 2$$

2.6 Ratio, Proportion, and Percent

Classroom Examples, Now Try Exercises

1. **(a)** To find the ratio of 3 days to 2 weeks, first convert 2 weeks to days.

$2 \text{ weeks} = 2 \cdot 7 = 14 \text{ days}$

The ratio of 3 days to 2 weeks is then

$$\frac{3 \text{ days}}{2 \text{ weeks}} = \frac{3 \text{ days}}{14 \text{ days}} = \frac{3}{14}.$$

(b) $4 \text{ days} = 4 \cdot 24 = 96 \text{ hours}$

The ratio of 12 hr to 4 days is then

$$\frac{12 \text{ hr}}{4 \text{ days}} = \frac{12 \text{ hr}}{96 \text{ hr}} = \frac{12}{96} = \frac{1}{8}.$$

N1. **(a)** The ratio of 7 inches to 4 inches is

$$\frac{7 \text{ inches}}{4 \text{ inches}} = \frac{7}{4}.$$

(b) $45 \text{ seconds} = \frac{45}{60} = \frac{3}{4} \text{ minute}$

The ratio of 45 seconds to 2 minutes is then

$$\frac{45 \text{ seconds}}{2 \text{ minutes}} = \frac{3}{4} \div 2 = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}.$$

2. The results in the following table are rounded to the nearest thousandth.

Size	**Unit Cost** (dollars per oz)
12 oz	$\frac{\$1.89}{12} = \$0.158$
24 oz	$\frac{\$2.79}{24} = \$0.116$
36 oz	$\frac{\$3.89}{36} = \$0.108\ (*)$

Because the 36-oz size produces the lowest unit cost, it is the best buy. The unit cost, to the nearest thousandth, is \$0.108 per oz.

N2. The results in the following table are rounded to the nearest thousandth.

Size	**Unit Cost** (dollars per oz)
75 oz	$\frac{\$8.94}{75} = \$0.119\ (*)$
100 oz	$\frac{\$13.97}{100} = \$0.140$
150 oz	$\frac{\$19.97}{150} = \$0.133$

Because the 75-oz size produces the lowest unit cost, it is the best buy. The unit cost, to the nearest thousandth, is \$0.119 per oz.

3. **(a)** $\frac{21}{15} = \frac{62}{45}$

Compare the cross products.

$21 \cdot 45 = 945$

$15 \cdot 62 = 930$

The cross products are *different*, so the proportion is *false.*

(b) $\frac{13}{17} = \frac{91}{119}$

Check to see whether the cross products are equal.

$13 \cdot 119 = 1547$

$17 \cdot 91 = 1547$

The cross products are *equal*, so the proportion is *true.*

N3. **(a)** $\frac{1}{3} = \frac{33}{100}$

Compare the cross products.

$1 \cdot 100 = 100$

$3 \cdot 33 = 99$

The cross products are *different*, so the proportion is *false.*

(b) $\frac{4}{13} = \frac{16}{52}$

Check to see whether the cross products are equal.

$4 \cdot 52 = 208$

$13 \cdot 16 = 208$

The cross products are *equal*, so the proportion is *true.*

4. $\frac{x}{6}=\frac{35}{42}$

$42x = 6\cdot 35$ Cross products

$x=\frac{6\cdot 35}{42}$ Divide by 42.

$=\frac{6\cdot 5\cdot 7}{6\cdot 7}$ Factor.

$=5$ Cancel.

The solution set is $\{5\}$.

Note: We could have multiplied $6\cdot 35$ to get 210 and then divided 210 by 42 to get 5. This may be the best approach if you are doing these calculations on a calculator. The factor and cancel method is preferable if you're not using a calculator.

N4. $\frac{9}{7}=\frac{x}{56}$

$7x = 9\cdot 56$ Cross products

$x=\frac{9\cdot 56}{7}$ Divide by 7.

$=\frac{9\cdot 7\cdot 8}{7}$ Factor.

$=72$ Cancel.

The solution set is $\{72\}$.

Note: We could have multiplied $9\cdot 56$ to get 504 and then divided 504 by 7 to get 72. This may be the best approach if you are doing these calculations on a calculator. The factor and cancel method is preferable if you're not using a calculator.

5. $\frac{x+6}{2}=\frac{2}{5}$

$5(x+6) = 2(2)$ Cross products

$5x+30 = 4$ Distributive property

$5x = -26$ Subtract 30.

$x=-\frac{26}{5}$ Divide by 5.

The solution set is $\left\{-\frac{26}{5}\right\}$.

N5. $\frac{k-3}{6}=\frac{3k+2}{4}$

$4(k-3) = 6(3k+2)$ Cross products

$4k-12 = 18k+12$ Distributive property

$-14k-12 = 12$ Subtract $18k$.

$-14k = 24$ Add 12.

$k=-\frac{24}{14}$ Divide by -14.

$=-\frac{12}{7}$

The solution set is $\left\{-\frac{12}{7}\right\}$.

6. Let x = the cost for 16.5 gallons.

$\frac{\$49.68}{12}=\frac{x}{16.5}$

$12x = 16.5(49.68)$ Cross products

$12x = 819.72$ Multiply.

$x = 68.31$ Divide by 12.

It would cost \$68.31.

N6. Let x = the cost for 27 gallons.

$\frac{\$69.80}{20}=\frac{x}{27}$

$20x = 27(69.80)$ Cross products

$20x = 1884.60$ Multiply.

$x = 94.23$ Divide by 20.

It would cost \$94.23.

7. **(a)** What is 3% of 80?

$3\%\cdot 80 = 0.03\cdot 80 = 2.4$

(b) 16% of what number is 12?

As in Example 7(b), let n denote the number.

$0.16\cdot n = 12$

$n=\frac{12}{0.16}$ Divide by 0.16.

$=75$ Simplify.

16% of 75 is 12.

(c) What percent of 75 is 90?

As in Example 7(c), let p denote the percent.

$90 = p\cdot 75$

$p=\frac{90}{75}=1.2=120\%$

90 is 120% of 75.

N7. **(a)** What is 20% of 70?
$20\% \cdot 70 = 0.20 \cdot 70 = 14$

(b) 40% of what number is 130?
As in Example 7(b), let n denote the number.
$0.40 \cdot n = 130$

$$n = \frac{130}{0.40} \quad \text{Divide by 0.40.}$$
$$= 325 \quad \text{Simplify.}$$

40% of 325 is 130.

(c) 121 is what percent of 484?
As in Example 7(c), let p denote the percent.
$121 = p \cdot 484$

$$p = \frac{121}{484} = 0.25 = 25\%$$

121 is 25% of 484.

8. We can think of this problem as "What percent of \$3800 is \$912?"
Let p denote the number.
$p \cdot 3800 = 912$

$$p = \frac{912}{3800} \quad \text{Divide by 3800.}$$
$$= 0.24 = 24\% \quad \text{Simplify.}$$

Mark's rent is 24% of his monthly income.

N8. We can think of this problem as "48 is what percent of 120?"
Let p denote the percent.
$48 = p \cdot 120$

$$p = \frac{48}{120} = 0.40 = 40\%$$

The coat costs 40% of the regular price, so the savings is $100\% - 40\% = 60\%$ of the regular price.

Exercises

1. **(a)** 75 to 100 is $\frac{75}{100} = \frac{3}{4}$, or 3 to 4.
The answer is C.

(b) 5 to 4, or $\frac{5}{4} = \frac{5 \cdot 3}{4 \cdot 3} = \frac{15}{12}$, or 15 to 12.
The answer is D.

(c) $\frac{1}{2} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{50}{100}$, or 50 to 100.
The answer is B.

(d) 4 to 5, or $\frac{4}{5} = \frac{4 \cdot 20}{5 \cdot 20} = \frac{80}{100}$, or 80 to 100.
The answer is A.

2. Since there are 14 days in 2 weeks, the ratio of 4 days to 2 weeks can be represented by $\frac{4}{14}$, choice C, or $\frac{2}{7}$, choice E.

3. The ratio of 40 miles to 30 miles is
$\frac{40 \text{ miles}}{30 \text{ miles}} = \frac{40}{30} = \frac{4}{3}$.

4. The ratio of 60 feet to 70 feet is
$\frac{60 \text{ feet}}{70 \text{ feet}} = \frac{60}{70} = \frac{6}{7}$.

5. The ratio of 120 people to 90 people is
$\frac{120 \text{ people}}{90 \text{ people}} = \frac{4 \cdot 30}{3 \cdot 30} = \frac{4}{3}$.

6. The ratio of 72 dollars to 220 dollars is
$\frac{72 \text{ dollars}}{220 \text{ dollars}} = \frac{72}{220} = \frac{18 \cdot 4}{55 \cdot 4} = \frac{18}{55}$.

7. To find the ratio of 20 yards to 8 feet, first convert 20 yards to feet.

$$20 \text{ yards} = 20 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = 60 \text{ feet}$$

The ratio of 20 yards to 8 feet is then
$\frac{60 \text{ feet}}{8 \text{ feet}} = \frac{60}{8} = \frac{15 \cdot 4}{2 \cdot 4} = \frac{15}{2}$.

8. First convert 8 feet to inches.
$8 \text{ feet} = 8 \cdot 12 = 96 \text{ inches}$
The ratio of 30 inches to 8 feet is then
$\frac{30 \text{ inches}}{96 \text{ inches}} = \frac{30}{96} = \frac{5 \cdot 6}{16 \cdot 6} = \frac{5}{16}$.

9. Convert 2 hours to minutes.

$$2 \text{ hours} = 2 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}}$$
$$= 120 \text{ minutes}$$

The ratio of 24 minutes to 2 hours is then
$\frac{24 \text{ minutes}}{120 \text{ minutes}} = \frac{24}{120} = \frac{1 \cdot 24}{5 \cdot 24} = \frac{1}{5}$.

10. To find the ratio of 16 minutes to 1 hour, first convert 1 hour to minutes.
1 hour = 60 minutes
The ratio of 16 minutes to 1 hour is then
$\frac{16 \text{ minutes}}{60 \text{ minutes}} = \frac{16}{60} = \frac{4 \cdot 4}{15 \cdot 4} = \frac{4}{15}$.

11. 2 yards = $2 \cdot 3 = 6$ feet

6 feet = $6 \cdot 12 = 72$ inches

The ratio of 60 inches to 2 yards is then

$$\frac{60 \text{ inches}}{72 \text{ inches}} = \frac{5 \cdot 12}{6 \cdot 12} = \frac{5}{6}.$$

12. 1 hour = 3600 seconds

The ratio of 720 seconds to 1 hour is then

$$\frac{720 \text{ seconds}}{3600 \text{ seconds}} = \frac{1 \cdot 720}{5 \cdot 720} = \frac{1}{5}.$$

13. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
4 lb	\$3.29	$\frac{\$3.29}{4} = \$0.823$
10 lb	\$7.49	$\frac{\$7.49}{10} = \$0.749$ (*)

The 10-lb size is the best buy.

14. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
23 oz	\$1.99	$\frac{\$1.99}{23} \approx \$0.087$
48 oz	\$3.49	$\frac{\$3.49}{48} \approx \$0.073$ (*)

The 48-oz size is the best buy.

15. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
64 oz	\$2.99	$\frac{\$2.99}{64} \approx \$0.047$ (*)
89 oz	\$4.79	$\frac{\$4.79}{89} \approx \$0.054$
128 oz	\$6.49	$\frac{\$6.49}{128} \approx \$0.051$

The 64-oz size is the best buy.

16. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
8 oz	\$1.69	$\frac{\$1.69}{8} \approx \$0.211$
16 oz	\$1.97	$\frac{\$1.97}{16} \approx \$0.123$ (*)
36 oz	\$5.99	$\frac{\$5.99}{36} \approx \$0.166$

The 16-oz size is the best buy.

17. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
8.5 oz	\$5.79	$\frac{\$5.79}{8.5} \approx \$0.681$
12.5 oz	\$7.99	$\frac{\$7.99}{12.5} \approx \$0.639$
32 oz	\$16.99	$\frac{\$16.99}{32} \approx \$0.531$ (*)

The 32-oz size is the best buy.

18. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
16.9 oz	\$3.39	$\frac{\$3.39}{16.9} \approx \$0.201$
33.8 oz	\$3.49	$\frac{\$3.49}{33.8} \approx \$0.103$ (*)
50.7 oz	\$5.29	$\frac{\$5.29}{50.7} \approx \$0.104$

The 33.8-oz size is the best buy.

19. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
32 oz	\$1.79	$\frac{\$1.79}{32} \approx \$0.056$ (*)
36 oz	\$2.69	$\frac{\$2.69}{36} \approx \$0.075$
40 oz	\$2.49	$\frac{\$2.49}{40} \approx \$0.062$
64 oz	\$4.38	$\frac{\$4.38}{64} \approx \$0.068$

The 32-oz size is the best buy.

20. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
12 oz	\$1.05	$\frac{\$1.05}{12} \approx \$0.088$
18 oz	\$1.73	$\frac{\$1.73}{18} \approx \$0.096$
32 oz	\$1.84	$\frac{\$1.84}{32} \approx \$0.058$ (*)
48 oz	\$2.88	$\frac{\$2.88}{48} = \$0.06$

The 32-oz size is the best buy.

21. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
87 oz	\$7.88	$\frac{\$7.88}{87} \approx \$0.091$
131 oz	\$10.98	$\frac{\$10.98}{131} \approx \$0.084$
263 oz	\$19.96	$\frac{\$19.96}{263} \approx \$0.076$ (*)

The 263-oz size is the best buy.

22. Find the unit price for each size.

Size	Price	**Unit Cost** (dollars per lb)
14 oz	\$1.79	$\frac{\$1.79}{14} \approx \$0.128$
24 oz	\$1.77	$\frac{\$1.77}{24} \approx \$0.074$ (*)
48 oz	\$3.65	$\frac{\$3.65}{48} \approx \$0.076$

The 24-oz size is the best buy.

23. Check to see whether the cross products are equal.

$5 \cdot 56 = 280$

$35 \cdot 8 = 280$

The cross products are *equal*, so the proportion is *true*.

24. Check to see whether the cross products are equal.

$4 \cdot 21 = 84$

$12 \cdot 7 = 84$

The cross products are *equal*, so the proportion is *true*.

25. Check to see whether the cross products are equal.

$120 \cdot 10 = 1200$

$82 \cdot 7 = 574$

The cross products are *different*, so the proportion is *false*.

26. Check to see whether the cross products are equal.

$27 \cdot 110 = 2970$

$160 \cdot 18 = 2880$

The cross products are *different*, so the proportion is *false*.

27. Check to see whether the cross products are equal.

$\frac{1}{2} \cdot 10 = 5$

$5 \cdot 1 = 5$

The cross products are *equal*, so the proportion is *true*.

28. Check to see whether the cross products are equal.

$\frac{1}{3} \cdot 18 = 6$

$6 \cdot 1 = 6$

The cross products are *equal*, so the proportion is *true*.

29. $20k = 4(175)$ Cross products

$20k = 700$

$\frac{20k}{20} = \frac{700}{20}$ Divide by 20.

$k = 35$

The solution set is $\{35\}$.

30. $\frac{x}{6} = \frac{18}{4}$

$x \cdot 4 = 6 \cdot 18$ Cross products

$4x = 108$

$\frac{4x}{4} = \frac{108}{4}$ Divide by 4.

$x = 27$

The solution set is $\{27\}$.

31. $\frac{49}{56} = \frac{z}{8}$

$56z = 49(8)$ Cross products

$56z = 392$

$\frac{56z}{56} = \frac{392}{56}$ Divide by 56.

$z = 7$

The solution set is $\{7\}$.

32. $\frac{20}{100} = \frac{z}{80}$

$100 \cdot z = 20 \cdot 80$ Cross products are equal.

$100z = 1600$

$\frac{100z}{100} = \frac{1600}{100}$ Divide by 100.

$z = 16$

The solution set is $\{16\}$.

33. $\dfrac{x}{24}=\dfrac{15}{16}$

$16x=24(15)$ Cross products

$16x=360$

$\dfrac{16x}{16}=\dfrac{360}{16}$ Divide by 16.

$x=\dfrac{45\cdot 8}{2\cdot 8}=\dfrac{45}{2}$

The solution set is $\left\{\dfrac{45}{2}\right\}$.

34. $\dfrac{x}{4}=\dfrac{12}{30}$

$30x=4(12)$ Cross products

$30x=48$

$\dfrac{30x}{30}=\dfrac{48}{30}$ Divide by 30.

$x=\dfrac{8\cdot 6}{5\cdot 6}=\dfrac{8}{5}$

The solution set is $\left\{\dfrac{8}{5}\right\}$.

35. $\dfrac{z}{2}=\dfrac{z+1}{3}$

$3z=2(z+1)$ Cross products

$3z=2z+2$ Distributive property

$z=2$ Subtract $2z$.

The solution set is $\{2\}$.

36. $\dfrac{m}{5}=\dfrac{m-2}{2}$

$2m=5(m-2)$ Cross products

$2m=5m-10$ Distributive property

$-3m=-10$ Subtract $5m$.

$m=\dfrac{10}{3}$ Divide by -3.

The solution set is $\left\{\dfrac{10}{3}\right\}$.

37. $\dfrac{3y-2}{5}=\dfrac{6y-5}{11}$

$11(3y-2)=5(6y-5)$ Cross products

$33y-22=30y-25$ Distributive property

$3y-22=-25$ Subtract $30y$.

$3y=-3$ Add 22.

$y=-1$ Divide by 3.

The solution set is $\{-1\}$.

38. $\dfrac{2r+8}{4}=\dfrac{3r-9}{3}$

$3(2r+8)=4(3r-9)$ Cross products

$6r+24=12r-36$ Distributive property

$-6r+24=-36$ Subtract $12r$.

$-6r=-60$ Subtract 24.

$r=10$ Divide by -6.

The solution set is $\{10\}$.

39. $\dfrac{5k+1}{6}=\dfrac{3k-2}{3}$

$3(5k+1)=6(3k-2)$ Cross products

$15k+3=18k-12$ Distributive property

$-3k+3=-12$ Subtract $18k$.

$-3k=-15$ Subtract 3.

$k=5$ Divide by -3.

The solution set is $\{5\}$.

40. $\dfrac{x+4}{6}=\dfrac{x+10}{8}$

$8(x+4)=6(x+10)$ Cross products

$8x+32=6x+60$ Distributive property

$2x+32=60$ Subtract $6x$.

$2x=28$ Subtract 32.

$x=14$ Divide by 2.

The solution set is $\{14\}$.

41. $\dfrac{2p+7}{3}=\dfrac{p-1}{4}$

$4(2p+7)=3(p-1)$ Cross products

$8p+28=3p-3$ Distributive property

$5p+28=-3$ Subtract $3p$.

$5p=-31$ Subtract 28.

$p=-\dfrac{31}{5}$ Divide by 5.

The solution set is $\left\{-\dfrac{31}{5}\right\}$.

42. $\dfrac{3m-2}{5}=\dfrac{4-m}{3}$

$3(3m-2)=5(4-m)$ Cross products

$9m-6=20-5m$ Distributive property

$14m-6=20$ Add $5m$.

$14m=26$ Add 6.

$m=\dfrac{26}{14}=\dfrac{13}{7}$ Divide by 14.

The solution set is $\left\{\dfrac{13}{7}\right\}$.

43. $\dfrac{2(x-4)}{3}=\dfrac{4(x-3)}{5}$

$\dfrac{2x-8}{3}=\dfrac{4x-12}{5}$ Distributive property

$5(2x-8)=3(4x-12)$ Cross products

$10x-40=12x-36$ Dist. prop.

$-40=2x-36$ Subtract $10x$.

$-4=2x$ Add 36.

$x=\dfrac{-4}{2}=-2$ Divide by 2.

The solution set is $\{-2\}$.

44. $\dfrac{9(x-3)}{6}=\dfrac{6(x-2)}{2}$

$\dfrac{9x-27}{6}=\dfrac{6x-12}{2}$ Distributive property

$2(9x-27)=6(6x-12)$ Cross products

$18x-54=36x-72$ Dist. prop.

$-54=18x-72$ Subtract $18x$.

$18=18x$ Add 72.

$x=1$ Divide by 18.

The solution set is $\{1\}$.

45. Let x = the cost of 24 candy bars.
Set up a proportion.

$\dfrac{x}{24}=\dfrac{\$20.00}{16}$

$16x=24(20)$

$16x=480$

$x=30$

The cost of 24 candy bars is \$30.00.

46. Let x = the cost of 8 ring tones.
Set up a proportion.

$\dfrac{x}{8}=\dfrac{\$30.00}{12}$

$12x=8(30)$

$12x=240$

$x=20$

The cost of 8 ring tones is \$20.00.

47. Let x = the cost of 5 quarts of oil.
Set up a proportion.

$\dfrac{x}{5}=\dfrac{\$14.00}{8}$

$8x=5(14)$

$8x=70$

$x=8.75$

The cost of 5 quarts of oil is \$8.75.

48. Let x = the cost of 7 tires.
Set up a proportion.

$\dfrac{x}{7}=\dfrac{\$398.00}{4}$

$4x=7(398)$

$4x=2786$

$x=696.50$

The cost of 7 tires is \$696.50.

49. Let x = the cost of 5 pairs of jeans.

$\dfrac{9\text{ pairs}}{\$121.50}=\dfrac{5\text{ pairs}}{x}$

$9x=5(121.50)$

$9x=607.5$

$\dfrac{9x}{9}=\dfrac{607.5}{9}$

$x=67.5$

The cost of 5 pairs is \$67.50.

50. Let x = the cost of 11 shirts.

$\dfrac{7\text{ shirts}}{\$87.50}=\dfrac{11\text{ shirts}}{x}$

$7x=11(87.50)$

$7x=962.5$

$\dfrac{7x}{7}=\dfrac{962.5}{7}$

$x=137.5$

The cost of 11 shirts is \$137.50.

51. Let x = the cost for filling a 15-gallon tank.
Set up a proportion.

$$\frac{x \text{ dollars}}{\$22.56} = \frac{15 \text{ gallons}}{6 \text{ gallons}}$$
$$6x = 15(22.56)$$
$$6x = 338.4$$
$$x = 56.4$$

It would cost \$56.40 to completely fill a 15-gallon tank.

52. Let x = the sales tax on a \$120.00 DVD player.

$$\frac{\$1.32}{\$x} = \frac{\$16}{\$120}$$
$$\frac{1.32}{x} = \frac{16}{120}$$
$$16x = 120(1.32)$$
$$16x = 158.4$$
$$x = 9.90$$

The sales tax on a \$120 DVD player would be \$9.90.

53. Let x = the number of fish in North Bay.
Set up a proportion with one ratio involving the sample and the other involving the total number of fish.

$$\frac{7 \text{ fish}}{700 \text{ fish}} = \frac{500 \text{ fish}}{x \text{ fish}}$$
$$7x = (700)(500)$$
$$7x = 350,000$$
$$x = 50,000$$

We estimate that there are 50,000 fish in North Bay.

54. Let x = the number of fish in West Okoboji Lake.
Set up a proportion with one ratio involving the sample and the other involving the total number of fish.

$$\frac{18 \text{ fish}}{1000 \text{ fish}} = \frac{840 \text{ fish}}{x \text{ fish}}$$
$$18x = (1000)(840)$$
$$18x = 840,000$$
$$x = 46,666.\overline{6}$$

The approximate fish population of West Okoboji Lake, to the nearest hundred, is 46,700 fish.

55. Let x = the distance between Memphis and Philadelphia on the map (in feet).
Set up a proportion with one ratio involving map distances and the other involving actual distances.

$$\frac{x \text{ feet}}{2.4 \text{ feet}} = \frac{1000 \text{ miles}}{600 \text{ miles}}$$
$$\frac{x}{2.4} = \frac{1000}{600}$$
$$600x = (2.4)(1000)$$
$$600x = 2400$$
$$x = 4$$

The distance on the map between Memphis and Philadelphia would be 4 feet.

56. Let x = the number of inches between Mexico City and Cairo on the map.
Set up a proportion.

$$\frac{11 \text{ inches}}{x \text{ inches}} = \frac{3300 \text{ miles}}{7700 \text{ miles}}$$
$$\frac{11}{x} = \frac{3300}{7700}$$
$$3300x = 11(7700)$$
$$3300x = 84,700$$
$$x = \frac{84,700}{3300}$$
$$= \frac{847}{33} = \frac{77}{3}, \quad \text{or} \quad 25\frac{2}{3}$$

Mexico City and Cairo are $25\frac{2}{3}$ inches apart on the map.

57. Let x = the number of inches between St. Louis and Des Moines on the map.
Set up a proportion.

$$\frac{8.5 \text{ inches}}{x \text{ inches}} = \frac{1040 \text{ miles}}{333 \text{ miles}}$$
$$1040x = 8.5(333)$$
$$1040x = 2830.5$$
$$x \approx 2.72$$

St. Louis and Des Moines are about 2.7 inches apart on the map.

58. Let x = the number of inches between Milwaukee and Seattle on the map.
Set up a proportion.

$$\frac{8.0 \text{ inches}}{x \text{ inches}} = \frac{912 \text{ miles}}{1940 \text{ miles}}$$

$$912x = 8.0(1940)$$

$$912x = 15{,}520$$

$$x \approx 17.0$$

Milwaukee and Seattle are about 17.0 inches apart on the map.

59. Let x = the number of inches between Moscow and Berlin on the globe.
Set up a proportion.

$$\frac{12.4 \text{ inches}}{x \text{ inches}} = \frac{10{,}080 \text{ km}}{1610 \text{ km}}$$

$$10{,}080x = 12.4(1610)$$

$$10{,}080x = 19{,}964$$

$$x \approx 1.98$$

Moscow and Berlin are about 2.0 inches apart on the globe.

60. Let x = the number of inches between Paris and Stockholm on the globe.
Set up a proportion.

$$\frac{21.5 \text{ inches}}{x \text{ inches}} = \frac{17{,}615 \text{ km}}{1605 \text{ km}}$$

$$17{,}615x = 21.5(1605)$$

$$17{,}615x = 34{,}507.5$$

$$x \approx 1.96$$

Paris and Stockholm are about 2.0 inches apart on the globe.

61. Let x = the number of cups of cleaner.
Set up a proportion with one ratio involving the number of cups of cleaner and the other involving the number of gallons of water.

$$\frac{x \text{ cups}}{\frac{1}{4} \text{ cup}} = \frac{10\frac{1}{2} \text{ gallons}}{1 \text{ gallons}}$$

$$x \cdot 1 = \frac{1}{4}\left(10\frac{1}{2}\right)$$

$$x = \frac{1}{4}\left(\frac{21}{2}\right) = \frac{21}{8}$$

The amount of cleaner needed is $2\frac{5}{8}$ cups.

62. Let x = the number of cups of cleaner.
Set up a proportion with one ratio involving the number of cups of cleaner and the other involving the number of gallons of water.

$$\frac{x \text{ cups}}{\frac{1}{2} \text{ cup}} = \frac{15\frac{1}{2} \text{ gallons}}{1 \text{ gallons}}$$

$$x \cdot 1 = \frac{1}{2}\left(15\frac{1}{2}\right)$$

$$x = \frac{1}{2}\left(\frac{31}{2}\right) = \frac{31}{4}$$

The amount of cleaner needed is $7\frac{3}{4}$ cups.

63. Let x = the number of U.S. dollars Ashley exchanged.
Set up a proportion.

$$\frac{\$1.3492}{x \text{ dollars}} = \frac{1 \text{ euro}}{300 \text{ euros}}$$

$$x \cdot 1 = 1.3492(300)$$

$$x = 404.76$$

She exchanged \$404.76.

64. Let x = the number of pesos that can be obtained for \$65.
Set up a proportion.

$$\frac{103.0 \text{ pesos}}{x \text{ pesos}} = \frac{8 \text{ dollars}}{65 \text{ dollars}}$$

$$x \cdot 8 = 103.0(65)$$

$$8x = 6695$$

$$x = 836.875$$

To the nearest tenth, one can obtain 836.9 pesos for \$65.

65. $\frac{x}{12} = \frac{3}{9}$

$9x = 12 \cdot 3 = 36$

$x = 4$

Other possibilities for the proportion are $\frac{12}{x} = \frac{9}{3}$, $\frac{x}{12} = \frac{5}{15}$, and $\frac{12}{x} = \frac{15}{5}$.

66. $\frac{6}{x} = \frac{8}{12}$

$8x = 6 \cdot 12$

$8x = 72$

$x = 9$

Other possibilities for the proportion are $\frac{6}{x} = \frac{4}{6}$, $\frac{x}{6} = \frac{6}{4}$, and $\frac{x}{6} = \frac{12}{8}$.

67. $\frac{x}{2}=\frac{12}{3}$

$3x = 2\cdot 12 = 24$

$x = 8$

68. $\frac{x}{4}=\frac{6}{8}$

$8x = 4\cdot 6 = 24$

$x = 3$

69. $\frac{x}{15}=\frac{12}{8}$ $\qquad$ $\frac{y}{17}=\frac{12}{8}$

$8x = 15(12) = 180$ $\qquad$ $8y = 17(12) = 204$

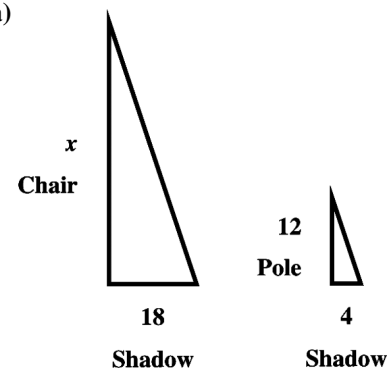
$x = 22.5$ $\qquad$ $y = 25.5$

70. $\frac{x}{6}=\frac{7}{14}$ $\qquad$ $\frac{y}{11}=\frac{7}{14}$

$14x = 7\cdot 6 = 42$ $\qquad$ $14y = 7\cdot 11 = 77$

$x = 3$ $\qquad$ $y = 5.5$

71. (a)



(b) These two triangles are similar, so their sides are proportional.

$\frac{x}{12}=\frac{18}{4}$

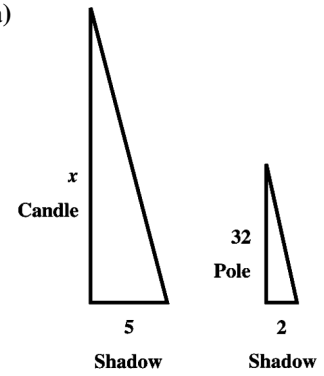
$4x = 18(12)$

$4x = 216$

$x = 54$

The chair is 54 feet tall.

72. (a)



(b) These two triangles are similar, so their sides are proportional.

$\frac{x}{32}=\frac{5}{2}$

$2x = 5(32)$

$2x = 160$

$x = 80$

The candle was 80 feet tall.

73. Let x = the 2001 price of electricity.

$$\frac{\text{1999 price}}{\text{1999 index}}=\frac{\text{2001 price}}{\text{2001 index}}$$

$$\frac{225}{166.6}=\frac{x}{177.1}$$

$$166.6x = 225(177.1)$$

$$x=\frac{225(177.1)}{166.6}\approx \$239$$

The 2001 price would be about \$239.

74. Let x = the 2003 price of electricity.

$$\frac{\text{1999 price}}{\text{1999 index}}=\frac{\text{2003 price}}{\text{2003 index}}$$

$$\frac{225}{166.6}=\frac{x}{184.0}$$

$$166.6x = 225(184.0)$$

$$x=\frac{225(184.0)}{166.6}\approx \$248$$

The 2003 price would be about \$248.

75. Let x = the 2007 price of electricity.

$$\frac{\text{1999 price}}{\text{1999 index}}=\frac{\text{2007 price}}{\text{2007 index}}$$

$$\frac{225}{166.6}=\frac{x}{207.3}$$

$$166.6x = 225(207.3)$$

$$x=\frac{225(207.3)}{166.6}\approx \$280$$

The 2007 price would be about \$280.

76. Let x = the 2011 price of electricity.

$$\frac{\text{1999 price}}{\text{1999 index}}=\frac{\text{2011 price}}{\text{2011 index}}$$

$$\frac{225}{166.6}=\frac{x}{224.9}$$

$$166.6x = 225(224.9)$$

$$x=\frac{225(224.9)}{166.6}\approx \$304$$

The 2011 price would be about \$304.

77. **(a)** Find the total amount of medication by multiplying.

$$(375 \text{ mg/day})(7 \text{ days}) = 2625 \text{ mg}$$

(b) Let x = the number of mL of suspension.

$$\frac{\text{mg of Amoxil}}{\text{mL of suspension}} = \frac{\text{total mg of Amoxil}}{\text{total mL of suspension}}$$

$$\frac{125 \text{ mg}}{5 \text{ mL}} = \frac{2625 \text{ mg}}{x}$$

(c) Solve the proportion.

$$\frac{125 \text{ mg}}{5 \text{ mL}} = \frac{2625 \text{ mg}}{x}$$

$$125x = 5(2625)$$

$$x = \frac{5(2625)}{125} = 105 \text{ mL}$$

Logan's pharmacist will make 105 mL of Amoxil suspension for the total course of treatment.

78. **(a)** Find the total amount of medication by multiplying.

$$(900 \text{ mg/day})(10 \text{ days}) = 9000 \text{ mg}$$

(b) Let x = the number of mL of suspension.

$$\frac{\text{mg of Amoxil}}{\text{mL of suspension}} = \frac{\text{total mg of Amoxil}}{\text{total mL of suspension}}$$

$$\frac{250 \text{ mg}}{5 \text{ mL}} = \frac{9000 \text{ mg}}{x}$$

(c) Solve the proportion.

$$\frac{250 \text{ mg}}{5 \text{ mL}} = \frac{9000 \text{ mg}}{x}$$

$$250x = 5(9000)$$

$$x = \frac{5(9000)}{250} = 180 \text{ mL}$$

Ava's pharmacist will make 180 mL of Amoxil suspension for the total course of treatment.

79. What is 18% of 780?

$$18\% \cdot 780 = 0.18 \cdot 780 = 140.4$$

80. What is 23% of 480?

$$23\% \cdot 480 = 0.23 \cdot 480 = 110.4$$

81. 42% of what number is 294?
As in Example 7(b), let n denote the number.

$$0.42 \cdot n = 294$$

$$n = \frac{294}{0.42} \quad \text{Divide by 0.42.}$$

$$= 700 \quad \text{Simplify.}$$

42% of 700 is 294.

82. 18% of what number is 108?
As in Example 7(b), let n denote the number.

$$0.18 \cdot n = 108$$

$$n = \frac{108}{0.18} \quad \text{Divide by 0.18.}$$

$$= 600 \quad \text{Simplify.}$$

18% of 600 is 108.

83. 120% of what number is 510?
As in Example 7(b), let n denote the number.

$$1.20 \cdot n = 510$$

$$n = \frac{510}{1.20} \quad \text{Divide by 1.20.}$$

$$= 425 \quad \text{Simplify.}$$

120% of 425 is 510.

84. 140% of what number is 315?
As in Example 7(b), let n denote the number.

$$1.40 \cdot n = 315$$

$$n = \frac{315}{1.40} \quad \text{Divide by 1.40.}$$

$$= 225 \quad \text{Simplify.}$$

140% of 225 is 315.

85. 4 is what percent of 50?
As in Example 7(c), let p denote the percent.

$$4 = p \cdot 50$$

$$p = \frac{4}{50} = 0.08 = 8\%$$

4 is 8% of 50.

86. 8 is what percent of 64?
As in Example 7(c), let p denote the percent.

$$8 = p \cdot 64$$

$$p = \frac{8}{64} = 0.125 = 12.5\%$$

8 is 12.5% of 64.

87. What percent of 30 is 36?
As in Example 7(c), let p denote the percent.

$$36 = p \cdot 30$$

$$p = \frac{36}{30} = 1.2 = 120\%$$

36 is 120% of 30.

88. What percent of 48 is 96?
As in Example 7(c), let p denote the percent.
$96 = p \cdot 48$

$$p = \frac{96}{48} = 2 = 200\%$$

96 is 200% of 48.

89. 48 is what percent of 60?
As in Example 7(c), let p denote the percent.
$48 = p \cdot 60$

$$p = \frac{48}{60} = 0.08 = 80\%$$

He earned 80% of the total points.

90. 63 is what percent of 75?
As in Example 7(c), let p denote the percent.
$63 = p \cdot 75$

$$p = \frac{63}{75} = 0.84 = 84\%$$

She scored 84% of the total points.

91. Find the amount of the savings.
$\$700 - \$504 = \$196$
What percent of \$700 is \$196?
As in Example 7(b), let n denote the number.
$n \cdot 700 = 196$

$$n = \frac{196}{700} \quad \text{Divide by 700.}$$
$$= 0.28 \quad \text{Simplify.}$$

$0.28 = 28(0.01) = 28 \cdot 1\% = 28\%$

92. Find the amount of the savings.
$\$980 - \$833 = \$147$
What percent of \$980 is \$147?
As in Example 7(b), let n denote the number.
$n \cdot 980 = 147$

$$n = \frac{147}{980} \quad \text{Divide by 980.}$$
$$= 0.15 \quad \text{Simplify.}$$

$0.15 = 15(0.01) = 15 \cdot 1\% = 15\%$

93. What percent of \$1500 is \$480?
As in Example 7(b), let n denote the number.
$n \cdot 1500 = 480$

$$n = \frac{480}{1500} \quad \text{Divide by 1500.}$$
$$= 0.32 \quad \text{Simplify.}$$

$0.32 = 32(0.01) = 32 \cdot 1\% = 32\%$

Tyler pays 32% of his income in rent.

94. What percent of \$2200 is \$154?
As in Example 7(b), let n denote the number.
$n \cdot 2200 = 154$

$$n = \frac{154}{2200} \quad \text{Divide by 2200.}$$
$$= 0.07 \quad \text{Simplify.}$$

$0.07 = 7(0.01) = 7 \cdot 1\% = 7\%$

Lily budgeted 7% of her income for entertainment.

95. 65% of what number is 1950?
As in Example 7(b), let n denote the number.
$0.65 \cdot n = 1950$

$$n = \frac{1950}{0.65} \quad \text{Divide by 0.65.}$$
$$= 3000 \quad \text{Simplify.}$$

She needs \$3000 for the car.

96. 70% of what number is 525?
As in Example 7(b), let n denote the number.
$0.70 \cdot n = 525$

$$n = \frac{525}{0.70} \quad \text{Divide by 0.70.}$$
$$= 750 \quad \text{Simplify.}$$

She needs \$750 for the apartment deposit.

2.7 Further Applications of Linear Equations

Classroom Examples, Now Try Exercises

1. **(a)** The amount of pure acid in 40 L of a 16% acid solution is

40 L	×	0.16	= 6.4 L.
↑		↑	↑
Amount of solution		Rate of concentration	Amount of pure acid

(b) If \$5000 is invested for one year at 4% simple interest, the amount of interest earned is

\$5000	×	0.04	= \$200.
↑		↑	↑
Principal		Interest rate	Interest earned

N1. **(a)** The amount of pure alcohol in 70 L of a 20% alcohol solution is

70 L	×	0.20	=	14 L.
↑		↑		↑
Amount of solution		Rate of concentration		Amount of pure alcohol

(b) If \$3200 is invested for one year at 2% simple interest, the amount of interest earned is

\$3200	×	0.02	=	\$64.
↑		↑		↑
Principal		Interest rate		Interest earned

2. Let x = the number of kilograms of metal that is 40% copper.

Then $x+80$ = the number of kilograms of metal that is 50% copper.

Copper in 40% metal	plus	copper in 70% metal	is	copper in 50% metal.
↓	↓	↓	↓	↓
$0.40x$	+	$0.70(80)$	=	$0.05(x+80)$

Solve the equation.

$$0.40x+0.70(80)=0.50x+40$$

Multiply by 10 to clear decimals.

$$4x+7(80)=5x+400$$
$$4x+560=5x+400$$
$$560=x+400$$
$$160=x$$

160 kilograms of metal that is 40% copper is needed.

Check $x=160$:

LS and RS refer to the left side and right side of the original equation.

LS: $0.40(160)+0.70(80)=64+56=120$

RS: $0.50(160+80)=0.50(240)=120$

N2. Let x = the number of ounces of seasoning that is 70% salt.

Then $x+30$ = the number of ounces of seasoning that is 50% salt.

Salt in 70% seasoning	plus	salt in 10% seasoning	is	salt in 50% seasoning.
↓	↓	↓	↓	↓
$0.70x$	+	$0.10(30)$	=	$0.50(x+30)$

Solve the equation.

$$0.70x+0.10(30)=0.50x+15$$

Multiply by 10 to clear decimals.

$$7x+1(30)=5x+150$$
$$7x+30=5x+150$$
$$2x+30=150$$
$$2x=120$$
$$x=60$$

60 ounces of seasoning that is 70% salt is needed.

Check $x=60$:

LS and RS refer to the left side and right side of the original equation.

LS: $0.70(60)+0.10(30)=42+3=45$

RS: $0.50(60+30)=0.50(90)=45$

3. Let x = the number of gallons of 12% indicator solution.

Then $10-x$ = the number of gallons of 20% indicator solution.

12% solution	plus	20% solution	is	14% solution.
↓	↓	↓	↓	↓
$0.12x$	+	$0.2(10-x)$	=	$0.14(10)$

Solve the equation.

$$0.12x+0.2(10-x)=0.14(10)$$

Multiply by 100 to clear decimals.

$$12x+20(10-x)=14(10)$$
$$12x+200-20x=140$$
$$-8x+200=140$$
$$-8x=-60$$
$$x=\frac{60}{8}=\frac{15}{2}=7\frac{1}{2}$$

There needs to be $7\frac{1}{2}$ gallons of the 12% indicator solution.

Check $x=7\frac{1}{2}$:

LS and RS refer to the left side and right side of the original equation.

LS: $12\left(7\frac{1}{2}\right)+20\left(10-7\frac{1}{2}\right)=90+50=140$

RS: $14(10)=140$

N3. Let x = the number of liters of 25% saline solution.
Then $15 - x$ = the number of liters of 10% saline solution.

25% solution		10% solution		15% solution.
↓	↓	↓	↓	↓
$0.25x$	+	$0.1(15-x)$	=	$0.15(15)$

Solve the equation.

$$0.25x + 0.1(15 - x) = 0.15(15)$$

Multiply by 100 to clear decimals.

$$25x + 10(15 - x) = 15(15)$$
$$25x + 150 - 10x = 225$$
$$15x + 150 = 225$$
$$15x = 75$$
$$x = 5$$

There needs to be 5 liters of the 25% saline solution.

Check $x = 5$:

LS and RS refer to the left side and right side of the original equation.

LS: $25(5) + 10(15 - (5)) = 125 + 100 = 225$

RS: $15(15) = 225$

4. Let x = the amount invested at 5%.
Then $2x + 3000$ = the amount invested at 8%.

Amount Invested (in dollars)	Rate of Interest	Interest for One Year
x	0.05	$0.05x$
$2x + 3000$	0.08	$0.08(2x + 3000)$

Since the total annual interest was \$1710, the equation is

$$0.05x + 0.08(2x + 3000) = 1710.$$

Multiply by 100 to eliminate the decimals.

$$5x + 8(2x + 3000) = 100(1710)$$
$$5x + 16x + 24,000 = 171,000$$
$$21x + 24,000 = 171,000$$
$$21x = 147,000$$
$$x = 7000$$

The engineer invested \$7000 at 5%.

N4. Let x = the amount invested at 3%.
Then $x + 5000$ = the amount invested at 4%.

Amount Invested (in dollars)	Rate of Interest	Interest for One Year
x	0.03	$0.03x$
$x + 5000$	0.04	$0.04(x + 5000)$

Since the total annual interest is \$410, the equation is $0.03x + 0.04(x + 5000) = 410$.

Multiply by 100 to eliminate the decimals.

$$3x + 4(x + 5000) = 100(410)$$
$$3x + 4x + 20,000 = 41,000$$
$$7x + 20,000 = 41,000$$
$$7x = 21,000$$
$$x = 3000$$

The financial advisor should invest \$3000 at 3% and \$8000 at 4%.

5. Let x = the number of quarters.
Then $x + 9$ = the number of nickels.

The value of quarters	plus	the value of nickels	is	\$2.55.
↓	↓	↓	↓	↓
$0.25x$	+	$0.05(x + 9)$	=	2.55

Multiply by 100 to eliminate the decimals.

$$25x + 5(x + 9) = 255$$
$$25x + 5x + 45 = 255$$
$$30x + 45 = 255$$
$$30x = 210$$
$$x = 7$$

Since $x = 7$, $x + 9 = 16$.

The man has 16 nickels and 7 quarters.

N5. Let x = the number of dimes.
Then $x + 10$ = the number of quarters.

The value of dimes	plus	the value of quarters	is	\$5.65.
↓	↓	↓	↓	↓
$0.10x$	+	$0.25(x + 10)$	=	\$5.65

Multiply by 100 to eliminate the decimals.

$$10x + 25(x + 10) = 565$$
$$10x + 25x + 250 = 565$$
$$35x + 250 = 565$$
$$35x = 315$$
$$x = 9$$

Since $x = 9$, $x + 10 = 19$.

Clayton has 19 quarters and 9 dimes.

6. $r = \dfrac{d}{t} = \dfrac{100 \text{ meters}}{9.63 \text{ seconds}} \approx 10.3842$

His rate was about 10.38 meters per second.

N6. $r = \dfrac{d}{t} = \dfrac{400 \text{ miles}}{6 \text{ hours}} \approx 66.6667$

The rate was about 66.67 miles per hour.

7. Let t = the time it takes for the planes to be 3290 miles apart. Use the formula $d = rt$.

$$d_{\text{faster}} + d_{\text{slower}} = d_{\text{total}}$$
$$530t + 410t = 3290$$
$$940t = 3290$$
$$t = \frac{3290}{940} = 3.5$$

It will take 3.5 hours for the planes to be 3290 miles apart.

N7. Let t = the time it takes for the bicyclists to be 5 miles apart. Use the formula $d = rt$.

$$d_{\text{faster}} - d_{\text{slower}} = d_{\text{total}}$$
$$20t - 18t = 5$$
$$2t = 5$$
$$t = \frac{5}{2} = 2.5$$

It will take 2.5 hours for the bicyclists to be 5 miles apart.

8. Let x = the rate of the slower bus.

Then $x + 10$ = the rate of the faster bus.

Use the formula $d = rt$ and the fact that each bus travels for $\frac{1}{5}$ hour.

$$d_{\text{slower}} + d_{\text{faster}} = d_{\text{total}}$$
$$(x + 10)\left(\frac{1}{5}\right) + x\left(\frac{1}{5}\right) = 12$$
$$\frac{1}{5}x + 2 + \frac{1}{5}x = 12$$
$$\frac{2}{5}x + 2 = 12$$
$$\frac{2}{5}x = 10$$
$$\frac{5}{2}\left(\frac{2}{5}x\right) = \frac{5}{2}(10)$$
$$x = 25$$

Since $x = 25$, $x + 10 = 35$. The slower bus had a rate of 25 mph and the faster bus had a rate of 35 mph.

Check: The slower bus traveled $25\left(\frac{1}{5}\right) = 5$ miles and the faster bus traveled $35\left(\frac{1}{5}\right) = 7$ miles. The total miles traveled is $5 + 7 = 12$, as required.

N8. Let x = the rate of the slower car.

Then $x + 6$ = the rate of the faster car.

Use the formula $d = rt$ and the fact that each car travels for $\frac{1}{4}$ hour.

$$d_{\text{slower}} + d_{\text{faster}} = d_{\text{total}}$$
$$(x + 6)\left(\frac{1}{4}\right) + x\left(\frac{1}{4}\right) = 35$$
$$\frac{1}{4}x + \frac{3}{2} + \frac{1}{4}x = \frac{70}{2}$$
$$\frac{2}{4}x + \frac{3}{2} = \frac{70}{2}$$
$$\frac{1}{2}x = \frac{67}{2}$$
$$\frac{2}{1}\left(\frac{1}{2}x\right) = \frac{2}{1}\left(\frac{67}{2}\right)$$
$$x = 67$$

Since $x = 67$, $x + 6 = 73$. The slower car had a rate of 67 mph and the faster car had a rate of 73 mph.

Check: The slower car traveled $67\left(\frac{1}{4}\right)=\frac{67}{4}$ miles and the faster car traveled $73\left(\frac{1}{4}\right)=\frac{73}{4}$ miles. The total miles traveled is $\frac{67}{4}+\frac{73}{4}=\frac{140}{4}=35,$ as required.

Exercises

1. The amount of pure alcohol in x liters of a 75% alcohol solution is 0.75 times the volume of solution, or $0.75x$ liters. So choice A is the correct answer.

2. Each quarter is worth \$0.25. The value of x quarters is $0.25x$ dollars. So choice C is the correct answer.

3. Use $d=rt,$ where $r=55$ and t is the number of hours. $d=(55)t=55t$ miles.
So choice C is the correct answer.

4. Use $d=rt,$ where $t=6$ and r is the rate of the car. $d=r(6)=6r$ miles.
So choice D is the correct answer.

5. The concentration of the new solution could not be more than the strength of the stronger of the original solutions, so the correct answer is D, since 32% is stronger than both 20% and 30%.

6. Because pure alcohol (100% concentration) is to be added, the new solution must be stronger than the original one. Therefore, the concentration of the new solution must be greater than 24%, so the correct answer is A, 22%.

7. To estimate the average rate of the trip, round 405 to 400 and 8.2 to 8.
Use $r=\frac{d}{t}$ with $d=405$ and $t=8.2.$
$r=\frac{d}{t}=\frac{405}{8.2}\approx 49.4$
The best estimate is choice A, 50 mph.

8. The distance traveled cannot be found by multiplying 45 and 30 because the rate is given in miles per hour, while the time is given in minutes. To find the correct distance, start by converting the time to hours.

$$30 \text{ minutes}=\frac{1}{2} \text{ hour}$$
$$d=rt$$
$$=45\left(\frac{1}{2}\right)$$
$$=22.5$$

The car traveled 22.5 $\left(\text{or } 22\frac{1}{2}\right)$ miles.

9. The amount of pure alcohol in 150 liters of a 30% alcohol solution is

150	×	0.30	= 45 liters.
↑		↑	↑
Amount of solution		Rate of concentration	Amount of pure alcohol

10. The amount of pure acid in 250 milliliters of a 14% acid solution is

250	×	0.14	= 35 milliliters.
↑		↑	↑
Amount of solution		Rate of concentration	Amount of pure acid

11. If \$25,000 is invested at 3% simple interest for one year, the amount of interest earned is

\$25,000	×	0.03	×	1	= \$750.
↑		↑		↑	↑
Principal		Interest rate		Time	Interest earned

12. If \$10,000 is invested for one year at 3.5% simple interest, the amount of interest earned is

\$10,000	×	0.035	= \$350.
↑		↑	↑
Principal		Interest rate	Interest earned

13. The monetary value of 35 half-dollars is

35	×	\$0.50	= \$17.50.
↑		↑	↑
Number of coins		Denomination	Monetary value

14. The monetary value of 283 nickels is

$$\begin{array}{ccccc} 283 & \times & \$0.05 & = & \$14.15. \\ \uparrow & & \uparrow & & \uparrow \\ \text{Number} & & \text{Denomination} & & \text{Monetary} \\ \text{of coins} & & & & \text{value} \end{array}$$

15. *Step 2*
Let x = the number of liters of 25% acid solution to be used.
Step 3
Use the box diagram in the textbook to write the equation.

$$\begin{array}{ccccccc} \text{Pure acid} & & \text{pure acid} & & \text{pure acid} \\ \text{in 25\%} & & \text{in 40\%} & & \text{in 30\%} \\ \text{solution} & \text{plus} & \text{solution} & \text{is} & \text{solution.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.25x & + & 0.40(80) & = & 0.30(x+80) \end{array}$$

Step 4
Multiply by 100 to clear decimals.

$$\begin{aligned} 25x + 40(80) &= 30(x+80) \\ 25x + 3200 &= 30x + 2400 \\ 25x + 800 &= 30x \\ 800 &= 5x \\ 160 &= x \end{aligned}$$

Step 5
160 liters of 25% acid solution must be added.
Step 6
25% of 160 liters plus 40% of 80 liters is 40 liters plus 32 liters, or 72 liters, of pure acid; which is equal to 30% of $(160+80)$ liters.
$[0.30(240) = 72]$

16. *Step 2*
Let x = the number of gallons of 50% solution needed.
Then $x+80$ = the number of gallons of 40% solution.
Step 3
Use the box diagram in the textbook to write the equation.

$$\begin{array}{ccccccc} \text{Pure} & & \text{pure} & & \text{pure} \\ \text{antifreeze} & & \text{antifreeze} & & \text{antifreeze} \\ \text{in 50\%} & \text{plus} & \text{in 20\%} & \text{is} & \text{in 40\%} \\ \text{solution} & \downarrow & \text{solution} & \downarrow & \text{solution.} \\ \downarrow & + & \downarrow & = & \downarrow \\ 0.50x & & 0.20(80) & & 0.40(x+80) \end{array}$$

Step 4
Solve the equation.

$$0.50x + 0.20(80) = 0.40(x+80)$$

Multiply by 10 to clear decimals.

$$\begin{aligned} 5x + 2(80) &= 4(x+80) \\ 5x + 160 &= 4x + 320 \\ x + 160 &= 320 \\ x &= 160 \end{aligned}$$

Step 5
160 gallons of 50% antifreeze is needed.
Step 6
50% of 160 gallons plus 20% of 80 gallons is 80 gallons plus 16 gallons, or 96 gallons, of pure antifreeze, which is equal to 40% of $(160+80)$ gallons. $[0.40(240) = 96]$

17. Let x = the number of liters of 5% drug solution.

$$\begin{array}{ccccccc} \text{Pure drug} & & \text{pure drug} & & \text{pure drug} \\ \text{in 10\%} & & \text{in 5\%} & & \text{in 8\%} \\ \text{solution} & \text{plus} & \text{solution} & \text{is} & \text{solution.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.10(20) & + & 0.05x & = & 0.08(x+20) \end{array}$$

Solve the equation.

$$\begin{aligned} 0.10(20) + 0.05x &= 0.08(x+20) \\ 10(20) + 5x &= 8(x+20) \\ 200 + 5x &= 8x + 160 \\ 200 &= 3x + 160 \\ 40 &= 3x \\ x &= \frac{40}{3} = 13\frac{1}{3} \end{aligned}$$

The pharmacist needs $13\frac{1}{3}$ liters of 5% drug solution.

Check $x = 13\frac{1}{3}$:

LS and RS refer to the left side and right side of the original equation.

$$\text{LS: } 0.10(20) + 0.05\left(13\frac{1}{3}\right) = 2\frac{2}{3}$$

$$\text{RS: } 0.08\left(13\frac{1}{3} + 20\right) = 2\frac{2}{3}$$

18. Let x = the number of kilograms of metal that is 20% tin.
Then $x+80$ = the number of kilograms of metal that is 50% tin.

Tin in 20% metal		tin in 70% metal		tin in 50% metal.
↓	↓	↓	↓	↓
$0.20x$	+	$0.70(80)$	=	$0.50(x+80)$

Solve the equation.

$$0.20x+0.70(80)=0.50x+40$$

Multiply by 10 to clear decimals.

$$2x+7(80)=5x+400$$
$$2x+560=5x+400$$
$$560=3x+400$$
$$160=3x$$
$$x=\frac{160}{3}=53\frac{1}{3}$$

$53\frac{1}{3}$ kilograms of metal that is 20% tin is needed.

Check $x=53\frac{1}{3}$:

LS and RS refer to the left side and right side of the original equation.

$$\text{LS: } 0.20\left(53\frac{1}{3}\right)+0.70(80)=66\frac{2}{3}$$

$$\text{RS: } 0.50\left(53\frac{1}{3}+80\right)=66\frac{2}{3}$$

19. Let x = the number of liters of the 20% alcohol solution.
Complete the table.

Strength	Liters of Solution	Liters of Pure Alcohol
12%	12	$0.12(12)=1.44$
20%	x	$0.20x$
14%	$x+12$	$0.14(x+12)$

From the last column, we can formulate an equation that compares the number of liters of pure alcohol. The equation is
$1.44+0.20x=0.14(x+12)$.

Solve the equation.

$$1.44+0.20x=0.14(x+12)$$
$$1.44+0.20x=0.14x+1.68$$
$$0.06x=0.24$$
$$x=4$$

4 L of the 20% alcohol solution is needed.

20. Let x = the number of liters of the 10% solution.

Strength	Liters of Solution	Liters of Pure Alcohol
10%	x	$0.10x$
50%	40	$0.50(40)=20$
40%	$x+40$	$0.40(x+40)$

Alcohol in 10% solution		alcohol in 50% solution		alcohol in 40% solution.
↓	↓	↓	↓	↓
$0.10x$	+	20	=	$0.40(x+40)$

Multiply by 10 to clear decimals.

$$1x+200=4(x+40)$$
$$x+200=4x+160$$
$$-3x=-40$$
$$x=\frac{-40}{-3}=\frac{40}{3}, \text{ or } 13\frac{1}{3}$$

$13\frac{1}{3}$ L of 10% alcohol solution should be added.

21. Let x = the amount of water to be added.
Then $20+x$ = the amount of 2% solution.
There is no minoxidil in water.

Pure minoxidil in x mL solution		pure minoxidil in 4% solution		pure minoxidil in 2% solution.
↓	↓	↓	↓	↓
$0(x)$	+	$0.04(20)$	=	$0.02(20+x)$

Solve the equation.

$$0x+0.04(20)=0.02(20+x)$$
$$4(20)=2(20+x)$$
$$80=40+2x$$
$$40=2x$$
$$20=x$$

20 milliliters of water should be used.

Check $x=20$:

$$\text{LS: } 0(20)+0.04(20)=0.8$$
$$\text{RS: } 0.02(20+20)=0.8$$

This answer should make common sense—that is, equal amounts of 0% and 4% solutions should produce a 2% solution.

22. Let x = the number of milliliters of 4% solution.

Pure minoxidil in 1% solution		pure minoxidil in 4% solution		pure minoxidil in 2% solution.
↓	↓	↓	↓	↓
$0.01(50)$	+	$0.04x$	=	$0.02(50+x)$

Multiply by 10 to clear decimals.

$$1(50)+4x=2(50+x)$$
$$50+4x=100+2x$$
$$50+2x=100$$
$$2x=50$$
$$x=25$$

The pharmacist must add 25 milliliters of 4% solution.

Check $x=25$:

LS: $0.01(50)+0.04(25)=1.5$

RS: $0.02(50+25)=1.5$

23. Let x = the number of liters of 60% acid solution.

Then $20-x$ = the number of liters of 75% acid solution.

Pure acid in 60% solution		pure acid in 75% solution		pure acid in 72% solution.
↓	↓	↓	↓	↓
$0.60x$	+	$0.75(20-x)$	=	$0.72(20)$

Solve the equation.

$$0.60x+0.75(20-x)=0.72(20)$$
$$60x+75(20-x)=72(20)$$
$$60x+1500-75x=1440$$
$$1500-15x=1440$$
$$-15x=-60$$
$$x=4$$

4 liters of 60% acid solution must be used.

Check $x=4$:

LS: $0.60(4)+0.75(20-4)=14.4$

RS: $0.72(20)=14.4$

24. Let x = the number of gallons of 50% juice fruit drink.

Then $12-x$ = the number of gallons of 20% juice fruit drink.

Juice in 50% drink		juice in 20% drink		juice in 40% drink.
↓	↓	↓	↓	↓
$0.5x$	+	$0.20(12-x)$	=	$0.40(12)$

Multiply by 10 to clear decimals.

$$5x+2(12-x)=4(12)$$
$$5x+24-2x=48$$
$$3x+24=48$$
$$3x=24$$
$$x=8$$

8 gallons of the 50% juice fruit drink must be added.

Check $x=8$:

LS: $0.5(8)+0.20(12-8)=4.8$

RS: $0.4(12)=4.8$

25. Let x = the amount invested at 5% (in dollars).

Then $x-1200$ = the amount invested at 4% (in dollars).

Amount Invested (in dollars)	Rate of Interest	Interest for One Year
x	0.05	$0.05x$
$x-1200$	0.04	$0.04(x-1200)$

Since the total annual interest was \$141, the equation is

$$0.05x+0.04(x-1200)=141.$$
$$5x+4(x-1200)=100(141)$$
$$5x+4x-4800=14{,}100$$
$$9x-4800=14{,}100$$
$$9x=18{,}900$$
$$x=2100$$

Since $x=2100$, $x-1200=900$.

Arlene invested \$2100 at 5% and \$900 at 4%.

26. Let x = the amount invested at 2%.
Then $x+3000$ = the amount invested at 3%.

Amount Invested (in dollars)	Rate of Interest	Interest for One Year
x	0.02	$0.02x$
$x+3000$	0.03	$0.03(x+3000)$

Since the total annual interest was \$390, the equation is $0.02x+0.03(x+3000)=390$.

Multiply by 100 to clear decimals.

$$2x+3(x+3000)=39,000$$
$$2x+3x+9,000=39,000$$
$$5x+9,000=39,000$$
$$5x=30,000$$
$$x=6000$$

Since $x=6000$, $x+3000=9000$.

Margaret deposited \$6000 at 2% and \$9000 at 3%.

27. Let x = the amount invested at 6%.
Then $3x+6000$ = the amount invested at 5%.

$$0.06x+0.05(3x+6000)=825$$
$$6x+5(3x+6000)=100(825)$$
$$6x+15x+30,000=82,500$$
$$21x+30,000=82,500$$
$$21x=52,500$$
$$x=2500$$

Since $x=2500$, $3x+6000=13,500$.

The artist invested \$2500 at 6% and \$13,500 at 5%.

28. Let x = the amount invested at 3%.
Then $2x+30,000$ = the amount invested at 4%.

$$0.03x+0.04(2x+30,000)=5600$$
$$3x+4(2x+30,000)=100(5600)$$
$$3x+8x+120,000=560,000$$
$$11x+120,000=560,000$$
$$11x=440,000$$
$$x=40,000$$

Since $x=40,000$, $2x+30,000=110,000$.

The actor invested \$40,000 at 3% and \$110,000 at 4%.

29. Let x = the amount Jamal invested at 8%.
Then $2500-x$ = the amount invested at 2%.

$$0.08x+0.02(2500-x)=152$$
$$8x+2(2500-x)=15,200$$
$$8x+5000-2x=15,200$$
$$6x+5000=15,200$$
$$6x=10,200$$
$$x=1700$$

Since $x=1700$, $2500-x=800$.

Jamal invested \$1700 at 8% and \$800 at 2%.

30. Let x = the amount Carter invested at 1%.
Then $9000-x$ = the amount invested at 4%.

$$0.01x+0.04(9000-x)=285$$
$$1x+4(9000-x)=28,500$$
$$x+36,000-4x=28,500$$
$$-3x+36,000=28,500$$
$$-3x=-7500$$
$$x=2500$$

Since $x=2500$, $9000-x=6500$.

Carter invested \$2500 at 1% and \$6500 at 4%.

31. Let x = the number of nickels.
Then $x+2$ = the number of dimes.

The value of nickels		the value of dimes		
↓	↓	↓	↓	↓
$0.05x$	+	$0.10(x+2)$	=	1.70

$$5x+10(x+2)=100(1.70)$$
$$5x+10x+20=170$$
$$15x+20=170$$
$$15x=150$$
$$x=10$$

The collector has 10 nickels.

32. Let x = the number of \$5 bills.
Then $x+5$ = the number of \$20 bills.

The value of fives		the value of twenties		
↓	↓	↓	↓	↓
$5x$	+	$20(x+5)$	=	725

$$5x+20x+100=725$$
$$25x+100=725$$
$$25x=625$$
$$x=25$$

The teller has twenty-five \$5 bills.

33. Let x = the number of 46-cent stamps.
Then $45 - x$ = the number of 20-cent stamps.
The value of the 46-cent stamps is $0.46x$ and the value of the 20-cent stamps is $0.20(45 - x)$.
The total value is \$15.50, so

$$0.46x + 0.20(45 - x) = 15.50$$
$$46x + 20(45 - x) = 1550$$
$$46x + 900 - 20x = 1550$$
$$26x + 900 = 1550$$
$$26x = 650$$
$$x = 25$$

Since $x = 25$, $45 - x = 20$.
She bought twenty-five 46-cent stamps valued at \$11.50 and twenty 20-cent stamps valued at \$4.00, for a total value of \$15.50.

34. Let x = the number of adult tickets sold.
Then $600 - x$ = the number of children's tickets sold.
The value of the adult tickets is $8x$ and the value of the children's tickets is $5(600 - x)$. The total value was \$4116, so

$$8x + 5(600 - x) = 4116.$$
$$8x + 3000 - 5x = 4116$$
$$3x + 3000 = 4116$$
$$3x = 1116$$
$$x = 372$$

Since $x = 372$, $600 - x = 228$.
There were 372 adult tickets valued at \$2976 and 228 children's tickets valued at \$1140, for a total value of \$4116.

35. Let x = the number of pounds of Colombian Decaf beans.
Then $2x$ = the number of pounds of Arabian Mocha beans.

	Number of Pounds	Cost per Pound	Total Value (in \$)
Colombian Decaf	x	\$8.00	$8x$
Arabian Mocha	$2x$	\$8.50	$8.5(2x)$

The total value is \$87.50, so

$$8x + 8.5(2x) = 87.50.$$
$$8x + 17x = 87.50$$
$$25x = 87.50$$
$$x = 3.5$$

Since $x = 3.5$, $2x = 7$.
She can buy 3.5 pounds of Colombian Decaf and 7 pounds of Arabian Mocha.

36. Let x = the number of pounds of Italian Espresso beans.
Then $4x$ = the number of pounds of Kona Deluxe beans.

	Number of Pounds	Cost per Pound	Total Value (in \$)
Italian Espresso	x	\$9.00	$9.00x$
Kona Deluxe	$4x$	\$11.50	$11.50(4x)$

The sum of the values of the Kona Deluxe beans and the Italian Espresso beans must equal the value of the mixture, so

$$9.00x + 11.50(4x) = 247.50$$
$$90x + 115(4x) = 2475$$
$$90x + 460x = 2475$$
$$550x = 2475$$
$$x = 4.5$$

Since $x = 4.5$, $4x = 18$.
The customer can buy 4.5 pounds of Italian Espresso beans and 18 pounds of Kona Deluxe beans.

37. Use the formula $d = rt$ with $r = 53$ and $t = 10$.

$$d = rt$$
$$= (53)(10)$$
$$= 530$$

The distance between Memphis and Chicago is 530 miles.

38. Use $d = rt$ with $r = 164$ and $t = 2$.

$$d = rt$$
$$= 164(2)$$
$$= 328$$

The distance from Warsaw to Rome is 328 miles.

39. Use $d = rt$ with $d = 500$ and $r = 187.433$.

$$d = rt$$
$$500 = 187.433t$$
$$t = \frac{500}{187.433} \approx 2.668$$

His time was about 2.668 hours.

40. Use $d = rt$ with $d = 400$ and $r = 153.485$.

$$d = rt$$
$$400 = 153.485t$$
$$t = \frac{400}{153.485} \approx 2.606$$

His time was about 2.606 hours.

41. $r = \frac{d}{t} = \frac{200 \text{ meters}}{21.88 \text{ seconds}} \approx 9.14$

Her rate was about 9.14 meters per second.

42. $r = \frac{d}{t} = \frac{400 \text{ meters}}{49.55 \text{ seconds}} \approx 8.07$

Her rate was about 8.07 meters per second.

43. $r = \frac{d}{t} = \frac{110 \text{ meters}}{12.92 \text{ seconds}} \approx 8.51$

His rate was about 8.51 meters per second.

44. $r = \frac{d}{t} = \frac{200 \text{ meters}}{19.32 \text{ seconds}} \approx 10.35$

His rate was about 10.35 meters per second.

45. Let t = the number of hours Marco and Celeste traveled.

Make a chart using the formula $d = rt$.

	r	***t***	***d***
Marco	10	t	$10t$
Celeste	12	t	$12t$

Marco's distance minus Celeste's distance is 15.

$$12t - 10t = 15$$
$$2t = 15$$
$$t = \frac{15}{2} \quad \text{or} \quad 7\frac{1}{2}$$

They will be 15 miles apart in $7\frac{1}{2}$ hours.

46. Let t = the number of hours until the steamboats will be 9 miles apart.

Make a chart using the formula $d = rt$.

	r	***t***	***d***
Slower Boat	18	t	$18t$
Faster Boat	24	t	$24t$

Distance traveled by faster boat minus distance traveled by slower boat is 9.

$$24t - 18t = 9$$
$$6t = 9$$
$$t = \frac{3}{2} \quad \text{or} \quad 1\frac{1}{2}$$

In $1\frac{1}{2}$ hours, the steamboats will be 9 miles apart.

47. Let t = the number of hours until John and Pat meet.

The distance John travels and the distance Pat travels total 440 miles.

John's distance and Pat's distance is total distance.

$$60t + 28t = 440$$
$$88t = 440$$
$$t = 5$$

It will take 5 hours for them to meet.

48. Let t = the time each plane travels.

Use the chart in the text to help write the equation.

Distance of plane leaving Portland plus distance of plane leaving St. Louis is distance between Portland and St. Louis.

$$90t + 116t = 2060$$

Solve the equation.

$$90t + 116t = 2060$$
$$206t = 2060$$
$$t = \frac{2060}{206}$$
$$t = 10$$

It will take the planes 10 hours to meet.

49. Let t = the number of hours until the trains are 315 kilometers apart.

Distance of northbound train	plus	distance of southbound train	is	total distance.
↓	↓	↓	↓	↓
$85t$	+	$95t$	=	315

Solve the equation.

$$85t + 95t = 315$$
$$180t = 315$$
$$t = \frac{315}{180} = \frac{7}{4}$$

It will take $1\frac{3}{4}$ hours for the trains to be 315 kilometers apart.

50. Let t = the number of hours until the steamers are 110 miles apart.
Each steamer will travel $22t$ miles and the total distance traveled will be 110 miles.

$$22t + 22t = 110$$
$$44t = 110$$
$$t = \frac{110}{44} = \frac{5}{2}$$

It will take $2\frac{1}{2}$ hours for the steamers to be 110 miles apart.

51. Let x = the rate of the westbound plane.
Then $x - 150$ = the rate of the eastbound plane.
Using the formula $d = rt$ and the chart in the text, we see that

$$d_{\text{west}} + d_{\text{east}} = d_{\text{total}}$$
$$x(3) + (x - 150)(3) = 2250$$
$$3x + 3x - 450 = 2250$$
$$6x = 2700$$
$$x = 450$$

Since $x = 450,\ x - 150 = 300.$
The rate of the westbound plane is 450 mph and the rate of the eastbound plane is 300 mph.

52. Let x = the rate of the northbound train.
Then $x + 20$ = the rate of the southbound train.
Using the formula $d = rt$ and the chart in the text, we see that

$$d_{\text{north}} + d_{\text{south}} = d_{\text{total}}$$
$$x(2) + (x + 20)(2) = 280$$
$$2x + 2x + 40 = 280$$
$$4x = 240$$
$$x = 60$$

Since $x = 60,\ x + 20 = 80.$
The rate of the northbound train is 60 mph and the rate of the southbound train is 80 mph.

53. Let x = the rate of the slower car.
Then $x + 20$ = the rate of the faster car.
Use the formula $d = rt$ and the fact that each car travels for 4 hours.

$$d_{\text{faster}} + d_{\text{slower}} = d_{\text{total}}$$
$$(x + 20)(4) + (x)(4) = 400$$
$$4x + 80 + 4x = 400$$
$$8x = 320$$
$$x = 40$$

The rate of the slower car is 40 mph and the rate of the faster car is 60 mph.

54. Let x = the rate of the faster car.
Then $x - 15$ = the rate of the slower car.
Use the formula $d = rt$ and the fact that each car travels for 2 hours.

$$d_{\text{faster}} + d_{\text{slower}} = d_{\text{total}}$$
$$x(2) + (x - 15)(2) = 230$$
$$2x + 2x - 30 = 230$$
$$4x = 260$$
$$x = 65$$

Since $x = 65,\ x - 15 = 50.$
The rate of the faster car is 65 km per hour and the rate of the slower car is 50 km per hour.

55. Let x = Bob's current age.
Then $3x$ = Kevin's current age.
Three years ago, Bob's age was $x - 3$ and Kevin's age was $3x - 3,$ and this sum was 22.

$$(x - 3) + (3x - 3) = 22$$
$$4x - 6 = 22$$
$$4x = 28$$
$$x = 7$$

Bob is 7 years old and Kevin is $3(7) = 21$ years old.

56. Let x = the number of pint cartons.
Then $6x$ = the number of quart cartons.

Since 1 quart = 2 pints, 1 pint $= \frac{1}{2}$ quart, and $\frac{1}{2}x$ is the number of quarts contained in pint cartons. The total number of quarts is 39, so

$$\frac{1}{2}x + 6x = 39.$$

$$\frac{1}{2}x + \frac{12}{2}x = 39$$

$$\frac{13}{2}x = 39$$

$$\frac{2}{13}\left(\frac{13}{2}x\right) = \frac{2}{13}(39)$$

$$x = 6$$

There are 6 pint cartons and $6(6) = 36$ quart cartons.

57. Let w = the width of the table.
Then $3w$ = the length of the table.
If we subtract 3 feet from the length $(3w - 3)$ and add 3 feet to the width $(w + 3)$, then the length and the width would be equal.

$$3w - 3 = w + 3$$

$$3w = w + 6$$

$$2w = 6$$

$$w = 3$$

The width is 3 feet and the length is $3(3) = 9$ feet.

58. Let x = the number of hours worked.
Her gross pay (pay before deductions) is $8x$.
gross pay − deduction = take-home pay

$$8x - 0.25(8x) = 450$$

$$8x - 2x = 450$$

$$6x = 450$$

$$x = \frac{450}{6} = 75$$

She must work 75 hours to take home \$450.

59. Let x = her gross pay (pay before deductions).
gross pay − deduction = take-home pay

$$x - 0.10(x) = 585$$

$$0.90x = 585$$

$$x = \frac{585}{0.90} = 650$$

She was paid \$650 before deductions.

60. Let x = the amount of the sales.
sales + tax = total

$$x + 0.05x = 2394$$

$$1.05x = 2394$$

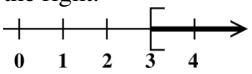
$$x = \frac{2394}{1.05} = 2280$$

The amount of sales was \$2280.

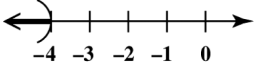
2.8 Solving Linear Inequalities

Classroom Examples, Now Try Exercises

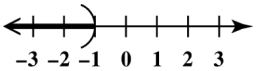
1. **(a)** The statement $x \geq 3$ says that x can represent any number greater than or equal to 3. The interval is written as $[3, \infty)$ (the parenthesis at ∞ shows that ∞ is *not* part of the graph). To graph the inequality, place a bracket at 3 and draw an arrow extending to the right.



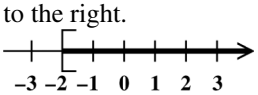
(b) The statement $-4 > x$ is the same as $x < -4$. The interval is written as $(-\infty, -4)$. Graph this inequality by placing a parenthesis at -4 on a number line and drawing an arrow to the left.



N1. **(a)** The statement $x < -1$ says that x can represent any number less than -1. The interval is written as $(-\infty, -1)$. Graph this inequality by placing a parenthesis at -1 on a number line and drawing an arrow to the left.



(b) The statement $-2 \leq x$ is the same as $x \geq -2$. The interval is written as $[-2, \infty)$. Graph this inequality by placing a bracket at -2 on a number line and drawing an arrow to the right.



2. $-1+8x<7x+2$

$-1+8x+1<7x+2+1$ Add 1.

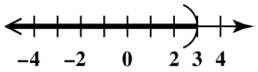
$8x<7x+3$

$8x-7x<7x+3-7x$ Subtract $7x$.

$x<3$

Graph the solution set $(-\infty, 3)$.

To graph this inequality, place a parenthesis at 3 on a number line and draw an arrow to the left.



N2. $5+5x\geq 4x+3$

$5+5x-5\geq 4x+3-5$ Subtract 5.

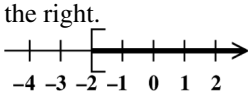
$5x\geq 4x-2$

$5x-4x\geq 4x-2-4x$ Subtract $4x$.

$x\geq -2$

Graph the solution set $[-2, \infty)$.

To graph this inequality, place a bracket at -2 on a number line and draw an arrow to the right.

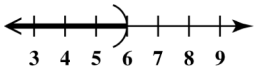


3. $-2r>-12$

$\frac{-2r}{-2}<\frac{-12}{-2}$ Divide by -2.

$r<6$

Graph the solution set $(-\infty, 6)$.

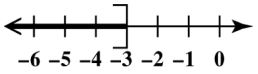


N3. $-5k\geq 15$

$\frac{-5k}{-5}\leq\frac{15}{-5}$ Divide by -5.

$k\leq -3$

Graph the solution set $(-\infty, -3]$.



4. $5x-x+2<7x-5$

$4x+2<7x-5$

$4x+2-7x<7x-5-7x$ Subtract $7x$.

$-3x+2<-5$

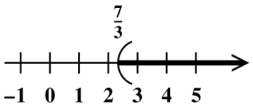
$-3x+2-2<-5-2$ Subtract 2.

$-3x<-7$

$\frac{-3x}{-3}>\frac{-7}{-3}$ Divide by -3.

$x>\frac{7}{3}$

Graph the solution set $\left(\frac{7}{3}, \infty\right)$.



N4. $6-2t+5t\leq 8t-4$

$3t+6\leq 8t-4$

$3t+6-8t\leq 8t-4-8t$ Subtract $8t$.

$-5t+6\leq -4$

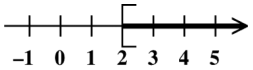
$-5t+6-6\leq -4-6$ Subtract 6.

$-5t\leq -10$

$\frac{-5t}{-5}\geq\frac{-10}{-5}$ Divide by -5.

$t\geq 2$

Graph the solution set $[2, \infty)$.



5. $4(x-1)-3x>-15-(2x+1)$

$4x-4-3x>-15-2x-1$ Dist. Prop.

$x-4>-16-2x$

$x-4+2x>-16-2x+2x$ Add $2x$.

$3x-4>-16$

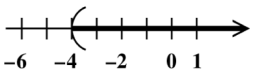
$3x-4+4>-16+4$ Add 4.

$3x>-12$

$\frac{3x}{3}>\frac{-12}{3}$ Divide by 3.

$x>-4$

Graph the solution set $(-4, \infty)$.



N5. $2x-3(x-6)<4(x+7)$

$2x-3x+18<4x+28$ Dist. prop.

$-x+18<4x+28$

$-x+18+x<4x+28+x$ Add x.

$18<5x+28$

$18-28<5x+28-28$ Subtract 28.

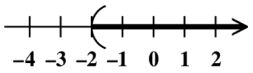
$-10<5x$

$\frac{-10}{5}<\frac{5x}{5}$ Divide by 5.

$-2<x$

$x>-2$

Graph the solution set $(-2,\infty)$.



6. $\frac{1}{2}(3x-1)>\frac{1}{5}(x+4)$

$5(3x-1)>2(x+4)$ Multiply by 10.

$15x-5>2x+8$ Dist. prop.

$15x-5-2x>2x+8-2x$ Subtract $2x$.

$13x-5>8$

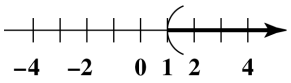
$13x-5+5>8+5$ Add 5.

$13x>13$

$\frac{13x}{13}>\frac{13}{13}$ Divide by 13.

$x>1$

Graph the solution set $(1,\infty)$.



N6. $\frac{1}{8}(x+4)\ge\frac{1}{6}(2x+8)$

$3(x+4)\ge 4(2x+8)$ Multiply by 24.

$3x+12\ge 8x+32$ Dist. prop.

$3x+12-3x\ge 8x+32-3x$ Subtract $3x$.

$12\ge 5x+32$

$12-32\ge 5x+32-32$ Subtract 32.

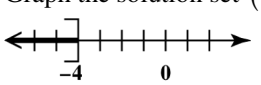
$-20\ge 5x$

$\frac{-20}{5}\ge\frac{5x}{5}$ Divide by 5.

$-4\ge x$

$x\le -4$

Graph the solution set $(-\infty,-4]$.



7. Let x = Maggie's score on the fourth test.

The average is at least 90.

$\frac{98+86+88+x}{4}\ge 90$

Solve the inequality. Combine like terms in the numerator, and multiply by 4 to eliminate the fraction.

$4\left(\frac{272+x}{4}\right)\ge 4(90)$

$272+x\ge 360$

$272+x-272\ge 360-272$ Subtract 272.

$x\ge 88$ Combine terms.

She must score 88 or more on the fourth test to have an average of *at least* 90.

N7. Let x = Will's score on the third test.

The average is at least 90.

$\frac{98+85+x}{3}\ge 90$

Solve the inequality. Combine like terms in the numerator, and multiply by 3 to eliminate the fraction.

$3\left(\frac{183+x}{3}\right)\ge 3(90)$

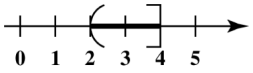
$183+x\ge 270$

$183+x-183\ge 270-183$ Subtract 183.

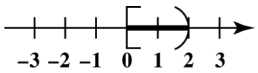
$x\ge 87$ Combine terms.

He must score 87 or more on the third test to have an average of *at least* 90.

8. The statement $2<x\le 4$ says that x can represent any number between 2 and 4, excluding 2 and including 4. To graph the inequality, place a parenthesis at 2 and a bracket at 4 and draw a line segment between them. The interval is written as $(2, 4]$.



N8. The statement $0\le x<2$ says that x can represent any number between 0 and 2, including 0 and excluding 2. To graph the inequality, place a bracket at 0 and a parenthesis at 2 and draw a line segment between them. The interval is written as $[0, 2)$.



9. $2 \le 3x-1 \le 8$

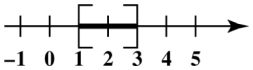
$2+1 \le 3x-1+1 \le 8+1$ Add 1.

$3 \le 3x \le 9$

$\frac{3}{3} \le \frac{3x}{3} \le \frac{9}{3}$ Divide by 3.

$1 \le x \le 3$

Graph the solution set $[1, 3]$.



N9. $-4 \le \frac{3}{2}x-1 \le 0$

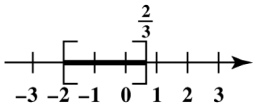
$-4+1 \le \frac{3}{2}x-1+1 \le 0+1$ Add 1.

$-3 \le \frac{3}{2}x \le 1$

$\frac{2}{3}(-3) \le \frac{2}{3}\left(\frac{3}{2}x\right) \le \frac{2}{3}(1)$ Multiply by $\frac{2}{3}$.

$-2 \le x \le \frac{2}{3}$

Graph the solution set $\left[-2, \frac{2}{3}\right]$.



Exercises

1. When graphing an inequality, use a parenthesis if the inequality symbol is $<$ or $>$. Use a square bracket if the inequality symbol is $\leqq$ or $\geqq$.
Examples:
A parenthesis would be used for the inequalities $x < 2$ and $x > 3$. A square bracket would be used for the inequalities $x \le 2$ and $x \ge 3$.

2. *False.* A parenthesis is always used with the symbols $-\infty$ and ∞.

3. In interval notation, the set $\{x \mid x > 0\}$ is written $(0, \infty)$.

4. In interval notation, the set of all real numbers is $(-\infty, \infty)$.

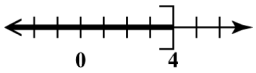
5. The set of numbers graphed corresponds to the inequality $x > -4$.

6. The set of numbers graphed corresponds to the inequality $x \ge -4$.

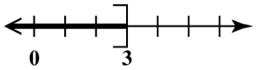
7. The set of numbers graphed corresponds to the inequality $x \le 4$.

8. The set of numbers graphed corresponds to the inequality $x < 4$.

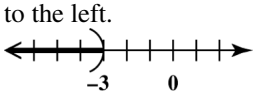
9. The statement $z \le 4$ says that z can represent any number less than or equal to 4. The interval is written as $(-\infty, 4]$. To graph the inequality, place a square bracket at 4 (to show that 4 is part of the graph) and draw an arrow extending to the left.



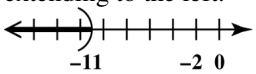
10. The statement $x \le 3$ can be written as $(-\infty, 3]$.



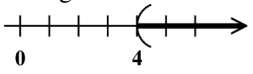
11. The statement $x < -3$ says that x can represent any number less than -3. The interval is written as $(-\infty, -3)$. To graph the inequality, place a parenthesis at -3 (to show that -3 is *not* part of the graph) and draw an arrow extending to the left.



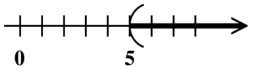
12. The statement $r < -11$ says that r can represent any number less than -11. The interval is written $(-\infty, -11)$. To graph the inequality, place a parenthesis at -11 (to show that -11 is *not* part of the graph) and draw an arrow extending to the left.



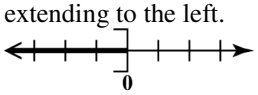
13. The statement $t > 4$ says that t can represent any number greater than 4. The interval is written $(4, \infty)$. To graph the inequality, place a parenthesis at 4 (to show that 4 is *not* part of the graph) and draw an arrow extending to the right.



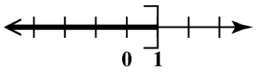
14. The statement $m > 5$ can be written as $(5, \infty)$.



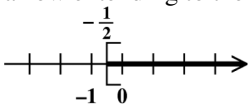
15. The statement $0 \geq x$ (or $x \leq 0$) says that x can represent any number less than or equal to 0. The interval is written as $(-\infty, 0]$. To graph the inequality, place a bracket at 0 (to show that 0 is part of the graph) and draw an arrow extending to the left.



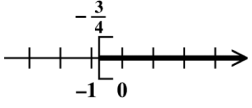
16. The statement $1 \geq x$ (or $x \leq 1$) says that x can represent any number less than or equal to 1. The interval is written as $(-\infty, 1]$. To graph the inequality, place a bracket at 1 (to show that 1 is part of the graph) and draw an arrow extending to the left.



17. The statement $-\frac{1}{2} \leq x$ $\left(\text{or } x \geq -\frac{1}{2}\right)$ says that x can represent any number greater than or equal to $-\frac{1}{2}$. The interval is written $\left[-\frac{1}{2}, \infty\right)$. To graph the inequality, place a bracket at $-\frac{1}{2}$ (to show that $-\frac{1}{2}$ is part of the graph) and draw an arrow extending to the right.



18. The statement $-\frac{3}{4} \leq x$ $\left(\text{or } x \geq -\frac{3}{4}\right)$ says that x can represent any number greater than or equal to $-\frac{3}{4}$. The interval is written $\left[-\frac{3}{4}, \infty\right)$. To graph the inequality, place a bracket at $-\frac{3}{4}$ (to show that $-\frac{3}{4}$ is part of the graph) and draw an arrow extending to the right.

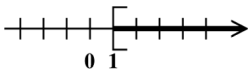


19. $z - 8 \geq -7$

$z - 8 + 8 \geq -7 + 8$ Add 8.

$z \geq 1$

Graph the solution set $[1, \infty)$.

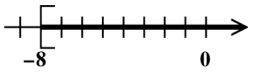


20. $p - 3 \geq -11$

$p - 3 + 3 \geq -11 + 3$ Add 3.

$p \geq -8$

Graph the solution set $[-8, \infty)$.



21. $2k + 3 \geq k + 8$

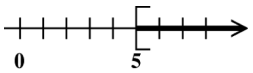
$2k + 3 - k \geq k + 8 - k$ Subtract k.

$k + 3 \geq 8$

$k + 3 - 3 \geq 8 - 3$ Subtract 3.

$k \geq 5$

Graph the solution set $[5, \infty)$.



22. $3x + 7 \geq 2x + 11$

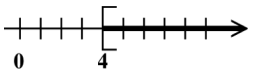
$3x + 7 - 2x \geq 2x + 11 - 2x$ Subtract $2x$.

$x + 7 \geq 11$

$x + 7 - 7 \geq 11 - 7$ Subtract 7.

$x \geq 4$

Graph the solution set $[4, \infty)$.



23. $3n + 5 < 2n - 6$

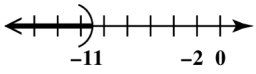
$3n - 2n + 5 < 2n - 2n - 6$ Subtract $2n$.

$n + 5 < -6$

$n + 5 - 5 < -6 - 5$ Subtract 5.

$n < -11$

Graph the solution set $(-\infty, -11)$.



24. $5x - 2 < 4x - 5$

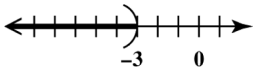
$5x - 2 - 4x < 4x - 5 - 4x$ Subtract $4x$.

$x - 2 < -5$

$x - 2 + 2 < -5 + 2$ Add 2.

$x < -3$

Graph the solution set $(-\infty, -3)$.



25. The inequality symbol must be reversed when one is multiplying or dividing by a negative number.

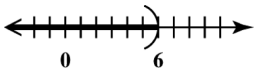
26. For choice A, the second part of the multiplication property of inequality states that when both sides of an inequality are multiplied by a negative number, the inequality symbol is reversed. Since r is negative, this statement is false.
Look at choice B. Both sides of the inequality $p < q$ have been multiplied by the negative number r and the inequality symbol has been reversed, so this statement is true.
Look at choice C. By the addition property of inequality, this statement is true. When the same number is added to both sides of an inequality, the inequality symbol is not reversed.
Look at choice D. When the same number is subtracted from both sides of an inequality, the inequality symbol is not reversed. The statement is true.
Therefore, only statement A is false.

27. $3x < 18$

$\frac{3x}{3} < \frac{18}{3}$ Divide by 3.

$x < 6$

Graph the solution set $(-\infty, 6)$.

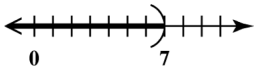


28. $5x < 35$

$\frac{5x}{5} < \frac{35}{5}$ Divide by 5.

$x < 7$

Graph the solution set $(-\infty, 7)$.

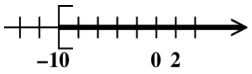


29. $2y \geq -20$

$\frac{2y}{2} \geq \frac{-20}{2}$ Divide by 2.

$y \geq -10$

Graph the solution set $[-10, \infty)$.

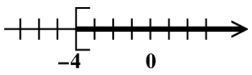


30. $6m \geq -24$

$\frac{6m}{6} \geq \frac{-24}{6}$ Divide by 6.

$m \geq -4$

Graph the solution set $[-4, \infty)$.

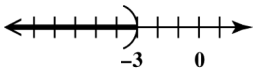


31. $-8t > 24$

$\frac{-8t}{-8} < \frac{24}{-8}$ Divide by -8.

$t < -3$

Graph the solution set $(-\infty, -3)$.

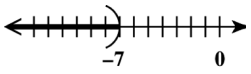


32. $-7x > 49$

$\frac{-7x}{7} < \frac{49}{-7}$ Divide by -7.

$x < -7$

Graph the solution set $(-\infty, -7)$.



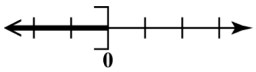
33. $-x \geq 0$

$-1x \geq 0$

$\frac{-1x}{-1} \leq \frac{0}{-1}$ Divide by -1.

$x \leq 0$

Graph the solution set $(-\infty, 0]$.



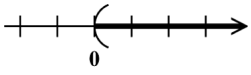
34. $-k < 0$

$-1k < 0$

$\frac{-1k}{-1} > \frac{0}{-1}$ Divide by -1.

$k > 0$

Graph the solution set $(0, \infty)$.



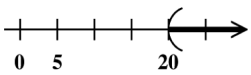
35. Multiply by $-\frac{4}{3}$, the reciprocal of $-\frac{3}{4}$; reverse the inequality sign.

$$-\frac{3}{4}r < -15$$

$$\left(-\frac{4}{3}\right)\left(-\frac{3}{4}r\right) > \left(-\frac{4}{3}\right)(-15)$$

$$r > 20$$

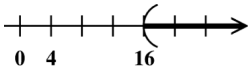
Graph the solution set $(20, \infty)$.



36. Multiply by $-\frac{8}{7}$, the reciprocal of $-\frac{7}{8}$, and reverse the inequality symbol.

$$-\frac{7}{8}t < -14$$
$$\left(-\frac{8}{7}\right)\left(-\frac{7}{8}t\right) > \left(-\frac{8}{7}\right)(-14)$$
$$t > 16$$

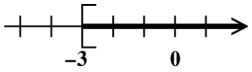
Graph the solution set $(16, \infty)$.



37. $-0.02x \le 0.06$

$$\frac{-0.02x}{-0.02} \ge \frac{0.06}{-0.02} \quad \text{Divide by } -0.02.$$
$$x \ge -3$$

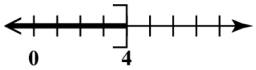
Graph the solution set $[-3, \infty)$.



38. $-0.03v \ge -0.12$

$$\frac{-0.03v}{-0.03} \le \frac{-0.12}{-0.03} \quad \text{Divide by } -0.03.$$
$$v \le 4$$

Graph the solution set $(-\infty, 4]$.



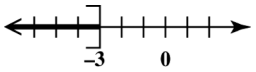
39. $8x + 9 \le -15$

$8x \le -24$ Subtract 9.

$\frac{8x}{8} \le \frac{-24}{8}$ Divide by 8.

$x \le -3$

Graph the solution set $(-\infty, -3]$.



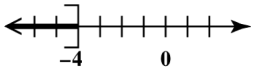
40. $6x + 7 \le -17$

$6x \le -24$ Subtract 7.

$\frac{6x}{6} \le \frac{-24}{6}$ Divide by 6.

$x \le -4$

Graph the solution set $(-\infty, -4]$.



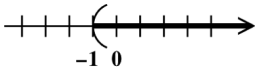
41. $-4x - 3 < 1$

$-4x < 4$ Add 3.

$\frac{-4x}{-4} > \frac{4}{-4}$ Divide by -4.

$x > -1$

Graph the solution set $(-1, \infty)$.



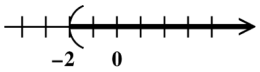
42. $-5x - 4 < 6$

$-5x < 10$ Add 4.

$\frac{-5x}{-5} > \frac{10}{-5}$ Divide by -5.

$x > -2$

Graph the solution set $(-2, \infty)$.



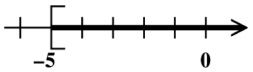
43. $5r + 1 \ge 3r - 9$

$2r + 1 \ge -9$ Subtract $3r$.

$2r \ge -10$ Subtract 1.

$r \ge -5$ Divide by 2.

Graph the solution set $[-5, \infty)$.



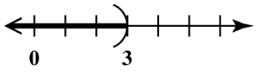
44. $6t + 3 < 3t + 12$

$3t + 3 < 12$ Subtract $3t$.

$3t < 9$ Subtract 3.

$r < 3$ Divide by 3.

Graph the solution set $(-\infty, 3)$.



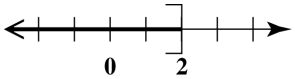
45. $5x - 2 \le -x + 10$

$6x - 2 \le 10$ Add x.

$6x \le 12$ Add 2.

$x \le 2$ Divide by 6.

Graph the solution set $(-\infty, 2]$.



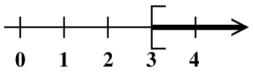
46. $3x-9\ge -2x+6$

$5x-9\ge 6$ Add $2x$.

$5x\ge 15$ Add 9.

$x\ge 3$ Divide by 5.

Graph the solution set $[3,\infty)$.



47. $-7x+4>-3x-2$

$4>4x-2$ Add $7x$.

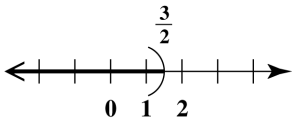
$6>4x$ Add 2.

$\frac{6}{4}>x$ Divide by 4.

$\frac{3}{2}>x$

$x<\frac{3}{2}$

Graph the solution set $\left(-\infty,\frac{3}{2}\right)$.



48. $-8x+1<-4x+11$

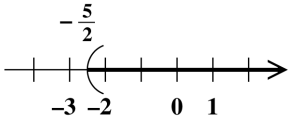
$1<4x+11$ Add $8x$.

$-10<4x$ Subtract 11.

$-\frac{10}{4}<x$ Divide by 4.

$x>-\frac{5}{2}$

Graph the solution set $\left(-\frac{5}{2},\infty\right)$.



49. $6x+3+x<2+4x+4$

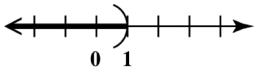
$7x+3<4x+6$ Combine like terms.

$3x+3<6$ Subtract $4x$.

$3x<3$ Subtract 3.

$x<1$ Divide by 3.

Graph the solution set $(-\infty,1)$.



50. $-4w+12+9w\ge w+9+w$

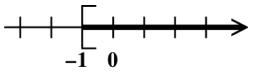
$5w+12\ge 2w+9$ Combine terms.

$3w+12\ge 9$ Subtract $2w$.

$3w\ge -3$ Subtract 12.

$w\ge -1$ Divide by 3.

Graph the solution set $[-1,\infty)$.



51. $-x+4+7x\le -2+3x+6$

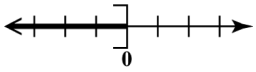
$6x+4\le 4+3x$

$3x+4\le 4$

$3x\le 0$

$x\le 0$

Graph the solution set $(-\infty,0]$.



52. $14x-6+7x>4+10x-10$

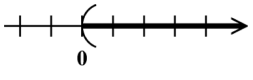
$21x-6>10x-6$

$11x-6>-6$

$11x>0$

$x>0$

Graph the solution set $(0,\infty)$.



53. $5(t-1)>3(t-2)$

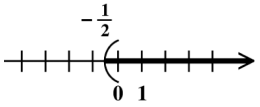
$5t-5>3t-6$

$2t-5>-6$

$2t>-1$

$t>-\frac{1}{2}$

Graph the solution set $\left(-\frac{1}{2},\infty\right)$.



54. $7(m-2) < 4(m-4)$

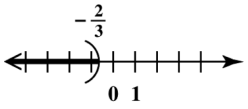
$7m-14 < 4m-16$

$3m-14 < -16$

$3m < -2$

$m < -\frac{2}{3}$

Graph the solution set $\left(-\infty, -\frac{2}{3}\right)$.



55. $5(x+3)-6x \le 3(2x+1)-4x$

$5x+15-6x \le 6x+3-4x$

$-x+15 \le 2x+3$

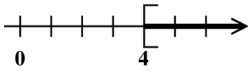
$-3x+15 \le 3$

$-3x \le -12$

$\frac{-3x}{-3} \ge \frac{-12}{-3}$ Divide by -3.

$x \ge 4$

Graph the solution set $[4, \infty)$.



56. $2(x-5)+3x < 4(x-6)+1$

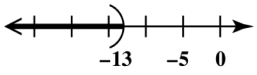
$2x-10+3x < 4x-24+1$

$5x-10 < 4x-23$

$x-10 < -23$

$x < -13$

Graph the solution set $(-\infty, -13)$.



57. $\frac{1}{3}(5x-4) \ge \frac{2}{5}(x+3)$

$15\left(\frac{1}{3}\right)(5x-4) \ge 15\left(\frac{2}{5}\right)(x+3)$

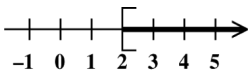
$5(5x-4) \ge 6(x+3)$

$25x-20 \ge 6x+18$

$19x \ge 38$

$x \ge 2$

Graph the solution set $[2, \infty)$.



58. $\frac{5}{12}(5x-7) < \frac{5}{6}(x-5)$

$12\left(\frac{5}{12}\right)(5x-7) < 12\left(\frac{5}{6}\right)(x-5)$

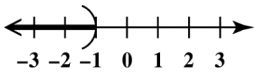
$5(5x-7) < 10(x-5)$

$25x-35 < 10x-50$

$15x < -15$

$x < -1$

Graph the solution set $(-\infty, -1)$.



59. $\frac{2}{3}(p+3) > \frac{5}{6}(p-4)$

$6\left(\frac{2}{3}\right)(p+3) > 6\left(\frac{5}{6}\right)(p-4)$

$4(p+3) > 5(p-4)$

$4p+12 > 5p-20$

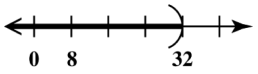
$-p+12 > -20$

$-p > -32$

$\frac{-p}{-1} < \frac{-32}{-1}$ Divide by -1.

$p < 32$

Graph the solution set $(-\infty, 32)$.



60. $\frac{7}{9}(x-4) \le \frac{4}{3}(x+5)$

$9\left(\frac{7}{9}\right)(x-4) \le 9\left(\frac{4}{3}\right)(x+5)$

$7(x-4) \le 12(x+5)$

$7x-28 \le 12x+60$

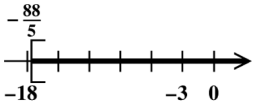
$-5x-28 \le 60$

$-5x \le 88$

$\frac{-5x}{-5} \ge \frac{88}{-5}$ Divide by -5.

$x \ge -\frac{88}{5}$

Graph the solution set $\left[-\frac{88}{5}, \infty\right)$.



61. $\frac{4}{5}x-\frac{1}{2}(x+3)\le\frac{3}{10}$

$10\left[\frac{4}{5}x-\frac{1}{2}(x+3)\right]\le 10\left(\frac{3}{10}\right)$

$8x-5(x+3)\le 3$

$8x-5x-15\le 3$

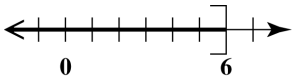
$3x-15\le 3$

$3x\le 18$

$\frac{3x}{3}\le\frac{18}{3}$

$x\le 6$

Graph the solution set $(-\infty, 6]$.



62. $\frac{1}{6}x+\frac{1}{3}(x-1)>\frac{1}{2}$

$6\left[\frac{1}{6}x+\frac{1}{3}(x-1)\right]>6\left(\frac{1}{2}\right)$

$x+2(x-1)>3$

$x+2x-2>3$

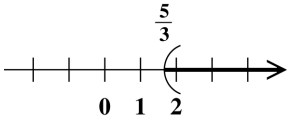
$3x-2>3$

$3x>5$

$\frac{3x}{3}>\frac{5}{3}$

$x>\frac{5}{3}$

Graph the solution set $\left(\frac{5}{3},\infty\right)$.



63. $4x-(6x+1)\le 8x+2(x-3)$

$4x-6x-1\le 8x+2x-6$

$-2x-1\le 10x-6$

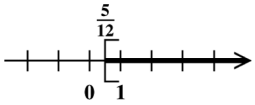
$-12x-1\le -6$

$-12x\le -5$

$\frac{-12x}{-12}\ge\frac{-5}{-12}$ Divide by -12.

$x\ge\frac{5}{12}$

Graph the solution set $\left[\frac{5}{12},\infty\right)$.



64. $2z-(4z+3)>6z+3(z+4)$

$2z-4z-3>6z+3z+12$

$-2z-3>9z+12$

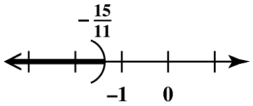
$-11z-3>12$

$-11z>15$

$\frac{-11z}{-11}<\frac{15}{-11}$ Divide by -11.

$z<-\frac{15}{11}$

Graph the solution set $\left(-\infty,-\frac{15}{11}\right)$.



65. $5(2k+3)-2(k-8)>3(2k+4)+k-2$

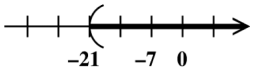
$10k+15-2k+16>6k+12+k-2$

$8k+31>7k+10$

$k+31>10$

$k>-21$

Graph the solution set $(-21,\infty)$.



66. $2(3z-5)+4(z+6)\ge 2(3z+2)+3z-15$

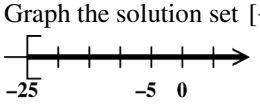
$6z-10+4z+24\ge 6z+4+3z-15$

$10z+14\ge 9z-11$

$z+14\ge -11$

$z\ge -25$

Graph the solution set $[-25,\infty)$.



67. The statement "You must be at least 18 yr old to vote" translates as $x \geq 18$.

68. The statement "Less than 1 in. of rain fell" translates as $x < 1$.

69. The statement "Chicago received more than 5 in. of snow" translates as $x > 5$.

70. The statement "A full-time student must take at least 12 credits" translates as $x \geq 12$.

71. The statement "Tracy could spend at most \$20 on a gift" translates as $x \leq 20$.

72. The statement "The car's rate exceeded 60 mph" translates as $x > 60$.

73. Let x = the score on the third test.

$$\begin{array}{lll} \text{The average of the three tests} & \text{is at least} & 80. \\ \downarrow & \downarrow & \downarrow \\ \dfrac{76+81+x}{3} & \geq & 80 \end{array}$$

Then solve the inequality.

$$\frac{157+x}{3} \geq 80$$

$$3\left(\frac{157+x}{3}\right) \geq 3(80)$$

$$157+x \geq 240$$

$$x \geq 83$$

In order to average at least 80, Christy's score on her third test must be 83 or greater.

74. Let x = the score on the third test.

$$\begin{array}{lll} \text{The average of the three tests} & \text{is at least} & 90. \\ \downarrow & \downarrow & \downarrow \\ \dfrac{96+86+x}{3} & \geq & 90 \end{array}$$

Then solve the inequality.

$$\frac{182+x}{3} \geq 90$$

$$3\left(\frac{182+x}{3}\right) \geq 3(90)$$

$$182+x \geq 270$$

$$x \geq 88$$

In order to average at least 90, Joseph's score on his third test must be 88 or greater.

75. Let x = the amount of precipitation in December.

$$\begin{array}{lll} \text{The average precipitation} & \text{exceeds} & 4.6 \text{ in.} \\ \downarrow & \downarrow & \downarrow \\ \dfrac{5.7+4.3+x}{3} & > & 4.6 \end{array}$$

Then solve the inequality.

$$\frac{10+x}{3} > 4.6$$

$$3\left(\frac{10+x}{3}\right) > 3(4.6)$$

$$10+x > 13.8$$

$$x > 3.8$$

In order for the average monthly precipitation to exceed 4.6 in., more than 3.8 in. must fall in December.

76. Let x = the amount of precipitation in August.

$$\begin{array}{lll} \text{The average precipitation} & \text{exceeds} & 6.7 \text{ in.} \\ \downarrow & \downarrow & \downarrow \\ \dfrac{8.1+5.7+x}{3} & > & 6.7 \end{array}$$

Then solve the inequality.

$$\frac{13.8+x}{3} > 6.7$$

$$3\left(\frac{13.8+x}{3}\right) > 3(6.7)$$

$$13.8+x > 20.1$$

$$x > 6.3$$

In order for the average monthly precipitation to exceed 6.7 in., more than 6.3 in. must fall in August.

77. Let n = the number.

"When 2 is added to the difference between six times a number and 5, the result is greater than 13 added to five times the number" translates to $(6n-5)+2 > 5n+13$.

Solve the inequality.

$$6n-5+2 > 5n+13$$

$$6n-3 > 5n+13$$

$$n-3 > 13 \qquad \text{Subtract } 5n.$$

$$n > 16 \qquad \text{Add } 3.$$

All numbers greater than 16 satisfy the given condition.

78. Let n = the number.
"When 8 is subtracted from the sum of three times a number and 6, the result is less than 4 more than the number" translates to $(3n+6)-8<n+4$.

Solve the inequality.

$$\begin{aligned} 3n+6-8 &< n+4 \\ 3n-2 &< n+4 \\ 2n-2 &< 4 \quad \text{Subtract } n. \\ 2n &< 6 \quad \text{Add 2.} \\ n &< 3 \quad \text{Divide by 2.} \end{aligned}$$

All numbers less than 3 satisfy the given condition.

79. The Fahrenheit temperature must correspond to a Celsius temperature that is greater than or equal to $-25°$.

$$\begin{aligned} C &= \frac{5}{9}(F-32) \ge -25 \\ \frac{9}{5}\left[\frac{5}{9}(F-32)\right] &\ge \frac{9}{5}(-25) \\ F-32 &\ge -45 \\ F &\ge -13 \end{aligned}$$

The temperature in Minneapolis on a certain winter day is never less than $-13°$ Fahrenheit.

80. The Celsius temperature must give a Fahrenheit temperature that is less than or equal to 122°.

$$\begin{aligned} F &= \frac{9}{5}C+32 \le 122 \\ \frac{9}{5}C &\le 90 \\ \frac{5}{9}\left(\frac{9}{5}C\right) &\le \frac{5}{9}(90) \\ C &\le 50 \end{aligned}$$

The temperature of Phoenix has never exceeded $50°$ Celsius.

81. $P = 2L + 2W;\ P \ge 400$
From the figure, we have $L = 4x+3$ and $W = x+37$. Thus, we have the inequality $2(4x+3)+2(x+37) \ge 400$.

Solve this inequality.

$$\begin{aligned} 8x+6+2x+74 &\ge 400 \\ 10x+80 &\ge 400 \\ 10x &\ge 320 \\ x &\ge 32 \end{aligned}$$

The rectangle will have a perimeter of at least 400 if the value of x is 32 or greater.

82. $P = a+b+c;\ P \ge 72$
From the figure, we have $a = x,\ b = x+11,$ and $c = 2x+5$. Thus, we have the inequality $x+(x+11)+(2x+5) \ge 72$.

Solve this inequality.

$$\begin{aligned} 4x+16 &\ge 72 \\ 4x &\ge 56 \\ x &\ge \frac{56}{4} = 14 \end{aligned}$$

The triangle will have a perimeter of at least 72 if the value of x is 14 or greater.

83.

$$\begin{aligned} 2+0.30x &\le 5.60 \\ 10(2+0.30x) &\le 10(5.60) \\ 20+3x &\le 56 \\ 3x &\le 36 \\ x &\le 12 \end{aligned}$$

Alan can use the phone for a maximum of 12 minutes.

84. Let x = the number of gallons.
The amount she spends can be represented by $\$3+\$3.60x$. This must be less than or equal to $48.00.

$$\begin{aligned} 3+3.6x &\le 48 \\ 3.6x &\le 45 \quad \text{Subtract 3.} \\ \frac{3.6x}{3.6} &\le \frac{45}{3.6} \quad \text{Divide by 3.6.} \\ x &\le 12.5 \end{aligned}$$

She can purchase 12.5 gallons of gasoline.

85. "Revenue from the sales of the DVDs is $5 per DVD less sales costs of $100" translates to $R = 5x-100,$ where x represents the number of DVDs to be produced.

86. "Production costs are $125 plus $4 per DVD" translates to $C = 125+4x$.

87.

$$\begin{aligned} P &= R-C \\ &= (5x-100)-(125+4x) \\ &= 5x-100-125-4x \\ &= x-225 \end{aligned}$$

We can use this expression for P to solve the inequality.

$$\begin{aligned} P &> 0 \\ x-225 &> 0 \\ x &> 225 \end{aligned}$$

88. To make a profit, more than 225 DVDs must be produced and sold.

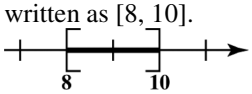
89. The graph corresponds to the inequality $-1 < x < 2$, excluding both -1 and 2.

90. The graph corresponds to the inequality $-1 \le x < 2$.

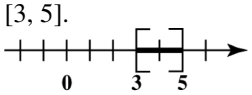
91. The graph corresponds to the inequality $-1 < x \le 2$, excluding -1 but including 2.

92. The graph corresponds to the inequality $-1 \le x \le 2$.

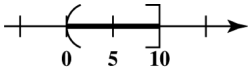
93. The statement $8 \le x \le 10$ says that x can represent any number between 8 and 10, including 8 and 10. To graph the inequality, place brackets at 8 and 10 (to show that 8 and 10 are part of the graph) and draw a line segment between the brackets. The interval is written as [8, 10].



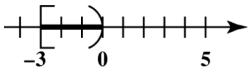
94. The statement $3 \le x \le 5$ can be written as [3, 5].



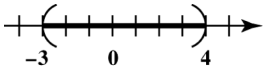
95. The statement $0 < y \le 10$ says that y can represent any number between 0 and 10, excluding 0 and including 10. To graph the inequality, place a parenthesis at 0 and a bracket at 10 and draw a line segment between them. The interval is written as (0, 10].



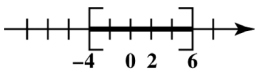
96. The statement $-3 \le x < 0$ can be written as $[-3, 0)$.



97. The statement $4 > x > -3$ can be written as $-3 < x < 4$. Graph the solution set $(-3, 4)$.

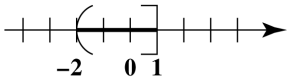


98. The statement $6 \ge x \ge -4$ can be written as $-4 \le x \le 6$. Graph the solution set $[-4, 6]$.



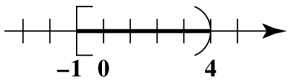
99.
$$\begin{aligned} -8 < 4x &\le 4 \\ \frac{-8}{4} < \frac{4}{4}x &\le \frac{4}{4} \quad \text{Divide by 4.} \\ -2 < x &\le 1 \end{aligned}$$

Graph the solution set $(-2, 1]$.



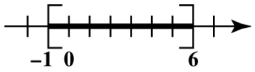
100.
$$\begin{aligned} -3 \le 3x &< 12 \\ \frac{-3}{3} \le \frac{3}{3}x &< \frac{12}{3} \quad \text{Divide by 3.} \\ -1 \le x &< 4 \end{aligned}$$

Graph the solution set $[-1, 4)$.



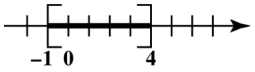
101.
$$\begin{aligned} -5 \le 2x - 3 &\le 9 \\ -5 + 3 \le 2x - 3 + 3 &\le 9 + 3 \quad \text{Add 3.} \\ -2 \le 2x &\le 12 \\ \frac{-2}{2} \le 2x &\le \frac{12}{2} \quad \text{Divide by 2.} \\ -1 \le x &\le 6 \end{aligned}$$

Graph the solution set $[-1, 6]$.



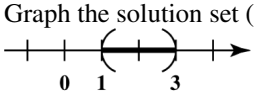
102.
$$\begin{aligned} -7 \le 3x - 4 &\le 8 \\ -7 + 4 \le 3x - 4 + 4 &\le 8 + 4 \quad \text{Add 4.} \\ -3 \le 3x &\le 12 \\ -1 \le x &\le 4 \quad \text{Divide by 3.} \end{aligned}$$

Graph the solution set $[-1, 4]$.



103.
$$\begin{aligned} 10 < 7p + 3 &< 24 \\ 7 < 7p &< 21 \quad \text{Subtract 3.} \\ 1 < p &< 3 \quad \text{Divide by 7.} \end{aligned}$$

Graph the solution set (1, 3).

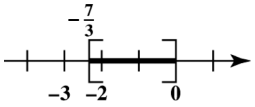


104. $-8 \le 3r-1 \le -1$

$-7 \le 3r \le 0$ Add 1.

$-\frac{7}{3} \le r \le 0$ Divide by 3.

Graph the solution set $\left[-\frac{7}{3}, 0\right]$.



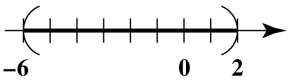
105. $-4 < -2x < 12$

$\frac{-4}{-2} > \frac{-2x}{-2} > \frac{12}{-2}$ Divide by -2.

$2 > x > -6$

$-6 < x < 2$

Graph the solution set $(-6, 2)$.



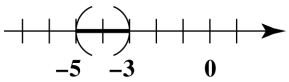
106. $9 < -3x < 15$

$\frac{9}{-3} > \frac{-3x}{-3} > \frac{15}{-3}$ Divide by -3.

$-3 > x > -5$

$-5 < x < -3$

Graph the solution set $(-5, -3)$.



107. $5 < 1-6m < 12$

$5-1 < 1-6m-1 < 12-1$ Subtract 1.

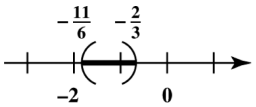
$4 < -6m < 11$

$\frac{4}{-6} > \frac{-6m}{-6} > \frac{11}{-6}$ Divide by -6.

$-\frac{2}{3} > m > -\frac{11}{6}$

$-\frac{11}{6} < m < -\frac{2}{3}$

Graph the solution set $\left(-\frac{11}{6}, -\frac{2}{3}\right)$.



108. $-1 < 1-5q \le 16$

$-1-1 \le 1-5q-1 \le 16-1$ Subtract 1.

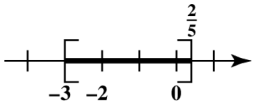
$-2 \le -5q \le 15$

$\frac{-2}{-5} \ge \frac{-5q}{-5} \ge \frac{15}{-5}$ Divide by -5.

$\frac{2}{5} \ge q \ge -3$

$-3 \le q \le \frac{2}{5}$

Graph the solution set $\left[-3, \frac{2}{5}\right]$.



109. $6 \le 3(x-1) < 18$

$6 \le 3x-3 < 18$ Distributive prop.

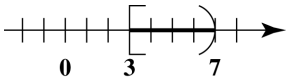
$6+3 \le 3x-3+3 < 18+3$ Add 3.

$9 \le 3x < 21$

$\frac{9}{3} \le \frac{3x}{3} < \frac{21}{3}$ Divide by 3.

$3 \le x < 7$

Graph the solution set $[3, 7)$.



110. $-4 < 2(x+1) \le 6$

$-4 < 2x+2 \le 6$ Distributive prop.

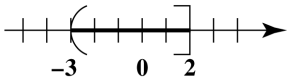
$-4-2 < 2x+2-2 \le 6-2$ Subtract 2.

$-6 < 2x \le 4$

$\frac{-6}{2} < \frac{2x}{2} \le \frac{4}{2}$ Divide by 2.

$-3 < x \le 2$

Graph the solution set $(-3, 2]$.



111. $-12 \le \frac{1}{2}z+1 \le 4$ Multiply by 2.

$2(-12) \le 2\left(\frac{1}{2}z+1\right) \le 2(4)$

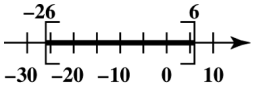
$-24 \le z+2 \le 8$ Dist. prop.

$-24-2 \le z+2-2 \le 8-2$ Subtract 2.

$-26 \le z \le 6$

Note: We could have started this solution by subtracting 1 from each part.

Graph the solution set $[-26, 6]$.



112. $-6 \le \frac{1}{3}x + 3 \le 5$

$-6 - 3 \le \frac{1}{3}x + 3 - 3 \le 5 - 3$ Subtract 3.

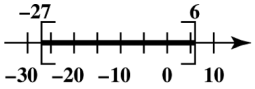
$-9 \le \frac{1}{3}x \le 2$

$3(-9) \le 3\left(\frac{1}{3}x\right) \le 3(2)$ Multiply by 3.

$-27 \le x \le 6$

Note: We could have started this solution by multiplying each part by 3.

Graph the solution set $[-27, 6]$.



113. $1 \le 3 + \frac{2}{3}p \le 7$

$3(1) \le 3\left(3 + \frac{2}{3}p\right) \le 3(7)$ Multiply by 3.

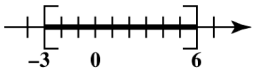
$3 \le 9 + 2p \le 21$ Dist. prop.

$3 - 9 \le 9 + 2p - 9 \le 21 - 9$ Subtract 9.

$-6 \le 2p \le 12$ Divide by 2.

$-3 \le p \le 6$

Graph the solution set $[-3, 6]$.



114. $2 < 6 + \frac{3}{4}x < 12$

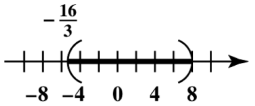
$4(2) < 4\left(6 + \frac{3}{4}x\right) < 4(12)$ Multiply by 4.

$8 < 24 + 3x < 48$ Distributive prop.

$-16 < 3x < 24$ Subtract 24.

$-\frac{16}{3} < x < 8$ Divide by 3.

Graph the solution set $\left(-\frac{16}{3}, 8\right)$.



115. $-7 \le \frac{5}{4}r - 1 \le -1$

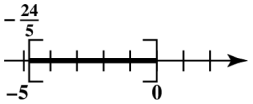
$-7 + 1 \le \frac{5}{4}r - 1 + 1 \le -1 + 1$ Add 1.

$-6 \le \frac{5}{4}r \le 0$

$\frac{4}{5}(-6) \le \frac{4}{5}\left(\frac{5}{4}r\right) \le \frac{4}{5}(0)$ Multiply by $\frac{4}{5}$.

$-\frac{24}{5} \le r \le 0$

Graph the solution set $\left[-\frac{24}{5}, 0\right]$.



116. $-12 \le \frac{3}{7}x + 2 \le -4$

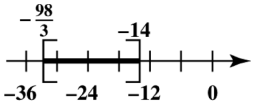
$7(-12) \le 7\left(\frac{3}{7}x + 2\right) \le 7(-4)$ Multiply by 7.

$-84 \le 3x + 14 \le -28$ Dist. prop.

$-98 \le 3x \le -42$ Subtract 14.

$-\frac{98}{3} \le x \le -14$ Divide by 3.

To graph the solution set $\left[-\frac{98}{3}, -14\right]$, note that $-\frac{98}{3} = -32\frac{2}{3}$.

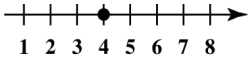


117. $3x + 2 = 14$

$3x = 12$

$x = 4$

Solution set: $\{4\}$

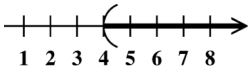


118. $3x + 2 > 14$

$3x > 12$

$x > 4$

Solution set: $(4, \infty)$

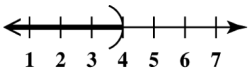


119. $3x+2<14$

$3x<12$

$x<4$

Solution set: $(-\infty, 4)$



120. If you were to graph all the solutions from Exercises 117–120 on the same number line, the graph would be the complete number line, that is, all real numbers.

Chapter 2 Review Exercises

1. $x-5=1$

$x=6$ Add 5.

The solution set is $\{6\}$.

2. $x+8=-4$

$x=-12$ Subtract 8.

The solution set is $\{-12\}$.

3. $3t+1=2t+8$

$t+1=8$ Subtract $2t$.

$t=7$ Subtract 1.

The solution set is $\{7\}$.

4. $5z=4z+\frac{2}{3}$

$z=\frac{2}{3}$ Subtract $4z$.

The solution set is $\left\{\frac{2}{3}\right\}$.

5. $(4r-2)-(3r+1)=8$

$(4r-2)-1(3r+1)=8$

$4r-2-3r-1=8$ Distributive property

$r-3=8$

$r=11$ Add 3.

The solution set is $\{11\}$.

6. $3(2x-5)=2+5x$

$6x-15=2+5x$ Distributive property

$x-15=2$ Subtract $5x$.

$x=17$ Add 15.

The solution set is $\{17\}$.

7. $7x=35$

$x=5$ Divide by 7.

The solution set is $\{5\}$.

8. $12r=-48$

$r=-4$ Divide by 12.

The solution set is $\{-4\}$.

9. $2p-7p+8p=15$

$3p=15$

$p=5$ Divide by 3.

The solution set is $\{5\}$.

10. $\frac{x}{12}=-1$

$x=-12$ Multiply by 12.

The solution set is $\{-12\}$.

11. $\frac{5}{8}q=8$

$\frac{8}{5}\left(\frac{5}{8}q\right)=\frac{8}{5}(8)$ Multiply by $\frac{8}{5}$.

$q=\frac{64}{5}$

The solution set is $\left\{\frac{64}{5}\right\}$.

12. $12m+11=59$

$12m=48$ Subtract 11.

$m=4$ Divide by 12.

The solution set is $\{4\}$.

13. $3(2x+6)-5(x+8)=x-22$

$6x+18-5x-40=x-22$

$x-22=x-22$

This is a true statement, so the solution set is {all real numbers}.

14. $5x+9-(2x-3)=2x-7$

$5x+9-2x+3=2x-7$

$3x+12=2x-7$

$x+12=-7$

$x=-19$

The solution set is $\{-19\}$.

15. $\frac{1}{2}r-\frac{r}{3}=\frac{r}{6}$

$6\left(\frac{1}{2}r\right)-6\left(\frac{r}{3}\right)=6\left(\frac{r}{6}\right)$ Multiply by 6.

$3r-2r=r$

$r=r$

This is a true statement, so the solution set is {all real numbers}.

16. Multiply by 10 to clear decimals.

$$10[0.1(x+80)+0.2x]=10(14)$$
$$(x+80)+2x=140 \quad \text{Dist. prop.}$$
$$3x+80=140$$
$$3x=60$$
$$x=20$$

The solution set is $\{20\}$.

17.
$$3x-(-2x+6)=4(x-4)+x$$
$$3x+2x-6=4x-16+x$$
$$5x-6=5x-16$$
$$-6=-16$$

This statement is false, so there is no solution set, symbolized by $\varnothing$.

18. Multiply both sides by 6 to eliminate fractions.

$$6\left[\frac{1}{2}(x+3)-\frac{2}{3}(x-2)\right]=6(3)$$
$$6\left[\frac{1}{2}(x+3)\right]-6\left[\frac{2}{3}(x-2)\right]=18$$
$$3(x+3)-4(x-2)=18$$
$$3x+9-4x+8=18$$
$$-x+17=18$$
$$-x=1$$
$$x=-1$$

The solution set is $\{-1\}$.

19. Let x represent the number.

$$5x+7=3x$$
$$7=-2x \quad \text{Subtract } 5x.$$
$$-\frac{7}{2}=x \quad \text{Divide by } -2.$$

The number is $-\frac{7}{2}$.

20. *Step 2*

Let x = the number of Republicans.

Then $x+24$ = the number of Democrats.

Step 3

$$x+(x+24)=118$$

Step 4

$$2x+24=118$$
$$2x=94$$
$$x=47$$

Step 5

Since $x=47$, $x+24=71$.

There were 71 Democrats and 47 Republicans.

Step 6

There are 24 more Democrats than Republicans and the total is 118.

21. *Step 2*

Let x = the land area of Rhode Island.

Then $x+5213$ = the land area of Hawaii.

Step 3

The areas total 7637 square miles, so

$$x+(x+5213)=7637.$$

Step 4

$$2x+5213=7637$$
$$2x=2424$$
$$x=1212$$

Step5

Since $x=1212$, $x+5213=6425$.

The land area of Rhode Island is 1212 square miles and that of Hawaii is 6425 square miles.

Step 6

The land area of Hawaii is 5213 square miles greater than the land area of Rhode Island and the total is 7637 square miles.

22. *Step 2*

Let x = the height of Twin Falls.

Then $\frac{5}{2}x$ = the height of Seven Falls.

Step 3

The sum of the heights is 420 feet, so

$$x+\frac{5}{2}x=420.$$

Step 4

$$2\left(x+\frac{5}{2}x\right)=2(420)$$
$$2x+5x=840$$
$$7x=840$$
$$x=120$$

Step 5

Since $x=120$, $\frac{5}{2}x=\frac{5}{2}(120)=300$.

The height of Twin Falls is 120 feet and that of Seven Falls is 300 feet.

Step 6

The height of Seven Falls is $\frac{5}{2}$ the height of Twin Falls and the sum is 420.

23. *Step 2*
Let x = the measure of the angle.
Then $90 - x$ = the measure of its complement and $180 - x$ = the measure of its supplement.
Step 3
$180 - x = 10(90 - x)$
Step 4
$180 - x = 900 - 10x$
$9x + 180 = 900$
$9x = 720$
$x = 80$
Step5
The measure of the angle is $80°$.
Its complement measures $90° - 80° = 10°$, and its supplement measures $180° - 80° = 100°$.
Step 6
The measure of the supplement is 10 times the measure of the complement.

24. *Step 2*
Let x = lesser odd integer.
Then $x + 2$ = greater odd integer.
Step 3
$x + 2(x + 2) = (x + 2) + 24$
Step 4
$x + 2x + 4 = x + 26$
$3x + 4 = x + 26$
$2x + 4 = 26$
$2x = 22$
$x = 11$
Step5
Since $x = 11$, $x + 2 = 13$.
The consecutive odd numbers are 11 and 13.
Step 6
The lesser plus twice the greater is $11 + 2(13) = 37$, which is 24 more than the greater.

25. Solve for h.
$$A = \frac{1}{2}bh$$
$$44 = \frac{1}{2}(8)h$$
$$44 = 4h$$
$$11 = h$$

26. Solve for A.
$$A = \frac{1}{2}h(b + B)$$
$$A = \frac{1}{2}(8)(3 + 4)$$
$$= \frac{1}{2}(8)(7)$$
$$= (4)(7)$$
$$A = 28$$

27. Solve for r.
$$C = 2\pi r$$
$$29.83 = 2(3.14)r$$
$$29.83 = 6.28r$$
$$\frac{29.83}{6.28} = \frac{6.28r}{6.28}$$
$$4.75 = r$$

28. Solve for V.
$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}(3.14)(6)^3$$
$$= \frac{4}{3}(3.14)(216)$$
$$= \frac{4}{3}(678.24)$$
$$V = 904.32$$

29. Solve for h.
$$\frac{A}{b} = \frac{bh}{b} \quad \text{Divide by } b.$$
$$\frac{A}{b} = h, \quad \text{or} \quad h = \frac{A}{b}$$

30. Solve for h.
$$2A = 2\left[\frac{1}{2}h(b + B)\right] \quad \text{Multiply by 2.}$$
$$2A = h(b + B)$$
$$\frac{2A}{(b + B)} = \frac{h(b + B)}{(b + B)} \quad \text{Divide by } b + B.$$
$$\frac{2A}{b + B} = h, \quad \text{or} \quad h = \frac{2A}{b + B}$$

31. Solve for y.
$$x + y = 11$$
$$y = -x + 11$$

32. Solve for y.

$$3x-2y=12$$
$$-2y=-3x+12$$
$$y=\frac{3}{2}x-6$$

33. Because the two angles are supplementary, $(8x-1)+(3x-6)=180$.

$$11x-7=180$$
$$11x=187$$
$$x=17$$

Since $x=17$, $8x-1=135$, and $3x-6=45$.

The measures of the two angles are $135°$ and $45°$.

34. The angles are vertical angles, so their measures are equal.

$$3x+10=4x-20$$
$$10=x-20$$
$$30=x$$

Since $x=30$, $3x+10=100$ and $4x-20=100$.

Each angle has a measure of $100°$.

35. Let W = the width of the rectangle.

Then $W+12$ = the length of the rectangle.

"The perimeter of the rectangle is 16 times the width" can be written as $2L+2W=16W$ since the perimeter is $2L+2W$.

Because $L=W+12$, we have

$$2(W+12)+2W=16W.$$
$$2W+24+2W=16W$$
$$4W+24=16W$$
$$-12W+24=0$$
$$-12W=-24$$
$$W=2$$

The width is 2 cm and the length is $2+12=14$ cm.

36. The sum of the three marked angles in the triangle is $180°$.

$$45°+(x+12.2)°+(3x+2.8)°=180°$$
$$4x+60=180$$
$$4x=120$$
$$x=30$$

Since $x=30$, $(x+12.2)°=42.2°$ and $(3x+2.8)°=92.8°$.

37. The ratio of 60 centimeters to 40 centimeters is

$$\frac{60\text{ cm}}{40\text{ cm}}=\frac{3\cdot 20}{2\cdot 20}=\frac{3}{2}.$$

38. To find the ratio of 90 inches to 10 feet, first convert 10 feet to inches.

$10\text{ feet}=10\cdot 12=120\text{ inches}$

Thus, the ratio of 90 inches to 10 feet is

$$\frac{90}{120}=\frac{3\cdot 30}{4\cdot 30}=\frac{3}{4}.$$

39.

$$\frac{p}{21}=\frac{5}{30}$$
$30p=105$ Cross products

$\frac{30p}{30}=\frac{105}{30}$ Divide by 30.

$$p=\frac{105}{30}=\frac{7\cdot 15}{2\cdot 15}=\frac{7}{2}$$

The solution set is $\left\{\frac{7}{2}\right\}$.

40.

$$\frac{5+x}{3}=\frac{2-x}{6}$$
$6(5+x)=3(2-x)$ Cross products

$30+6x=6-3x$ Distributive property

$30+9x=6$ Add $3x$.

$9x=-24$ Subtract 30.

$$x=\frac{-24}{9}=-\frac{8}{3}$$

The solution set is $\left\{-\frac{8}{3}\right\}$.

41. Let x = the tax on a \$36.00 item.

Set up a proportion with one ratio involving sales tax and the other involving the costs of the items.

$$\frac{x\text{ dollars}}{\$2.04}=\frac{\$36}{\$24}$$
$$24x=(2.04)(36)=73.44$$
$$x=\frac{73.44}{24}=3.06$$

The sales tax on a \$36.00 item is \$3.06.

42. Let x = the actual distance between the second pair of cities (in kilometers).

Set up a proportion with one ratio involving map distances and the other involving actual distances.

$$\frac{x\text{ kilometers}}{150\text{ kilomters}}=\frac{80\text{ centimeters}}{32\text{ centimeters}}$$
$$32x=(150)(80)=12,000$$
$$x=\frac{12,000}{32}=375$$

The cities are 375 kilometers apart.

43. Let x = the number of gold medals earned by Italy.

$$\frac{x \text{ gold medals}}{28 \text{ medals}} = \frac{2 \text{ gold medals}}{7 \text{ medals}}$$

$$7x = 2(28) = 56$$

$$x = 8$$

At the 2012 Olympics, 8 gold medals were earned by Italy.

44. To find the best buy, divide the price by the number of units to get the unit cost. Each result was found by using a calculator and rounding the answer to three decimal places. The best buy (based on price per unit) is the smallest unit cost. The results in the following table are rounded to the nearest thousandth.

Size	Price	Unit Cost (dollar per oz)
9 oz	\$3.49	$\frac{\$3.49}{9} \approx \$0.388$
14 oz	\$3.99	$\frac{\$3.99}{14} = \$0.285$
18 oz	\$4.49	$\frac{\$4.49}{18} \approx \$0.249$ (*)

Because the 18-oz size produces the lowest unit cost, it is the best buy. The unit cost, to the nearest thousandth, is \$0.249 per oz.

45. What percent of 12 is 21?
Let p denote the percent.

$$21 = p \cdot 12$$

$$p = \frac{21}{12} = 1.75 = 175\%$$

21 is 175% of 12.

46. 36% of what number is 900?
Let n denote the number.

$$0.36 \cdot n = 900$$

$$n = \frac{900}{0.36} \text{ Divide by 0.36.}$$

$$= 2500 \text{ Simplify.}$$

Thus, 36% of 2500 is 900.

47. Let x = the number of liters of the 60% solution to be used.
Then $x + 15$ = the number of liters of the 20% solution.

10% solution	plus	60% solution	is	20% solution.
↓	↓	↓	↓	↓
$0.10(15)$	+	$0.60(x)$	=	$0.20(x+15)$

Multiply by 10 to clear decimals.

$$1(15) + 6x = 2(x+15)$$

$$15 + 6x = 2x + 30$$

$$15 + 4x = 30$$

$$4x = 15$$

$$x = \frac{15}{4} = 3.75$$

3.75 liters of 60% solution are needed.

48. Let x = the amount invested at 5%.
Then $10,000 - x$ = the amount invested at 3%.

Interest at 5%	plus	interest at 3%	equals	\$400.
↓	↓	↓	↓	↓
$0.05x$	+	$0.03(10,000 - x)$	=	400

Solve the equation.

$$5x + 3(10,000 - x) = 100(400)$$

$$5x + 30,000 - 3x = 40,000$$

$$2x = 10,000$$

$$x = 5000$$

Robert invested \$5000 at 5% and $10,000 - 5000 = \$5000$ at 3%.

49. Use the formula $d = rt$ or $r = \frac{d}{t}$.

$$r = \frac{d}{t} = \frac{3150}{384} \approx 8.203$$

Rounded to the nearest tenth, the *Yorkshire's* average rate was 8.2 mph.

50. Let t = the number of hours until the planes are 1925 miles apart.

The distance one plane flies north	plus	the distance the other plane flies south	equals	the distance between the planes.
↓	↓	↓	↓	↓
$350t$	+	$420t$	=	1925

Solve the equation.

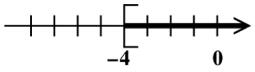
$$350t + 420t = 1925$$

$$770t = 1925$$

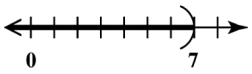
$$t = \frac{1925}{770} = \frac{5}{2} = 2\frac{1}{2}$$

The planes will be 1925 miles apart in $2\frac{1}{2}$ hours.

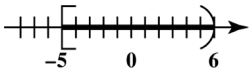
51. The statement $x \geq -4$ can be written as $[-4, \infty)$.



52. The statement $x < 7$ can be written as $(-\infty, 7)$.



53. The statement $-5 \leq x < 6$ can be written as $[-5, 6)$.

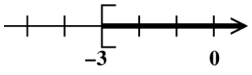


54. By examining the choices, we see that $-4x \leq 36$ is the only inequality that has a negative coefficient of x. Thus, B is the only inequality that requires a reversal of the inequality symbol when it is solved.

55. $x + 6 \geq 3$

$x \geq -3$ Subtract 6.

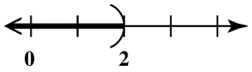
Graph the solution set $[-3, \infty)$.



56. $5x < 4x + 2$

$x < 2$ Subtract $4x$.

Graph the solution set $(-\infty, 2)$.

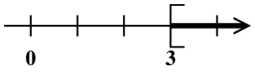


57. $-6x \leq -18$

$\dfrac{-6x}{-6} \geq \dfrac{-18}{-6}$ Divide by -6.

$x \geq 3$

Graph the solution set $[3, \infty)$.



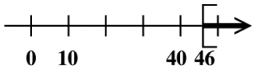
58. $8(x-5) - (2+7x) \geq 4$

$8x - 40 - 2 - 7x \geq 4$

$x - 42 \geq 4$

$x \geq 46$

Graph the solution set $[46, \infty)$.



59. $4x - 3x > 10 - 4x + 7x$

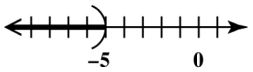
$x > 10 + 3x$

$-2x > 10$

$\dfrac{-2x}{-2} < \dfrac{10}{-2}$ Divide by -2.

$x < -5$

Graph the solution set $(-\infty, -5)$.



60. $3(2x+5) + 4(8+3x) < 5(3x+7)$

$6x + 15 + 32 + 12x < 15x + 35$

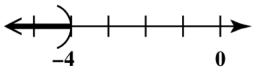
$18x + 47 < 15x + 35$

$3x + 47 < 35$

$3x < -12$

$x < -4$

Graph the solution set $(-\infty, -4)$.

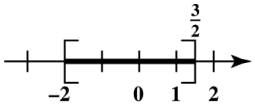


61. $-3 \leq 2x + 1 \leq 4$

$-4 \leq 2x \leq 3$ Subtract 1.

$-2 \leq x \leq \dfrac{3}{2}$ Divide by 2.

Graph the solution set $\left[-2, \dfrac{3}{2}\right]$.

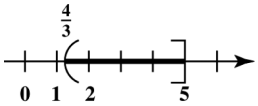


62. $9 < 3x + 5 \leq 20$

$4 < 3x \leq 15$ Subtract 5.

$\dfrac{4}{3} < x \leq 5$ Divide by 3.

Graph the solution set $\left(\dfrac{4}{3}, 5\right]$.



63. Let x = the score on the third test.

The average of the three tests	is at least	90.
$\downarrow$	$\downarrow$	$\downarrow$
$\frac{94+88+x}{3}$	$\geq$	90

$$\frac{182+x}{3} \geq 90$$

$$3\left(\frac{182+x}{3}\right) \geq 3(90)$$

$$182+x \geq 270$$

$$x \geq 88$$

In order to average at least 90, Awilda's score on her third test must be 88 or more.

64. Let n = the number.

"If nine times a number is added to 6, the result is at most 3" can be written as $9n+6 \leq 3$.

Solve the inequality.

$9n+6 \leq 3$

$9n \leq -3$ Subtract 6.

$n \leq \frac{-3}{9}$ Divide by 9.

$n \leq -\frac{1}{3}$

All numbers less than or equal to $-\frac{1}{3}$ satisfy the given condition.

Chapter 2 Mixed Review Exercises

1. $\frac{x}{7} = \frac{x-5}{2}$

$2x = 7(x-5)$ Cross products

$2x = 7x-35$

$-5x = -35$

$x = 7$

The solution set is $\{7\}$.

2. Solve $I = prt$ for r.

$\frac{I}{pt} = \frac{prt}{pt}$ Divide by pt.

$\frac{I}{pt} = r$, or $r = \frac{I}{pt}$

3. $-2x > -4$

$\frac{-2x}{-2} < \frac{-4}{-2}$ Divide by -2.

$x < 2$

The solution set is $(-\infty, 2)$.

4. $2k-5 = 4k+13$

$-2k-5 = 13$ Subtract $4k$.

$-2k = 18$ Add 5.

$k = -9$ Divide by -2.

The solution set is $\{-9\}$.

5. $0.05x + 0.02x = 4.9$

To clear decimals, multiply both sides by 100.

$100(0.05x+0.02x) = 100(4.9)$

$5x+2x = 490$

$7x = 490$

$x = 70$

The solution set is $\{70\}$.

6. $2-3(x-5) = 4+x$

$2-3x+15 = 4+x$

$17-3x = 4+x$

$17-4x = 4$

$-4x = -13$

$x = \frac{-13}{-4} = \frac{13}{4}$

The solution set is $\left\{\frac{13}{4}\right\}$.

7. $9x-(7x+2) = 3x+(2-x)$

$9x-7x-2 = 3x+2-x$

$2x-2 = 2x+2$

$-2 = 2$

Because $-2 = 2$ is a false statement, the given equation has no solution, symbolized by $\varnothing$.

8. $\frac{1}{3}s+\frac{1}{2}s+7 = \frac{5}{6}s+5+2$

$\frac{1}{3}s+\frac{1}{2}s = \frac{5}{6}s$ Subtract 7.

The least common denominator is 6.

$6\left(\frac{1}{3}s+\frac{1}{2}s\right) = 6\left(\frac{5}{6}s\right)$

$2s+3s = 5s$

$5s = 5s$

Because $5s = 5s$ is a true statement, the solution set is {all real numbers}.

9. Let x = the number of calories a 175-pound athlete can consume.
Set up a proportion with one ratio involving calories and the other involving pounds.

$$\frac{x \text{ calories}}{50 \text{ calories}} = \frac{175 \text{ pounds}}{2.2 \text{ pounds}}$$

$$2.2x = 50(175)$$

$$x = \frac{8750}{2.2} \approx 3977.3$$

To the nearest hundred calories, a 175-pound athlete in a vigorous training program can consume 4000 calories per day.

10. Let x = the sales for DiGiorno, in millions of dollars.
Then $x - 399.9$ the sales for Red Baron, in millions of dollars.

$$x + (x - 399.9) = 937.5$$

$$2x - 399.9 = 937.5$$

$$2x = 1337.4$$

$$x = 668.7$$

Since $x = 668.7$, $x - 399.9 = 268.8$.

DiGiorno sold \$668.7 million worth of frozen pizza and Red Baron sold \$268.8 million worth of frozen pizza.

11. The results in the following table are rounded to the nearest thousandth.

Size	Price	Unit Cost (dollar per oz)
50 oz	\$3.99	$\frac{\$3.99}{50} \approx \$0.080$
100 oz	\$7.29	$\frac{\$7.29}{100} \approx \$0.073$
160 oz	\$9.99	$\frac{\$9.99}{160} \approx \$0.062$ (*)

Because the 160-oz size produces the lowest unit cost, it is the best buy. The unit cost, to the nearest thousandth, is \$0.062 per oz.

12. The angles make up a right angle, so the sum of their measures is 90°.

$$(3x)^\circ + (8x + 2)^\circ = 90^\circ$$

$$11x + 2 = 90$$

$$11x = 88$$

$$x = 8$$

Since $x = 8$, $(3x)^\circ = 24^\circ$ and $(8x + 2)^\circ = 66^\circ$.

13. Use the formula $d = rt$, or $t = \frac{d}{r}$, where $d = 819$ and $r = 63$.

$$t = \frac{d}{r} = \frac{819}{63} = 13$$

It took Janet 13 hours to drive from Louisville to Dallas.

14. Let x = the rate of the slower train.
Then $x + 30$ = the rate of the faster train.

	r	*t*	*d*
Slower Train	x	3	$3x$
Faster Train	$x + 30$	3	$3(x + 30)$

The sum of the distances traveled by the two trains is 390 miles, so $3x + 3(x + 30) = 390$.

Solve the equation.

$$3x + 3(x + 30) = 390$$

$$3x + 3x + 90 = 390$$

$$6x + 90 = 390$$

$$6x = 300$$

$$x = 50$$

Since $x = 50$, $x + 30 = 80$.

The rate of the slower train is 50 miles per hour and the rate of the faster train is 80 miles per hour.

15. Let x = the length of the first side.
Then $2x$ = the length of the second side.
Use the formula for the perimeter of a triangle, $P = a + b + c$, with perimeter 96 and third side 30.

$$x + 2x + 30 = 96$$

$$3x + 30 = 96$$

$$3x = 66$$

$$x = 22$$

The sides have lengths 22 meters, 44 meters, and 30 meters. The length of the longest side is 44 meters.

16. Let s = the length of a side of the square. The formula for the perimeter of a square is $P = 4s$.

The perimeter	cannot be greater than	200.
↓	↓	↓
$4s$	$\le$	200

$$4s \le 200$$
$$s \le 50$$

The length of a side is 50 meters or less.

Chapter 2 Test

1. $5x + 9 = 7x + 21$

$-2x + 9 = 21$ Subtract $7x$.

$-2x = 12$ Subtract 9.

$x = -6$ Divide by -2.

The solution set is $\{-6\}$.

2. $$-\frac{4}{7}x = -12$$

$$\left(-\frac{7}{4}\right)\left(-\frac{4}{7}x\right) = \left(-\frac{7}{4}\right)(-12)$$
$$x = 21$$

The solution set is $\{21\}$.

3. $7 - (x - 4) = -3x + 2(x + 1)$

$7 - x + 4 = -3x + 2x + 2$

$-x + 11 = -x + 2$

$11 = 2$

Because the last statement is false, the equation has no solution set, symbolized by $\varnothing$.

4. To clear decimals, multiply both sides by 100.

$100[0.06(x + 20) + 0.08(x - 10)] = 100(4.6)$

$6(x + 20) + 8(x - 10) = 460$

$6x + 120 + 8x - 80 = 460$

$14x + 40 = 460$

$14x = 420$

$x = 30$

The solution set is $\{30\}$.

5. $-8(2x + 4) = -4(4x + 8)$

$-16x - 32 = -16x - 32$

Because the last statement is true, the solution set is {all real numbers}.

6. $2 - 3(x - 5) = 3 + (x + 1)$

$2 - 3x + 15 = 3 + x + 1$ Distributive property

$-3x + 17 = 4 + x$

$17 = 4 + 4x$ Add $3x$.

$13 = 4x$ Subtract 4.

$$\frac{13}{4} = x$$

The solution set is $\left\{\frac{13}{4}\right\}$.

7. Let x = the number of games the Cardinals lost.

Then $2x - 33$ = the number of games the Cardinals won.

The total number of games played was 162.

$x + (2x - 33) = 162$

$3x - 33 = 162$

$3x = 195$

$x = 65$

Since $x = 65$, $2x - 33 = 97$.

The Cardinals won 97 games and lost 65 games.

8. Let x = the area of Kauai (in square miles).

Then $x + 177$ = the area of Maui (in square miles), and $(x + 177) + 3293 = x + 3470$ = the area of Hawaii.

$x + (x + 177) + (x + 3470) = 5300$

$3x + 3647 = 5300$

$3x = 1653$

$x = 551$

Since $x = 551$, $x + 177 = 728$ and $x + 3470 = 4021$.

The area of Hawaii is 4021 square miles, the area of Maui is 728 square miles, and the area of Kauai is 551 square miles.

9. Let x = the measure of the angle.

Then $90 - x$ = the measure of its complement, and $180 - x$ = the measure of its supplement.

$180 - x = 3(90 - x) + 10$

$180 - x = 270 - 3x + 10$

$180 - x = 280 - 3x$

$180 + 2x = 280$

$2x = 100$

$x = 50$

The measure of the angle is $50°$. The measure of its supplement, $130°$, is $10°$ more than three times its complement, $40°$.

10. *Step 2*
Let x = the lesser even integer.
Then $x+2$ = the greater even integer.
Step 3
$3x = 20 + 2(x+2)$
Step 4
$3x = 20 + 2(x+2)$
$3x = 20 + 2x + 4$
$3x = 24 + 2x$
$x = 24$
Step5
Since $x = 24$, $x+2 = 26$.
The consecutive even numbers are 24 and 26.
Step 6
Three times the lesser is $3(24) = 72$, and 20 more than twice the greater is $20 + 2(26) = 72$.

11. **(a)** Solve $P = 2L + 2W$ for W.
$P - 2L = 2W$
$\frac{P-2L}{2} = W$, or $W = \frac{P-2L}{2}$

(b) Substitute 116 for P and 40 for L in the formula obtained in part (a).
$W = \frac{P-2L}{2}$
$= \frac{116-2(40)}{2}$
$= \frac{116-80}{2} = \frac{36}{2} = 18$

12. $5x - 4y = 8$
$5x - 4y - 5x = 8 - 5x$ Subtract $5x$.
$-4y = 8 - 5x$
$\frac{-4y}{-4} = \frac{8-5x}{-4}$ Divide by -4.
$y = -\frac{8-5x}{4}$, or $y = \frac{5}{4}x - 2$
(There are other correct forms.)

13. The angles are vertical angles, so their measures are equal.
$3x + 15 = 4x - 5$
$15 = x - 5$
$20 = x$
Since $x = 20$, $3x + 15 = 75$ and $4x - 5 = 75$.
Both angles have measure $75°$.

14. $\frac{z}{8} = \frac{12}{16}$
$16z = 8(12)$ Cross products
$16z = 96$
$\frac{16z}{16} = \frac{96}{16}$ Divide by 16.
$z = 6$
The solution set is $\{6\}$.

15. $\frac{x+5}{3} = \frac{x-3}{4}$
$4(x+5) = 3(x-3)$
$4x + 20 = 3x - 9$
$x + 20 = -9$
$x = -29$
The solution set is $\{-29\}$.

16. What percent of 65 is 26?
Let p denote the percent.
$26 = p \cdot 65$
$p = \frac{26}{65} = 0.4 = 40\%$
26 is 40% of 65.

17. The results in the following table are rounded to the nearest thousandth.

Size	Price	Unit Cost (dollar per oz)
8 oz	\$2.99	$\frac{\$2.99}{8} \approx \$0.374$
16 oz	\$3.99	$\frac{\$3.99}{16} \approx \$0.249$ (*)
48 oz	\$14.69	$\frac{\$14.69}{48} \approx \$0.306$

Because the 16-oz size produces the lowest unit cost, it is the best buy. The unit cost, to the nearest thousandth, is \$0.249 per oz.

18. Let x = the actual distance between Seattle and Cincinnati.
$\frac{x \text{ miles}}{1050 \text{ miles}} = \frac{92 \text{ inches}}{42 \text{ inches}}$
$42x = 92(1050) = 96,600$
$x = \frac{96,600}{42} = 2300$
The actual distance between Seattle and Cincinnati is 2300 miles.

19. Let x = the amount invested at 3%.
Then $x + 6000$ = the amount invested at 4.5%.

Amount Invested (in dollars)	Rate of Interest	Interest for One Year
x	0.03	$0.03x$
$x + 6000$	0.045	$0.045(x + 6000)$

$$0.03x + 0.045(x + 6000) = 870$$

To clear decimals, multiply both sides by 1000.

$$30x + 45x(x + 6000) = 870,000$$
$$30x + 45x + 270,000 = 870,000$$
$$75x + 270,000 = 870,000$$
$$75x = 600,000$$
$$x = 8000$$

Since $x = 8000$, $x + 6000 = 14,000$.
Carlos invested \$8000 at 3% and \$14,000 at 4.5%.

20. Use the formula $d = rt$ and let t be the number of hours they traveled.

	r	t	d
First Car	50	t	$50t$
Second Car	65	t	$65t$

First car's distance and second car's distance is total distance.

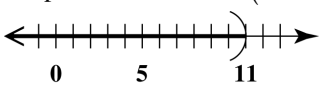
$$50t + 65t = 460$$
$$115t = 460$$
$$t = 4$$

The two cars will be 460 miles apart in 4 hours.

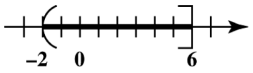
21. **(a)** The set of numbers graphed corresponds to the inequality $x < 0$.

(b) The set of numbers graphed corresponds to the inequality $-2 < x \le 3$.

22. $-3x > -33$
$x < 11$ Divide by -3.
Graph the solution set $(-\infty, 11)$.

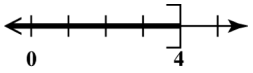


23. $-10 < 3x - 4 \le 14$
$-6 < 3x \le 18$ Add 4.
$-2 < x \le 6$ Divide by 3.
Graph the solution set $(-2, 6]$.



24. $-4x + 2(x - 3) \ge 4x - (3 + 5x) - 7$
$-4x + 2x - 6 \ge 4x - 3 - 5x - 7$
$-2x - 6 \ge -x - 10$
$-x - 6 \ge -10$
$-x \ge -4$
$\frac{-1x}{-1} \le \frac{-4}{-1}$
$x \le 4$

Graph the solution set $(-\infty, 4]$.



25. Let x = the score on the third test.

The average of the three tests is at least 80.

$$\frac{76 + 81 + x}{3} \ge 80$$
$$\frac{157 + x}{3} \ge 80$$
$$3\left(\frac{157 + x}{3}\right) \ge 3(80)$$
$$157 + x \ge 240$$
$$x \ge 83$$

In order to average at least 80, Susan's score on her third test must be 83 or more.

Chapters R–2 Cumulative Review Exercises

1. $\frac{5}{6} + \frac{1}{4} - \frac{7}{15} = \frac{50}{60} + \frac{15}{60} - \frac{28}{60}$
$= \frac{65 - 28}{60}$
$= \frac{37}{60}$

2. $\frac{9}{8}\cdot\frac{16}{3}\div\frac{5}{8}=\frac{9}{8}\cdot\frac{16}{3}\cdot\frac{8}{5}$

$=\frac{3\cdot3\cdot16\cdot8}{8\cdot3\cdot5}$

$=\frac{48}{5}$

3. $4.8+12.5+16.73=34.03$

4. "The difference of half a number and 18" is written $\frac{1}{2}x-18$.

5. "The quotient of 6 and 12 more than a number is 2" is written $\frac{6}{x+12}=2$.

6. $\frac{8(7)-5(6+2)}{3\cdot5+1}\ge1$

$\frac{8(7)-5(8)}{3\cdot5+1}\ge1$

$\frac{56-40}{15+1}\ge1$

$\frac{16}{16}\ge1$

$1\ge1$

The statement is true.

7. $\frac{-4(9)(-2)}{-3^2}=\frac{-36(-2)}{-1\cdot3^2}$

$=\frac{72}{-9}$

$=-8$

8. $(-7-1)(-4)+(-4)=(-8)(-4)+(-4)$

$=32+(-4)$

$=28$

9. $\frac{6(-4)-(-2)(12)}{3^2+7^2}=\frac{6(-4)-(-2)(12)}{9+19}$

$=\frac{-24-(-24)}{9+19}$

$=\frac{0}{28}=0$

10. Let $x=-2$, $y=-4$, and $z=3$.

$\frac{3x^2-y^3}{-4z}=\frac{3(-2)^2-(-4)^3}{-4(3)}$

$=\frac{3(4)-(-64)}{-12}$

$=\frac{12+64}{-12}$

$=\frac{76}{-12}$

$=-\frac{19}{3}$

11. $7(p+q)=7p+7q$

The multiplication of 7 is distributed over the sum, which illustrates the distributive property.

12. $7+(-7)=0$

A number added to its opposite is equal to 0. This illustrates the inverse property (of addition).

13. $3.5(1)=3.5$

A number multiplied by 1 is equal to itself. This illustrates the identity property (of multiplication).

14. $2r-6=8r$

$-6=6r$

$-1=r$

Check $r=-1$: $-8=-8$ True

The solution set is $\{-1\}$.

15. $4-5(s+2)=3(s+1)-1$

$4-5s-10=3s+3-1$

$-5s-6=3s+2$

$-8s-6=2$

$-8s=8$

$s=-1$

Check $s=-1$: $-1=-1$ True

The solution set is $\{-1\}$.

16. $\frac{2}{3}x+\frac{3}{4}x=-17$

$12\left(\frac{2}{3}x+\frac{3}{4}x\right)=12(-17)$ LCD = 12

$8x+9x=-204$

$17x=-204$

$x=-12$

Check $x=-12$: $-17=-17$ True

The solution set is $\{-12\}$.

17. $\frac{2x+3}{5}=\frac{x-4}{2}$

$(2x+3)(2)=(5)(x-4)$

$4x+6=5x-20$

$6=x-20$

$26=x$

Check $x=26$: $11=11$ True

The solution set is $\{26\}$.

18. Solve $3x+4y=24$ for y.

$4y=24-3x$

$y=\frac{24-3x}{4}$

19. Solve $P=a+b+c+B$ for c.
Subtract a, b, and B.

$P=a+b+c+B$

$P-a-b-B=c$

20. $6(r-1)+2(3r-5)\le -4$

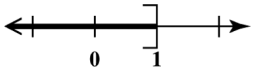
$6r-6+6r-10\le -4$

$12r-16\le -4$

$12r\le 12$

$r\le 1$

Graph the solution set $(-\infty, 1]$.

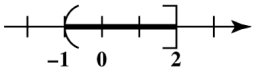


21. $-18\le -9z<9$

$2\ge z>-1$ Divide; reverse the symbols.

or $-1<z\le 2$

Graph the solution set $(-1, 2]$.



22. Let x = the length of the middle-sized piece.
Then $3x$ = the length of the longest piece, and
$x-5$ = the length of the shortest piece.

$x+3x+(x-5)=40$

$5x-5=40$

$5x=45$

$x=9$

The length of the middle-sized piece is 9 centimeters, that of the longest piece is 27 centimeters, and that of the shortest piece is 4 centimeters.

23. Let r = the radius and use 3.14 for π.
Using the formula for circumference, $C=2\pi r$, and $C=78$, we have

$2\pi r=78$

$r=\frac{78}{2\pi}\approx 12.4204$

To the nearest hundredth, the radius is 12.42 cm.

24. Let x = the rate of the slower car.
Then $x+20$ = the rate of the faster car.
Use the formula $d=rt$.

$d_{\text{slower}}+d_{\text{faster}}=d_{\text{total}}$

$(x)(4)+(x+20)(4)=400$

$4x+4x+80=400$

$8x+80=400$

$8x=320$

$x=40$

The rates are 40 mph and 60 mph.

25. **(a)** The segment of the circle representing white cars is 19% of the circle. What is 19% of 2.8 million?
$19\%\cdot 2.8=0.19\cdot 2.8=0.532$
0.532 million cars, or 532,000 cars, are white.

(b) The segment of the circle representing silver cars is 18% of the circle. What is 18% of 2.8 million?
$18\%\cdot 2.8=0.18\cdot 2.8=0.504$
0.504 million cars, or 504,000 cars, are silver.

(c) The segment of the circle representing red cars is 12% of the circle. What is 12% of 2.8 million?
$12\%\cdot 2.8=0.12\cdot 2.8=0.336$
0.336 million cars, or 336,000 cars, are red.